MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 24/1989

Ringe und Moduln

4.6. bis 10.6.1989

This meeting was organized by Gerhard O. Michler (Essen) and Lance W. Small (San Diego); it was attended by 38 participants from eight countries.

Since the last Oberwolfach conference on Rings and Modules in 1986, there has been progress in some well-established areas as well as developments in new directions. Recent results on embeddings of rings, the localizations of rings, rings with polynomial identities and enveloping algebras of Lie algebras were presented. Several reports were given on group actions, crossed products and Hopf algebras and their connection to Galois theory and other parts of ring theory. Furthermore, some talks were devoted to new results on finite dimensional algebras and orders. The organizers also gave emphasis to recent progress in ring theory with applications to other fields in mathematics or to physics. In particular, we mention the investigation of rings of differential operators, the study of quantum groups and the new contributions to rationality problems. The Russian colleagues gave a survey on recent research on ring theory in the USSR.

The informal discussions formed an invaluable part of the meeting which deepened the insights gained by the lectures and sometimes led to new results. The success of the conference was supported by the friendly atmosphere of the institute.

Vortragsauszüge

M. van den Bergh: Cohen-Macaulayness of invariant modules for tori

Let T be a torus, W a finite dimensional representation and χ be a character of T. Let $R = k[W]$ and define $R_\chi = \{f \in R \mid \tau \cdot f = \chi(\tau) f\}$. Then $R = \oplus_\chi R_\chi$ defines a grading on R with $X(T)$.

Stanley gave a complete answer to the question when R_χ is Cohen-Macaulay in terms of certain polyhedral complexes of dimension $O(\dim W)$. We also give a complete answer in terms of certain spherical complexes with dimension $\leq \dim T - 2$. We analyze the cases $\dim T = 1, 2$ and we give some consequences for the functional equation.

C. Bessenrodt: Modular representation theory of finite groups and the Auslander-Reiten quiver

Let G be a finite group and F a field of characteristic $p > 0$. We consider the stable Auslander-Reiten quiver for FG. In 1982, P. Webb gave a list of possible tree types for its connected components, namely the (finite or infinite) Dynkin diagrams and the Euclidean trees. Later Okuyama proved that for $p \neq 2$ the tree class is never Euclidean. For $p = 2$, the Euclidean diagrams $\tilde{A}_{12}$ and [illegible] were known to occur. We show now, that in fact these are the only possible Euclidean trees. Moreover, the block containing such a component has a Klein four group as a defect group. As a corollary we obtain that the heart of a projective indecomposable FG-module decomposes into at most 3 indecomposable summands (this bound is sharp).

Furthermore, the methods used can also be applied for obtaining the main results about the modular representations of blocks with cyclic defect groups.

L.A. Bokut: Some new results in ring theory

1. A positive solution of Restricted Burnside Problem (RBP) (E.I. Zel'manov)
2. Local Specht problem over a field of finite characteristic (A.R. Kemer)
3. Some new results on embedding into matrix rings (A.Z. Anan'in, N.G. Nesterenko)
4. Groups with standard basis (the diamond lemma in group theory)

The Zel'manov result consists of two parts - a reduction of RBP for primary order p^k to the problem of local nilpotency of any Lie algebra with Engel identity, and a proof of the last result. The method of the proof of this last result may have influence on the study of identities of Lie and associative algebras of characteristic $p > 0$.

New A.R. Kemer results (§ 2) show that the main problem about identities of associative algebras of characteristic p (the analogy of Specht problem) can be reduced to the question - is it true that all identities of associative algebras of characteristic p follow from identities with finite number of variables?

K. Bongartz: A geometric version of the Morita duality

Two basic observations on the schemes Mod_A^n of n-dimensional representations of a finite-dimensional algebra A are proved.

<u>Theorem 1</u> Let A and B be two finite-dimensional algebras over a commutative field k. Then the following are equivalent:

a) A and B are isomorphic (as k-algebras).

b) For all n, the k-schemes Mod_A^n and Mod_B^n are isomorphic in a GL_n-equivariant way.

c) For $n \in \{\dim A, \dim B\}$, the reduced k-schemes $(Mod_A^n)_{red}$ and $(Mod_B^n)_{red}$ are isomorphic in a GL_n-equivariant way.

<u>Theorem 2</u> Let A be a finite dimensional algebra over an alge-

braically closed field. Then the connected components of Mod_A^n are fibre bundles (with respect to the Zariski-topology in a schematic sense) over the smooth scheme of semisimples having as typical fibre the corresponding bound representations of the Gabriel quiver of A .

Some easy corollaries and examples are given.

In characteristic 0 , theorem 1 can be deduced easily from Procesi's work. In characteristic p , the results are new. Moreover, the proofs are direct and based on elementary facts from the representation theory of finite dimensional algebras and on base change in the sense of linear algebra!

A. Braun: Accessible points

The notion of accessible point in maxspec R is discussed where $R = F\{x_1,\dots,x_p\}$ is a prime affine P.I. (polynomial identity) ring. Two basic questions are raised:

Problem 1 A recognition question, namely what ring theoretic properties determine the accessibility of a point M in R .

Problem 2 Find some good properties of accessible points.

Partial answers to these questions are discussed. A major technique in solving them seems to be the notion of completion at a maximal ideal M of a ring R . This technique seems to shed some light on related questions in Ug , g is finite dimensional solvable $/\mathbb{C}$.

K.A. Brown: The finite dimensional algebras associated with differential operators on singular curves

The ring $D(X)$ of differential operators on the affine curve X has a unique minimal non-zero ideal $J(X)$, and $H(X) := D(X)/J(X)$ is a finite dimensional $\mathbb{C}$-algebra, with

$$H(X) = \sum_{x \in \text{Sing } X}^{\oplus} H(X,x) ,$$

with each $H(X,x)$ indecomposable. We describe how $H(X,x)$ is determined by the singularity x, and how the structure of $H(X)$ influences the structure of $D(X)$.

C. Dean: The ideal structure of the enveloping algebra of the Virasoro algebra

The enveloping algebra of the Virasoro algebra is discussed, and a new family of irreducible representations for the Virasoro algebra is constructed.

P. Dräxler: On indecomposable modules over directed algebras

Let k be a field and A a basic finite dimensional k-algebra which is representation directed. The following result is due to Bongartz in the case k is algebraically closed and to Dlab and Ringel if A is hereditary.

Theorem If U is a not simple indecomposable A-module then there is an exact sequence $0 \to U_1 \to U \to U_2 \to 0$ such that the A-modules U_i are indecomposable and U_1 or U_2 are simple.

We give a proof of this theorem which works for any field k with more than 2 elements.

A.W. Goldie: Embeddings in artinian rings

It is known (Blair-Small) that the theorem of Schofield can be applied to show that a noetherian ring R has a flat simple artinian extension provided that R has an exact additive rank function. This requires the assumption that $\text{Ass } R_R$ consists of minimal primes. In this talk the relationship between such functions on R and Gabriel topologies with artinian closure is pointed out (a theorem due to

Günter Krause) and the extension ring $S = \varinjlim_{F \in \mathcal{F}} \mathrm{Hom}_R(F,R)$, where the right ideals $F \in \mathcal{F}$ form such a topological set of right ideals, is studied. The structure of S is obtained by using the injective hull V_R of R_R , its endomorphism ring H and the bicommutator T , which is the dense or maximal ring of quotients of R . It is shown that -

Theorem $_HV$ is an artinian module. H is a semi-primary ring with index of nilpotency $\leq$ the length of the $\mathcal{F}$-closed lattice of right ideals.

For any extension ring $S = R_{\mathcal{F}}$ it is proved: -

Theorem Any right ideal I of S has the form $I = eS \oplus (1-e)I$, with $(1-e)I$ nilpotent. Consequently, S is a semi-primary ring (which contains R).

In discussion it was noticed that counter-examples existed which showed that the maximal ring of quotients T need not be artinian even when R is artinian! Whether there exists $\mathcal{F}$ such that $R_{\mathcal{F}}$ has the required properties (artinian and $_RS$ flat) is left unsettled.

K. Goodearl: Avoiding infinite sets of primes

We shall discuss a version of Prime Avoidance for infinite sets X of prime ideals in noetherian rings R , namely the Intersection Condition studied by Jategaonkar, Müller, and others: given a one-sided ideal I that contains no elements regular modulo all primes in X , there should exist P in X such that I contains no elements regular modulo P . Most known cases of the Intersection Condition for infinite sets of primes have been proved using an uncountable central subfield; however, Müller has verified it for arbitrary cliques of primes in affine noetherian P.I. rings. We shall present a variation on his method which yields the Intersection Condition for cliques of primes in a class of iterated differential operator rings including smash products $R\#U(g)$ where R is a commutative noetherian algebra over a field k of characteristic zero and g is a finite-dimensional solvable Lie algebra over k.

D. Happel: Hochschild cohomology of finite dimensional algebras

Let A be a finite-dimensional algebra over a field k and let M be an A-bimodule. Let A^e be the enveloping algebra of A. Then the Hochschild cohomology groups $H^i(A,M)$ are defined as $\mathrm{Ext}^i_{A^e}(A,M)$, where A and M are considered as left A^e-modules.
Let $H(A)$ be the Hochschild-cohomology-algebra, which is a $\mathbb{Z}$-graded algebra. (The low dimensional cohomology groups are interpreted in terms of derivations, extensions and deformations.)
Some results to compute $H^i(A) = H^i(A,A)$ are presented:

<u>Theorem:</u> Let P be a finite poset and $I(P)$ be its incidence algebra. Let Σ_P be the simplicial complex of chains in P. Then $H^i(I(P)) = H^i(\Sigma_P,k)$.
(Compare also an article by Gerstenhaber-Schack.)

<u>Theorem:</u> Let A be as above and let ${}_AN$ be a finite-dimensional left A-module and let $B = A[N]$ be the one-point extension algebra. Then there exists a long exact sequence
$0 \to H^0(B) \to H^0(A) \to \mathrm{End}_A N/k \to H^1(B) \to H^1(A) \to \mathrm{Ext}^1_A(N,N) \to \dots$.
This implies that a representation-directed algebra satisfies $H^i(A) = 0$ for $i \geq 2$.

<u>Theorem:</u> Let A be as above and ${}_AM$ be an r-tilting module with $B = \mathrm{End}_A M$. Then $H(A) \cong H(B)$ as $\mathbb{Z}$-graded algebras.

The proof of this result makes use of the fact that under the assumptions of this theorem the derived categories of bounded complexes of A-modules and B-modules are triangle-equivalent.

T. Hodges: Noncommutative Kleinian singularities

A family of algebras is studied whose associated graded rings are the commutative algebras $\mathbb{C}[X,Y,Z]/(XY-Z^n)$. The main structure theorem states that these algebras are the intersections of copies of the rings of twisted differential operators on the projective line "glued together" along a copy of the Weyl Algebra $A_1(\mathbb{C})$. A version of the Bernstein-Beilinson theorem for $U(sl(2))$ is proved for these algebras and from this result, facts about the global dimension and the Grothendieck group are deduced.

R.S. Irving: Categories of Harish-Chandra modules like the category $\mathcal{O}$

Let G be a real, semisimple linear Lie group with Lie algebra $\underline{g}$. Form the category HC_χ of Harish-Chandra modules with generalized infinitesimal character $\chi : Z(g_{\mathbb{C}}) \to \mathbb{C}$. There is a computable finite poset D such that for $\delta \in D$ one can associate a "generalized principal series" module $M(\delta)$ in HC_χ with simple top $L(\delta)$. Langlands classification is that $\{L(\delta) \mid \delta \in D\}$ are all of the irreducibles in HC_χ . We search for a full subcategory $\widetilde{HC}_\chi$ of HC_χ which has properties of the category $\mathcal{O}$: projective covers, BGG reciprocity, finite global dimension, etc. Such a category would allow us to interpret the Lusztig-Vogan polynomials of G via $\mathrm{Ext}^i_{\widetilde{HC}_\chi}(M(\gamma),L(\delta))$'s for instance. The list of requirements on $\widetilde{HC}_\chi$ leads to the restriction that G has 1 conjugacy class of Cartan subgroups; e.g., G is complex or $SL_n(\mathbb{H})$. Work of Bernstein-Gelfand, Joseph, Enright yields a solution for G complex, that doesn't generalize in any obvious way. Using a different approach, David Collingwood and I construct the desired $\widetilde{HC}_\chi$ for all groups G with 1 conjugacy class of Cartan subgroups.

A. Joseph: Some criteria for deciding when an ideal is induced

Let $\mathfrak{g}$ be a complex semisimple Lie algebra. We discuss some results concerning the set $\mathrm{Prim}_c U(\mathfrak{g})$ of completely prime, primitive ideals in the enveloping algebra $U(\mathfrak{g})$. First a positivity property of Goldie rank polynomials implies that any $I \in \mathrm{Prim}_c U(\mathfrak{g})$, with integral and regular central character, is induced. Secondly we show that for $I \in \mathrm{Max}\, U(\mathfrak{g})$ one has I is induced $\Leftrightarrow$ the associated variety of I is induced (in a suitable sense). Here one should like to replace the hypothesis $I \in \mathrm{Max}\, U(\mathfrak{g})$ by $I \in \mathrm{Prim}_c U(\mathfrak{g})$ and thereby obtain what should be the correct formulation of Moeglin's theorem (valid so far for $\mathfrak{g} \cong \mathfrak{sl}(n)$). Finally let $L(\lambda)$ be a homomorphic image of a Verma module and $I(\lambda)$ the annihilator of its canonical generator. One easily shows that $\mathrm{gr}\, I(\lambda)$ being prime implies $\mathrm{Ann}\, L(\lambda) \in \mathrm{Prim}_c U(\mathfrak{g})$. Moreover $\mathrm{gr}\, I(\lambda)$ is then the ideal of definition of an orbital variety and one can ask if all such ideals so occur. In $\mathfrak{sl}(6)$ a student of mine E. Benlolo (Haifa) has shown that this fails (in just two cases) if one imposes that $L(\lambda)$ be simple.

F. Kasch: The total in modules and rings

Let R be a ring with $1 \in R, M, N$ unitary R-right modules and $S := \mathrm{End}_R(N)$. Then $f \in \mathrm{Hom}_R(M,N)$ is called a total nonisomorphism $\Leftrightarrow \forall 0 \neq A \mid M,\ B \mid N,\ f(A) \subset B : f|A : A \to B$ is not an isomorphism.

This is equivalent to:
$\forall g \in \mathrm{Hom}_R(N,M)$ [fg is not an idempotent $\neq 0$ (in S)].

If f is not a total nonisomorphism, then there exists a $g \in \mathrm{Hom}_R(N,M)$ such that fg is an idempotent $\neq 0$ (in S). We call then f partially invertible (= pi).
First question: For which M,N is $\mathrm{Tot}(M,N) :=$ set of all total nonisomorphisms $f \in \mathrm{Hom}_R(M,N)$, closed under addition? If $\mathrm{Tot}(M) := \mathrm{Tot}(M,M)$ is closed under addition, then M is called a total module.

<u>Theorem</u> ([3]) M is a total module ⟺ M has d2-exchange property.

The d2-EP is weaker than the 2-EP hence each 2-EP module is a total module. Total modules can be considered as generalizations of LE-modules. For the special case that $M_R = R_R$, we identify $End(R_R)$ with R . It is easy to see that Tot(R) is independent of the side.

<u>Theorem</u> ([1]) Rad(R)+Tot(R) = Tot(R) , hence Rad(R) ⊂ Tot(R) .

<u>Theorem</u> ([1]) If Tot(R/Rad(R)) = 0 and idempotents can be lifted from R/Rad(R) to R , then Rad(R) = Tot(R) .

<u>Ex.:</u> 1) Semi-perfect rings
2) F-semi-perfect rings (Oherst-Schneider) = semi-regular rings (Nicholson).

There are more results in the special case of rings and also in the general case. Especially there are relations between pi-elements and regular elements (def. by fgf = f). The total is also a good notion to construct ideals in the category of all R-right modules.

<u>Lit.:</u> [1] F. Kasch, Algebra-Berichte 60, München 88
[2] F. Kasch, The total in the category of modules, Report on Conf. in Krems 1988, will appear Wien 1989
[3] W. Schneider, Algebra-Berichte 55, München 87

V. Kharchenco: Actions of groups and Hopf algebras on rings

Def.: The set of skew derivations $\{\delta_1,\dots,\delta_k\}$ is called reduced if

a) different automorphisms corresponding to these derivations are different modulo X-inner subgroup

b) if $\delta_{i_1},\dots,\delta_{i_k}$ are different s-derivations (corresponding to the same automorphism s) then they are lineary independent modulo subspace int L_s of inner s-derivations (recall that s-derivation $x \to ax-x^s a$ is called inner) for Martindale ring of quotients.

Theorem 1: If prime ring R satisfies multilinear generalized identity of the form $F(x_j^{h_i \delta_k}, x_j^{h_i}) = 0$, where $\{\delta_k\}$ - reduced set of skew derivations, $F(z_j^{(i,k)}, y_{ij})$ - generalized polynomial with non-commutative coefficients from Martindale ring of quotients. Then the ring R satisfies the identity $F(z_j^{(i,k)}, y_{ij}) = 0$.

Theorem 2: Let us suppose that the base automorphism acts trivially on all skew derivations of reduced set $\{\delta_1,\ldots,\delta_k\}$ and $\{\Delta_1,\ldots,\Delta_m\}$ the set of all right products (in Lie sense). Then if given prime ring R satisfies multilinear identity $F(x_j^{h_i \Delta_k}) = 0$ then it satisfies an identity $F(z_j^{(i,k)}) = 0$.

These theorems were proved by A.Z. Popov and V.K. Kharchenco. In conference there was also discussed the question on algebraic dependences between skew derivations in general case (when group acts nontrivially) with the help of Hopf algebra language.

L. Le Bruyn: On stable rationality of centers of generic division algebras

(Joint work with Christine Bessenrodt)

A representation theoretic method is given to determine whether almost free actions of PGL_n on a vectorspace have stable rational quotient varieties. In particular, stable rationality is proved for such PGL_p-quotients for $p = 2,3,5,7$.

H. Lenzing: Homological epimorphisms of rings

A ring homomorphism $\varphi: R \to S$ is called a homological epimorphism if

(a) $S \otimes_R S \xrightarrow{\sim} S$, i.e. φ is an epimorphism and

(b) $\mathrm{Tor}_i^R(S,S) = 0$ for all $i \geq 1$.

It is equivalent to assume that for all left (resp. right) S-modules M,N the mapping $\varphi_*^i : \mathrm{Ext}_S^i(M,N) \to \mathrm{Ext}_R^i(M,N)$ is an isomorphism or to assume that $\varphi_* : \mathrm{Mod}(S) \to \mathrm{Mod}(R)$ induces a full embedding $\mathfrak{D}^b(\varphi_*) : \mathfrak{D}^b(\mathrm{Mod}(S)) \to \mathfrak{D}^b(\mathrm{Mod}(R))$.

Theorem 1: Let R be right noetherian, $\varphi : R \to S$ a ring homomorphism, which is injective and a homological epimorphism, S_R finitely generated. $\psi : R \to \mathrm{End}({}_R(S/R))^{op} = T$ induced by multiplication. Then

1) $S \oplus S/R$ is a tilting module in $\mathrm{mod}(R)$.
2) $\varphi_* : \mathrm{mod}(S) \to \mathrm{mod}(R)$, $\psi_* : \mathrm{mod}(T) \to \mathrm{mod}(R)$

are full embeddings inducing an isomorphism $K_0(S) \oplus K_0(T) \xrightarrow{\sim} K_0(R)$.

Theorem 2: Let Λ, Λ' be tame hereditary connected finite-dimensional algebras over an algebraically closed field. There exists a homological epimorphism from Λ into an algebra Morita-equivalent to Λ' if and only if the Dynkin type of Λ dominates the Dynkin type of Λ' .

Joint work with W. Geigle.

E. Letzter: Noetherian ring extensions and Lie superalgebras

This talk examines an abstraction of the injective homomorphism of associative algebras $U(g_0) \to U(g_0 \oplus g_1)$ arising from a completely solvable Lie superalgebra $g_0 \oplus g_1$. The results obtained are applied to give a detailed description of the prime spectrum of the enveloping algebra of a finite dimensional nilpotent Lie superalgebra. A partial description is given for the non-nilpotent completely solvable case.

T. Levasseur: Lifting differential operators on orbit spaces

Let G be a reductive group (over $\mathbb{C}$) and $G \to GL(V)$ a finite dimensional representation of G . Denote by $D(V)^G$ the ring of invariant differential operators on V , and by D(V/G) the ring of differential operators on the orbit space V/G . We give some sufficient conditions for the natural map $D(V)^G \to D(V/G)$ to be surjective.

L.S. Levy: Cancellation and direct summands in dimension 1

Let Λ be a module-finite algebra over a commutative Noetherian ring of Krull dimension 1. We extend Roiter's direct-summand theorem to arbitrary finitely generated Λ-modules, obtaining a sharpened form of Serre's direct-summand theorem in this setting.
We also obtain a cancellation theorem that extends Bass's and Drozd's cancellation theorems, in this setting.
A corollary is that, over commutative Noetherian rings, cancellation holds in every genus of finitely generated modules. In the noncommutative case, cancellation is shown to be a condition involving division algebras.

M. Lorenz: Crossed products: characters, cyclic homology, and Grothendieck groups

Let $S = R*G = \bigoplus_{g\in G} S_g$ be a crossed product of the group G over the ring $R = S_1$. In this survey talk, we explain how the cyclic homology of S and Moody's induction theorem for $G_0(S)$ can be used to derive ring theoretic information on S .

<u>Cyclic homology:</u> By work of Feigin and Tsygan, the cyclic homology $HC_*(S)$ of a skew group ring $S = R_\alpha G$ $(\alpha : G \to \mathrm{Aut}\ R)$ decomposes as $HC_*(S) = \bigoplus_{[g]} HC_*(S)_{[g]}$, where [g] runs over the conjugacy classes of G . In the case where R is a separable k-algebra (k some commutative ring) and $\mathrm{order}(g) = \infty$, one has
$HC_*(S)_{[g]} \cong H_*(\mathbb{C}_G(g)/\langle g\rangle, S_g/[S_g,S_1])$. As a consequence, it follows that for f.g. projective modules P over the group ring $S = RG$, where R is a separable k-algebra with $\mathbb{Q} \subseteq k$ and G is a

torsion-free solvable group with $hd_{\mathbb{Q}}(G) < \infty$, the Hattori-Stallings character $\chi(P)$ can be computed by $\chi(P) = \chi(P/P\omega G \otimes_R S)$ (ωG = augmentation ideal).

Moody's induction theorem states that, for $S = R*G$ a crossed product with R right Noetherian and G polycyclic-by-finite, $G_0(S)$ is generated by the images of all $G_0(R*X)$ under induction, where X ranges over the finite subgroups of G . This leads to a formula for the Goldie rank of S in the case where S is prime. As another application, one obtains a formula for the rank of $G_0(kG)$ which extends classical results of Frobenius and Brauer for finite groups.

S. Montgomery: Crossed products of Hopf algebras and Galois extensions

H is a finite-dimensional Hopf algebra, with dual H^* . We review the notion of H-Galois extensions as defined by Kreimer-Takeuchi: if A is an H-comodule algebra via $\zeta: A \to A \otimes H$, the map $\gamma: A \otimes_{A^{coH}} A \to A \otimes H$ given by $\gamma(a \otimes b) = (a \otimes 1)\zeta(b)$ is bijective. As an example, a recent theorem of Blattner-Montgomery is that any crossed product $A\#_\sigma H$ is H-Galois over $A\#1$; this uses the fact that such extensions are cleft. As a corollary, we prove a duality theorem: $(A\#_\sigma H)\#H^* \cong M_n(A)$, for $n = \dim_k H$. Next we discuss recent joint work with Cohen and Fischman:

Theorem: $A/_{A^H}$ is H^*-Galois $\Leftrightarrow$ the map $A \otimes_{A^H} A \to A\#H$ given by $a \otimes b \to (a\#t)(b\#1)$, for $t \in \int_h$, is surjective $\Leftrightarrow$ for any left $A\#H$-module M , $A \otimes_{A^H} M^H \cong M$ as left $A\#H$-modules.

Corollary: If $A\#H$ is simple, then $A/_A H$ is H^*-Galois.

Theorem: If $A\#H$ is simple Artinian, then $A/_A H$ has the normal basis property. These results are applied to determine when a divi-

sion ring D is Galois over D^H ; namely $D/_D H$ is Galois $\Leftrightarrow$ $[D:D^H] = \dim H \Leftrightarrow D/_D H$ has a normal basis $\Leftrightarrow D \cong D^H \#_\sigma H^*$, a crossed product $\Leftrightarrow$ D#H is simple.
This extends classical results for fields.

F. van Oystaeyen: Global dimension, regularity and Gabber's Purity Theorem for filtered rings

For Zariskian filtrations the global dimension of a filtered ring may be calculated in terms of the global dimension of the associated graded ring; under a suitable finiteness condition this relation may also be obtained for more general filtrations including a.o. Fuchsian filtrations on the Weyl algebras. Similar results may be studied in connection with Auslander regularity. A somewhat stronger result concerning Auslander regularity of the Rees ring of the filtration has been established by Li Huishi, F.V.O., giving a positive answer to a question of J.-E. Björk and allowing to obtain an extension of Gabber's "purity" result to the case of Zariskian filtrations. Throughout the use of the Rees ring has provided a unifying tool for studying filtrations e.g. in order to establish equivalence of several properties hitherto considered to be different in the literature.

B. Pareigis: Canonical generators of group rings under faithfully flat descent

Faithfully flat descent can change group rings to more general rings called twisted group rings which are not isomorphic to group rings. The group itself gives rise to canonical generators for these twisted group rings, which are also new generators for the group rings in question. This construction will be studied and the way how the new relations arise from the group structure and descent data. These techniques give also a construction of a left adjoint (free construction) K_Γ : Groups $\rightarrow$ Algebras for the functor $U(L \otimes_K -)$: Algebras $\rightarrow$ Groups, where $L:K$ is a (free) Galois extension and U denotes the group of units.

D. Passman: Three simple observations

We discuss three unrelated problems.

1. E.A. Whelan posed several questions on modules induced from a primitive ring to its Martindale ring of quotients. We settle two of these in the negative.

2. Hales-Luthar-Passi obtained an interesting inequality on the coefficients of algebraic elements in complex group rings of finite groups. We extend this to the infinite case.

3. S. Montgomery asked whether prime skew group rings could be characterized in terms of the action of the group on the hereditary subrings of the ring. We discuss this question, but do not settle it.

D. Quinn: Integrality questions for noncommutative rings

The Paré-Schelter result that a matrix ring is Schelter integral over the coefficient ring of bounded degree (depending only on the size of the matrices) has been extended in several ways. This has led to some interesting applications. Here we give a brief account of some of these and some new results on integrality questions. In particular we consider subnormalizing extensions, algebraic automorphisms and finally Hopf algebra actions, where some obvious questions still lack answers.

C.M. Ringel: From representations of quivers to quantum groups via Hall algebras and Hall polynomials

Let R be a representation-finite hereditary ring, say of type Δ (a Dynkin diagram). Let $H(R;\Lambda,q)$ be the Hall algebra of R with coefficients in Λ and evaluation at $q \in \Lambda$, where Λ is some commutative ring, thus $H(R;\Lambda,q)$ is the free Λ-module with basis $(u_{[M]})_{[M]}$ indexed by the isomorphism classes of R-modules of finite length, with multiplication

$$u_{[N_1]}u_{[N_2]} = \sum_{[M]} \varphi^M_{N_1N_2}(q)u_{[M]} ,$$

where $\varphi^M_{N_1N_2} \in \mathbb{Z}[T]$ is a polynomial counting (for R finite) filtrations of M with factors N_1 and N_2. Let $S_1,\ldots,S_s$ be the simple R-modules. There are derivations δ_i on $H(R;\Lambda,q)$ defined by $\delta_i(u_{[M]}) = (\underline{\dim}\ M)_i u_{[M]}$, and we form the twisted polynomial ring

$$H'(R;\Lambda,q) = H(R;\Lambda,q)[T_i,\delta_i]_i .$$

Let g be the complex semisimple Lie algebra with triangular decomposition $g = u_- \oplus h \oplus u_+$. Then $H(R;\mathbb{C},1) \cong U(u_+)$, $H'(R;\mathbb{C},1) \cong U(b_+)$, where $b_+ = h \oplus u_+$. Finally, consider

$$\hat{H'}(R) := \varprojlim_m H'(R;\mathbb{C}[q]/(q-1)^m,q) .$$

Then $\hat{H'}(R) \cong U_q(b_+)$, the quantization of the enveloping algebra of b_+ in the sense of Drinfeld.

S.P. Smith: Duality for quantum $O_q(SL(n))$ and $U_q(sl(n))$

(Joint work with J.T. Stafford)

There is a non-degenerate pairing $<,> : O_q(SL(n)) \times U_q(sl(n)) \to \mathbb{C}$ such that

1) the image of the induced map $O_q \to U_q^*$ equals $\{f \in U_q^* \mid \ker f \supseteq \mathrm{Ann}\ V^{\otimes k}$ for some $k>>o\}$ where V is "the" basic n-dimensional simple $U_q(sl(n))$-module ;
2) the image of the induced map $U_q \to O_q^*$ is contained in $\{\varphi \in O_q^* \mid \ker \varphi \supseteq$ an ideal of finite codimension$\}$;
3) the induced map $U_q \to \mathrm{End}_{\mathbb{C}} O_q$ makes O_q a U_q-module algebra with U_q acting as "right invariant differential operators".

One key step in the proof is that $O_q(SL(n))$ satisfies a certain universal property with respect to quantum n-space. The algebra

$A_q = \mathbb{C}[X_1,\ldots,X_n]$ with $X_iX_j = q^{-2}X_jX_i$ for $1 \leq j < i \leq n$. The result allows $U_q(sl(n))$ to be defined as the full continuous dual of $O_q(SL(n))$ for a suitable topology. Since $O_q(SL(n))$ is defined (via the universal property) in a natural way from quantum n-space, it follows that $U_q(sl(n))$ may be defined in a natural way from quantum n-space.

N. Vonessen: Actions of linearly reductive groups on affine PI-algebras

Extending the theory of actions of finite groups on non-commutative rings and commutative invariant theory, I am studying actions of a linearly reductive group G on an affine PI-algebra R . One of the major results is that the fixed ring R^G is affine provided that R is Noetherian. Other topics include localization and a "lying over" result for $R^G \subseteq R$. Many of the results actually characterize linearly reductive groups and are in particular in prime characteristic false for actions of reductive groups which are not linearly reductive.

Berichterstatterin: Christine Bessenrodt

Tagungsteilnehmer

Prof. Dr. M. van den Bergh
Dept. of Mathematics and
Computer Science
University of Antwerp (UIA)
Universiteitsplein 1

B-2610 Wilrijk-Antwerp

Prof. Dr. K. A. Brown
Dept. of Mathematics
University of Glasgow
University Gardens

GB- Glasgow , G12 8QW

Dr. C. Bessenrodt
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 103 764

4300 Essen 1

Prof. Dr. C. Dean
Dept. of Mathematics
University of Chicago
5734 University Ave.

Chicago , IL 60637
USA

Prof. Dr. L. A. Bokut
Institute of Mathematics
Siberian Academy of Sciences of the
USSR
Universitetskii pr. 4

Novosibirsk 630090
USSR

Dr. P. Dräxler
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. K. Bongartz
Fachbereich 7: Mathematik
der Universität/Gesamthochschule
Wuppertal, Gaußstr. 20
Postfach 10 01 27

5600 Wuppertal 1

Prof. Dr. A. W. Goldie
School of Mathematics
University of Leeds

GB- Leeds , LS2 9JT

Prof. Dr. A. Braun
Dept. of Mathematics
University of Haifa

Haifa 31 999
ISRAEL

Prof. Dr. K. R. Goodearl
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Dr. D. Happel
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. F. Kasch
Mathematisches Institut
der Universität München
Theresienstr. 39

8000 München 2

Prof. Dr. A. Hodges
Dept. of Mathematical Sciences
University of Cincinnati

Cincinnati , OH 45221-0025
USA

Prof. Dr. V. Kharchenco
Institute of Mathematics
Novosibirsk State University

630090 Novosibirsk 90
USSR

Prof. Dr. R. S. Irving
Dept. of Mathematics
University of Washington
C138 Padelford Hall, GN-50

Seattle , WA 98195
USA

Prof. Dr. L. Le Bruyn
Dept. of Mathematics
Universitaire Instelling Antwerpen
Universiteitsplein 1

B-2610 Wilrijk

Prof. Dr. C. U. Jensen
Matematisk Institut
Kobenhavns Universitet
Universitetsparken 5

DK-2100 Kobenhavn

Prof. Dr. T. Lenagan
Dept. of Mathematics
University of Edinburgh
James Clerk Maxwell Bldg.
Mayfield Road

GB- Edinburgh , EH9 3JZ

Prof. Dr. A. Joseph
Dept. of Mathematics
The Weizmann Institute of Science
P. O. Box 26

Rehovot 76 100
ISRAEL

Prof. Dr. H. Lenzing
Fachbereich Mathematik/Informatik
der Universität Paderborn
Postfach 1621
Warburger Str. 100

4790 Paderborn

Prof. Dr. E. Letzter
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Prof. Dr. M. P. Malliavin
Institut de Mathematiques Pures et Appliquees, UER 47
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Prof. Dr. T. Levasseur
Dept. de Mathematiques
Universite de Bretagne Occidentale
6, Avenue Victor Le Gorgeu

F-29287 Brest Cedex

Prof. Dr. G. Michler
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 103 764

4300 Essen 1

Prof. Dr. L. S. Levy
c/o Prof. Dr. G. Michler
Fachbereich 6 Mathematik
Universität GHS Essen
Universitätsstr. 3

4300 Essen 1

Prof. Dr. S. Montgomery
Dept. of Mathematics, DRB 306
University of Southern California
University Park

Los Angeles , CA 90089-1113
USA

Prof. Dr. M. Lorenz
Dept. of Mathematics
Northern Illinois University

DeKalb , IL 60115
USA

Prof. Dr. F. van Oystaeyen
Dept. of Mathematics
Universitaire Instelling Antwerpen
Universiteitsplein 1

B-2610 Wilrijk

Prof. Dr. L. Makar-Limanov
Department of Mathematics
Wayne State University

Detroit , MI 48202
USA

Prof. Dr. B. Pareigis
Mathematisches Institut
der Universität München
Theresienstr. 39

8000 München 2

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Prof. Dr. D. Passman
Department of Mathematics
University of Wisconsin-Madison
Van Vleck Hall

Madison WI, 53705
USA

Prof. Dr. L. W. Small
Dept. of Mathematics
University of California, San Diego

La Jolla , CA 92093
USA

Prof. Dr. D. Quinn
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Prof. Dr. S. P. Smith
Dept. of Mathematics
University of Washington
C138 Padelford Hall, GN-50

Seattle , WA 98195
USA

Prof. Dr. C.M. Ringel
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof. Dr. J. T. Stafford
School of Mathematics
University of Leeds

GB- Leeds , LS2 9JT

Prof. Dr. K.W. Roggenkamp
Mathematisches Institut B
der Universität Stuttgart
Pfaffenwaldring 57
Postfach 80 11 40

7000 Stuttgart 80

Dr. N. Vonessen
Dept. of Mathematics, DRB 306
University of Southern California
University Park

Los Angeles , CA 90089-1113
USA

Prof. Dr. W. Schelter
Dept. of Mathematics
University of Texas at Austin

Austin , TX 78712
USA