MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 18/1989

Riesz Spaces and Operator Theory

23.4. bis 29.4.1989

Die Tagung "Riesz Spaces and Operator Theory" fand unter Leitung von Herrn Professor W.A.J. Luxemburg (Pasadena) und Herrn Professor H.H. Schaefer (Tübingen) statt. In 37 Vorträgen wurden aktuelle Forschungsresultate vorgestellt. Ferner wurde die Tagung zum Knüpfen von Kontakten, zu intensivem Gedankenaustausch und zur Zusammenarbeit genutzt.

Das Spektrum der Vorträge war weit gefächert. Ein großer Teil der Vorträge hatte die Struktur von Riesz-Räumen und von Operatoren zwischen Riesz-Räumen zum Gegenstand. Ergebnisse zur Struktur von Banachverbänden und Funktionenräumen, positiven und regulären Operatoren, Operatorenklassen, Tensorprodukten und Verbandsalgebren wurden vorgestellt. Ferner wurden Beiträge zur Ergodentheorie und zu Operatorhalbgruppen geleistet.

In mehreren Vorträgen wurde über spektraltheoretische Untersuchungen bei positiven Operatoren und in Banachverbandsalgebren berichtet. Darüber hinaus gab es Beiträge aus der nicht-kommutativen Theorie, zur Beweisbarkeit bestimmter Resultate mit und ohne Auswahlaxiom und zur Äquivalenz von Darstellungssätzen im Axiomensystem ZF.

Schließlich wurden Fragen aus der Maßtheorie, der Ökonomie, der Topologie sowie zu invarianten Teilräumen, Sätzen vom Hahn-Banach-Typ, konvexen Funktionen und der Berechnung von Operatornormen behandelt.

Abstracts

YU.A. ABRAMOVICH

On the Daugavet equation

The following two statements provide a unified approach (ideologically going back to G. Lozanovsky) allowing one to describe all previously known classes of operators satisfying the Daugavet equation (DE). Let X be an $AL-$ or an $AM-$space and let $T : X \to X$ be a bounded operator.

Proposition 1. If the identity operator I and T are disjoint, then T satisfies (DE), *i.e.*, $\|I + T\| = 1 + \|T\|$.

Proposition 2. If X is, additionally, atomless, then each weakly compact operator T is disjoint with I.

Corollary 3. (Daugavet, Lozanovsky, Foias-Singer, Krasnoselsky, Babenko-Pichugov, Holub, Kamowitz, Chauveheid) If X is atomless, then every weakly compact operator on X satisfies (DE).

The following theorem describes a new class of spaces with the same property. Let $Z = L_1(\mu) \oplus_\infty L_1(\mu)$ or $Z = L_\infty(\mu) \oplus_1 L_\infty(\mu)$, where μ is atomless.

Theorem 4. Every weakly compact operator on Z satisfies (DE).

We conclude with the following result which is a slight generalization of a corresponding result due to Holub.

Theorem 5. For each $T : X \to X$, where X is as above, there exists $\gamma \in \{-1, 1\}$ such that $\|I + \gamma T\| = 1 + \|T\|$.

C.D. ALIPRANTIS

Inductive limits of AM-spaces and the overlapping generations model

This work deals with the overlapping generations (OLG) model of P.A. Samuelson. In this model there are a countable number of overlapping generations and each generation consists of a finite number of finitely lived agents. In the simplest case, a generation consists of a single agent who lives for two periods. Each agent is endowed with an infinite consumption stream ω_t which is zero in every period except possibly the two periods in which it is alive, *i.e.*, $\omega_t = (0, 0, ..., \omega_t^t, \omega_t^{t+1}, 0, 0, ...)$. The model presents a new commodity-price space duality for the OLG model that also allows for an interpretation of prices as rates of interest.
The construction of the commodity space for the whole economy consists of "gluing" together the overlapping commodity spaces in a coherent fashion. The natural construction is to form the inductive limit of the individual commodity spaces which as it turns out are $AM-$spaces. The price space is then taken to be the projective limit of the $AM-$spaces. The main result of this work establishes that if the Riesz dual system in each period is symmetric, then the OLG model has a competitive equilibrium that can be supported by an order continuous positive price.
(This is a joint work with D.J. Brown and O. Burkinshaw.)

W. ARENDT

Positive semigroups generated by elliptic operators

(based on joint work with C. Batty and D. Robinson.)
Let $A_1, \dots A_N$ be generators of positive unitary groups on $L^2(\Omega, \mu)$ and $a_{ij} \in L^\infty(\Omega, \mu)$ satisfying

$$\sum_{i,j=1}^N a_{ij}(x)\xi_i\xi_j \geq n|\xi|^2 \quad \mu - a.e.$$

for all $\xi \in \mathbb{R}^N$ where $n > 0$. Assume that $\bigcap D(A_j)$ is dense in L^2. Then the operator

$$A = \sum_{i,j=1}^N A_i a_{ij} A_j$$

can be defined properly. A generates a positive semigroup T on L^2. If the groups generated by A_j interpolate on L^p, then T interpolates as well.
This abstract setting allows one to define elliptic operators on $L^p(G)$ where G is a Lie group. In that case regularity results can be obtained, so that a positive semigroup is induced on $C_0(G)$ as well.

S. BERNAU

Archimedean almost f-algebras

It is well known that an Archimedean f-algebra is necessarily associative and commutative, and that these facts have elementary proofs. Recently it has been shown that if we consider almost f-algebras (requiring only that $a \wedge b = 0$ implies $ab = 0$) the Archimedean property does not force associativity but still forces commutativity. In the talk I will discuss an elementary proof of this last fact. The discussion is based on a recent joint paper with C.B. Huijsmans.

G. BUSKES

Ultrapowers of a Riesz space

We define the ultrapower $F_{\mathcal{V}}$ of an Archimedean Riesz space F over a filter $\mathcal{V}$. We use these ultrapowers to show – in ZF – the equivalence of certain extension theorems. As a result we obtain that some representation theorems are equivalent in ZF. As an example of the latter we show that the following are equivalent:

Stone Representation Theorem for Boolean Algebras.
Kakutani Representation Theorem for AM-spaces with unit.
Gelfand Representation Theorem for Gelfand algebras.
Gelfand-Naimark-Segal Representation Theorem.
Maeda-Ogasawara for semisimple f-algebras.
Maeda-Ogasawara for Archimedean Riesz spaces with unit.

V. CASELLES

Continuity of the spectral radius in the order topology

Let us suppose that $0 \leq T$ is an irreducible operator on a Banach lattice E and $r(T)$ is a Riesz point of $\sigma(T)$. Let $0 \leq T_n$ be a sequence of operators on E such that (T_n) order converges to T and $\|(T_n - T)^+\| \to 0$. Then $r(T_n) \to r(T)$. Moreover if

$0 \neq v_n \geq 0$ is a norm one solution of $T_n v_n = r(T_n) v_n$ and $Tv = r(T)v$, $0 \neq v \geq 0$, $\|v\| = 1$, then $v_n \to v$. To prove this we assume that T is AM-compact. We should not worry to get a solution of $T_n v_n = r(T_n) v_n$ because we show that for n sufficiently large $r(T_n)$ is also a Riesz point of $\sigma(T_n)$. In all this work we suppose that E is a Banach lattice with order continuous norm. Applications are given to the computations of eigenvalues and eigenvectors for infinite positive matrices and integral kernels.

P.G. DODDS

Non-commutative Köthe duality

We consider duality theory (in the sense of Köthe) for (a class of) symmetric operator spaces $L_\gamma(\widetilde{\mathcal{M}})$ where $\widetilde{\mathcal{M}}$ is the space of τ-measurable operators (in the sense of Nelson) affiliated with a given semi-finite von Neumann algebra $\mathcal{M}$, equipped with a normal faithful semi-finite trace τ.
(This is joint work with Theresa K.-Y. Dodds and Ben de Pagter.)

T. DODDS

General Markus inequality

Let $\mathcal{M}$ be a von Neumann algebra on a Hilbert space H, with a normal faithful semi-finite trace τ. Denote by $\widetilde{\mathcal{M}}$ the set of all τ-measurable operators in the sense of E. Nelson. For $x \in \widetilde{\mathcal{M}}$ the *generalized singular value* $\mu_.(x)$ is defined by:

$$\mu_t(x) = \inf\{s \geq 0 : \tau(\chi_{(s,\infty)}(|x|)) \leq t\} \ , \ t \geq 0.$$

The following inequality holds: for $x, y \in \widetilde{\mathcal{M}}$,

$$\sup_{|E| \leq \alpha} \int_E |\mu_t(x) - \mu_t(y)| dt \ \leq \ \int_0^\alpha \mu_t(x-y) dt \quad \text{for all } \alpha \geq 0.$$

This generalizes the results of:
(1) A.S. Markus (1964) for the case that $\mathcal{M} = L(H)$, x, y are compact,
(2) F. Hiai and Y. Nakamura (1987) for the case that the trace τ is finite,
(3) G.G. Lorentz and T. Shimogaki (1968) for the case that $\mathcal{M}$ is commutative.

(This is joint work with Ben de Pagter and Peter Dodds.)

G. GODEFROY

Hypercyclic operators

In this joint work with J. Shapiro, we show that large classes of operators, modelled on the backward shift on Hilbert spaces, are such that there exists a dense linear subspace such that every non-zero vector in this subspace has a dense orbit. Among other results, we obtain a very general result which extends previous theorems of Mac Lane and Birkhoff.

J.J. GROBLER

Results about cyclicity of the peripheral spectrum

Let A be a unital Banach lattice algebra and let $z \in A_+$ such that $\|z\| \leq 1$. We prove that the peripheral spectrum of z is a finite union of non-trivial finite subgroups of the circle group if and only if there exists an integer $k_0 \geq 0$ such that $\|z^n - z^{n+k_0}\| \to 0$ as $n \to \infty$ and $\|z^n - z^{n+k}\| = 2$ for all k with $1 \leq k < k_0$ and for all $n \geq 1$. Moreover, k_0 is the least common multiple of the orders of the finite groups of which the peripheral spectrum is the union. In order to do this we prove a necessary condition for a positive element to have a cyclic peripheral spectrum. The main result is a consequence of the zero-two law for positive elements in the unit ball of A.

R. GRZASLEWICZ

On extreme positive operators

Let $1 < p < \infty$. And let $0 \neq T \in \mathcal{L}_+(l^p, l^p)$. We say that the entries of T are maximal if $\|(T_{ij} + \gamma\delta_{ii}\delta_{jj})\| > 1$ for every $\gamma > 0$. Put

$$M(T) = \{(x_i) \geq 0 : \sum_j t_{ij}(\sum_k t_{jk}x_k)^{p-1} = x_i^{p-1}\}.$$

Theorem. T is an extreme positive contraction if and only if
(i) the entries of T are maximal, and
(ii) $M(T) \neq \emptyset$ and $y_j^{p-1}t_{ji}x_i$ is an extreme doubly stochastic matrix with respect to (y_j^p), (x_i^p), where $(x_i) \in M(T)$ and $y_j = \sum_k t_{jk}x_k$.

W. HACKENBROCH

Bands of invariantly extensible measures

Let $\mathcal{A} \subset \mathcal{B}$ denote two σ-algebras of subsets of some set Ω; G denotes a semigroup of $\mathcal{A}$- and $\mathcal{B}$-measurable transformations of Ω. For a given $\mu \in M(\mathcal{A})_G$ (set of finite G-invariant measures on $\mathcal{A}$) we study the convex structure of the set $M(\mathcal{B};\mu)_G$ of G-invariant measure extensions of μ; in particular we ask for those properties of extensions such that the set of all $\mu \in M(\mathcal{A})_G$ admitting such extensions forms (the positive cone of) a band in the vector lattice of G-invariant measures on $\mathcal{A}$.

Theorem. $C \subset M(\mathcal{A})_G$ is a band in $M(\mathcal{A})_G$ iff the following two implications hold:
(a) $\mu \in C \Rightarrow \mu' \in C$ for each $\mu' \ll \mu$ in $M(\mathcal{A}_G)$;
(b) $\mu \in M(\mathcal{A})_G$, $(Z_n)_{n \in \mathbb{N}}$ disjoint sequence in A with union Z, then $\mu_Z \in C$ if all $\mu_{Z_n} \in C$.

Using this characterization it is shown that the set C of all $\mu \in M(\mathcal{A})_G$, admitting invariant extensions, such that each invariant extension $\nu \in M(\mathcal{B};\mu)_G$ has a barycentric decomposition $\nu = \int_{ex M(\mathcal{B};\mu)} \nu' \rho(d\nu')$, forms a band in $M(\mathcal{A})_G$.

F.L. HERNANDEZ

On l^p-complemented subspaces in Orlicz function spaces

A Banach function space $X([0,1]) \equiv X$ contains a singular l^p-complemented copy if there exists a complemented subspace H in X isomorphic to l^p, and H is not the span of a sequence of pairwise disjoint characteristic functions.
For any $p > 1$ we prove the existence of an Orlicz function space $L^F[0,1]$ with index p containing a singular l^p-complemented copy. This extends a previous result of N. Kalton given for Orlicz sequence spaces l^F. For $p = 1$ there is no separable Orlicz function space $L^F[0,1]$ containing a singular l^1-complemented copy. Before we characterize when the inclusion map between Orlicz function spaces is a disjointly singular operator.
(This is part of a joint work with B. R.-Salinas.)

C.B. HUIJSMANS

The second order dual of f-algebras

Let A be an f-algebra with separating dual A'. Then the order bidual A'' is again an f-algebra with respect to the Arens multiplication. An essential ingredient of the proof is the following: if $(A')'_n$ denotes the order continuous dual of A' and $(A')'_s$ the singular dual of A', then $F \cdot G = G \cdot F = 0$ for all $F \in A''$, $G \in (A')'_s$, and hence $F \cdot G = G \cdot F \in (A')'_n$ for all $F, G \in A''$.
An interesting corollary is that $G^2 = 0$ for all $G \in (A')'_s$, so if A'' is semiprime with respect to the Arens multiplication (which is certainly the case if A, or even A'', has a unit element) then $A'' = (A')'_n$.
Finally, if for $F \in A''$ the mapping $\nu_F : A' \to A'$ is defined by $\nu_F(f) = F \cdot f$ for all $f \in A'$, then $\nu_F \in Orth(A')$. The mapping $\nu : A'' \to Orth(A')$ defined by $\nu(F) = \nu_F$ is an algebra and a Riesz homomorphism. It can be shown that ν is injective iff A'' is semiprime and that ν is surjective (equivalently, ν is bijective) iff A'' has a unit element.

A. IWANIK

Multiple recurrence for Markov operators

For a single Markov operator T and its associated Markov process ξ_n there exists a point x in the phase space X (which is assumed metric compact) such that for every $l \geq 1$ and every neighbourhood U of x one has $P_x(\xi_m \in U,\ \xi_{2m} \in U, ..., \ \xi_{lm} \in U) > 0$ for some $m \geq 1$.
If Φ is a family of Markov operators on $C(X)$ then we say that x is *multiply recurrent* w.r.t. Φ if there exists a sequence $n_k \to \infty$ such that for any $0 \neq f \geq 0$, $f(x) > 0$, and any $T \in \Phi$ the inequality $T^{n_k} f(x) > 0$ holds for all sufficiently large k. The above result shows that multiply recurrent points exist whenever Φ is a cyclic semigroup. A stronger result, obtained jointly with T. Downarowicz, says that such points exist if Φ is a commutative semigroup generated by two operators. The main tool is the Furstenberg-Szemeredi recurrence theorem.

H. KÖNIG

The abstract Hahn-Banach theorem due to Rodé

More than ten years ago Rodé [Arch. Math. **31** (1978), 474–481] found an abstract Hahn-Banach theorem which is powerful and simple and comprehends the familiar and less familiar "concrete" versions of that theorem. But his proof of the abstract theorem was complicated and could not be simplified over the years. The actual reason for the present talk is that the speaker recently found an adequately simple proof of Rodé's theorem [Aequat. Math. **34** (1987), 89–95]. The short talk formulates the theorem and indicates how the proof proceeds, then how to derive concrete versions from it. As an application which seems to be not attainable by the usual Hahn-Banach versions a result of Kuhn on partially convex functions [General Inequalities IV, Oberwolfach 1983] is discussed.

S. KOSHI

Duals of convex functions and convex operators

1. Let (Ω, μ) be a measure space and $f(.,.) : \mathbb{R}^d \times \Omega \to \mathbb{R} \cup \{\infty\}$ a function which is convex on $\mathbb{R}^d$ for a fixed $t \in \Omega$. We will show the exact formula of the dual F^* of F which is defined as

$$F(a) = \int_\Omega f(a,t)d\nu(t) \quad \text{for } a \in \mathbb{R}^d.$$

2. Let E and F be Dedekind complete Riesz spaces and ϕ be a convex operator from E to F. We will define the dual of ϕ in a reasonable way in accordance with the duality theorem.

C.C. LABUSCHAGNE

Tensor products of Riesz spaces

The Riesz tensor product $E\bar{\otimes}F$ of Archimedean Riesz spaces E and F was introduced by D.H. Fremlin in 1972. We prove:

If $h \in (E\bar{\otimes}F)_+$, then there exists $(x,y) \in E_+ \times F_+$ such that for every $\varepsilon > 0$ there exists $v_\varepsilon \in E_+ \otimes F_+ := \{\sum_{i=1}^n x_i \otimes y_i : x_i \in E_+, y_i \in F_+, n \in \mathbb{N}\}$ such that $0 \le h - v_\varepsilon < \varepsilon x \otimes y$.

This property improves the approximation of elements in the Riesz tensor product by elements of the vector space tensor product as was done by D.H. Fremlin, H.H. Schaefer and A.R. Schep.

S. LEVI

Topologies on hyperspaces

Let (X,d) be a metric space and consider the hyperspace of X, $cL(X) = \{F \subset X : F$ is closed and nonempty$\}$. There are several ways to topologize $cL(X)$. The Hausdorff metric induces a topology on $cL(X)$ that can be split into its "upper part" H_d^+ and its "lower part" H_d^-. Similarly for the Vietoris topology $V = V^+ \vee V^-$, the Wijsman topology $W_d = W_d^+ \vee W_d^-$ and Kuratowski convergence $K = K^+ \vee K^-$. It is known that $V^- = W_d^- = K^-$ and that $V^+ \Rightarrow H_d^+ \Rightarrow W_d^+ \Rightarrow K^+$.

Theorem 1. i) $V^+ = \sup\{W_p^+ : p \text{ is a metric equivalent to } d\}$.
ii) $H_d^+ = \sup\{W_p^+ : p \text{ is a metric uniformly equivalent to } d\}$.

Theorem 2. $H_d^+ = W_d^+$ if and only if (X, d) is totally bounded.

Theorem 3. The suprema of Theorem 1 are actually maxima if and only if X is compact in case i) and (X, d) is totally bounded in case ii).

An open question is to characterize $\inf\{W_p^+ : p \text{ is equivalent to } d\}$ and we conjecture that this is the weakest topology on $cL(X)$ which is stronger than upper Kuratowski convergence.

Z. LIPECKI

Ideals and sublattices in linear lattices and F-lattices

The talk is based on joint work with Yu.A. Abramovich.
Let X be an F-lattice, *i.e.*, a topological linear lattice the topology of which is complete and metrizable. Then:
(1) Every ideal in X of finite codimension is closed. (For X being a Banach lattice and codimension 1 this result can be found in H.H. Schaefer's monograph, Banach lattices and positive operators.)
(2) If X is infinite-dimensional, then it contains (a) a prime ideal of codimension at least $2^{\aleph_0}$ (and so non-closed), (b) a dense (linear) sublattice of an arbitrary codimension between 1 and $2^{\aleph_0}$. (For codimension 1 part (b) was previously obtained by the authors jointly with G.Y. Rotkovich).
Both (1) and (2) fail without the assumption of metric completeness. In particular, every prime ideal of the linear lattice $S(2^\Omega)$ of all real-valued functions on a set Ω with finite range is of codimension 1, and every sublattice of $S(2^\Omega)$ is uniformly closed.

H.P. LOTZ

On the dual of weak L_1

Let (X, Σ, μ) be a non-atomic separable measure space. We give a representation theorem for the dual of the Lorentz space $W = L_{1,\infty}(X, \Sigma, \mu)$. Let W'_m be the subspace of W' generated by the maximal elements of the unit ball. W'_m is a closed sublattice and an AL-space. The ideal generated by the extreme points of the unit ball of W' is a weak* dense band and lattice isometric to a space $(\sum \oplus_{\alpha \in A} L^\infty(Y_\alpha))_{l^1(A)}$ where card $A = 2^c$.

W.A.J. LUXEMBURG

Superreflexivity of Orlicz spaces

The standard part or nonstandard hull of an enlargement of a Banach space has a finite dimensional subspace structure that is closely related to that of the given Banach space. For instance, the nonstandard hull is finitely representable in the given Banach space. As a consequence we have the theorem of Henson and Moore, that a Banach space is superreflexive iff its nonstandard hull is reflexive. This can be used to show that certain Banach function spaces such as Orlicz spaces have the property that they are superreflexive if and only if they are reflexive. A fact well-known for the L^p-spaces, $1 < p < \infty$.

B. DE PAGTER

Norm convergence in rearrangement invariant Banach function spaces

Given a (σ-finite) measure space (Ω, Σ, μ) we denote by f^* the decreasing rearrangement of $|f|$ for a function $f \in L^0(\Omega, \mu)$, *i.e.*,

$$f^*(t) = \inf\{\lambda \geq 0 : \mu(|f| > \lambda) \leq t\}, \; t \geq 0.$$

Let $\rho : L^0(\mathbb{R}_+, m)_+ \to [0, \infty]$ (m = Lebesgue measure) be a function norm which is rearrangement invariant (*i.e.* $f^* = g^*$ implies $\rho(f) = \rho(g)$) and has the Riesz-Fischer property. Furthermore we assume that $0 \leq f_n \to f$ a.e., $\rho(f)$, $\rho(f_n) < \infty$, implies that $\liminf \rho(f_n) \geq \rho(f)$. For a measure space (Ω, Σ, μ) we now define $L_\rho(\Omega, \Sigma, \mu) = \{f \in L^0(\Omega, \mu) : \; \rho(f^*) < \infty\}$, which is a rearrangement invariant Banach function space with respect to the norm $\|f\|_\rho = \rho(f^*)$. The main result in this talk is:

Theorem. Suppose in addition that ρ is order continuous. For $f, f_n \in L_\rho(\Omega, \mu)$ ($n = 1, 2, \ldots$) the following statements are equivalent:
(i) $\|f - f_n\|_\rho \to 0$ ($n \to \infty$).
(ii) $f_n \to f$ weakly and $\rho(f_n^* - f^*) \to 0$ ($n \to \infty$).

F. RÄBIGER

Dunford-Pettis operators between certain classes of Banach lattices

We introduce the class of weak Schur spaces, *i.e.*, Banach lattices in which relatively weakly compact sets and almost order bounded sets coincide. There follows a detailed study of Banach lattices in which every semi-normalized, order bounded, weakly null sequence contains a subsequence satisfying a lower resp. an upper 2-estimate. As a consequence we obtain that each bounded sequence in a weak Schur space has a subsequence which either converges or satisfies a lower 2-estimate. This generalizes a result of D.J. Aldous and D.H. Fremlin. From the previous results we obtain conditions under which non-Dunford-Pettis operators between certain classes of Banach lattices fix a copy of l_2. This generalizes results of H.P. Rosenthal and of N. Ghoussoub and H.P. Rosenthal.

A. VAN ROOIJ

Representation theorems without the axiom of choice

Most representation theorems for Riesz spaces, such as the Yosida, Maeda-Ogasawara and Kakutani theorems, are based upon the Axiom of Choice or the Ultrafilter Theorem. We present a systematic way to obtain consequences of these representation theorems within the system ZF. To this end we prove a representation theorem for "small" Riesz subspaces and then proceed to an inductive limit.

H.H. SCHAEFER

Dual characterization of order continuity

We "dualize" the concept of order continuity (or sequential order continuity) for linear operators between Riesz spaces. The general hypothesis is that the range space be separated by its order continuous (or sequentially order continuous) linear functionals. As applications, we prove rather strong closure properties of the respective sets of positive and of general order continuous linear operators in $L(E,F)$ and $L^r(E,F)$, respectively, where E and F are Riesz spaces. This entails, in particular, a characterization of one-parameter semigroups on $L^\infty(\mu)$ which are adjoints of C_0-semigroups on $L^1(\mu)$.

E. SCHEFFOLD

Bidual von F-Banachverbandsalgebren

Es wird berichtet, daß der Bidual einer solchen Algebra wieder eine F-Banachverbandsalgebra ist und somit direkte Summe seines Annulatorbandes und dessen orthogonalen Komplements ist. Ferner werden diese beiden Summanden charakterisiert. Unter anderem wird gezeigt, daß die Banachverbandsalgebren $C_0(X)$ die einzigen F-Banachverbandsalgebren sind, deren Bidual ein algebraisches Einselement mit Norm 1 besitzt.

A.R. SCHEP

Norms of positive operators on L^p-spaces

Let $0 \leq T : L^p \to L^q$, $1 \leq p,q \leq \infty$, and let $\|T\|_{p,q}$ denote the operator norm. We show that solving $T^*(Tf)^{q-1} = \lambda f^{p-1}$ can be used to compute the exact value of $\|T\|_{p,q}$ and that solving $T^*(Tf)^{q-1} \leq \lambda f^{p-1}$ can be used to bound $\|T\|_{p,q}$. As an application we compute the exact value of $\|V\|_{p,q}$, where $Vf(x) = \int_0^x f(t)dt$ is the Volterra operator from $L^p([0,1]) \to L^q([0,1])$. Other applications consist of a new proof of Maurey's factorization theorem for positive linear operators, interpolation of positive linear operators, etc.

K.D. SCHMIDT

Daugavet's equation and orthomorphisms

Let E be a Banach space. A bounded linear operator $T : E \to E$ satisfies the Daugavet equation if $\|I + T\| = 1 + \|T\|$ holds, where I denotes the identity operator. It is remarkable that in most of the existing results on Daugavet's equation E is actually a Banach lattice, whereas Banach lattice methods have only been used by Lozanovskii (1966) and Synnatzschke (1978). The aim of this talk is to show that orthomorphisms in Banach lattices provide a useful tool for either extending known results on Daugavet's equation or for unifying their proofs. In particular it is shown that every Dunford-Pettis operator in an AL-space or an order complete AM-space with unit which (in either case) is assumed to have no discrete elements, satisfies Daugavet's equation.

G. SWANEPOEL

Semi-Carleman operators in Riesz spaces

We define semi-Carleman operators in Riesz spaces and consider the duality between Carleman operators and semi-Carleman operators.

L.M. VENTER

The peripheral spectrum in Banach lattice algebras

In 1980 Scheffold published the following theorem:

If E is a Banach lattice algebra for which the set $\mathcal{M}$ of non-trivial multiplicative linear functionals on $E+iE$ is modulus-invariant, then $\mathcal{M}$ is cyclic.
If E is commutative and unital, and $z \in E+iE$ with $r(z)=1$, the following holds:
(i) $z \geq 0 \Rightarrow \sigma_r(z)$ is cyclic.
(ii) $z > 0 \Rightarrow \sigma_r(z) = \{1\}$.
(iii) z has period s $\Rightarrow \alpha^{sk+1} \in \sigma(z),\ \alpha^{sk} \in \sigma(|z|),\ \alpha \in \sigma_r(z), k \in \mathbb{Z}$.
We show that this result may also be obtained without using representation theory.

J. VOIGT

Approximation of multipliers by regular operators

Let G be a locally compact Abelian group, $\hat{G}$ its dual group, and $\lambda, \hat{\lambda}$ the respective Haar measures. For $1<p<\infty$, let $\mathcal{L}_i(L_p(G))$ denote the operators commuting with translations. Recall $\mathcal{L}_i(L_p(G)) \subset \mathcal{L}_i(L_2(G))$, and recall that, via Fourier transformation, every operator $T \in \mathcal{L}_i(L_2(G))$ corresponds to a multiplication operator by a function $\hat{T} \in L_\infty(\hat{G})$.

Theorem. Let $1<p<\infty$, $T \in \mathcal{L}_i(L_p(G))$, and assume $T \in \overline{\mathcal{L}^r(L_p(G))}$ (the operator norm closure of the regular operators $\mathcal{L}^r = \text{lin}\mathcal{L}_+$). Then $\hat{T} \in \overline{B(\hat{G})}$ the (uniform closure of the Fourier-Stieltjes algebra on $\hat{G}$).

Example. The Hilbert transformation on $L_p(\mathbb{R})$, $l_p(\mathbb{Z})$, $L_p(\Pi)$ $(1<p<\infty)$, does not belong to $\overline{\mathcal{L}^r}$.

As a consequence one obtains that, for any infinite dimensional L_p-space $(1<p<\infty)$, $\mathcal{L}^r(L_p)$ is not dense in $\mathcal{L}(L_p)$.

A.W. WICKSTEAD

Positive numerical range

If E is a (real) Banach lattice and $T \in L(E)$ then the *positive numerical range* of T is $V^+(T) = \{f(Tx) : x \in E_+, f \in E_+^*, \|f\| = \|x\| = f(x) = 1\}$, which is a non-empty bounded interval in $\mathbb{R}$. The *positive numerical radius* of T is $v^+(T) = \sup\{|\lambda| : \lambda \in V^+(T)\}$. My work on this is still in progress but results include:

1. If $T \geq 0$ then $V^+(T) \subseteq \mathbb{R}_+$, but the converse is false.
2. If $T \geq 0$ then $\|T\| \geq v^+(T) \geq \|T\|/e$. This does not extend to non-positive operators even if E is 2-dimensional.
3. If $T \geq 0$ and E is Dedekind σ-complete or an M-space, then $V^+(T) \subseteq V^+(T^*) \subseteq \overline{V^+(T)}$.

4. With hypotheses of (3) if there exists $\alpha > 0$ such that $\lambda \geq \alpha$ for all $\lambda \in V^+(T)$ then $T \geq \alpha I_E$.
5. If E is any Banach lattice, $T \in L(E)$, $e^T \geq 0$ and $\|T\| < \log 2$ then there exists $\alpha > 0$ such that $e^T \geq \alpha I_E$.
6. If A is a real 2×2-matrix and $e^A \geq 0$ then the diagonal entries in e^A are non-zero.

F. WIID

Choice free representations of Riesz spaces

We prove that Yosida's Representation theorem for uniformly complete Archimedean Riesz spaces with strong unit can be appropriately rephrased to give a Representation theorem not inclosing the axiom of choice. We have:

The category of uniformly complete, unital, Archimedean Riesz spaces is dually equivalent to the category of completely regular compact locales.

Our methods are applicable to the other known representation theorems in Riesz space theory. Applications to tensor products are given.

G. WITTSTOCK

The ideal of completely positive compact maps on non-commutative L^p-spaces

This is a report of results due to E. Neuhardt (Saarbrücken).
We show that the completely positive compact maps from $L^p(\mathcal{M})$ into $L^q(\mathcal{N})$, $\mathcal{M}$ and $\mathcal{N}$ von Neumann algebras, form an order ideal whenever $p > 1$ and $q < \infty$. This means that every completely positive map which is dominated by a compact map is itself compact. Since a positive map from $L^p(\mathcal{M})$ into $L^q(\mathcal{N})$ is already completely positive if $\mathcal{M}$ or $\mathcal{N}$ is abelian, this includes the abelian result. The main idea to the non-commutative extension consists in replacing formulas for the infimum of two linear operators by representation theorems for algebras and and linear functionals. The order ideal of completely positive compact maps from $\mathcal{M}$ into $L^q(\mathcal{N})$, $q < \infty$, is monotone closed. Hence every completely positive map from $\mathcal{M}$ into $L^q(\mathcal{N})$ is the unique sum of two completely positive maps of which one is compact and the other dominates no other compact map. This can be considered as a non-commutative analogue of the band decomposition.

M. WOLFF

On the dilation of strongly continuous semigroups of contractions on L^p

Let $E = L^p(\Omega, \mu)$ where (Ω, μ) is a finite measure space, and let $(T_t)_{t\geq 0}$ be a C_0-semigroup of positive contractions on E (satisfying an additional hypothesis in case $p > 1$). Then there exists another measure space $(\hat{\Omega}, \hat{\mu})$, a C_0-group $(U_t)_{t\in\mathbf{R}}$ of lattice isomorphisms on $F = L^p(\hat{\Omega}, \hat{\mu})$, an embedding $j : E \to F$, and a positive contraction P from F onto E such that the following diagram commutes for all $t \geq 0$

$$\begin{array}{ccc} E & \xrightarrow{\;T_t\;} & E \\ {\scriptstyle j}\downarrow & & \uparrow{\scriptstyle P} \\ F & \xrightarrow[\;U_t\;]{} & F \end{array}$$

This theorem answers a question raised by I. Prigogine.

R. ZAHAROPOL

Ergodic decompositions of Banach lattices

We will be concerned with the description and the properties of two ergodic decompositions which we have recently obtained: an extension of the Hopf ergodic decomposition and another decomposition which we call the $\Omega\Pi$-decomposition. As an application, we will discuss a characterization of KB-spaces among the order continuous Banach lattices; the characterization is stated in terms of subinvariant elements for positive operators and of the two above-mentioned ergodic decompositions.

Let E be a Banach lattice, let E' be the dual of E, let $T : E \to E$ be a positive operator, and let T' be the dual of T. We will define two conservative ideals I_C, B_C in E, E', respectively, and two dissipative ideals I_D, B_D in E, E', respectively. It turns out that $E' = B_C \oplus B_D$; if E has order continuous norm, then $E = I_C \oplus I_D$. Under fairly general conditions the conservative and the dissipative ideals have all the properties the conservative and the dissipative parts (in the classical Hopf decomposition) have.

Given a Banach lattice E and a positive operator $T : E \to E$, we will define two bands: $\Pi(T)$ and $\Omega(T)$. If E has the projection property, then $E = \Pi(T) \oplus \Omega(T)$ (we say that E has an $\Omega\Pi$-decomposition generated by T). The band $\Pi(T)$ plays an important role in the study of T-subinvariant elements in E and in a characterization of the KB-spaces among the Banach lattices having order continuous norm.

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