MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 19/1989

Finite Geometries

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The conference was led by Professor F. Buekenhout (Brussels) and Professor D.R Hughes (London). There were three main strands of interest. Many of the questions considered concerned diagram geometries. Of particular interest was the construction of new geometries with given groups of automorphisms, for instance using graphs on which such groups are known to act or by constructing extensions of preexisting geometries. Another problem associated with diagram geometries was that of classifying the geometries for a given diagram, usually subject to additional constraints needed to make the problem tractable. Further contributions established the existence or nonexistence of structures such as block designs and generalized quadrangles with prescribed parameters and properties. Also discussed were problems concerning various subsets and configurations in classical geometries. Throughout the subject there is a rich interplay between the methods of group theory and of combinatorics.

Abstracts

A. Delandtsheer:

Flag-transitive linear spaces, maximal (v, k)-arcs and semiovals.

Current work of Buekenhout, Delandtsheer, Doyen, Kleidmann, Liebeck and Saxl aiming at classification of the finite linear spaces S with a flag-transitive automorphism group G provides presently a complete list of such pairs (S, G) which are not of affine type (i.e., G has no elementary abelian minimal normal subgroup acting regularly on the points of S).

We derive from this partial result a classification of

(i) the secant-transitive maximal (v, k)-arcs;

(ii) the thick semiovals transitive on their incident point-secant pairs in finite projective planes.

W. Haemers:

Partial circle geometries.

A partial circle geometry (the name is still under discussion) is a one-point extension of a partial geometry such that the triples that are on a circle (= block) form a regular 2-graph.

Examples are provided by some known structures:
- extended generalized quadrangles with $s = 2$;
- half of the circles of a Möbius plane;
- the "nice" 2-(16, 6, 2)-design.

Our new example can be constructed from the strongly regular graph on 176 vertices having M_{22} as its group of automorphisms. By this example we have a better description of the corresponding partial geometry (which was constructed earlier by the author).

A. Pasini:

On geometries which are locally polar spaces.

I give a sketch of one of the possible ways to prove uniqueness statements as the final step in the classification of a given class of geometries.
This method relies on the fact that a flag-transitive geometry Γ is simply connected iff Aut(Γ) is the amalgamated product of the stabilizers in Aut(Γ) of the elements of a chamber. I apply this trick to the case of flag-transitive locally classical C_2 geometries. We get a classification of this class of geometries, putting together contributions by several people (Buekenhout-Hubaut, Weiss-Yoshihara, Del Fra-Ghinelli-Meixner-Pasini).
I show one way to write a unified version of the proof of that classification theorem (being understood that other ways exist).

D. Ghinelli:

Extensions of C_2 geometries.

Let S be an extended generalized quadrangle of order (s, t) (i.e., a connected incidence structure such that every residue is a GQ(s, t)). Recently it has been proved that the diameter Δ of the point-graph of S is at most s+1.
We show that $\Delta = s+1$ if and only if one of the following occurs:
(i) $t = 1$ and S is isomorphic to the Johnson geometry on $\binom{2(s+1)}{s+1}$ points ($s>0$);
(ii) $s = 2$, $t = 2$ or 4 and S is isomorphic to the affine polar space of order 2 and type A_2 or D_2^-, on 32 and 56 points respectively.
(iii) $s = 1$ and S is complete tripartite on $3(t+1)$ points ($t > 1$).
(Joint work with A. Del Fra.)

S. Löwe:

Groups, strongly regular graphs, and stars.

A subset P of a finite group G is called a (k, l, m)-star (or partial addition set) if

$$P^2 = (k-m)1 + (l-m)P + mG$$

in the group algebra $K[G]$. Let e be the exponent of G and ξ be a primitive e'th root of unity. Set $K := \mathbb{Q}(\xi)$. If π is a prime coprime with e, there exists an automorphism $f_\pi : K \to K$ induced by $\xi \to \xi^\pi$.

Theorem Let $d := ((l-m)/2)^2 + (k-m)$ and assume that P is a union of conjugacy classes.

(i) $P^{[\pi]} = \{p^\pi \mid p \in P\} = P \iff f_\pi(\sqrt{d}) = \sqrt{d}$;

(ii) $f_\pi(\sqrt{d}) = -\sqrt{d} \Rightarrow P^{[\pi]} \cap P = \varnothing$.

In case (i), G can be extended by at least the group of squares mod e (if each of them acts as a morphism or an automorphism).

F. Mazzocca:

On $(q+t, t)$-arcs of type $(0, 2, t)$ in PG(2, q).

Let T denote a $(q+t)$-point set in PG(2, q) such that the intersection numbers with respect to lines are $0, 2, t$ $(t = 0, 1, 2)$, i.e., a so-called $(q+t, t)$-arc of type $(0, 2, t)$.

The results in the following have been obtained in a joint work with Gabor Korchmaros.

A simple counting argument shows that every point in T lies on just one t-secant and, consequently, t divides q and q is even.

A class of examples is found for every $q = 2^r$ with $r>2$, and for every $t = 2^h$ with $(r-h) \mid r$. Moreover, such examples have the remarkable property that all the t-secants are concurrent. We conjecture that the same property holds for every T. This conjecture is proved to be true except for $q = 2^r$, $r = (b+1)c$, $t = 2^h$, $h = bc+1$ with $c>2$, in which case the problem is still open.

Finally, a canonical representation of T is provided in case all the t-secants be concurrent. Using this representation, all sets T are classified for $q = 8, 16$ and $t = 4$.

F. Zara:

The generalized quadrangle of order (2,2) in an unusual setting.

Let $K = \mathbb{Z}[i]$ and M be a free K-module of rank 4. Let $\phi : M \times M \to K$ be an hermitian form such that there exists a basis $(a_1,\dots,a_4)$ of M with the following properties: $\phi(a_i, a_i) = 2$, $\phi(a_1, a_2) = \phi(a_3, a_4) = -1$, $\phi(a_2, a_3) = 1+i$, and $\phi(a_i, a_j) = 0$, for all $1 \le i \ne j \le 4$. Then ϕ is positive definite.

We put $M_2 = \langle m \mid m \in M, \phi(m, m) = 2\rangle$. If $m \in M$, let $m^* = \{em; e$ a unit of $\mathring{K}\}$, and $M_2{}^* = \langle m^* \mid m \in M_2\rangle$.

In $M_2{}^*$ there are pairwise orthogonal systems of cardinality 4 which are determined by each of their elements. Let $F = \langle E \mid E$ an orthogonal system as described above$\rangle$. Then $|F| = 15$ and there is a natural structure of GQ of order (2, 2) on F.

We deduce from this that $|M_2{}^*| = 60$, $|M_2| = 240$ and Aut(M, ϕ) is a nonsplit extension of S_6 by $2_+^{1+4} * C_4$ with amalgamated subgroup of order 2.

J. Hirschfeld:

Subsets of a finite plane.

If $P(\lambda)$ is the conic in PG(2,q) with equation $yz - x^2 = \lambda z^2$, let $K(A) = \bigcup_{\lambda \in A} P(\lambda)$ where A is a subset of GF(q).

Theorem 1: For q an odd prime and ζ a nonsquare in GF(q), the set $K(A)$ is a Buekenhout-Metz unital when $A = \zeta \mathrm{GF}(\sqrt{q})$.

Using the idea that $P(\lambda)$ is contained in $E(\lambda)$ (the internal points of P(λ)) if and only if λ-μ is a nonzero square and similarly $P(\lambda)$ is contained in $I(\lambda)$ (the internal points of $P(\lambda)$) if λ-μ is a nonsquare, it follows that, if B, A are subsets of GF(q) with B contained in A and s is sufficiently small with respect to q, the set $\bigcap_{\lambda \in B} E(\lambda) \cap \bigcap_{\lambda \in A \setminus B} I(\lambda)$ has size $\sim q/2^s$, where $s = |A|$. A disjoint union of such sets gives the following result:

Theorem 2: For every $c \in (0, 1)$ and every $\varepsilon > 0$, there is $q_0(\varepsilon)$ such that, for any $q > q_0(\varepsilon)$, in PG(2, q) there is a (k; λ)-arc where $n < cq$ and $k > (c-\varepsilon)q^2$.

C. Lefevre-Percsy:

New geometries for finite groups.

Let Γ be a transversal geometry on a set Δ. Let {0, 1} be a subset of Δ and suppose the rank 2 residue Γ_{01} is an (undirected) graph without loops or multiple edges. If Γ satisfies some further (weak) condition then we can construct a new geometry $\Gamma^{\sim}(0,1)$ with the same rank as Γ and whose diagram is derived from that of Γ.

Furthermore, if G is a chamber-transitive group of automorphisms on Γ then G also acts chamber-transitively on $\Gamma^{\sim}(0,1)$.

A generalization of the theorem of Neumaier allows us to build from $\Gamma^{\sim}(0,1)$ another geometry Γ', which is chamber-transitive if $\Gamma^{\sim}(0,1)$ is.

Some examples of these constructions are discussed.

J.H. van Lint:

A new design: 2-(28, 10, 5).

We construct an (up to 1988 unknown) block design 2-(28, 10, 5). The only unknown design with smaller v is 2-(22, 8, 4). This design disproves the following conjecture: "If a quasiresidual design with $k < v/2$ exists then the corresponding symmetric design also exists."
(Since 2-(43, 15, 5) does not exist, the conjecture is false.)
Our main theorem is: Suppose a 2-($9m$+1, $3m$+1, ($3m$+1)/2) design exists, with an automorphism f of order 3 fixing one point and the maximum number of blocks, i.e., 3($3m$-1)/2. Then the cycles of f considered as points and the fixed blocks form a 2-($3m$, m, (m-1)/2) design and the incidence structure of the nonfixed points and blocks is a partially balanced divisible design with group of size 3, $\lambda_1 = 1$ and $\lambda_2 = m+1$.

Z. Janko:

Some new triplanes.

A triplane is a symmetric design with parameters (v, k, 3). The largest known triplanes have parameters (71, 15, 3) and in fact with these parameters therre are known exactly 4 triplanes up to isomorphism and duality and all these 4 triplanes are non-self-dual (Haemers).
We discuss here the following two new theorems
Theorem 1. Let T be a triplane for (71, 15, 3) with a Frobenius group of order 21 as a collineation group. Then there exist up to isomorphism and duality exactly 15 triplanes of which 13 are non-self-dual and 2 are self-dual. Also, two of these fifteen triplanes possess an elation of order 2 and four further triplanes possess an involution which has a centre but no axis.
Theoem 2. Let T be a self-dual triplane with parameters (71, 15, 3) on which operates a Frobenius group of order 21 as a collineation group. Then there are exactly two such triplanes and in each case the full collineation group is of order 21.
Three of the 15 triplanes in Theorem 1 were known before.
It is also mentioned that if a triplane with parameters (81, 16, 3) exists then its full collineation group must be a 2-group. It is very probable that such a triplane in fact possesses an involutory homology.

J. Bierbrauer:

A new family of block designs.

Theorem. Let $q = 2^f$, f odd, $f \geq 3$. Then there is a $P\Gamma L_2(q)$-invariant block design with parameters

4-(q+1, 6, 10) defined on the projective line.
Its blocks are the $P\Gamma L_2(q)$-orbits of the sets $\{a, 1/a, 1+a, 1+1/a, 1/(a+1), 1+1/(a+1)\}$ for $a \in GF(q)\backslash\{0, 1\}$.
Block designs with block size 6 defined on projective planes of even characteristic are also considered.
Theorem. There is exactly one $P\Gamma L_3(8)$-invariant 4-(73, 6, λ)-design (up to taking the complement in the complete design). It has $\lambda = 330$.
Families of 3-designs are easily constructed. For example there is a $PGL_3(q)$-invariant design with parameters 3-(q^2+q+1, 6, $12(q-1)^2(q-2)^2$) whenever $q = 2^f, f \equiv 1 \pmod 4), f \geq 9$.
One can also use the plane to glue together designs which are defined on the lines. Only one example:
Theorem. Let $q = 2^f, f \equiv 1 \pmod 4$. If there is a 4-($q$+1, 6, $3(q-1)$)-design, then there is also a 4-(q^2+q+1, 6, $3(q-1)$)-design.
Lastly I give an easy construction of a 4-(21, 6, 16)-design defined on the projective plane of order 4.

M.de Resmini:

On scattered derivation.

The Lorimer-Rahilly and Johnson-Walker planes (LR and JW) share many properties: both are translation planes of order 16 obtained by derivation from the semifield plane with kern GF(2); in both the full collineation group has two orbits on l_∞ of lengths 3 and 14; both admit shears, and all shears determine the same involution on l_∞ and have centres in the short orbit. I have shown that they share l_∞ and nine pencils, with centres on l_∞. The finite points of a line not common to both planes are the affine points of a Baer subplane in the other plane.

To get LR from JW and conversely we may use a procedure similar to classical derivation and called scattered derivation. Eight sets of five points of l_∞, none containing the centre of any of the common pencils, behave as classical derivation sets and each of these provides sixteen Baer subplanes, the affine points of which become the affine points of the unshared lines in the other plane. Applying this procedure twice one gets back to the original plane.

The link between JW and LR can also be viewed in terms of latin squares: by "breaking up" eight of the latin squares for one of the planes and reassembling them in a different way one gets the latin squares for the other plane, leaving the remaining squares unchanged.

S. V. Tsaranov:

Two-graphs, related groups and root-lattices.

Let $\Gamma = (V, E)$ be a graph having vertex set $V = \{1,...,n\}$; the edge set E will be represented by the (1, -1) adjacency matrix $E(\Gamma)$ with $e_{ii} = 0$, $e_{ij} = -1$ for $(i, j) \in E$ and $e_{ij} = 1$ for $(i, j) \notin E$. Define the group

$$G(\Gamma) = \langle x_1,...,x_n : x_i^3 = 1, (x_i x_j^{e_{ij}})^2 = 1 \ (i \neq j) \rangle.$$

The following statements hold:

Theorem. (1) $G(\Gamma)$ is finite iff all eigenvalues of $E(\Gamma)$ are greater than -3.
(2) If $G(\Gamma)$ is finite then it is isomorphic to the index 2 subgroup of even elements of the Weyl group of type A_n, D_n, E_6, E_7 or E_8.
(3) If $E(\Gamma)$ has unique least eigenvalue -3 then $G(\Gamma)$ is isomorphic to the index 2 subgroup of even elements of a Weyl group of extended type $\hat{D}_n$, $\hat{E}_6$, $\hat{E}_7$ or $\hat{E}_8$.

A. Blokhuis:

Classification of complete external sets.

A complete external set with respect to a conic C in the desarguesian projective plane $PG(2, q)$, q odd, is a set of $(q+1)/2$ external points such that the line joining any two of them is an exterior line, i.e., misses the conic. We prove the following
Theorem (A. Blokhuis, A. Seress, H.A. Wilbrink) If $q \equiv 1 \pmod 4$ then a complete external set consists of the $(q+1)/2$ external points on an exterior line.
The restriction to $q \equiv 1 \pmod 4$ is in a way essential since there exist non-collinear complete external sets for $q = 7, 11, 19, 23, 27, 31$. However, I conjecture that for sufficiently large q all complete external sets are collinear.

N. Percsy:

Zara graphs and locally polar spaces.

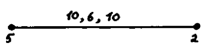
F. Zara has initiated the study of finite graphs in which, for any maximal clique M and any vertex $p \in M$, the set $p^\perp \cap M$ of vertices adjacent to p has a constant cardinality (independent of M and p). A main step in the study of these graphs is a theorem of A. Blokhuis, T. Kloks and H. Wilbrink stating that such a graph is (except for trivial cases) the collinearity graph of some locally polar space.
New ideas of proof enable us to generalize this result and obtain a unified proof of it together with the Buekenhout-Shult theorem on polar spaces and the Buekenhout and Johnson-Shult theorem on incidence structures S of points and lines in which $p^\perp$ satisfies the Buekenhout-Shult axiom for any point $p \notin S^\perp$:

Theorem. Assume G is a (not necessarily finite) connected and non-complete graph with at least one maximal clique of finite dimension, satisfying the following generalization of Zara's condition:
(GZ) for any maximal clique M and any vertex $p \notin M$ such that $p^{\perp} \cap M \neq \emptyset$, there is a maximal clique M' containing p such that $M \cap M'$ is maximal in the set $\{X \cap M \mid X \neq M$ is a maximal clique$\}$.
Then apart from trivial canonically described cases, G is the collinearity graph of some locally polar geometry.

M. Hermand:

Computing intersection matrices with CAYLEY.

Intersection numbers have been defined by D. Higman. Let G be a finite transitive permutation group acting on Ω. For each a in Ω let $\Gamma_1(a), \ldots, \Gamma_r(a)$ be the G_a-orbits on Ω, where G_a denotes the stabilizer of a in G. The number r is called the rank of G on Ω; the notation is chosen so that for all a in Ω, g in G, we have $\Gamma_i(ag) = (\Gamma_i(a))g$. The mappings Γ_i taking a to $\Gamma_i(a)$ are called the orbitals of G. For each orbital we define $\Gamma'(a) = \{ag^{-1};\ ag \in \Gamma(a)\}$. Wielandt has proved that Γ' is an orbital of the same length as Γ, $\Gamma'' = \Gamma$ and $a \in \Gamma'(b)$ iff $b \in \Gamma(a)$. If $\Gamma = \Gamma'$ then Γ is called a self-paired orbital. For a particular orbital Γ_i we define a (directed) graph on the points of Ω: (a, b) is an edge iff $b \in \Gamma_i(a)$. This graph is undirected iff Γ is a self-paired orbital. Intersection numbers relative to Γ_i are defined by: $m_{jk} = |\Gamma_i(b) \cap \Gamma_j(a)|$ ($b \in \Gamma_j(a)$). The $(r \times r)$ matrix $M_i = (m_{jk})$ is the intersection matrix of Γ_i.
We have written a little CAYLEY program to compute these matrices when G and G_a are given. As an application we have computed all intersection matrices for primitive representations of the Hall-Janko group, except when $G_a = A_5$ (in this case the rank is 191). We found interesting semi-linear spaces such as a



which is very probably embedded in PG(5, 5).

A. Prince:

Permutations of quadratic residues and finite projective planes.

The problem of constructing a projective plane of order $p+1$, admitting a collineation of order p with three fixed points, is considered.
Let $p \equiv 3$ (4). Denote by Q the set of quadratic residues mod p. Let g be a permutation of Q. For each $d \in \mathbf{Z}_{/p}{}^*$, let $X_d = \{(a, b) \in Q \times Q \mid a - b = d\}$, so that $|X_d| = (p-3)/4$. Let $A_d(g) = \{\pm(g(a)-g(b)),\ \pm(g(a)+g(b)) \mid (a, b) \in X_d\}$, so that $|A_d| \leq p-3$.
Theorem. If there is a permutation g of Q satisfying both of

(I) $|A_d(g)| = p-3$ for all $d \in \mathbf{Z}_{/p}{}^*$;

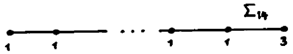
(II) $d_1 - d \notin A_d(g)$ for all $d \in \mathbf{Z}_{/p}{}^*$

then there is a projective plane of order $p+1$.
The permutation $g(a) = \sqrt{a}$ (where $\sqrt{a}$ denotes the unique square root of a in Q) satisfies (I) (but not (II)), and gives rise to the completion of a $p^2 \times p^2$ block in the incidence matrix (which cannot, however, be extended fully).

L. Soicher:

Geometries for Co_1 and the Suzuki sequence groups.

We discuss strongly-connected flag-transitive geometries having diagrams of the type



(*)

where Σ_{14} is the incidence graph of the (7, 4, 2)-biplane. A certain chain of geometries of this type with diagrams (*) having 2, 3, 4, 5, 6, 7 nodes have respective automorphism groups $L_3(2)$:2, $U_3(3)$:2, J_2:2, $G_2(4)$:2, 3·Suz:2 and Co_1.

A.E. Brouwer:

Shult's theorem on graphs with odd cocliques.

In a graph Γ, denote by $X^{\perp} = \{x\} \cup \Gamma(x)$ the set of vertices consisting of x and all its neighbours. A subset A is called odd when $|x^{\perp} \cap A|$ is odd for all x. Consider the following two conditions.
$(CC)_d$ Any (d-1)-coclique is contained in an odd d-coclique.
$(C1)_d$ There exists an odd d-coclique C and a vertex $x \notin C$ such that $|x^{\perp} \cap C| = 1$.
Theorem Let Γ satisfy $(CC)_d$ and $(C1)_d$ for some d. Then Γ is known.
More precisely we have
Theorem Let $d \geq 3$ and let Γ be a reduced coconnected graph satisfying $(CC)_d$ and $(C1)_d$. Then Γ is one of the graphs $VO^{\varepsilon}(m, 2)$, $O^{\varepsilon}(m, 2)$, $TO^{\varepsilon}(m, 2)$, $Sp(2n, 2)$, $N^{\varepsilon}(2n, 2)$, or the complement of $T(n)$.

A. Beutelspacher:

Subspaces of a subspace.

We discuss the proof and generalizations of the following
Theorem. Let L be a set of lines in PG(3,q) having the following properties.

(1) Any plane contains at least one line of L.
(2) If a point P is on at leas two lines of L then any plane on P contains at least one line of L/P (:= $\{l \in L \mid P \in L\}$). Such a point is called thick.

(3) There exists at least one thick point.

Then $|L| \geq q^2+q+1$ with equality if and only if one of two possibilities occurs:

(a) L is the set of lines of a plane, or

(b) the thick points are the points $P_0,\ldots,P_{q+1}$ on a line l and L is the union of the pencils of lines through P_i in π_i, where $\pi_0,\ldots,\pi_{q+1}$ are the planes through l.

J. Siemons:

The spectrum of an incidence structure.

Let $L = (P, B; \mathrm{I})$ be a finite incidence structure with point set P, block set B, incidence I and incidence matrix S. The spectrum of L is the spectrum of SS^T. If its distinct values are $\lambda_0>\ldots>\lambda_t\geq 0$, put $f(x) = (x-\lambda_1)\ldots(x-\lambda_t)$. We shall say that L is (r, k)-regular provided each point is incident with r blocks and each block with k points. J denotes the all-one matrix.

Theorem 1. Let L be (r, k)-regular. Then L is connected if and only if $f(SS^T) = |P|^{-1} f(rk) J$.

Let T be a tactical decomposition of the points and blocks of L and denote by C^- the matrix whose i'th row counts the number of blocks in each class incident with a point in the i'th point-class. Define C^+ dually and call the eigenvalues of C^-C^+ the spectrum of T. Let $f_T(x) = (x-\lambda_{1'})\ldots(x-\lambda_{t'})$ where the product includes only those values $\lambda_{i'} \neq \lambda_0$ which occur in Spec(T). Let U denote the matrix of constant rows whose entries are the numbers of points in each point-class of T.

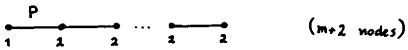
Theorem 2. In a connected (r, k)-regular structure let T be a tactical decomposition with matrices C^+ and C^-. Then $f_T(C^+C^-) = |P|^{-1} f_T(rk) N$.

We have complete information about spectral distributions for subset and subspace incidence structures (in finite affine and projective spaces) and their tactical decompositions.

A. A. Ivanov:

On universal covers of certain P-geometries

The talk presents joint work with S.V. Shpectorov on flag-transitive geometries G with the diagram

P^m: 

where P stands for the geometry of the Petersen graph.

Result 1 (S.V. Shpectorov) If $m = 1$ then $G = G(M_{22})$ or $G(3{\cdot}M_{22})$.

Result 2 (S.V. Shpectorov) If $m = 2$ then the universal 2-cover of G is isomorphic to that of $G(M_{23})$, $G(Co_2)$ or $G(J_4)$.

The geometries $G(M_{23})$, $G(Co_2)$ and $G(J_4)$ contain subgeometries H isomorphic to $G(A_7)$, $G(Sp_6(2))$ and $G(M_{24})$ respectively. Let $G^{\sim}$ be the universal 2-cover of G and $H^{\sim}$ be a connected component of the preimage of H in $G^{\sim}$.

Result 3: (i) If $G = G(M_{23})$ then $G^{\sim} = G$;

(ii) if $G = G(Co_2)$ then either $G^{\sim} = G$ or $H^{\sim} = G(3^7 \cdot Sp_6(2))$;

(iii) if $G = G(J_4)$ then $H^{\sim} = H$.

In the case $G = G(Co_2)$, $G^{\sim} = G(3^7 \cdot Sp_6(2))$ the automorphism group of G must have the form $3^{23} \cdot Co_2$ (a nonsplit extension). We do not know whether such a group exists.

W. Lempken:

Amalgams of type (Alt_6, Sym_6).

We consider the following group-theoretical situation:

Hyp(p): G is a group containing two finite subgroups P_1 and P_2 such that

(1) $G = \langle P_1, P_2 \rangle \neq P_i$ for i = 1, 2;

(2) $B := P_1 \cap P_2$ contains no nontrivial normal subgroup of G.

(3) There exists a prime p such that each p-Sylow of B is also a Sylow of P_i, i = 1, 2.

(4) $Q_i := O_p(P_i)$ contains its own centralizer in P_i for i = 1, 2.

This is the standard set-up for the study of p-local amalgams of rank 2. We then prove the

Theorem Suppose that (G, P_1, P_2) satisfies **Hyp**(2) such that $P_1/Q_1 \cong Alt_6$ and $P_2/Q_2 \cong Sym_3$. Then $B = Q_1Q_2$ and is a 2-Sylow of both P_1 and P_2, and one of the following occurs:

(I) $Q_1 \cong Ex^+(2^7)$ and $Q_1/Z(Q_1)$ is isomorphic to a 6-dimensional permutation module over F_2 for P_1/Q_1; B is of type Co_3 and in particular $|B| = 2^{10}$.

(II) Q_1 is elementary abelian of order 2^4 or 2^5 such that the centralizer in P_1 of Q_1 is trivial and $[Q_1, P_1] \cong E(2^4)$ is an irreducible P_1/Q_1-module; in particular, $|B| \in \{2^7, 2^8\}$.

Moreover, the structure is known in each case.

$U_4(3)$ and $U_4(3){:}2_3$ (ATLAS-notation) provide examples of finite completions in case (II) if P_1 splits over Q_1. We don't know of any finite completions in case (I).

M. Liebeck:

Minimal degrees of primitive permutation groups.

For G a primitive permutation group on a set Ω of size n, the minimal degree $\mu(G)$ is $\max\{|supp(g)|: g \in G\backslash\{1\}\}$, where $supp(g) = \{\omega \in \Omega : \omega g \neq \omega\}$. Suppose first that $G \leq Aut(L)$ containing L as normal subgroup, for some simple group L.

Theorem 1. If $L = L(q)$ is of Lie type over F_q then $\mu(G) \geq (1-(4/3q))n$, with some exceptions when $L = L_2(q)$, $L_4(2)$ or $PSp_4(3)$.

Theorem 2. If $L = A_m$ then either $\mu(G) \geq n/3$ or Ω is the set of k-subsets of m for some k.

Theorem 3. If L is sporadic then $\mu(G) \geq n/2$.

Corollary. For arbitrary G, either $\mu(G) \geq n/3$ or $(A_m)^r < G \leq S_m$ wr S_r and $n = \binom{m}{k}^r$.

There is an application: if $G = \langle x_1,...,x_r \rangle$ is a transitive subgroup of S_n with $x_1 x_2 .. x_r = 1$ then G is isomorphic to the monodromy group of a Riemann surface of genus g, where $2g+2(n-1)$ is equal to the sum of the numbers $\text{ind}(x_i) := n$-(number of orbits of $\langle x_i \rangle$). If $g = 0$ we call G a group of genus zero. It can be shown that some x_i fixes at least $n/85$ points if G has genus zero, and then Theorem 1 gives

Theorem 4. If $L = L(q)$ is a composition factor of a group of genus zero then $q < 113$ (if $L \neq L_2(q)$) or $q < 85^2$ ($L = L_2(q)$).

J.A Thas:

New flocks, new GQs, new planes, a characterization of all classical GQs, the complete classification of all minimal external sets and a remark on spreads and partial spreads of hermitian varieties.

Let F_0 be a flock of the quadratic cone K of PG(3, q), q odd. Then Bader, Lunardon and Thas prove that from F_0 q flocks can be derived. Applying this process of derivation to a class of flocks discovered by Kantor, they prove that there arises a new class of flocks for $q = 5^e$, $e>1$. Applying derivation to other known classes of flocks, Johnson discovered a few days ago two further new classes of flocks, one for $q = 3^{4e}$. By the connection between flocks, planes and GQ due to Thas, there arise new classes of GQ of order (q, q^2) and new classes of planes of order q^2. Recently Thas and Van Maldeghem obtained a new combinatorial characterization of all classical GQ. They prove a generalization to GQ of Baer's theorem on (p, L)-transitivity of projective planes and then apply the Moufang theorem of Tits. Also characterizations of classes of non-classical GQ are obtained.

I prove that the hyperbolic quadric $Q^+(5, q)$, q odd, admits no maximal exterior set (MES), achieving the complete classification of all MES of the hyperbolic quadrics.

If S is a partial spread of H(3, q^2) then I show that the size of S is at most q^3-q^2+q+1; it follows that H(3, q^2) has no spread (a well-known result of Bruen and Thas). If S is a partial spread of H(5, q^2) then I show that the size of S is at most $q^2(q^2+q-1)$, whence H(5, q^2) has no spread.

S. Payne:

Generalized quadrangles of order (q^2, q).

Let $S = (P, B, I)$ be a generalized quadrangle of order (s, t). If p, x are distinct points of a line L, we say an (x, L, p)-symmetry is a collineation θ of S which fixes each line through x, each point on L and each line through p and such that if θ fixes a line M meeting L in some point y then θ fixes all lines through y. Under some additional hypothesis, for example that $(s, t) = (q, q^2)$, we have:

Theorem. Let L, M be distinct lines meeting at a point p. Assume that for each point x of $L \cap M$, $p \neq x$, there is a group $H(x)$ of (p, px, x)-symmetries with $|H(x)| = s$. Put $A = \langle H(x) \colon p \neq x \mathrm{I} L \rangle$, $B = \langle H(x) \colon p \neq x \mathrm{I} M \rangle$, $G = \langle A, B \rangle$. Then

(i) $|G| = s^2 t$; G acts regularly on $P \backslash p^{\perp}$ and fixes each line through p.

(ii) A and B are elementary abelian normal subgroups of G with $|A| = |B| = st$.

(iii) If $s > t$ then $C = A \cap B$ is the center of G; $|C| = t$; C is a group of symmetries about p (i.e., C acts trivially on $p^{\perp}$); p is regular.

(iv) For each point $z \in p^{\perp} \backslash \{p\}$, $H(z) = \{\theta \in G \colon \theta \text{ is a } (z, zp, p)\text{-symmetry}\}$ is a group of order s.

We say $S = (S^{(p)}, G)$ is a Moufang skew translation GQ.

Until recently there were known seven infinite families of GQ of order (q^2, q) associated with flocks and due to Thas, and one due to Tits. For $q = 3^e \geq 27$ we construct a family not direcly associated with a flock. For $q = 5^e \geq 25$, Bader, Lunardon and Thas have constructed a new family by a process of derivation of a flock to produce a new flock. N.L Johnson and I have imitated this process to obtain another new family with $q = 3^e \geq 27$. Hence there are now known eleven infinite families of GQ with order (q^2, q).

C. Hering:

On the cardinality of certain sets of geometric objects.

Many classical sets of geometric objects in finite buildings have a cardinality of the form

$$p^e \prod_{i=1}^{t} \Phi_{u_i}(q)$$

where $q \in \mathbb{N}$, $\Phi_n(x)$ is the n'th cyclotomic polynomial and $e, t, u_i \in \mathbb{N} \cup \{0\}$. A set of this kind tends to have exceptional properties if the prime divisors of the numbers $\Phi_n(q)$ are particularly small. This leads to the following problem: determine all pairs (n, x) of natural numbers such that $n \geq 3$ and $\Phi_n(x)$ is not divisible by any prime larger than $2n+1$. We present an algorithm to determine all integral solutions of

$$ax^2+bx+c = p_1^{y_1} \ldots p_s^{y_s}$$

for given integers a, b, c and primes $p_1, \ldots, p_s$. This allows to solve the above problem if n is a power of 2 or a power of 3 or if $n \in \{6, 18, 36\}$. The case $n = 5^r$, $r > 1$, can be handled using results of Nagell (1921) and others on diophantine equations of the form $(x^n-1)/(x-1) = y^2$. By a result of B. Mühlherr there is no solution if $n = 3^r 5^s$ where $r, s > 1$. If $n = 5$ or 15 there is no solution such that x is a prime power greater than 3.

D. Jungnickel:

Divisible difference sets.

Let G be a group of order mn with a normal subgroup N of order n. A k-subset of G is called a divisible difference set with parameters $(m, n, k, \lambda_1, \lambda_2)$ if each element of $N\backslash\{1\}$ has exactly λ_1 representations as a "difference" de^{-1} with $d, e \in D$, while each element of $G\backslash N$ has exactly λ_2 such representations. Then dev $D = (G,\{Dg: g \in G\})$ is a divisible design with the same parameters admitting G as a Singer group. An automorphism of G is called a multiplier for D if it is also an automorphism of dev D. We consider abelian DDD's for which -1 (the mapping $x \rightarrow x^{-1}$) is a multiplier. Note that this does not necessarily imply that any translate of D is fixed under -1. We obtain severe restrictions on the structure of D; for instance, at least one of the numbers $k-\lambda_1$, $k-\lambda_2 mn$ is a square and if p is a prime dividing the squarefree part of $k-\lambda_1$ then either $p = 2$ or $p \equiv 1$ (mod 4) and n is a power of p. If D is in fact reversible (i.e., fixed under -1) then only the case $p=2$ can arise. We determine all multipliers of a reversible DDS. A relative difference set with multiplier -1 (i.e., $\lambda_1 = 0$) has parameters $m = k = n\lambda$, m even (and thus belongs to a symmetric transversal design); if D is reversible then m is a perfect square. We also construct families of examples and partially characterize the cyclic reversible DDD's and the DDS's with $k-\lambda_1 = 1$. The proofs are mainly algebraic, using group algebras, characters both over the complex field and finite fields, and some algebraic number theory.

E. Shult:

Geometric Hyperplanes.

Let H be a geometric hyperplane of a point-line geometry $\Gamma = (P, L)$, that is, a subspace of Γ which intersects each line nontrivially. We say H arises from an embedding e: $\Gamma \rightarrow \mathbf{P}$, where $\mathbf{P} \cong PG(d, k)$, if H is the inverse image of $e(\mathbf{P}) \cap Y$ where Y is a projective hyperplane of $\mathbf{P}$. Joint work with B. Cooperstein shows that all geometric hyperplanes arise from an embedding if Γ is one of the geometries $A_{n,2}$, $A_{n,3}$ (not over GF(2)), $D_{5,5}$ and $E_{6,1}$. It is also shown that if H is a geometric hyperplane of a near hexagon with quads which meets each quad at a star, then H is a generalized hexagon. If Γ is finite with parameters, it is a dual polar space of type $\Omega(7, q)$ or $Sp(6,q)$, q even, and H is the generalized hexagon of type $G_2(q)$.

A. Camina:

On the automorphisms of a (91, 6) regular linear space.

There are four known (91, 6) regular linear spaces and each has a regular cyclic subgroup of automorphisms. V.D. Tonchev and Stoichev have calculated their complete automorphism groups.

Two of them are line-transitive but point -imprimitive. This is a rare phenomenon for block designs, as is shown by a beautiful result of Delandtsheer and Doyen. In fact the two (91, 6) linear spaces are the only known examples of regular linear spaces with this property. Lino Di Martino and I have proved that if L is a (91, 6) regular linear space and G = Aut L is point-primitive then G is soluble, $|\mathrm{Fit}(G)| = 91$ and $G/\mathrm{Fit}(G)$ is isomorphic to a cyclic group of order 12.

R. Scharlau:

Combinatorial methods for integral quadratic forms.

Consider an integral lattice L in a real vector space V, that is, a positive definite scalar product $(x|y)$ is given such that $(x|y)$ is an integer whenever x, y are in L. A primitve vector $v \in L$ is called a root of L if the corresponding reflection $s_v: x \to x - 2(x|v)/(v|v)v$ maps L into itself. The set $R(L)$ of all roots is a root system. L is called reflective if its root system has greatest possible rank, that is, the sublattice generated by roots is of finite index. Notice that if L is not unimodular the roots do not necessarily have length 1 or 2.

It turns out that, although the class of reflective lattices is much larger than the class of ordinary root lattices (generated by vectors of lengh 1 or 2), the reflective lattices can be classified in a fashion only depending on the combinatorial type of the root system. As an example, consider the following result in dimension 3, obtained by Britta Blaschke. We use the notation ${}^{\alpha}\mathrm{A}_n$ (roots of length 2α), ${}^{\alpha}\mathrm{B}_n$ (short roots of length α), ${}^{\alpha}\mathrm{C}_n$ (short roots of length α) for "scaled" root systems.

Theorem. The root systems of the indecomposable 3-dimensional reflective lattices are ${}^{\alpha}\mathrm{A}_1 \times {}^{\beta}\mathrm{A}_1 \times {}^{\gamma}\mathrm{A}_1$ $(\alpha<\beta<\gamma)$, ${}^{\alpha}\mathrm{A}_1 \times {}^{\beta}\mathrm{B}_2$ $(\alpha \equiv 0\ (2), \beta \neq \alpha \neq 2\beta)$, ${}^{\alpha}\mathrm{C}_3$, ${}^{\alpha}\mathrm{B}_3$ $(\alpha \equiv 0\ (4))$. For each of these there is a unique lattice, except for ${}^{2\alpha}\mathrm{A}_1 \times {}^{2\beta}\mathrm{A}_1 \times {}^{2\gamma}\mathrm{A}_1$, where there are two.

The point is that the values of α, β, γ do not play a rôle.

E. Moorhouse:

Planes, semibiplanes and related complexes.

From a projective plane Π with involutory homology τ one constructs a semibiplane $\Sigma = \Sigma(\Pi, \tau)$ whose "Points" and "Blocks" are the τ-orbits of length 2 on the points and lines of Π. We are interested in reversing this process: given Σ, what planes, if any, yield Σ in this way?

From S we construct a rank-2 cell complex $\Gamma = \Gamma(\Sigma)$ whose vertices are the points and blocks of Σ; its edges are the flags of Σ and its faces are the digons of Σ. Let $F = \mathrm{GF}(2)$, $C^i = C^i(\Gamma, F)$ the F-space freely spanned by the i-cells of G, $\delta: C^i \to C^{i+1}$ the coboundary operator. We show that

(i) "liftings" from Σ to planes correspond bijectively to "admissible" elements of C^1, which (if they exist) form a coset of $Z^1 = \ker \delta: C^1 \to C^2$;

(ii) if the cohomology $H^1(\Gamma, F) = 0$ then Σ lifts in at most one way up to equivalence;

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(iii) case (ii) occurs if Σ arises from PG(2, p); and

(iv) the equivalence class of pairs (Π, τ) such that $\Sigma(\Pi, \tau) \cong \Sigma$ correspond bijectively with the orbits of Aut Σ on the set of admissible elemnts of C^1, modulo $B^1 = \delta(C^0) \leq C^1$.

The translation planes of order 16 provide examples with dim $H^1(\Gamma, F) = 0, 1, 2, 4$. We also ask how far these ideas apply if $\langle\tau\rangle$ is replaced by some other collineation group, especially one of odd prime order.

E. Spence:

(45, 12, 3)-designs.

A (v, k, λ)-graph is a graph with v vertices and degree k in which every pair of vertices has λ neighbours. Thus a (v, k, λ)-graph G may be considered as a (v, k, λ)-design $D(G)$ where the points are the vertices of the graph and the blocks are the neighbours of the points. A (v, k, λ)-design is a (v, k, λ)-graph if it possesses a polarity having no absolute points. During investigations into the question of which (v, k, λ)-graphs G have the property that Aut(G)≠Aut($D(G)$) (joint work at various stages with F.C. Bussemaker, W.H. Haemers and J.J. Seidel) a new family of (45, 12, 3)-designs was discovered. They were found originally by looking at the adjacency matrix A of a (45, 12, 3)-graph for which Aut(G)≠Aut($D(G)$). It turns out that there must be an automorphism P of order 3 with either 3 or 9 fixed points, such that $PAP = A$. Further examination of A yields a certain block structure for A, which, with minor modification, gives rise to over 2550 designs (up to isomorphism) of which 550 have trivial automorphism group. They all have automorphism group with order of the form $2^a 3^b 5^c$.

D.R. Hughes:

Extended transversal designs and extended nets.

We attempt to find examples of, and to characterize, (finite) extended transversal designs and (finite) extended nets.

Theorem. (i) An extended TD(s) is a 2-design or is a TD(s+1); (ii) if $s = 2$ then the extension is a 2-(7, 4, 2) or a 2-(22, 6, 4) (both unique), or is an extension of a dual affine plane of even order to a TD(3) (examples exist for all powers of 2); (iii) if $s = 3$ then there is no extension.

Nets are more difficult: we restrict to extensions which are partial geometries.

Theorem. If an extended net is a partial geometry then it is one of the following: (a) an inversive plane or "half-inversive plane"; (b) an extended TD(2) (examples exist for all prime powers); (c) an extension of a (n, n-1)-net, having $(n+1)^2$ points (examples exist for all prime powers); (d) one of an infinite class of possibilities extending (n, n-e)-nets (no examples are known).

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