MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 23/1989

Funktionalanalytische Methoden bei Evolutionsgleichungen

28.5. bis 3.6.1989

Diese Tagung wurde organisiert durch die Herren H. Amann und P. Hess (beide Universität Zürich). Das grosse Interesse an dieser Tagung spiegelt die hohe Zahl von 44 Teilnehmern aus 17 Ländern wider (wobei leider etliche weitere Interessenten nicht berücksichtigt werden konnten). In 38 Vorträgen wurde ein Überblick über die neuesten Entwicklungen auf diesem Teilgebiet der Mathematik gegeben, wobei sowohl abstrakte Fragestellungen in der Theorie der Evolutionsgleichungen als auch konkrete Anwendungen auf nichtlineare parabolische und hyperbolische Differentialgleichungen behandelt wurden. Das Vortragsprogramm wurde durch anregende Diskussionen und persönliche Gespräche in harmonischer Atmosphäre ergänzt.

Der Leitung des Institutes sowie dem freundlichen Personal des Hauses danken wir an dieser Stelle herzlich.

VORTRAGSAUSZÜGE

P. ACQUISTAPACE:

REGULARITY PROPERTIES OF FUNDAMENTAL SOLUTIONS OF ABSTRACT NON-AUTONOMOUS PARABOLIC EQUATIONS

We study the regularity properties of the evolution operator $V(t,s)$ relative to the Cauchy problem

$$u'(t) - A(t)u(t) = f(t), \quad t \in [0,T], \quad u(0) = x ,$$

under classical Kato-Tanabe's assumptions (but the domains of the $A(t)$'s may be not dense in the Banach space E). We show that if $t > s$ then there exists $\frac{d}{ds}U(t,s)x \quad \forall\, x \in E$, and study some features of this operator. We also give an application to boundary control of parabolic equations, in a Hilbert space setting.

F. ALI MEHMETI

GLOBAL EXISTENCE OF SOLUTIONS OF CERTAIN NONLINEAR EVOLUTION EQUATIONS WITH APPLICATIONS TO INTERACTION PROBLEMS

Consider n bounded domains of possibly different dimension and nonlinear wave equations on these domains. We use the interaction formalism of [1] to describe many different kinds of influence between the evolution on the different domains, including interface problems and transmission problems on ramified spaces (c.f. G. Lumer, S. Nicaise, J. von Below, B. Gramsch). This formalism furnishes a maximal accretive operator in a suitable Hilbert space, which allows to transform the interaction problem into an evolution equation.

Global existence and uniqueness of solutions are proved for locally Lipschitzian nonlinearities in a situation applicable to wave equations with damping. More singular nonlinearities are treated in the case of networks using results of J. Shatah and T. Kato.

Finally a common research with S. Nicaise is mentioned where convex analysis is used to study nonlinear interactions.

[1] Ali Mehmeti, F.: Regular solutions of transmission and interaction problems for wave equations; to appear in: Math. Meth. Appl. Sc.

S.B. Angenent

Parametric parabolic equations of quasilinear and fully nonlinear type

Let $V : \mathbf{R}^2 \times S^1 \times \mathbf{R} \longrightarrow \mathbf{R}$, $V = V(x, y, \theta k)$ be a given function which satisfies $\frac{\partial V}{\partial k} > 0$, as well as some more conditions.

Corresponding to this V, one has the following initial value problem: Given a plane immersed curve C_0, find a maximal familiy of smooth curves $C_t (0 \le t < T)$ which satisfy

(1) $$v_\perp = V(x, y, \theta k)$$

for $0 < t < T$, at each point $(x, y) \in C_t$. Here $v_\perp$ is the normal velocity of the curve, θ is the angle between the tangent to C_t at (x, y) and (say) the x-axis, and k is the signed curvature at this point.

Under a few technical assumptions (all of which are satisfied by $V(x, y, \theta) \equiv k$), one can prove that there exists a *global weak solution* to (1), which is smooth except at a discrete set of times $t_1, t_2, t_3, \ldots$. (This is our main result.) The proof involves a number of geometric arguments, based on a theorem of Sturm's , of 1836.

W. Arendt

Elliptic operators and positive semigroups

It is shown that a degenerate elliptic operator

$$A = \sum_{i,j=1}^{N} D_i\, a_{ij}\, D_j$$

generates a positive semigroup on $L^p(\Omega)$ $(1 \le p \le \infty)$ if $a_{ij} \in W^{1,\infty}$. We treat simultaneously the case where Ω is an open bounded set in $\mathbf{R}^n$ (and Dirichlet boundary conditions are imposed), or $\Omega = \mathbf{R}^N$ or where Ω is a Lie group.

Ph. Clement

Completely positive measures and Feller semigroups

We consider the problem $\frac{\partial}{\partial t} \left[b_0\, u(t,x) + \int_{-\infty}^{t} b_1(t-s)\, u(s,x) ds \right] =$

$c_\infty\, \Delta u(t,x) + \frac{\partial}{\partial t} \int_{-\infty}^{t} c_1(t-s)\, \Delta u(s,x) ds + f(t,x)$, $t \in \mathbf{R}$, $x \in \Omega$ with the boundary condition $u(t,x) = 0$, $t \in \mathbf{R}$, $x \in \Omega$, where Ω is a bounded domain in $\mathbf{R}^N$ with smooth boundary. We assume: $b_1, c_1 \in L^1(\mathbf{R}^+)$ nonnegative, nonincreasing, $b_0 + \int_0^\infty b_1(z) dz > 0$, $c_\infty > 0$, $c_1(0^+) < \infty$.

We prove that if $f \in L^p(\mathbf{R}; L^q(\Omega))$, $1 < p,\ q < \infty$, then problem (P) has one and only one solution $u \in L^p(\mathbf{R}, L^q(\Omega))$ satisfying $\Delta u \in L^p(\mathbf{R}; L^q(\Omega))$, $u \in L^p(\mathbf{R}; W_0^{1,q}(\Omega))$, $c_1 * \Delta u \in W^{1,p}(\mathbf{R}; L^q(\Omega))$, $b_0 + b_1 * u \in W^{1,p}(\mathbf{R}; L^p(\Omega))$, (where $*$ denotes the convolution).

For the proof we use an extension of a theorem of Dore and Venni, due to Prüss and Sohr, on the closedness of the sum of two closed operators, and an estimate on the norm of the imaginary powers of the negative generator of a semigroup on $L^p(\mathbf{R}; L^q(\Omega))$ "induced" by a Feller semigroup on $\mathbf{R}$. (This is joint work with J. Prüss.)

G. da Prato

Floquet exponents for periodic parabolic systems

We give some result on stabilizability of parabolic systems with coefficients periodic in time.

W. Desch

Essential growth rate of hyperbolic PDE's on an interval

The essential growth rate of a semigroup is the infimum of growth rates that can be obtained by finite dimensional feedback, thus it tells how much the system may be stabilized.

If the hyperbolic PDE

$$u_t(t,x) = A(x)u_x(t,x) + B(x)u(t,x) \text{ (+ suitable boundary cond.)}$$

is transformed to diagonal form ($A(x)$ is a diagonal matrix), then the essential growth rate of its solution semigroup depends only on $A(x)$ and the diagonal elements of $B(x)$.

G. Dore

Maximal regularity for parabolic equations in higher order spaces

We study the homogeneous IBVP for a parabolic equation such as $u' = \Delta u + f$ in Sobolev spa[illegible] through functional analytic methods.

The main tool is a theorem that allows to obtain maximal regularity for the abstract equation $Au + Bu = f$ in spaces related to $D(A^N)$, starting from maximal regularity in the space in which A and B are defined.

H.O. Fattorini

Semilinear parabolic equations with non-smooth nonlinearities

This work is motivated by the control theory of distributed parameter systems of the form

$$\begin{cases} y_t(t,x) & = \Delta y(t,x) + \Phi(t, y(t,x), u(t,x)) \\ y(t,x) & = 0 \, (x \in T) \end{cases}$$

where $x \in \Omega$, Ω *a* domain in m-dimensional Euclideon space $\mathbf{R}^m$ with boundary T.

For physical reasons, it is better to treat as an abstract differential equation in $L^\infty(\Omega)$. However, there are three difficulties:

(a) The semigroups $S(t)$ generated by Δ (with the corresponding boundary condition) is not strongly continuous at $t = 0$.

(b) The nonlinear control term, as a function of t with values on $L^\infty(\Omega)$ may not be strongly measurable. Example: the system

$$y_t(t,x) = y_{xx}(t,x) + u(t,x)$$

in $0 \leq t \leq 1$ with $\Omega = (0,1)$; if, for instance, $u(t,x)$ is the characteristic function of the triangle $0 \leq x \leq t \leq 1$ $t \to u(t,\cdot)$ is not strongly measurable.

(c) Linear functionals in $L^\infty(\Omega)$ (used in control theory) are difficult to characterize.

However, some of these difficulties are essentially due to the smoothing properties of the semigroup $S(t)$. We develop here a theory based on an abstract "two-spaces" scheme introduced in H.O. Fattorini, The time-optimal problem in Banach spaces, Appl. Math. Optimization 1(1974) 163-188: the two spaces here are $C(\overline{\Omega})$ and $L^\infty(\Omega)$. The theory applies without changes to the spaces $L^1(\Omega)$ and $\sum(\overline{\Omega})$ (regular Borel measures on $\overline{\Omega}$ with the total variation norm).

B. Fiedler

Complicated dynamics of scalar reaction diffusion equations with a non-local term

We consider the dynamics of scalar equations

$$u_t = u_{xx} + f(x,u) + c(x)\alpha(u), \quad 0 < x < 1 \, ,$$

where α denotes a weighted spatial average and Dirichlet boundary conditions are assumed. Prescribing f, c, α appropriately, it is shown that complicated dynamics can occur. Specifically, linearizations at equilibria can have any number of purely imaginary eigenvalues. Moreover, the higher order terms of the reduced vector field in an associated center manifold can be prescribed arbitrarily, up to any finite order. These results are in marked contrast with the case $\alpha = 0$, where bounded solutions are known to converge to equilibrium.

The above results are joint work with Peter Polacik.

H. Gajewski

On nonlinear evolution equations governing the carrier transport in semiconductors

Evolution equations modelling the transport of charge carriers in semiconductor devices were proposed already 1950 by van Roosbroeck. These equations form the basis of the numerical simulation of the behaviour of semiconductor devices, which nowadays is a powerful tool of device obsigners.

Inspite of their physical and technical relevance the device equations received relatively little attention from the side of the mathematical analysis for a long time. It is the aim of the lecture to direct the attention to these interesting equations and to state some basic results about existence, uniqueness and asymptotic behaviour of solutions.

J.A. Goldstein

Spin-polarized Thomas-Fermi theory

This is joint work with Ph. Bénilan and G.R. Rieder. Consider a quantum mechanical system of N electrons and Z protons with $Z \geq N$ and an external electric field. We established the existence and uniqueness of the ground state electron density in the appropriate version of Th. Fermi theory. The ground state is spin polarized, i.e. the densities of the spin up and spin down electrons differ. One of our tools is a new method for solving the elliptic system

$$\begin{aligned} -\Delta u + \beta_1(u) + \beta_2(v) &= f \\ -\Delta v + \beta_3(u) + \beta_e(v) &= g \end{aligned}$$

where β_i is a nondecreasing continuous function on $\mathbf{R}$ with $\beta_i = 0$ on $(-\infty, 0]$, and f, g are finite measures on $\mathbf{R}^3$.

G. Greiner

Weak spectral mapping theorems for functional differential equations

For a C_0-semigroup $(T(t))$ on a B-space X one can characterize the resolvent set of a single operator $T(t)$ in terms of the resolvent of the generator A as follows: "$e^{-\mu t} \in \rho(T(t))$ iff $\mu + i\frac{2\pi}{t}\mathbf{Z} \in \rho(A)$ and $\{(\lambda - A)^{-1} : \lambda \in \mu + i\frac{2\pi}{t}\mathbf{Z}\}$ is bounded in $L(X)$".

We sketch the proof of this result and give two applications:

1. One can give a simple proof of a result of Henry on neutral differential equations (J. Diff. Equ. 15 (1974). In this result the spectrum is described as zeros of a characteristic function.

2. For retarded equations $x(t) = \sum_{i=1}^{N} A_i X(t - r) + \int_{-r}^{0} A(S)X(t + s)ds$ one obtains a similar result by studying the associated C_0-semigroup in the space $L^1([r, 0], \mathbf{C}^n)$.

M. Grobbelaar

The Navier-Stokes equations with dynamic boundary conditions in L^2 and L^p

When a symmetric body performs a rotation in a viscous incompressible fluid, the system of governing equations consists of the conservation laws of linear momentum of the fluid (Navier-Stokes) equations and angular momentum of the rigid body. The latter equation which is of a dynamic nature can be considered as a boundary condition for the Navier-Stokes equations. In this Navier-Stokes problem with dynamic boundary conditions the unknowns are v, w and p with v the velocity field for the fluid, w the angular velocity of the rigid body and p the pressure field. Existence and uniqueness results are obtained by following a Hilbert space approach and using as main tools the theory of B-evolutions and the theory of fractional powers of a closed pair of operators.

We also report on the progress made in L^p.

A. Haraux

Anti-periodic solutions of some nonlinear evolution equations

Following a recent work of H. Okochi in the monotone setting, we point out that many quasi autonomous evolution equations of non monotone type associated to odd nonlinearities have some anti-periodic solutions provided the forcing term is anti-periodic. This comes from the fact that the space of anti-periodic functions is transversal to the kernel of the linear part and stable under the action of odd nonlinear operators. The proofs of our results combine strong a priori estimates independent of the nonlinearity with an application of Schauder's fixed point theorem to some related dissipative equations.

N. Kenmochi

Neumann problems for parabolic-elliptic equations; stability of periodic solutions

We treat a nonlinear evolution equation of singular and degenerate type

(1) $$u_t - \Delta\tilde{\beta} = f, \tilde{\beta} = \tilde{\beta}(t,x) \in \beta(u(t,x)), \quad \text{in } Q = (0,+\infty)\times\Omega$$

with Neumann boundary condition

(2) $$\partial_n\tilde{\beta} = h \quad \text{on } \sum = (0,+\infty)\times\partial\Omega.$$

Here Ω is a bounded domain in $R^N (N \geq 1)$ with smooth boundary $\partial\Omega$, β is a given maximal monotone graph in $R \times R$ and f, h are given data. In this talk, assuming that the domain of β is bounded and not a singleton, i.e.

$$\overline{D(\beta)} = [r_*, r^*] \quad \text{for some } -\infty < r_* < r^* < +\infty,$$

we show: (a) for any $u_0 \in L^\infty(\Omega)$ with $r_* \le u_0(x) \le r^*$ and

$$r_* < \frac{1}{|\Omega|}\Big\{\int_\Omega u_0 + \int_0^t \int_\Omega f + \int_0^t \int_{\partial\Omega} h\Big\} < r^* \quad \forall\, t \ge 0,$$

problem (1)-(2) has a unique solution u such that $u(0,\cdot) = u_0$; (b) if f and h are T-periodic (i.e. periodic in time with period T) and c_0 is a number with

$$r_* < \frac{1}{|\Omega|}\{c_0 + \int_0^t \int_\Omega f + \int_0^t \int_{\partial\Omega} h\} < r^* \quad \forall t \in [0,T]),$$

then problem (1)-(2) has a T-periodic solution w with

$$\int_\Omega w(0,x)dx = c_0;$$

(c) any solution of (1)-(2) converges to a certain T-periodic solution as t tends to $+\infty$.

G. Lumer

Applications of nonstandard analysis methods to the approximation and regularity of solutions of evolution equations

One establishes (easily) the following nonstandard exponential approximation formula (for a stable familiy of generators):

Lemma. Let X be a standard Banach space (with standard norm), $\{A_k\}$ a standard stable family of semigroup generators in X converging strongly in the (Kato) generalized sense to a densely defined operator. Then

$$e^{tA_k} \simeq \big(1 - \frac{t}{N}A_k\big)^{-N}$$

on the standard elements of X, for all k, $0 \le t$ limited, $0 < N$ infinitely large integer. Also

$$\big(1 - \frac{t}{N}A_k\big)^{-n} \simeq 1$$

on the standard elements of X, for $0 \le t$ infinitesimal, all integers $k \ge 0, n \ge 1$.

We derive several classical consequences: (i) Very short proofs of the Trotter-Kato semigroup convergence theorem (and its usual variants); (ii) A necessary and sufficient condition for the convergence of a very general abstract discrete approximation scheme, which is essentially given by the following result.

Theorem. Let $\{A_k\}$ be a stable sequence of generators in X, A a generator in X. Then $\big(1 - \frac{t}{n}A_k\big)^{-n} \to e^{tA}$ strongly on X, uniformly in t on t-compacta, as $n, k \to \infty$, iff $R(\lambda, A_k) \to R(\lambda, a)$ strongly on X for one (all) $\lambda > \omega$.

(iii) Using $\int_\delta^t \big(1 - \frac{t-s}{N}A\big)^{-N}F(s)ds$, $\delta > 0$ infinitesimal, N as in the lemma above, we give a very transparent and simple proof of a known regularity result (*), which sheds additional light into further regularity matters.

(*) <u>*Theorem.*</u> For A a generator in X, $du/dt = Au + f(t)$, $u(0) = u_0$, $t \in [0,a]$, has a unique classical solution $\forall\, u_0 \in D(A)$ if $F \in W^{1,1}(]0,a[,X)$.

A. LUNARDI

STABILIZABILITY OF INTEGRODIFFERENTIAL PARABOLIC EQUATIONS

We consider the stabilizability problem for an abstract parabolic integrodifferential equation. Under suitable hypotheses, we give a necessary and sufficient condition for stabilizability, generalizing the well known Hautus condition. Then we apply the abstract result to a classical parabolic integrodifferential equations in bounded domains, including the heat equation with memory. (Work written in collaboration with G. Da Prato.)

K. MASUDA

ASYMPTOTIC BEHAVIOR OF SOME REACTION-DIFFUSION EQUATIONS

I consider the reaction-diffusion system of the form:

$$\frac{\partial u_j}{\partial t} = d_j \Delta u_j - e_j u + b_0^{-1} \Big(\sum_{k=1}^{N} a_{jk} u_k\Big) u_j \quad (j = 1, \ldots, N) \tag{1}$$

where $d_j > 0$, $e_j, b_j > 0$, $a_{j,k}$ are given constant.

I am concerned with the asymptotic behavior of (1) with the homogeneous Neumann boundary condition.

J. MIERCZYŃSKI

THE KREIN-RUTMAN THEOREM FOR SEMIFLOWS ON BANACH BUNDLES AND ITS APPLICATIONS TO PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

Assume that $\varphi = \{\varphi_t\}$ is a flow on a compact metric space M. Let B be a Banach space, partially ordered by a closed normal cone B_+ with nonempty interior Int B_+. Let $\Phi = \{\Phi_t\}$ be a linear skew-product semiflow on a Banach bundle $M \times B$, covering φ and such that for each $t > 0, \Phi_t$ takes $B_+ \setminus \{0\}$ into its interior. Under some additional assumptions, it is proved that there is an invariant direct sum decomposition of the bundle $M \times B = V_1 \oplus V_2$, where V_1 is one-dimensional and contained in Int $B_+ \cup$ (-Int $B_+) \cup \{0\}$. A spectral statement is proved which is analogous to the Krein-Rutman theorem.

The above abstract theorem can be applied to the case where the semiflow $\{\Phi_t\}$ generated by a linearized parabolic partial differential equation of second order.

E. Mitidieri

Estimates from below of the solutions of a class of second order evolution equations

Let H be a real Hilbert space. Let $F : H \to]-\infty, +\infty]$ be a l.s.c. proper convex function. If

$$F(tx) = t^p F(x) \quad \forall t > 0, \quad p > 2, \quad x \in D(F)$$

then the solution to

$$\begin{aligned} &u'' \in \partial F(u) \\ &u(0) = u_0 \in D(F) \\ &\sup_{t>0} \|u(t)\| < +\infty \end{aligned}$$

satisfies

$$\|u(t)\| \geq K(u_0)(1+t)^{-\frac{1}{p-2}} \quad t \geq t_0$$

where K_0, t_0 are positive constants depending on u_0.

R. Nagel

Systems of linear evolution equations

We study systems of linear evolution equations which can be written in the form

$$\dot{u}(t) = Au(t), \quad u(0) = u_0$$

where $u(t) \in E^n$, E a Banach space, and $A = (p_{ij}(A))_{n \times n}, A$ a fixed closed densely defined operator on E and $p_{ij} \in \mathbb{C}[x]$. It is characterized under which conditions on the operator A and the polynomials p_{ij} the operator matrix A generates a strongly continuous semigroup on E^n, resp. on appropriate "energy spaces". The results have been obtained jointly with K.J. Engel.

J. Naumann

Differenzierbarkeit schwacher Lösungen parabolischer Systeme mit quadratischen Nichtlinearitäten

In einem zylindrischen Gebiet des $\mathbf{R}^{n+1}$ betrachten wir parabolische Systeme der Form

$$\frac{\partial u^i}{\partial t} - \frac{\partial}{\partial x_\alpha} A_i^\alpha(x,t,u,\nabla u) = B_i(x,t,u,\nabla u) \quad (i = 1, ..., N)$$

mit $|B_i(x,t,u,\xi)| \leq c(1 + |\xi|^2)$. Es wird gezeigt, dass für jede Hölder-stetige schwache Lösung u gilt: $\nabla u \in L^4_{\text{loc}}$. Dieses Resultat basiert wesentlich auf dem Nachweis einer gebrochenen Differenzierbarkeit von ∇u bzgl. t. Die lokale quadratische Integrierbarkeit der zweiten verallgemeinerten Ableitungen von u bezüglich x folgt dann mit Standardargumenten.

S. OHARU

THE FRACTIONAL STEP METHOD FOR ANISOTROPIC DIFFUSION EQUATIONS

Anisotropic diffusion equations of the form

$$\text{(ADE)} \qquad u_t = \sum_{i=1}^{N} \partial^2_{x_i} \Phi_i(u), \qquad t > 0,\ x = (x_1, ..., x_N) \in \mathbf{R}^N.$$

are discussed from the point of view of the approximation theory for nonlinear contraction semigroups in the Banach lattice $X = L^1(\mathbf{R}^N)$. Here $\Phi_i : \mathbf{R} \to \mathbf{R}$ are supposed to be nondecreasing and locally Lipschitz continuous on $\mathbf{R}$. A notion of generalized solution to the initial-value problem for (ADE) is introduced and a nonlinear contraction semigroup $G \equiv \{S(t) : t \geq 0\}$ which provides the generalized solutions is constructed in X. The generators A of G represents the anisotropic diffusion operator $\sum_{i=1}^{N} \partial^2_{x_i} \Phi_i(u)$ and satisfies the range condition $R(I - \lambda A) = L^1(\mathbf{R}^N) \cap L^\infty(\mathbf{R}^N)$ for $\lambda > 0$. The method is based on the use of product formulae for nonlinear semigroups and a system of one-dimensional finite-difference approximation of the form

$$h^{-1}[u_i^{n+1}(x) - u_i^n(x)] = l^{-2}[\Phi_i(u_i^n(x + le_i)) - 2\Phi_i(u_i^n(x)) + \Phi_i(u_i^n(x - le_i))],$$

$$u^{n+1}(x) = u_N^{n+1}(x), \quad u_0^n(x) = u^n(x), \quad i = 1, ..., N, \quad n = 0, 1, 2, ...$$

Consequently, two types of fractional step methods for constructing the generalized solutions of (ADE) are obtained.

J. PRÜSS

AN EXTENSION OF THE DORE-VENNI THEOREM AND APPLICATIONS

We present an extension of the Dore-Venni theorem on the closedness of the sum of two commuting operators, and discuss some of the tools needed for the proof. The result is illustrated by means of several applications to elliptic and parabolic PDE's as well as to Volterra convolution equations as arising in the theory of viscoelastic materials.

P. QUITTNER

VARIATIONAL INEQUALITIES

We investigate the stability of stationary solutions of inequalities of the type

$$u \in K : \left(\frac{du}{dt} - F(u),\ v - u\right) \geq 0 \quad \forall\, v \in K, \quad u(0) = u_0,$$

where K is a closed convex set in a real Hilbert space $V, (\cdot,\cdot)$ is the duality between V and its dual V' and $F : V \to V'$ is a (semilinear) map fulfilling some additional properties. We introduce conditions which are sufficient for the stability or the instability of a given stationary solution and we show that these conditions are in some sense optimal and that they can be in many applications easily verified.

R. Racke

Asymptotic behavior of (non-)linear dissipative systems

We describe the decay behavior of L^q-norms, $2 \leq q \leq \infty$, of solutions u to parabolic resp. damped hyperbolic systems of the following type: $u_t + L^k u = 0$ resp. $u_{tt} + L^k u + u_t = 0$, where $L = -\Delta$ or $L = -\partial_i\, a_{ik}(x)\partial_k$ elliptic with $L = -\Delta$ outside a ball, in an exterior domain $\Omega \subset \mathbb{R}^n$, together with initial and suitable boundary conditions. We use a generalized eigenfunction expansion and are led to the study of pointwise estimates of solutions of exterior boundary value problems for the operator L. - As further motivation we discuss the importance of these estimates for corresponding nonlinear systems.

B. Scarpellini

Singular solutions of singular boundary value problems

The following singular boundary value problem is investigated, which goes back to Brauner and Nicolaenko.

(*) $\Delta u = \lambda f(u)u^{-k}$ where $x \in S^n = \{x/x \in R^n \| x\| \leq 1\}$, $u = 1$ on ∂S_n.

An additional notion, due to Brauner-Nicolaenko, is introduced:

Definition: A function $u \in C^1([0,1])$ is a singular solution of (*) iff $u \in C^1([0,1]) \cap C^2([r_0,1])$ for some $r_0 \in [0,1)$ ($\equiv$ singularity radius), $u \equiv 0$ on $[0,r_0], (r^{n-1}u_r)_r = \lambda f(u)u^{-k}$ on $(r_0,1]$, and $u \equiv 1$ on ∂S_n.

Theorem: Under mild assumptions on u_1, u_2, u_3 : if u_1, u_2, u_3 are three distinct classical solutions of (*) for some $\lambda > 0$, then there exists a singular solution of (*).

[Remark: f is supposed to be smooth, and to satisfy $f \geq \mu_0 > c$ for some μ_0, and $f_u \leq c,\ f_{uu} \geq 0$.]

P.E. Sobolevskii

Investigation of a mathematical model of thermoelasticity

We consider the boundary value problem

$$\kappa^{-1}\frac{\partial\Theta}{\partial t} - \Delta\Theta + \eta\frac{\partial}{\partial t}\operatorname{div}\overline{v} = -\kappa^{-1}Q$$

$$\rho\frac{\partial^2\overline{v}}{\partial t^2} - \mu\Delta\overline{v} - (\lambda+\mu)\operatorname{grad}\operatorname{div}\overline{v} + \gamma\operatorname{grad}\Theta = \rho\overline{F}, \quad 0 \leq t \leq T,\ x \in \Omega;$$

$$\Theta = 0,\ \overline{v} = \overline{0}, \quad 0 \leq t \leq T,\ x \in \partial\Omega,$$

$$\Theta(0,x) = \Theta^0(x),\ \overline{v}(0,x) = \overline{v}^0(x), \quad x \in \overline{\Omega},$$

which describes the motion and the temperature change of a homogeneous isotropic elastic medium occupying the volume Ω with density ρ, Lame's constants λ, μ and thermal coefficients γ, κ, η.

Here Θ is a deviation of temperature, $\overline{v} = (v_1, v_2, v_3)$ is a vector of elastic displacements, Q is an outward flow of heat, $\overline{F} = (F_1, F_2, F_3)$ is a vector of volume density of outward forces, Ω is the domain of R^3 with boundary $\partial\Omega$ and $\overline{\Omega} = \Omega \cup \partial\Omega$.

Problem (I) is reduced to the Cauchy problem for the system of two differential equations of parabolic and hyperbolic types in Banach spaces. In this way the existence and uniqueness theorem of solution of problem (I) in the class

$$\frac{\partial\Theta}{\partial t}, \frac{\partial^2\Theta}{\partial x_i \partial x_k} \in L_p([0,T], L_2[\Omega]) \quad (p > 2);$$

$$\frac{\partial^2\overline{v}}{\partial t^2}, \frac{\partial^2\overline{v}}{\partial t \partial x_i}, \frac{\partial^2\overline{v}}{\partial x_i \partial x_k} \in C([0,T], L_2[\Omega]); \quad [x = (x_1, x_2, x_3)]$$

was obtained under the problem data conditions which are close to the necessary conditions.

J. SOLÀ-MORALES

INERTIAL MANIFOLDS AND THE SINGULAR LIMITS HYPERBOLIC-PARABOLIC AND ELLIPTIC-PARABOLIC

We have been interested in the dynamics defined by the equation

$$\varepsilon u_{tt} + u_t = u_{xx} + f(u)$$

(for $\varepsilon > 0$, for x in a bounded interval, and with boundary conditions at the ends), near the parabolic limit $\varepsilon = 0$. We have proved that this limit is no more singular when one restricts the flow to a suitable finite dimensional globally attracting (smooth) invariant manifold of finite dimension, embedded in the (function-) space of states (i.e., an inertial manifold).

We have also an example of non-existence of such a manifold for ε large.

This work can be considered as one part of a larger study developed by our group in Barcelona.

A. STAHEL

THE WAVE EQUATION WITH SEMILINEAR BOUNDARY CONDITIONS

We present a joint work with Irena Lasiecka. On a bounded smooth domain Ω in $\mathbf{R}^n (n \leq 3)$ we consider the initial value problem

$$(P) \quad \begin{cases} u_{tt} - \Delta u + u = 0 & \text{in} \quad [0,T) \times \Omega \\ \frac{\partial}{\partial\nu} u = g(u) & \text{on} \quad [0,T) \times \partial\Omega \\ u(0) = u_0, \; \dot{u}_{(0)} = \dot{u}_0 & \text{in} \quad \Omega \end{cases}$$

Using a result of Miyatake (1973) we obtain

Theorem:

$$g \in C^3(\mathbf{R}, \mathbf{R}), \; \frac{3}{2} < s \leq 2$$

$$u_0 \in H^s(\Omega), \; \dot{u}_0 \in H^{s-1}, \quad \frac{\partial}{\partial\nu} u_0 = g(u_0)$$

$$\Longrightarrow \; \exists! \; u \in C^0([0,T^+), H^s(\Omega)) \cap C^1([0,T^+), H^{s-1}(\Omega))$$

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and u is a solution of (P).

If we assume an H^1 energy estimate and some growth conditions on g we also obtain global solutions. This second result is based on a recent regularity result of Lasiecka, Triggiani.

H. Tanabe

Control theory for linear delay-differential equations in a Hilbert space

Let

(1) $$du(t)/dt = A_0u(t) + A_1u(t-h) + \int_{-h}^{0} a(s)A_2u(t+s)ds$$

be a linear delay-differential equation in a Hilbert space H. Here, A_0 is the operator associated with a sesquilinear form defined in $V \times V$ satisfying Gårding's inequality, where V is a Hilbert space such that $V \subset H \subset V^*$. A_1 and A_2 are bounded linear operators from V to V^*, and a is a real valued Hölder continuous function in $[-h,0]$. Let $S(t) = e^{tA}$ and F be the solution semigroup and structural operator for (1) respectively considered as an equation in V^*. Then, it is shown that $FS(t) = S_T^*(t)F$, $S^*(t)F^* = F^*S_T(t)$, where $S_T(t)$ is the solution semigroup for the adjoint equation. In the special case $A_1 = \gamma A_0$ and $A_2 = A_0$ with some real constant γ it can be shown that the generalized eigenvectors of A are complete in $H \times L^2(-h,0;V)$. Some applications are given.

B. Terreni

Boundary control for non-autonomous parabolic systems over infinite time-horizon

We consider a boundary control problem over infinite time horizon for a class of linear nonautonomous parabolic systems. We use a Hilbert space approach and some results from the theory of evolution operators. The cost functional is quadratic and we get a feedback formula for the optimal pair by a dynamic programming argument after a direct study of the Riccati equation related to the problem.

P. Vuillermot

Invariant manifolds for nonlinear Klein-Gordon equations on $\mathbf{R}^2$: Some recent results

Recent results concerning the existence of spatially localized almost-periodic oscillations to semilinear wave equations on $\mathbf{R}^2$ will be discussed within the framework of nonlinear functional analysis and dynamical system theory.

W. VON WAHL

THE INSTATIONARY NAVIER-STOKES EQUATIONS ON EXTERIOR DOMAINS

We consider the problem

$$(1)\quad \begin{cases} u' - \Delta u + u \cdot \nabla u + \nabla \pi &= f\,, \\ \nabla \cdot u &= 0\,, \\ u(0) &= \varphi\,, \\ u|\partial\Omega &= 0 \end{cases}$$

over a cylindrical domain $(0,+\infty) \times \Omega \subset \mathbf{R}^4$, where Ω is unbounded. This is precisely the situation where a body Ω^c is surrounded by a fluid with velocity u and pressure π. First we collect some a-priori information for any weak solution, in particular we show that $\pi \in L^{5/3}((0,T) \times \Omega)$, $T > 0$. Making strong use of this information we can construct a particular weak solution fulfilling a localized energy inequality and being bounded outside a compact region around the body (provided the external force dies out if $t \to \infty$). Thus all possible singularities are concentrated in this compact region. Moreover this solution is stable in the sense that $\|u(t)\|_{L^2(\Omega)}$ tends to 0 as $t \to \infty$. There is almost no hope to come by a singularity by numerical or physical experiments. The reason is that for any $T > 0$, any smooth f and $L^s((0,T), L^r(\Omega))$-neighbourhood of f with radius $\varepsilon > 0$ $(\frac{2}{s} + \frac{3}{r} > 4)$ there is a smooth f_ε, lying in this neighbourhood, such that (1) (with f replaced by f_ε) has a smooth solution on $[0,T] \times \overline{\Omega}$. The work reported was done jointly with H. Sohr and M. Wiegner.

E. ZEIDLER

NONLINEAR SYMMETRIC HYPERBOLIC SYSTEMS

Let us consider the following general nonlinear symmetric hyperbolic system of first order

$$(1)\qquad \frac{du}{dt} = F(u), \qquad u(0) = u_0$$

where

$$(Fu)(x) = f\big((x, u(x), \partial u(x)\big).$$

It is possible to prove a theorem for (1) concerning
(i) existence and uniqueness;
(ii) regularity, and
(iii) well-posedness.
The solution lives in the space

$$C([0,T], H^s) \cap C([0,T], H^{s-1}),$$

where

$$s \geq [n/2] + 3 \quad \text{and} \quad H^s = H^s(\mathbf{R}^n, \mathbf{R}^m).$$

(In the quasilinear case we only need $s \geq [n/2] + 2$.)

The fairly simple proof of this theorem due to M. Günther (Leipzig) is based on a substantial generalization of a recent abstract theorem by Kato and Lai (J. Funct. Anal. 1984) and on a new theorem about nonlinear interpolation. The detailed proof along with applications will be provided in the book Zeidler, Nonlinear Functional Analysis, Vol. 5.

It was discovered by K.O. Friedrichs in the 1950's that a large class of problems in mathematical physics can be reduced to problem (1). For example, this concerns magneto-hydrodynamic, compressible fluids, and general relativity.

ZHENG SONGMU

GLOBAL SMOOTH SOLUTIONS TO NONLINEAR EVOLUTIONS WITH DISSIPATION

This talk is concerned with the global existence of smooth solution to the following initial boundary value problems of nonlinear hyperbolic system:

$$\begin{cases} \vec{u}_{tt} = \sum_{i=1}^{n} \partial_i \left(\vec{a}_i(\Lambda\vec{u})\right) = \vec{f}_\Omega(x,t), & \text{in } \Omega \times (0,\infty) \\ \sum_{i=1}^{n} \nu_i \vec{a}_i(\Lambda\vec{u}) + \vec{b}(x,\partial_t\vec{u}) = \vec{f}_\Gamma(x,t), & \text{on } \Gamma \times (0,\infty) \\ \vec{u}(x,0) = \vec{u}_0(x), \ \ \vec{u}_t(x,0) = \vec{u}_1(x) & \text{in } \Omega \end{cases}$$

where Ω is a bounded domain in R^n with smooth boundry Γ, $\vec{u} = (u_1, \ldots, u_1)^T$ with $l = n$ or $l = 1; \Lambda\vec{u} = (\varepsilon_{ij})$ with $\varepsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$.
Let $\vec{b} = (b_1, \ldots, b_2)$, $\quad b_{ij} = \partial b_i / \partial(\partial_t u_j)$.
Under the assumption $\sum b_{ij}\xi_i\xi_j \geq \delta|\xi|^2$ with $\delta > 0$, which physically means that there is a damping force depending on the velocity, and other reasonable assumptions on $\vec{a}_i, \vec{b}$ it is proved jointly with Shibata that the problem admits a unique global solution provided the initial data and $\vec{f}_\Omega, \vec{f}_\Gamma$ are small. Two applications to nonlinear elastodynamic systems and nonlinear acoustic wave equations with damping boundary conditions are also shown.

BERICHTERSTATTER: H. Amann und P. Hess

Tagungsteilnehmer

Prof. Dr. P. Acquistapace
Dipartimento di Metodi e Modulli
Matematici per le Scienze Applicate
Via A. Scarpa 10

I-00161 Roma

Prof. Dr. Ph. Clement
Onderafdeling der Wiskunde en
Informatica
Technische Hogeschool Delft
Julianalaan 132

NL-2628 BL Delft

F. Ali-Mehmeti
Fachbereich Mathematik
der Universität Mainz
Postfach 3980
Saarstr. 21

6500 Mainz

Prof. Dr. G. Da Prato
Scuola Normale Superiore
Piazza dei Cavalieri, 7

I-56100 Pisa

Prof. Dr. H. Amann
Mathematisches Institut
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof. Dr. G.W. Desch
Institut für Mathematik
Karl-Franzens-Universität
Elisabethstr. 16

A-8010 Graz

Prof. Dr. S. B. Angenent
Department of Mathematics
University of Wisconsin-Madison
712 Van Vleck Hall
610 Walnut Street

Madison , WI 53705
USA

Dr. G. Dore
Dipartimento di Matematica
Universita degli Studi di Bologna
Piazza di Porta S. Donato, 5

I-40127 Bologna

Prof. Dr. W. Arendt
Laboratoire de Mathematiques
Universite de Franche-Comte
Route de Gray

F-25030 Besancon Cedex

J. Escher
Mathematisches Institut
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof. Dr. H. Fattorini
Dept. of Mathematics
University of California
405 Hilgard Avenue

Los Angeles , CA 90024
USA

Prof. Dr. M. Grobbelaar
Department of Mathematics and
Applied Mathematics
University of South Africa
P. O. Box 392

0001 Pretoria
SOUTH AFRICA

Dr. B. Fiedler
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg 1

Prof. Dr. A. Haraux
Laboratoire d'Analyse Numerique,
C.N.R.S., Universite P. et M. Curie
Tour 55 - 65 - 5eme Etage
4, Place Jussieu

F-75230 Paris

Prof. Dr. H. Gajewski
Akademie der Wissenschaften
Institut für Mathematik
Mohrenstr. 39

DDR-1086 Berlin

Prof. Dr. P. Hess
Mathematisches Institut
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof. Dr. J.A. Goldstein
Dept. of Mathematics
Tulane University

New Orleans , LA 70118
USA

Prof. Dr. N. Kenmochi
Dept. of Mathematics
Faculty of Education
Chiba University

Chiba
JAPAN

Dr. G. Greiner
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

P. Koch
Mathematisches Institut
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof. Dr. G. Lumer
Mathematiques
Faculte des Sciences
Universite de L'Etat a Mons
Av. Maistriau 15

B-7000 Mons

Prof. Dr. R. Nagel
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof. Dr. A. Lunardi
Dipartimento di Matematica
Universita
Via Ospedale 72

I-09100 Cagliari

Prof. Dr. B. Najman
Department of Mathematics
University of Zagreb
P. O. Box 187

YU-41000 Zagreb

Prof. Dr. K. Masuda
Dept. of Mathematics
Faculty of Science
University of Tokyo
Hongo 7-3 -, Bunkyo-ku

Tokyo 113
JAPAN

Prof. Dr. J. Naumann
Sektion Mathematik der
Humboldt-Universität Berlin
Postfach 1297
Unter den Linden 6

DDR-1086 Berlin

Dr. J. Mierczynski
Institute of Mathematics
Politechnika
Wyb. Wyspianskiego 27

50-370 Wroclaw
POLAND

Prof. Dr. S. Oharu
Dept. of Mathematics
Fac. of Integrated Arts + Sciences
Hiroshima University

730 Hiroshima
JAPAN

Prof. Dr. E. Mitidieri
Dipartimento di Scienze Matematiche
Universita di Trieste
Piazzale Europa 1

I-34100 Trieste (TS)

Prof. Dr. B. de Pagter
Onderafdeling der Wiskunde en
Informatica
Technische Hogeschool Delft
Julianalaan 132

NL-2628 BL Delft

Prof. Dr. J. Prüss
Fachbereich Mathematik/Informatik
der Universität Paderborn
Postfach 1621
Warburger Str. 100

4790 Paderborn

G. Simonett
Mathematisches Institut
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Dr. P. Quittner
Inst. of Physics of the Electro-
Physical research Centre
Slovak Academy of Sciences
Dubravska cesta 9

842 28 Bratislava
CZECHOSLOVAKIA

Prof. Dr. P. Sobolevskii
Department of Mathematics
Voronezskii University
Universitetskaja pl. 1

Voronez 394693
USSR

Dr. R. Racke
Institut für Angewandte Mathematik
der Universität Bonn
Wegelerstr. 10

5300 Bonn 1

Prof. Dr. J. Sola-Morales
Departamento de Matematicas
Universidad Autonoma de Barcelona
Campus Universitario

E-08193 Bellaterra Barcelona

Dr. R. Redlinger
Mathematisches Institut I
der Universität Karlsruhe
Englerstr. 2
Postfach 6380

7500 Karlsruhe 1

Prof. Dr. A. Stahel
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Prof. Dr. B. Scarpellini
Mathematisches Institut
Universität Basel
Rheinsprung 21

CH-4051 Basel

Prof. Dr. H. Tanabe
Dept. of Mathematics
Osaka University
Toyonaka

Osaka 560
JAPAN

Prof. Dr. B. Terreni
Dipartimento di Matematica
"Federigo Enriques"
Universita di Milano
Via Saldini 50

I-20133 Milano

A. Wyler
Mathematisches Institut
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof. Dr. P. A. Vuillermot
Mathematikdepartment
ETH-Zürich
Rämistrasse 101

CH-8092 Zürich

Prof. Dr. E. Zeidler
Sektion Mathematik
Karl-Marx-Universität
Karl-Marx-Platz

DDR-7010 Leipzig

Prof. Dr. W. von Wahl
Lehrstuhl für Angewandte Mathematik
Universität Bayreuth
Postfach 10 12 51

8580 Bayreuth

Prof. Dr. Zheng Songmu
Dept. of Mathematics
Fudan University

Shanghai
CHINA

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