MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 20/1989

Generalized Functions and Complex Analysis

7.5. bis 13.5.1989

The meeting was organized by P. Dierolf (Trier), V.S. Vladimirov (Moscow) and D. Vogt (Wuppertal). 53 mathematicians from 16 countries had met and 40 lectures in a broad spectrum were delivered. The atmosphere was described by all participants as friendly and congenial.

The generalized functions that were considered comprise classical distributions in the sense of L. Schwartz, ultradistributions, hyperfunctions, as well as other concepts. Some of the questions that were investigated are the solvability of partial differential and related operators, the continuity of solution operators, the propagation of singularities, various concepts for products of generalized functions, and many others. The conference offered the opportunity for fruitful discussions and exchange of ideas over a wide range of interesting subjects within the framework of generalized functions.

Abstracts

Berg, L.

Die mehrdimensionale Translationsgleichung

Die monotonen Lösungen der eindimensionalen Translationsgleichung $F(t,F(s,z)) = F(t+s,z)$ mit $F(O,z) = 0$ haben bekanntlich die Gestalt (i) $F(t,z) = f^{-1}(t+f(z))$. Im differenzierbaren Fall genügen sie den drei Differentialgleichungen von E. Jabotinsky (1963), aus denen man (i) umgekehrt durch Integration gewinnen kann. Ist t ein m- und si F und z beide n-dimensionale Vektoren, so hat man es mit einem System partieller Differentialgleichungen zu tun, wie es in der Theorie der Lie-Reihen auftritt. Durch Integration dieses Systems entsteht eine Verallgemeinerung von (i), die gleichzeitig auch die Lösungsdarstellung aus dem Buch J. Aczél, "Vorlesungen über Funktionalgleichungen und ihre Anwendungen", Berlin 1961, verallgemeinert. Der Vortrag ist auch eine Verallgemeinerung des Falls $m = 1$, der unter dem Thema "On nonlinear systems of ordinary differential equations" auf der Tagung "Generalized Functions, Convergence Structures, and Their Applications" im Juni 1987 in Dubrovnik vorgetragen wurde.

Braun, R.W. (reporting on joint work with R. Meise and D. Vogt)

Surjectivity of convolution and partial differential operators on classical Gevrey classes

Let $\Gamma^{\{d\}}(\mathbb{R}^N)$ denote the classical Gevrey class (Roumieu-type) of order $d > 1$. It is expained how the projective limit functor introduced by Palamodov can be used to characterize the surjectivity of convolution operators T_μ on $\Gamma^{\{d\}}(\mathbb{R})$ and of partial differential operators on $\Gamma^{\{d\}}(\mathbb{R}^N)$. In the case of convolution operators, the characterization is given by a condition concerning the location of the zeros of μ in the complex plane. In the case of partial differential operators, a "Phragmén-Lindelöf"-condition is stated. This extends previous results of Cattabriga and Zampieri. "Phragmén-Lindelöf"-conditions were introduced by Hörmander when he solved the analogous problem in the space of real analytic functions.

Burzyk, J.

The Fourier transform of boehmians

The following two theorems are proved:

Theorem 1: If $\omega : \mathbb{R} \to [0,\infty)$ is such a measurable function that: $\int_{-\infty}^{\infty} \frac{\omega(t)}{1+t^2} dt < \infty$, ess sup $|\omega(t+s) - \omega(t)| < \infty$ for each $s \in \mathbb{R}$, then for an arbitrary locally integrable function $f : \mathbb{R} \to \mathbb{R}$ such that $|f(t)| \leq e^{\omega(t)}$, there exists a transformable boehmian x such that $F(x) = f$.

Theorem 2: (a) If F(z) is an entire function such that:

(1) For every $\varepsilon > 0$ there exists a constant A_ε such that $|F(z)| < A_\varepsilon e^{(\sigma+\varepsilon)|z|}$

(2) $\int_{-\infty}^{\infty} \frac{\log_+ |F(t)|}{1+t^2} dt < \infty$

Then there exists a boehmian x such that supp $x < [-\sigma, \sigma]$ and $F(x) = F$

(a) Conversly, the Fourier transform of each boehmian whose support is contained in $[-\sigma, \sigma]$ satisfies conditions (1) and (2).

We give some applications of these theorems and prove a theorem about the existence of a solution of a convolution equation.

Cioranescu, I.

The abstract Cauchy problem in spaces of almost periodic distributions

The following result is presented:

Let X be a Banach space and A a linear and densely defined operator in X, with a non-void resolvent set; A is the infinitesimal generator of an almost periodic distribution group iff the spectrum of A lies on iR, the set of its eigenvectors is total in X and the resolvent function satisfies the following conditions:

(i) there exist $k \in N_o$ and $C > 0$ such that

$$\|R(z;A)\| \leq C \frac{(1+|z|^2)^k}{|\mathrm{Re}\ z|}, \quad |\mathrm{Re}\ z| \neq 0.$$

ii) $\sup_{\substack{\varepsilon>0 \\ s\varepsilon R}} \| \int_{-\infty}^{+\infty} e^{ist} (1+t^2)^{-(k+2)} [R(\varepsilon+it;A) - R(-\varepsilon+it;A)] dt \| < \infty$

Application is made to operators on L^2 coming from the perturbation of the operator $d/_{dt}$ by a potential.

Colombeau, J.F.

Multiplications of distributions in engineering and physics

We show systems of equations used by engineers and physicists, in which there appear products in the form of "multiplications of distributions". These products do not make sense within distribution theory (or within classical concepts of generalized functions). We expose how one can resolve this problem, using a nonclassical theory of generalized functions. The key lies in that this theory permits mathematical statements of physical laws that are deeper than usal; this permits to resolve ambiguities inherent in such products in their classical heuristic formulation. This method leads at once to (new) algebraic formulas and new numerical methods.

Dimovski, I.H.

Solution of the Mikusinski problem in non-Mikusinski's operational calculi

It still remains open the following problem for the field of Mikusinski: If $f(\lambda)$ is a differentiable parametric function in an interval I, i.e. it is a convolution quotient $f(\lambda) = \{\phi(t,\lambda)\} / \{\psi(t,\lambda)\}$ of continuously differentiable functions with respect to λ, then does the identity $f'(\lambda) = 0$ with $f'(\lambda) = \frac{\phi_\lambda * \psi - \phi * \psi_\lambda}{\psi^2}$ imply $f(\lambda) = \text{const}$ in I.
We show that if instead of the Duhamel convolution it is taken the convolution found by the author and L. Berg in 1974:

$$(1) \qquad (f*g)(t) = \Phi_\tau \{\int_\tau^t f(t+\tau-\sigma)\, g(\sigma)\, d\sigma\};\ \Phi \in [C[\alpha,\beta]]^*$$

then a sufficient condition for a positive answer of the Mikusinski problem is that the endpoints α and β should belong to the support of the functional Φ in (1).

Dwilewicz, R.

Geometry of hypersurfaces of finite type

Conditions for holomorphic extendability of CR-functions from hypersurfaces of finite type will be presented. Also qualitative conditions for holomorphic hulls of balls of the Nagel-Stein-Wainger type will be given.

Fisher, B. (paper joint with Li Chen Kuan)

<u>The product of generalized functions in m variables</u>

Let ρ be a fixed infinitely differentiable function of the single variable x with the properties:

(i) $\rho(x) = 0$, $|x| \geqq 1$, (ii) $\rho(x) \geqq 0$,

(iii) $\rho(x) = \rho(-x)$,

(iv) $\int_{-1}^{1} \rho(x)\,dx = 1.$

If now $x = (x_1, \ldots, x_m)$, the function $\delta_n(x)$ is defined by

$$\delta_n(x) = n^m \rho(nx_1) \ldots \rho(nx_m).$$

It follows that the sequence $\{\delta_n\}$ converges to the Dirac delta function δ.

The product of two distributions f and g in K_m' is defined to be the neutrix limit h of the sequence $\{fg_n\}$, provided h exists in the sense that

$$\underset{n\to\infty}{\text{N-lim}}\, (fg_n, \phi) = (h,\phi)$$

for all ϕ in K_m, where $g_n = g * \delta_n$. Some results are given.

Gesztelyi, E.

<u>Generalized functions as carriers of information in the theory of colour recognition</u>

We deal here with the modelling of input transformers of information carried by the light.

Let $X = \{\xi' \,|\, \xi \in NBV_+(\alpha,\beta)\}$ be the set of derivatives in generalized sense of increasing normed functions on an interval (α,β).

Let $A : X \to \mathbb{B}$ be a mapping. Then the relation defined by

$x \overset{(A)}{\equiv} y \Leftrightarrow A(x) = A(y)$ will be called x identical with y according to A. We proved previously that, using some conditions corresponding to Grassmann's laws, an input transformer may be written in the form

$$A(x) = \int_{\alpha}^{\beta} \underline{u}(t)\, d\xi(t) \quad (x=\xi').$$

An element x_o is said to be recognizable by A if

$$\forall x \in X : x \overset{(A)}{\equiv} x_o \Rightarrow x = x_o.$$

An input transformer is called normal if any Dirac-delta function in X is recognizable by A.

<u>Theorem:</u> An input transformer A is normal iff $\underline{u} = (u_1,u_2,u_3)$ and u_1,u_2,u_3 form a Chebichev-system on (α,β).

Gramsch, B.

<u>Topological algebras of some classes of pseudo-differential operators</u>

For applications and generalizations in the theory of distributions and in complex analysis (in K-theory, in perturbation theory and in differential geometry) it turned out that it is useful to consider Fréchet algebras ψ with the properties I and II);
I): The group of invertible elements of ψ is open; II): ψ is a countable projective limit of Banach algebras. Due also to results of R. Beals, Connes, Cordes, Schrohe, Schulze and Ueberberg (cf. E. Schrohe, Arch. Math. 51 (1988), J. Ueberberg, manuscr. math. 61 (1988)) many algebras of pseudo-differential operators are Fréchet spaces with the properties I and II). A recent result of Ueberberg, Wagner and the author shows that the Hörmander classes $\psi^0_{\rho,\delta}$, $0 \leq \delta \leq \rho \leq 1$, $\delta < 1$, have property II) (and I)). An L^p-version is also possible. With the concept of ψ^*-algebras (i.e. symmetric Fréchet subalgebras ψ of L(H), H Hilbert space, with the spectral invariance property $\psi \cap L(H)^{-1} = \psi^{-1}$ for the groups ψ^{-1} and $L(H)^{-1}$ of invertible elements, cf. Math. Ann. 269, 27-71 (1984)) it is possible to give connections to the division problem of distributions for real analytic Fredholm functions and operator valued distributions (Kaballo and the author, Int. Eq. Op. Th. 12 (1989)). The Arens-Royden theorem of Davie (1971) for Fréchet algebras can be generalized to certain holomorphic Fredholm valued operator functions in ψ^*-algebras with property II). Several problems are mentioned concerning Fréchet algebras, pseudo-differential operators and characterizations by C^∞-algebras using group representations.

Horváth, J.

Convolutions with singular elliptic pseudofunctions

If k is an integrable function on the unit sphere S_{n-1} of R^n, then the tempered distribution K_λ is defined by

$$\langle K_\lambda, \phi \rangle = \int_{R^n} k\left(\frac{x}{|x|}\right) |x|^\lambda \phi(x)\, dx$$

for Re $\lambda > -n$ and by analytic continuation or Hadamard finite parts for Re $\lambda \leq -n$. A classical theorem of Sobolev states that for $k(\sigma) \equiv 1$, and $-n < \mu = \text{Re } \lambda < 0$ the convolution operator $f \to K_\lambda * f$ is a continuous linear map from L^p into L^q, where $\frac{1}{q} = \frac{1}{p} - \frac{n}{\mu} - 1$. The talk will present similar continuity theorems for more general functions k, other values of λ, and involving Sobolev spaces $W^{s,p}$.

Jelinek, J., and Wawak, R.

Solution to a problem of M. Oberguggenberger

We give the answer to the question of equivalence of several definitions of the product of distributions asked by M. Oberguggenberger during the International Conference on Generalized functions, Convergence Structures and Their Applications, Dubrovnik 1987. He considered products of the type lim $(S*\Phi^\varepsilon)\cdot(T*\Phi^\varepsilon)$ where Φ^ε are taken from several classes of delta-nets. Several products are proved to be equal but the Tillmann product is shown to be more general.

Kamiński, A. (reporting on joint work with S. Pilipović and J. Uryga)

The convolution and the convolutors in the spaces $K'(M_p)$ of distributions of Gelfand-Shilov

The spaces $K(M_p)$ and their duals $K'(M_p)$, introduced by I.M. Gelfand and G.E. Shilov, are considered. It is proved that the definitions of the convolution in $K'(M_p)$, analogous to the definitions of C. Chevalley, R. Shiraishi and V.S. Vladimirov in the case of the convolution in $\mathcal{D}'$, are equivalent. Sufficient (and, in some sense, necessary) conditions are given in terms of the supports of the convolution factors. Several equivalent descriptions of the space $O'_c(M_p)$ of convolutors for $K'(M_p)$ (i.e. such $f \in K'(M_p)$ that $f * \phi \in K(M_p)$ for all $\phi \in K(M_p)$) are given. It is proved that $O'_c(M_p)$

is the dual of the space

$$O_c(M_p) = \bigcup_{k=1}^{\infty} S_k(M_p)$$

endowed with the inductive limit topology, where

$$S_k(M_p) = \{\phi \in C^\infty : \forall_{i\in N_o} \forall_{\varepsilon>0} \exists_{\rho_{i\varepsilon}>0} |\phi^{(i)}(x)|<\varepsilon M_k(x) \quad \text{for } |x| > \rho_{i\varepsilon}\}$$

with the topology given by the pseudonorms

$$q_k^i(\phi) = \sup_{k\varepsilon R} M_k^{-1}(x)| \ \phi^{(i)}(x) \ | \quad (i \ \varepsilon \ N).$$

Moreover, the equality $O_c(M_p) \cap O_c'(M_p) = K(M_p)$ is shown.

Kaneko, A.

<u>Analyticity of the minimal dimensional locus of singularity of real analytic solutions</u>

Let u be a real analytic solution of P(x,D)u = 0 defined outside a closed set C. If u cannot be extended even as a hyperfunction solution to C, we call C "locus of singularity" for u. Our problem is to determine the possible form of C completely. For that purpose minimal dimensional ones are important. If C is of minimal dimension and is contained in a real analytic non-characteristic hypersurface, we can prove that it becomes a real analytic subvariety.

Komatsu, H.

<u>Ultradistribution semi-groups of operators as inverse Laplace transforms</u>

As we showed two years ago, Laplace transforms $\hat{f}(\lambda)$ are most naturally defined for the class $B^{exp}_{[a,\infty)} := O^{exp}(\mathbb{C}\setminus[a,\infty))/O^{exp}(\mathbb{C})$ of hyperfunctions f(x) (called the Laplace hyperfunctions) on the extended real line $[-\infty,\infty]$. Here O^{exp} denotes the space of holomorphic functions of exponential type.

We give a sufficient (and almost necessary) condition on $\hat{f}(\lambda)$ in order that the restriction of f(x) to $\mathbb{R}$ belongs to the distributions $D'_{[a,\infty)}$ or the ultradistributions $D^{*\prime}_{[a,\infty)}$ of class *.

Further, extending the theory to the case of functions with values in a Banach space, we reconstruct the theory of abstract Cauchy problems and, in particular, of semi-groups of operators, due to Chazarain, Ouchi and others.

Korevaar, J.

Mixed derivatives lemma and applications

Our basic real variables lemma gives an estimate for mixed partial derivatives in terms of a family of directional derivatives of the same order. The sharp "polynomial approximation" constant $\beta(E)$ in the result is related to potential theory for $\mathbb{C}^n$. The lemma provides a unified approach to a number of problems in real and complex analysis.

Definition: For $E \subset \mathbb{C}^n$ bounded, $z^k = z_1^{k_1} \dots z_n^{k_n}$, $m = 1,2,\dots$

$$\beta_m(E) := \inf_{|k| = k_1+\dots+k_n = m} \inf_{b_j} \| z^k - \sum_{|j| = m, j \neq k} b_j z^j \|_E^{1/m} .$$

Properties: (i) $\beta_m(E) \to \beta(E) \overset{\text{def}}{=} \inf_s \beta_s(E)$, $m \to \infty$.

(ii) For nonempty open $E \subset S^{n-1} \subset \mathbb{R}^n$ one has $\beta(E) > 0$.

(iii) $\beta(E) = \beta(\overline{E}_c) = \sigma(\overline{E}_c)$, where σ is a $\mathbb{C}^n$ capacity on circular compacts in $\mathbb{C}^n$ $[E_c = \cup_\theta e^{i\theta} E]$. In particular, $\beta(E) = 0$ if and only if $\overline{E}_c$ is pluripolar in $\mathbb{C}^n$.

Lemma: (K-Wiegerinck). Let $E \subset S^{n-1} \subset \mathbb{R}^n \subset \mathbb{C}^n$ and let f be of class C^∞ around a in $\mathbb{R}^n$,

$$f(a+x) - f(a) \sim \sum_{|k|\geq 1} c_k x^k = \sum_{m\geq 1} \sum_{|k|=m} c_k x^k = \sum_{m\geq 1} q_m(x).$$

Then

$$\sup_{|k|=m} \frac{1}{k!} |D^k f(a)| = \sup_{|k|=m} |c_k| \leq \sup_E |q_m(y)| \Big/ \beta_m(E)^m$$

$$\leq \sup_{y\in E} \frac{1}{m!} \left| \left(\frac{d}{dt}\right)^m f(a+ty)\Big|_{t=0} \right| \Big/ \beta(E)^m.$$

Applications: 1. Recessed edge theorem of Behnke-Kneser type. 2. Support theorems for Radon transforms.

3. Real analyticity of certain functions in $\mathbb{R}^n$.
4. An edge-of-the-wedge theorem.
5. From analyticity of a function on a family of complex lines to n-dimensional analyticity.

Langenbruch, M.

Right inverses for partial differential operators in the space of tempered distributions

The class of partial differential operators P(D) admitting a linear continuous right inverse in the space of tempered distributions is characterized in the following way: Let $V_P := \{x \in \mathbb{R}^N \mid P(x) = 0\}$.

Theorem: P(D) has a right inverse in $\mathcal{S}'(\mathbb{R}^N)$ iff $P = Q_o Q_1 \dots Q_k$, where $Q_o \in \mathbb{C}[x_1, \dots, x_N]$, $V_{Q_o} = \emptyset$, and $Q_j \in \mathbb{R}[x_1,\dots,x_N]$ is irreducible and has a right inverse in $\mathcal{S}'(\mathbb{R}^N)$ for $j \geq 1$.

Theorem: Let $P \in \mathbb{R}[x_1,\dots,x_N]$ be irreducible.

The following are equivalent:

i) P(D) has a right inverse in $\mathcal{S}(\mathbb{R}^N)'$

ii) $P\, C^\infty(\mathbb{R}^N) = I\,(V_P)(\mathbb{R}^N) := \{f \in C^\infty(\mathbb{R}^N) \mid f|_{V_P} = 0\}$.

This is the "property of zeroes" of R. Thom.

Lelong, P.

Plurisubharmonic functions f with logarithmic singularities or points of density for the dd^cf. Application to a theory of the resultant for (f,g) in such classes

In a locally convex complex vector space E, we denote by $L(E) = \cup_{\sigma>0} L_\sigma(E)$ the class of the plurisubharmonic functions with logarithmic growth at infinity: $f \in L_\sigma(E)$ if

$$\sup_y l_f(x,y) = l_f(x) = \sigma \qquad 0 < \sigma < +\infty$$

where $l_f(x,y) = \limsup_{|u|\to\infty} [f(x+uy)](\log|u|)^{-1}$.

1°) First we prove: $f \in L_\sigma(E)$ if and only if there exists a Banach space M and $\pi : E \to M$ a linear continuous projection, $f_1 \in L_\sigma(M)$ and $f = f_1 \circ \pi(x)$. As a consequence if E is an F-space and $E = \lim\limits_{\leftarrow} \tilde{E}_k$, $\tilde{E}_k$ Banach spaces, $L_\sigma(E) = \oplus L_\sigma(\tilde{E}_k)$.

2°) A leading function is defined in a density point $\xi \in \Omega$ for $f \in PSH(\Omega)$ and $\xi \in \Omega$: $\tilde{f}_\xi(x) = \limsup\limits_{\zeta \to 0} f(\xi + x\zeta) - \nu_f(\xi) \log |\zeta|$. Same definition for $f \in L_\sigma(E)$ at infinity is given.

3°) A mapping $(f,g) \to \rho(f,g)$ is defined, which is the true generalization of $(P,Q) \to R(P,Q)$ or $(\log|P|, \log|Q|) \to \log|R(P,Q)|$, where $R(P,Q)$ is the Bezout resultant of $P(x,\xi)$ and $Q(x,\xi)$. (See a note to follow in CRAS Paris)

Meise, R.

Generalized Fourier expansion for zero-solutions of convolution equations

Let $\omega: [o,\infty[\to [o,\infty[$ denote a weight which can be used to define the non-quasianalytic classes $\mathcal{E}_{(\omega)}$, $\mathcal{E}_{\{\omega\}}$, $\mathcal{D}_{(\omega)}$ and $\mathcal{D}_{\{\omega\}}$ in the sense of Beurling. Denote by T_μ (resp. S_μ) the convolution operator induced by $\mu \in \mathcal{E}'_*(\mathbb{R})$ on $\mathcal{E}_*(\mathbb{R})$ (resp. $\mathcal{D}'_*(\mathbb{R})$). Assume that $\hat{\mu} : z \to \langle \mu_x, e^{-ixz} \rangle$ has the zero $(a_j)_{j \in \mathbb{N}}$, counted with multiplicities. Then we have:

Thm 1. If $T_\mu : \mathcal{E}_{(\omega)}(\mathbb{R}) \to \mathcal{E}_{(\omega)}(\mathbb{R})$ is surjective, then ker T_μ and ker S_μ have an absolute basis of exponential solutions and

(a) $\ker T_\mu \cong \{ (x_j)_{j\in\mathbb{N}} \in \mathbb{C}^{\mathbb{N}} \mid \forall k \in \mathbb{N} : \sum_{j=1}^{\infty} |x_j| \exp(k(|\operatorname{Im} a_j| + \omega(|a_j|))) < \infty \}$

(b) $\ker S_\mu \cong \{ (x_j)_{j\in\mathbb{N}} \in \mathbb{C}^{\mathbb{N}} \mid \forall k \in \mathbb{N}\ \exists m \in \mathbb{N} : \sum_{j=1}^{\infty} |x_j| \exp(k|\operatorname{Im} a_j| - m\,\omega(|a_j|)) < \infty \}$

Thm 2. If $S_\mu : \mathcal{D}'_{\{\omega\}}(\mathbb{R}) \to \mathcal{D}'_{\{\omega\}}(\mathbb{R})$ is surjective, then ker T_μ and ker S_μ have an absolute basis of exponential solutions and

(a) $\ker T_\mu \cong \{ (x_j)_{j\in\mathbb{N}} \in \mathbb{C}^{\mathbb{N}} \mid \forall k \in \mathbb{N}\ \exists m \in \mathbb{N} : \sum_{j=1}^{\infty} |x_j| \exp(k|\operatorname{Im} a_j| + \frac{1}{m}\omega(|a_j|)) < \infty \}$

(b) $\ker S_\mu \cong \{(x_j)_{j\in\mathbb{N}} \in \mathbb{C}^{\mathbb{N}} \mid \forall k\in\mathbb{N} : \sum_{j=1}^{\infty} |x_j| \exp(k\,|\mathrm{Im}\, a_j| - \frac{1}{k}\omega(|a_j|)) < \infty \}$.

These results have been obtained in joint work of the lecturer together with K. Schwerdtfeger and B.A. Taylor (Thm. 1 (a)), U. Franken (Thm 1. (b)) and R.W. Braun (Thm. 2 (b)).

Misra, O.P.

Some spaces of distributions for differential and functional differential equations

It is a well known fact that opinion of some mathematicians about functional analysis as a purely mathematical abstraction is true to some extent. In response to this question, Dieudonné [1] has referred to the applications of the theory of distributions created by the French mathematician Professor Laurent Schwartz in differential equations. The construction of the present lecture is as follows.
First we describe the relation between spaces of Schwartz (see $\mathbb{D}$ and $\mathbb{D}'$ defined in Misra and Lavoine [3]) and the spaces of Gelfand and Shilov (see S, S^α, S^β and S_α^β defined in [2]). Finally, we present a method to find out the unique solution of differential and functional differential equations in Schwartz space as indicated above.

References

[1] Dieudonné, J. Hist. Math., 2 (1975), 537-548

[2] Gelfand, I.M. and Shilov, G.E., Generalized Functions, Vol. 2 Academic Press, 1968

[3] Misra, O.P. and Lavoine, J., Transform Analysis of Generalized Functions, North Holland Publ. 1986.

Oberguggenberger, M.

Regularity of generalized solutions to partial differential equations in Colombeau algebras

Given an open subset Ω of $\mathbb{R}^n$, we construct a subalgebra $G^\infty(\Omega)$ of the Colombeau algebra $G(\Omega)$ of generalized functions with the property that

$$G^\infty(\Omega) \cap D'(\Omega) = C^\infty(\Omega).$$

The concept of $G^{\infty}(\Omega)$ allows us to describe the regularity of generalized solutions in $G(\Omega)$ to partial differential equations intrinsically, i.e. without recourse to a corresponding distributional solution (which, as a matter of fact, need not exist):
Proposition 1 (elliptic regularity): If $U \in G(\Omega)$ and $\Delta U \in G^{\infty}(\Omega)$, then $U \in G^{\infty}(\Omega)$.

Proposition 2 (propagation of singularities): Assume that A,B belong to $G(\mathbb{R}^n) \cap G^{\infty}(\mathbb{R}^n \setminus S)$ where S is some closed subset of $\mathbb{R}^n$. Let $U \in G(\mathbb{R}^{n+1})$ be the solution of the wave equation

$$\Box U = 0$$
$$U\,|\{t=0\} = A,\ \partial_t U|\{t=0\} = B.$$

Then U belong to G^{∞} off the union of light cones emanating from S, i.e. off the set $\{(x,t) \in \mathbb{R}^{n+1} : \exists y \in S, |x-y|^2 = t^2\}$.

Ortner, N.

Applying a multidimensional formula of "parameter integration": representation of a fundamental solution of a certain inhomogeneous elliptic differential operator

By means of the formula

$$(a^2-b^2-c^2)^{-1} = (2\pi)^{-1} \int_B (a+\lambda b+\mu c)^{-2} (1-\lambda^2-\mu^2)^{-1/2} \, d\lambda\, d\mu$$

and using Fourier transforms it is possible to prove the following
Proposition: Let $B = \{(\lambda,\mu) \in \mathbb{R}^2 | \lambda^2 + \mu^2 \leq 1\}$ and let $P_o(\partial)$, $P_1(\partial)$, $P_2(\partial)$ be differential operators with constant coefficients ($\partial = (\partial_1, \ldots, \partial_n)$). If $P_o(i\xi) + \lambda P_1(i\xi) + \mu P_2(i\xi) \neq 0$ ($\forall \xi \in \mathbb{R}^n$ and $\forall (\lambda,\mu) \in B$) then the uniquely determined, tempered fundamental solution E of the operator $P_o^2(\partial) - P_1^2(\partial) - P_2^2(\partial)$ is given by the formula $E = \frac{1}{2\pi} \int_B E_{\lambda,\mu} (1 - \lambda^2 - \mu^2)^{-1/2} \, d\lambda\, d\mu$, wherein $E_{\lambda,\mu}$ is the uniquely determined, tempered fundamental solution of the iterated operator $(P_o(\partial) + \lambda P_1(\partial) + \mu P_2(\partial))^2$. The proposition allows to represent "the" f.s. of the operator $(\partial x^2 + \partial y^2)^2 - a\,\partial_x^2 - b\,\partial_y^2$ as a simple definite integral over products of Bessel functions.

Petzsche, H.-J.

Extension of ultradifferentiable functions

The following problem was considered: Let K be a compact subset of $\mathbb{R}^N$, G(K) a vector subspace of C^∞-Whitney jets on K, U a neighbourhood of K.

Question 1. Is the restriction R : G(U) → G(K) surjective?

Question 2. Does there exist a continuous linear extension operator

E : G(U)→ G(K) ?

For G we considered $C^{\infty,r}_{(Mp)}$, $C^{\infty,r}_{\{Mp\}}$, non-quasianalytic classes of minimal or maximal type, respectively. r equals 2 or ∞, depending whether L^2- or supremum norms are used.

The following theorem was discussed for various sets K (we put $m_p := M_p/M_{p-1}$):

Theorem: (1) The following assertions are equivalent:

(i) (β_1) $\exists k \in \mathbb{N} \liminf_{p\to\infty} \frac{m_{kp}}{m_p} > k.$

(ii) r is onto for G ε $\{C^{\infty,r}_{(Mp)}, C^{\infty,r}_{\{Mp\}}\}$.

(iii) E exists for $C^{\infty,r}_{(Mp)}$.

(2) The following assertions are equivalent:

(i) (β_2) $\lim_{\beta\searrow\infty} \limsup_{p\to\infty} \max_{j\leq\beta_p} \left\{\left(\frac{M_p}{M_j}\right)^{\frac{1}{p-j}} \frac{1}{m_p}\right\} = 0$

(ii) E exists for $C^{\infty,r}_{\{Mp\}}$.

The Gevrey sequences $M_p = p!^s$, s > 1, satisfy (β_1) and not (β_2).

Pflug, P.

Explicit formulae for the Carathéodory distance

In Complex Analysis, invariant metrics as the Carathéodory - or the Kobayashi-metrics are important, for example, for classification problems. It was known how the Kobayashi-metric looks like on product domains (due to Royden). The analogous formula for the Carathéodory-metric is proven. In addition, there are explicit formulas for the Carathéodory-metric on special Reinhardt

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domains.
The results are contained in a common work together with M. Jarnicki (Kraków).

Schapira, P.

Analytic wave front set at the boundary

Let M be a real analytic manifold, Ω an open subset and X a complexification of M. Using the functor μhom (a generalization of the functor of Sato's microlocalization) one can define for a hyperfunction u on Ω its "wave front set at the boundary", $SS_\Omega(u)$, a closed conic subset of $\sqrt{-1}T^*M$. This set appears as a natural tool in the study of boundary value problems.
Some open problems concerning the vanishing of cohomology groups, related to μhom, are discussed.

Stanković, B.

Abelian theorems for some integral transformations of distributions

We define an integral transformation of distributions (S_ρ -transformation) which contains some well-known transformations. In this way the Abelian type theorems we prove for the S_ρ -transformation are valid also for many others. For the asymptotic behaviour of distributions we use the S-asymptotic (shift asymptotic).

First we prove a structural theorem for the distributions having S-asymptotic. This theorem we use to prove the existence of the S_ρ -transformation for some class of distributions and the Abelian type theorems for this class. At the end we give some examples which show to what extent our theorems are precise.

Szmydt, Z., and Ziemian, B.

Local regularity of solutions to singular partial differential equations

We study local existence and regularity of solutions of singular elliptic operators on manifolds with corner singularities by means of a pertubation technique based on the theory of multidimensional Mellin transformations. We establish relations with 2-(micro) local regularity.

Takači, A.

Asymptotic bounds for integral transforms

We observe the generalized Laplace transfrom as given by Zemanian (1968) and the generalized Stieltjes transform as given by Lavoine and Misra (1974). Using Landau symbols for distributions in the sense of S.E. Silva (1964) and quasiasymptotics (1987), we obtain asymptotic bounds for these integral transforms.

Theorem 1. If $f \in D'_+$ is O in S.E. Silva's sense of a power function $\rho(t) = t^\alpha$ at infinity, then

i) $L(f)(s) = O(s^{-\alpha-1})$ as $s \to o+$, provided that $f \in L'(\omega)$, $\omega \leq 0$ and $\alpha > -1$;

ii) $S_r(f)(s) = O(s^{\alpha-r})$ as $s \to \infty$, provided that $f \in I'(r)$, $-1 < \alpha < r$.

Theorem 2. If $f \in D'_+$ is O in the quasiasymptotical sense of a regularly varying function $\rho(t) = t^\alpha L(t)$ at infinity, then

i) $L(f)(s) = O(s^{-\alpha-1} L(\frac{1}{s}))$ as $s \to 0+$, provided that $f \in L'(\omega)$, $\omega \leq 0$ and $\alpha \in \mathbb{R}$;

ii) $S_r(f)(s) = O(s^{\alpha-r} L(s))$ as $s \to \infty$, provided that $f \in I'(r)$, $\alpha < r$.

Remark. Theorems 1 and 2 remain true if "infinity" is replaced with "zero", (and vice versa) and, also, if the symbol "O" is replaced with "o".

Takači, D.

Approximate solutions of the differential equations in the field of Mikusinski operators

We observe the equation (in the field of Mikusinski operators)

$$\sum_{\mu=0}^{m} a_\mu(s)\, x^{(\mu)}(\lambda) + e^{-\tau s} \sum_{\mu=0}^{m_1} b_\mu(s)\, x^{(\mu)}(\lambda) = f(\lambda)$$

where

$$a_\mu(s) = \sum_{\nu=0}^{n} \alpha_{\mu,\nu}\, s^\nu \quad , \quad \mu = 0,\dots m,$$

$$b_\mu(s) = \sum_{\nu=0}^{n_1} \beta_{\mu,\nu}\, s^\nu, \quad \mu = 0, \ldots m_1 ,$$

$f(\lambda)$ is a continuous operational function, and $0 \leq \lambda \leq 1$, $\tau > 0$.

Using the two dimensional operator calculus introduced by T. Ogata (1982) and the results of J. Wloka for ordinary differential-difference equations we construct the approximate solution.

Also, we estimate the error of approximation by using the results of J. Burzyk connected with the convergence of type I' in the Mikusinski operational field.

Taylor, B.A. (reporting on joint work with R. Meise and D. Vogt)

Phragmén-Lindelöf principles on algebraic varieties and right inverses for partial differential operators

Let $\mathcal{E}(\mathbb{R}^n)$ and $\mathcal{D}'(\mathbb{R}^n)$ denote, respectively, the space of infinitely differentiable functions and distribution on $\mathbb{R}^n$, provided with their usual topologies. For a polynomial P in n variables, $P(z) = \sum_{|\alpha| \leq m} a_\alpha z^\alpha$, the associated partial differential operator is $f \to P(D)f = \sum a_\alpha i^{-\alpha} f^{(\alpha)}$, where $f^{(\alpha)} = \frac{\partial^{|\alpha|} f}{\partial x^\alpha}$. It is well-known that for $P \neq 0$, the operator $P(D)$ is continuous and surjective on $\mathcal{E}(\mathbb{R}^n)$ and $\mathcal{D}'(\mathbb{R}^n)$.

Theorem: For a polynomial P in n variables, the following are equivalent.

(1) $P(D) : \mathcal{E}(\mathbb{R}^n) \to \mathcal{E}(\mathbb{R}^n)$ has a continuous, linear right inverse;

(2) $P(D) : \mathcal{D}'(\mathbb{R}^n) \to \mathcal{D}'(\mathbb{R}^n)$ has a continuous linear right inverse;

(3) On the variety $V = \{z \in \mathbb{C}^n : P(-z) = 0\}$, the following Phragmén-Lindelöf principle is valid:

there exists a constant $A > 0$ such that for all $\rho > 0$, there exists a constant $B = B(\rho)$ such that

$$u(z) \leq A|\mathrm{Im}\, z| + B \log(2 + |z|), \quad z \in V,$$

for all (weakly) psh functions u on V such that

$$u(z) \leq |\mathrm{Im}\, z| + O(\log(1+|z|)), \quad z \in V$$

and

$$u(z) \leq \rho|\mathrm{Im}\, z| , \quad z \in V.$$

Tonev, T.

Generalized analytic manifolds in uniform algebra spectra

Topological and algebraic conditions on a uniform algebra that assure existence of global and local generalized-analytic structures in algebra spectra are given and some old results due to Stoilow, Gleason, Browder, Bishop, Bear-Hile and others are extended for generalized-analytic functions in the sense of Arens-Singer.

Triebel, H.

Spaces of functions and distributions

1. Brief historical introduction: Sobolev, Hölder, Besov spaces
2. The late sixties and seventies: Interpolation methods in function spaces, Fourier-analytical approach
3. Some recent developments: Local means, spaces on manifolds.

Vladimirov, V.S.

Distributions over the field of p-adic numbers and some applications in mathematical physics

Some new results on analysis of complex valued functions and distributions of p-adic arguments are presented: integration, convolution, Fourier transformation, differentiation, differential and pseudo-differential operators, a complete and orthogonal set of eigenfunctions of some differential operators. Applications for construction of p-adic quantum mechanics with complex valued wave functions and p-adic coordinates, impulses and time. As a concrete example of the simplest quantum-mechanical system we consider a harmonic oscillator (and a free particle): the Weyl representation, evolution operator, vacuum states, p-adic Schrödinger type equation, spectrum, probability,(Euclidean) formulation of the p-adic quantum mechanics.

Vogt, D. (reporting on joint work with R. Meise and B.A. Taylor)

Right inverses for partial differential operators on subsets of $\mathbb{R}^n$ with boundary

Let P(D) be a linear partial differential operator with constant coefficients in n-variables, $P_m(D)$ its principal part. The following theorem was presented and explained.

Theorem: For P(D) the following are equivalent

(1) There exists a bounded open set Ω with C^1-boundary such that P(D) admits a right inverse in $D'(\Omega)$ of $C^\infty(\Omega)$.

(2) P(D) is hyperbolic with respect to every non characteristic direction

(3) P(D) has a right inverse in $D'(\Omega)$ and $C^\infty(\Omega)$ for every bounded open set Ω

(4) P and P_m are equally strong and P_m is proportional to a product of linear forms with real coefficients.

Wagner, P.

Multiplikation und Faltung von homogenen Distributionen

Obwohl im allgemeinen die Begriffe faltbar und ζ'-faltbar für temperierte Distributionen nicht äquivalent sind (vgl. (1)), gilt das, wenn beide homogen sind. (vgl. (2), Satz 8). Bei den verschiedenen Definitionen des multiplikativen Produktes von S,T in (3) können analoge Äquivalenzen gezeigt werden, falls S,T homogen sind. Genauere Charakterisierungen ergeben sich, wenn zusätzlich S oder T C^∞ in $\mathbb{R}^n \setminus 0$ vorausgesetzt werden. Dies verallgemeinert Satz 10 in (2), der unter der Annahme, daß sowohl S als auch T unendlich oft in $\mathbb{R}^n \setminus 0$ differenzierbar sind, die Faltbarkeit von S und T durch die Bedingung ((Summe der Homogenitätsgrade von S und T $< -n$) oder $(S(x) \cdot T(-x) = 0$ für $x \neq 0$) charakterisiert.

(1) P. Dierolf, J. Voigt, Convolution and ζ'-convolution of distributions, Collect. Math. 29 (1978)

(2) P. Wagner, Zur Faltung von Distributionen, Math. Ann. 276 (1987)

(3) A. Kamiński, Convolution, product and Fourier transform of distributions, Studia Math. 74 (1982)

Zampieri, G. (reporting on joint work with A. D'Agnolo)

Existence and continuation of holomorphic solutions of micro-hyperbolic differential equations

Let $M \cong \mathbb{R}^n \ni x$, $X \cong \mathbb{C}^n \ni z = x+iy$, $\Omega \subset M$ an open set, $\gamma \subset \overline{\Omega} \times i\mathring{\mathbb{R}}^n_y$ an open set with convex conic fibers.

Definition: $V \subset X$ is a Ω-tuboid with profile γ iff $\forall \gamma' \subset\subset \gamma\ \exists \varepsilon = \varepsilon_{\gamma'}$,

$$V \supset \{z \in \Omega \times_M \gamma' \mid |y| < \varepsilon \operatorname{dist}(x, \partial\Omega)\}.$$

For an operator $P = P(x,D)$ with C^ω-coefficients and for a point $(x_o; iy_o) \in T^{*}_{M}X$ $(x_o \in \partial\Omega)$, we shall assume:

(1) Any exterior conormal ϑ to Ω is microhyperbolic for P at $(x_o; iy_o)$ over $\overline{\Omega}$.

Or else:

(2) $\partial\Omega$ is C^ω and P is semihyperbolic in the sense of Kaneko-Kataoka (who gave results of type (3) below).

(Both (1) and (2) are non-invariant under C^ω-coordinate transformations)

Let U (resp. W) be the Ω-tuboids with profile γ in a neighborhood of x_o s.t.: $\gamma \supset \Omega \times i\mathring{\mathbb{R}}^n$ and γ°_x is contained in a small neighborhood of y_o (resp. $\gamma \supset \overline{\Omega} \times i\mathring{\mathbb{R}}^n$). Then:

Theorem: Let (1) or (2) hold. Then:

(3) $f \in \varinjlim_{U} \mathcal{O}_X(V)$, $Pf \in \varinjlim_{W} \mathcal{O}_X(W) \Rightarrow f \in \varinjlim_{W} \mathcal{O}_X(W)$.

(4) $\forall f \in \varinjlim_{U} \mathcal{O}_X(V)\ \exists u \in \varinjlim_{U} \mathcal{O}_X(V)$ s.t. $Pu - f \in \varinjlim_{W} \mathcal{O}_X(W)$.

One can rephrase (3)-(4) by saying that P is an isomorphism of $\pi^{-1}\Gamma_\Omega(A_M)$ in the sheaf of microfunctions at the boundary by Schapira. The method of the proof is the Cauchy-Kowalewsky theorem and a variant of the method of the non-characteristic deformation for non-convex sets (as the U's):

Lemma: Let V and $\{V_\alpha\}_{0 \le \alpha \le 1}$ verify:

(i) $V_\alpha \subset V_\beta$ for $\alpha < \beta$, $\bigcup_{\beta<\alpha} V_\beta = V_\alpha$, $\bigcap_{\beta>\alpha} V_\beta = \overline{V}_\alpha$

(ii) $\partial V_\alpha \setminus \overline{V_1 \setminus V} \subset\subset V_1$

(iii) ∂V_α is non-characteristic for P on $\overline{V_1 \setminus V}$.

Then: $f \in \mathcal{O}_X(V)$, $Pf \in \mathcal{O}_X(V \cup V_1) \Rightarrow f \in \mathcal{O}_X(V \cup V_1)$.

Zav'jalov, B.I.

Tauberian theorems for distributions and asymptotic properties of holomorphic functions

We consider the holomorphic functions in curved tubular domains, so-called tuboids, with polynomial growth near the boundary and study the connection between asymptotic behaviour of the real part of a boundary value of a function on the edge at some fixed point with asymptotics of the whole function from the inside of the tuboid. The consideration is based on some tauberian theorems for distributions with supports in cones.

Zemanian, A.H.

Transfinite graphs and electrical networks

Various physical situations desribed by the exterior problems of partial differential equations can be modeled by infinite electrical networks having finite-valued sources connected to extremities of the network at infinity. In fact, certain situations suggest the extension of the network "beyond infinity". This leads to a new kind of graph, one in which some pairs of nodes are connected by transfinite paths but not by finite paths. Correspondingly, there has been no theory for a transfinite electrical network. The present work establishes one.
An existence and uniqueness theorem is proven for countable transfinite resistive networks that have an infinity of voltage and current sources, which may connect between transfinitely separated nodes. The hypothesis of the theorem requires that the maximum power available from all the sources be finite. An example about well-logging in geophysical exploration is given to show how at least some of these new concepts can arise from physical applications.

Zharinov, V.V.

Some problems of infinite-dimensional analysis

Let I be a countable index set filtrated by its finite subsets I_n, n = 0,1,2, ... Let us put

$$X = \lim_{n \to \infty} \operatorname{proj} X_n, \quad X_n = \mathbb{R}^p \times \mathbb{R}^I$$

where p is a natural number. For an open set $O \subset X$ let us define the algebra

$$C^{\infty}(O) = \lim_{n\to\infty} \text{ind}\ C^{\infty}(\pi_n O),$$

where $\pi_n : X \to X_n$ are the natural projections. An operator $F : C^{\infty}(O) \to C^{\infty}(O)$ is called subordinated to the filtration $\{I_n\}$ if $F(C^{\infty}(\pi_n O)) \subset C^{\infty}(\pi_n O)$ for every number n and nonsubordinated in the other case. Nonsubordinated operators arise in many problems of the algebra-geometric theory of nonlinear PDE. No general results about such nonsubordinated operators are known up to now.

Zsidó, L.

The integrability of absolute values of derivatives

Let $f : (a,b) \to \mathbb{R}^n$ be differentiable. One can elementarily prove: if $(a,b) \supset [a',b'] \ni t \to |f'(t)|$ is Riemann integrable then f has bounded variation $V_f([a',b'])$ on $[a',b']$ and

$$V_f([a',b']) = \int_{a'}^{b'} |f'(t)|\, dt.$$

Since the Riemann integrability of the component functions $[a',b'] \ni t \to f_j'(t)$, $1 \leq j \leq n$, implies the Riemann integrability of $[a',b'] \ni t \to |f'(t)|$, it is natural to ask: does the converse implication hold or not? The answer is yes for $n = 1$ and no for $n \geq 2$!

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