MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 2/1990

Mathematische Optimierung
7.1. - 13.1.1990

Leitung: Bernhard Korte (Bonn)
Klaus Ritter (München)

Wie in den letzten Jahren ist auch diese Tagung über mathematische Optimierung in Oberwolfach auf besonderes Interesse im In- und Ausland gestoßen. Insgesamt 66 Teilnehmer aus 12 Ländern nahmen an dieser Tagung teil, davon kamen knapp die Hälfte aus Übersee. Der Themenbereich der 52 Vorträge war weit gespannt und sprach Probleme aus dem gesamten Spektrum der diskreten und nichtlinearen Optimierung an.

Verschiedene Aspekte von interior point Algorithmen, insbesondere sowohl theoretische als auch praktische Untersuchungen der Komplexität solcher Algorithmen, standen im Mittelpunkt einer Reihe von Vorträgen. Weitere Beiträge der stetigen Optimierung beschäftigten sich u.a. mit neuen Ansätzen zur globalen und nicht konvexen Optimierung, mit Dekompositionsverfahren und der Parallelisierung von Algorithmen. In den Vorträgen über diskrete Optimierung wurden neue Ergebnisse der Graphentheorie, der polyedrischen Kombinatorik, sowie neue Algorithmen zu verschiedenen Netzwerkfluß- und Scheduling-Problemen vorgestellt. Neben der methodischen Behandlung ganzer Problemklassen wurden dabei auch Lösungsansätze für konkrete Probleme aus der Praxis diskutiert.

An den Abenden fanden häufig Diskussionen und Voträge in kleineren Kreisen statt. Besonders lebhaftes Interesse fand eine — aus gegebenem Anlaß — geführte Diskussion über die Patentierfähigkeit von Algorithmen und/oder mathematischer Ideen.

Ein musischer Höhepunkt der Tagung war das Klavierkonzert am Freitag abend von Thèrèse Dussaut-Lemaréchal mit Werken von Rachmaninoff.

Der besondere Dank der Veranstalter und Teilnehmer dieser Tagung gilt dem Direktor des Mathematischen Forschungsinstitutes, Herrn Professor Dr. M. Barner, und seinen Mitarbeitern für deren Gastfreundschaft und die ausgezeichnete Betreuung.

Vortragsauszüge

E. Balas: *Projection and sequential convexification*

We relate the recent results of Lovasz and Schrijver on matrix cuts to the sequential convexification procedure for facial disjunctive programs (1974). In particular, given a $0-1$ programming problem in $\mathbf{R}^n$ whose linear programming relaxation is of the form $Ax \geq b$, $0 \leq x \leq 1$, written as $\tilde{A}x \geq \tilde{b}$, let $P_j(\tilde{A}x \geq \tilde{b})$ be the system of linear inequalities obtained from

$$(\tilde{A}x - \tilde{b})(1 - x_j) \geq 0$$
$$(\tilde{A}x - \tilde{b})x_j \geq 0$$

by setting $x_j^2 = x_j$ and eliminating all bilinear terms $x_i x_j$ (i.e., linearizing and projecting the system onto $\mathbf{R}^n$). Then

$$P_n(P_{n-1}(...(P_1(\tilde{A}x \geq \tilde{b})...) = \text{ conv } \{x \epsilon \{0,1\}^n \ : \ \tilde{A}x \geq \tilde{b}\}$$

R. E. Burkard: *Lexicographic bottleneck problems*

In this joint paper with F. Rendl we study bottleneck problems where in addition to minimizing the largest element of a feasible solution also the second largest, third largest and so on is as small as possible. Since scaling leads to impracticable large cost coefficients we suggest to solve this problem in an iterative way. A sequence of min-problems is recursively defined and solved. Using bisection we get as overall complexity for the lexicographic version of the bottleneck problem $T \cdot O(\log k \cdot \min(n^2, k^2))$, where T is the complexity for solving a sum-problem, k is the number of different cost coefficients and n is the size of a feasible solution.

A. R. Conn: *A projection-relaxation method for a location-allocation problem*

We propose a solution method for a minisum location-allocation problem involving l_p distances $(1 < p < \infty)$. The objective function is neither convex nor concave and is not everywhere differentiable. We relax the constraint that the allocations be $\{0,1\}$ and rewrite them to reflect this relaxation and also to eliminate degeneracy. Necessary and sufficient conditions for local minima of the relaxed problem are given. These conditions are constructive in that they lead directly to the development of an algorithm involving active set methods and orthogonal projections. In practise, local minima of the relaxed problem typically have $\{0,1\}$ allocations. In this case, the algorithm exhibits a quadratic rate of convergence. If a local minimum includes fractional allocations, the allocations are updated using a projected gradient technique whereas the locations use a projected Newton direction. The implementation employs efficient techniques that exploit the special structure of the relaxed problem. The necessary algebra to determine the projections and dual variables is essentially trivial. Numerical results are presented. (Joint work with P.H. Calamai, I. Bongartz).

W. H. Cunningham: *Bisubmodular polyhedra*

A bounded set $\mathcal{J} \subseteq \mathbb{Z}^S$ satisfies the *2-Step Axiom* (2SA) if $x, y \in \mathcal{J}$, $e_i \in st(x,y) \Rightarrow x + e_i \in \mathcal{J}$ or there exists $e_j \in st(x,y)$ with $x + e_i + e_j \in \mathcal{J}$. Here $st(x,y)$ means $\{e_i \;:\; d(x+e_i,y) = d(x,y) - 1,\; e_i$ a unit vector in $\mathbb{Z}^S\}$ and $d(x,y) = \sum_{i \in S} |x_i - y_i|$. Some examples are: the set of integral points in an integral polymatroid, the set of degree-sequences of b–matchings of a graph, and the set of incidence vectors of feasible sets of delta-matroids. A *bisubmodular polyhedron* is a set of the form

$$\{x \in \mathbb{R}^S \;:\; x(A) - x(B) \le f(A,B),\; A,B \subseteq S,\; A \cap B = \emptyset\}$$

where f satisfies the bisubmodular inequality

$$f(A,B)+f(A',B') \ge f(A\cap A', B\cap B')+f((A\cup A')\backslash(B\cup B'),(B\cup B')\backslash(A\cup A')).$$

If $\mathcal{J}$ satisfies 2SA, then $conv(\mathcal{J})$ is such a polyhedron. A weak converse is that the integral points in $P(f)$ for f integral satisfy 2SA. We give several constructions for sets satisfying 2SA. For some the corresponding f is well-characterized. For others it can be proved that f cannot be computed efficiently. (Joint work with A. Bouchet, Le Mans).

J.E. Dennis: *Some topics on nonlinear optimization*

This talk deals with some features of a mathematical programming approach to the numerical solution of inverse problems for ordinary differential equations. This class of problems is very important in science and engineering, and there are interesting features of the associated optimization problem: Function values and derivatives are expensive and difficult to obtain, and automatic differentiation may be very important here. Linearized constraints often are numerically degenerate. The problems may be very large. Some problems have nonlinear parameters that should belong to a finite set.

A. Frank: *Conservative weightings and ear-decomposition of graphs*

An edge-weighting w of a graph $G = (V, E)$ is called *conservative* if there is no circuit of negative total weight. We prove that the minimum of $w(E)$ over all conservative ± 1 weightings is equal to the maximum number of odd ears in an ear-decomposition of G. (G is supposed to be 2-edge-connected). This theorem provides an answer to a question of Solé and Zaslavsky about the biggest cardinality μ of a minimum $T-$join over all even subsets T of V. Namely, $\mu = \frac{1}{2}(\varphi + |V| - 1)$ where φ denotes the minimum number of edges the contraction of which makes the graph factor-critical. The proof of the main theorem gives rise to a polynomial-time algorithm to construct the optima in question.

D. Goldfarb: *Strongly polynomial simplex algorithms for minimum cost network flow algorithms*

We present two variants of the primal network simplex algorithm which solve the minimum cost network flow problem in at most $O(n^2m^2 \log n)$ pivots. Here we define the network simplex method as a method which proceeds from basis tree to adjacent basis tree regardless of the change in objective function value; i.e., the objective function is allowed to increase on some iterations. One of the methods is a "simplex adaptation" of the minimum augmenting cycle cancelling method of Goldberg and Tarjan. The other is a combination of a scaling technique with the strongly polynomial network simplex algorithm of Goldfarb and Hao for the maximum flow problem. We give a short proof that the latter algorithm solves an $m-$arc, $n-$node max flow problem in at most $n \cdot m$ pivots and $O(n^2m)$ time. Finally, we show that the diameter (in a graph theory sense) of the primal network flow polyhedron is bounded above by $2n^2m$. This is joint work with J. Hao and of GTE Labs. (Waltham, MA)

M. D. Grigoriadis: *A network simplex method for max flow: Theoretical and practical improvements*

We improve the $O(n^2m)$ time bound, obtained by the Goldfarb-Hao pivot selection rule, to $O(nm \log n)$ by extending the Sleator-Tarjan dynamic tree data structure. This bound is larger by a logarithmic factor than those of the fastest known algorithms for maximum flow. Our extension of dynamic trees may well have additional applications. We also discuss practical improvements, such as early termination criteria, and present extensive computational results with randomly-generated problems of sizes up to 8000 vertices and 40000 edges. (Joint work with A. Goldberg and R.E. Tarjan).

M. Grötschel: *On a scheduling problem in manufacturing*

We report about a problem in flexible manufacturing that is (often) called sequential ordering problem (SOP) and that can be formulated as a hamiltonian path problem in a digraph with certain precedence constraints. We discuss two integer programming formulations of this problem, outline polynomial time separation algorithms for some of the classes of inequalities arising in these models and present our computational experience with a cutting plane algorithm for the SOP. This work is joint with N. Ascheuer, L. Escudero and M. Stoer.

P. L. Hammer: *On the max cut polytope*

We present a one-to-one correspondence between the set of valid inequalities for the complete cut-polytope of K_{n+1} and the set of nonnegative quadratic functions over n Boolean variables. Based on this correspondence a new class of valid inequalities and facets for the cut-polytope is characterized. It is shown that this class includes many of the known classes of valid inequalities, e.g. the Deza-Laurent hypermetric and cyclic inequalities, triangle inequalities, etc. It includes also an exponential number of new facets. This is a joint work with Endre Boros.

J. B. Hiriart-Urruty: *New prospects in testing global optimality in 'hard' nonconvex optimization problems*

We consider nonconvex optimization problems like:

$$(P) \quad \begin{cases} \text{Maximize a convex function (say a quadratic convex function)} \\ \text{over a convex set (say a polyhedron),} \end{cases}$$

especially those arising from 0-1 linear (or quadratic) programming. After having recalled the *necessary and sufficient conditions for global optimality* in (P) [Hiriart-Urruty 1988] , we present some prospects on the complexity of these conditions and their potential use for algorithmic purposes.

M. Iri: *An approach to numerical-combinatorial algorithms with an example of Voronoi diagram construction*

In many algorithms for geometrical problems such as the Voronoi diagram construction, the topology of the solution is determined based on the signs of the results of numerical computation with rounding errors. So, a naive implementation of an algorithm which has been designed under the implicit assumption that arithmetic operations on real numbers could be performed in infinite precision will often fail to yield a solution (i.e., it will fail to stop or will stop with an output which could not be regarded as an approximate solution) due to unforeseen disturbance (or inconsistency) in topology occuring in the course of computation. With the problem of constructing a two-dimensional Voronoi diagram as an example, we propose an algorithm which lays more stress on topology than on numerical results thus yielding always a planar regular graph of degree 3. The algorithm has been tested against many large problems and proved to be highly robust. (With a number of exquisite reformulations of relevant expressions in numerical computation also incorporated, it solves million-point problems in single-precision arithmetic.) (This is joint work with Dr. K. Sugihara.)

E. L. Johnson: *Integer programming computation*

Some computational experiences with the new IBM OSL (Optimization Subroutine Library) are discussed. The new simplex LP code is faster and more stable than MPSX and competes well with the interior barrier code also in OSL. Padberg and Rinaldi have successfully used it in their TSP code on the 3080VF. Some experience with hard, set partitioning LP problems is given. Such an LP with 5.5 million columns has been solved in one hour on a 3090VF computer.

V. Kovacevic-Vujicic: *Global optimization*

This paper presents a new method for the global minimization of a continouos function subject to linear constraints:

$$\text{(1)} \quad \text{global } \min f(x), x \in X = \{x \in \mathbb{R}^n \mid a_i^T x \leq b_i,\ i = 1, \ldots, m\}.$$

The feasible set is first covered by a union of hypercubes:
$X \subset \bigcup_{x \in E_0} C(x, h^0)$, where $E_0 \subset \mathbf{R}^n$ is a finite set, $h^0 > 0$, $C(x, h^0) = [x_1 - h_1^0, x_1 + h_1^0] \times \cdots \times [x_n - h_n^0, x_n + h_n^0]$, and then some of them are eliminated using some global information about the objective function (Lipschitz constants, derivatives, etc.). The cover is then refined and the process is repeated. It is shown that the method generates a sequence whose cluster points are solutions to (1). Estimates of computational complexity are given for some classes of problems. Estimates are of the form: $\text{card}(E_k) \le M$, for some $M > 0$, meaning that the number of points generated by the algorithm grows linearly in k.
(joint work with Mizeslav D. Asic)

W. Krabs: *On quadratic optimization problems whose duals are decomposable*

The kind of problems to be considered arises in the approximate solution of quadratic optimal control problems. It consists of minimizing

$$f(x_1, ..., x_M) = \frac{1}{2} \sum_{i=1}^{M} x_i^T C_i x_i, \quad x_i \in \mathbf{R}^m,$$

subject to

$$\sum_{i=1}^{M} A_i x_i = b \text{ and } \gamma_i^1 \le x_i \le \gamma_i^2 \text{ for } i = 1, ..., M$$

where $C_1, ..., C_M$ are real symmetric, positive definite $m \times m-$matrices, $b \in \mathbf{R}^n$, and $\gamma_i^1, \gamma_i^2 \in \mathbf{R}^m$ are given vectors with $\gamma_{ih}^1 < \gamma_{ih}^2$ for $i = 1, ..., M$ and $h = 1, ..., m$; $A_1, ..., A_M$ are given $n \times m-$matrices such that the rank of the matrix $(A_1|...|A_M)$ is equal to n which is less than $m \cdot M$. The dual of this problem is decomposable which is used for deriving a dual iteration method for solving the problem. In the case $m = 1$ this method, generically, turns out to be Newton's method.

C. Kredler: *The role of automatic differentiation in sequential active set programming*

A comparison of the well known SQP-mehtod and so called sequential active set algorithms for general nonlinear programming problems has been presented. In the framework of active set methods fit gradient projections, augmented Lagrange as well as the so called Best-Bräuninger-Ritter-Robinson approach. For linear constraints a combined barrier-active-set method turned out to be very robust against degeneracy. The algorithms mentioned above start with a penalty-like phase 1 and then solve a sequence of linearly equality constrained subproblems. These again can be tackled efficiently by Newton-type unconstrained methods and automatic differentiation which incorporates the coordinates of the corresponding subspace in a natural and elegant way.

E. L. Lawler: *Searching sequences for patterns in sublinear time*

The following type of problem arises in molecular biology. T is a text and P is a pattern, each consisting of a sequence of characters from a finite alphabet, e.g. of size 4 in the case of DNA sequences and size 20 in the case of proteins, it is desired to find all approximate matches of the pattern in the text, where by "approximate" we mean that there should be no more than a specified constant number, κ, of character substitutions or insertions / deletions. Typically, the length n of T may be 10^7 and the length m of P may be 500. It is possible to solve the problem by straightforward dynamic programming in $O(mn)$ time, or by refined DP in $O(n\kappa)$ time. With William Chang, I have derived an algorithm with expected running time $O(n\kappa\frac{\log m}{m})$. The algorithm divides the text into about $\frac{n}{m}$ segments, and, with the aid of a suffix tree for the pattern, tests each segment independently for the possible existence of an approximate match with the pattern. The $O(n\kappa)$ dynamic programming algorithm is then applied in those segments for which a pattern may exist. For $\kappa \leq 15\% - 25\%$ of m, the expected running time is sublinear in n.

C. Lemarechal: *Some remarks on nonconvex decomposition. Application*

Consider the problem

(1) $$\min \sum_i f_i(x_i); \qquad x_i \in D_i; \qquad \sum_i g_i(x_i) = 0.$$

An application is to optimize the production of electricity while meeting the consumption: then, i indices power plants and runs from 1 to 10^2. Lagrangian relaxation consists of maximizing with respect to λ the dual function

(2) $$q(\lambda) := \sum_i \min_{x_i \in D_i} [f_i(x_i) + \lambda \cdot g_i(x_i)].$$

When a cutting plane approach is used (e.g. bundle method), each dual iteration yields $\{x_i^*(\lambda)\}$ having the property that

$$\sum_i g_i[x_i^*(\lambda)] \longrightarrow 0 \qquad \text{when } \lambda \text{ tends to a dual max.}$$

These $x_i^*(\lambda)$ can be used to obtain
- a primal optimum if (1) is convex (decomposition of linear programs)
- primal heuristics if (1) is not convex; they are based on adding in (2) a penalty of type $r||g_i(x_i) - g_i[x_i^*(\lambda)]||^2$.

T.M. Liebling: *Polycrystal growth and Laguerre duality on the torus*

Polycrystals, for example ceramics are composite materials made of monocrystalline grains, which can be assimilated to nearly convex cell complexes. Laguerre diagrams turn out to suitably approximate these structures. Polycrystals show normal grain growth, i.e. large grains tend to grow even larger at the expense of the small ones. Grain growth can be simulated by modifying these Laguerre diagrams according to a suitable law of motion, f.i. setting the rate of change of the defining parameters proportional to the gradient of energy, measured by total boundary area. Laguerre diagrams are defined by a set of spheres given on the unit torus to avoid boundary effects. The dual Delaunay partition is itself a Laguerre partition generated by spheres centered at the vertices of the original one and usually, iterated dualization will return the latter. A new paraboloid interpretation of such pairs of dual Laguerre diagrams leads to the efficient gradient computation and updating procedures during simulation. These are some results described in H. Telley's PhD thesis carried out under joint supervision of A. Mocellin and the speaker.

L. Lovász: *How to compute the volume*

Recently Dyer, Frieze and Kannan gave a randomized polynomial time algorithm to approximate the volume of a convex body. Two main ingredients of the method are a result of Sinclair and Jerrum on the mixing rate of Markov chains and an isoperimetric inequality. In a recent work with M. Siminovits, we improve both of these results. In particular, we can eliminate the time-reversibility condition in Sinclair's and Jerrum's result and improve the running time of the volume algorithm.

M. Lucertini: *Combinatorial models in flexible manufacturing systems*

The presentation deals with the modelling and the optimization of Flexible Manufacturing Systems (FMS) and Flexible Assembling Systems (FAS). Fully deterministic combinatorial formulation of particular classes of FMS and FAS are analyzed and the part flow management problem is formulated as a sequence of combinatorial optimization problems. We consider a system consisting of several multi-tool automated machines, each one equipped with a (different) tool set, and linked each other by a transportation system for part moving. The focused flow management problem is the problem of finding a workload balanced part routing and, successively, a minimum-completion-time scheduling of the operations on each machine. All those problems are formulated in terms of combinatorial optimization. The parts to be processed by the production system are organized in different sets (part types) and each set consists of parts with the same operation requirements; the sequence of part types is given and the parts are introduced in the working systems following the part type ordering. Two assembling problems are considered: the part routing in the working system and the scheduling of the operations on the workstations. The general aim of the optimization models is the minimization of the completion time. In order to achieve such goal, the routing problem is formulated in a Network Programming framework with the goal of balancing the workstations workload, the scheduling problem is solved via heuristic procedure with the aim of minimizing the completion time.

T. L. Magnanti: *A generalization of the minimal spanning tree problem*

One way to view the minimal spanning tree problem is a fixed cost network flow problem with (i) a fixed cost for installing any arc in the network, but with no variable (flow) cost, (ii) a positive amount of demand between every pair of nodes, and (iii) an infinite capacity for flow on any arc that we install. We consider a 2-facility capacitated "network loading" generalization of this problem which permits us to install integer multiples of either of two levels of capacity on each arc and which accomodates arbitrary demand patterns. We describe two sets of facets for this problems, show that they define the convex hull of a Lagrangian subproblem and a three node subproblem, and report on computational experience on a set of prototype telecommunication applications.

L. McLinden: *Monotone operator duality*

A perturbational duality theory is developed for problems involving maximal monotone multifunctions in reflexive Banach spaces. For such spaces the framework presented subsumes the well known theory for convex minimization and a similar one for convex-concave minimax problems. It also covers a variety of operator problems not of extremum type, such as variational inequalities, Walrasian economic equilibrium, and singular Hammerstein integral equations, under the monotonicity assumption on the relevant operators. The theory required first developing several sharp tool theorems which address the issues of the maximality and the range of the sum of two monotone operators. These tools, combined with the symmetries inherent in the framework, imply a broad duality theorem yielding existence of solutions, where the existence is stable under appropriate perturbations of the natural parameters involved. Additional uses of the tool theorems are possible in analyzing particular problems having specific structure. For example, for the parametric solution multifunction of the monotone variational inequality problem we develop a structure theorem from which follows a detailed stability analysis of that problem.

K. Mehlhorn: *Maximum network flow in* $O(n^3/\log n)$

Let $N = (V, E, cap)$ be a network with n vertices. We give a randomized algorithm which computes a maximum flow from s to t, where $s, t \in V$, in time $O(n^3/\log n)$. The algorithm is a refinement of the recent algorithm of Cheriyan and Hagerup (FOCS 89) which runs in time $O(nm + n^2(\log n)^2)$ where m is the number of edges. The main new idea is to not consider the entire network from the beginning but to add the edges in the order of decreasing capacity at appropriate moments during the execution. (Joint work with J. Cheriyan and T. Hagerup).

R. Meyer: *Parallel algorithms for large-scale network optimization*

There is currently substantial interest in the construction of algorithms capable of solving network optimization problems of hundreds of thousands or millions of arcs. In order to achieve reasonable solution times for such problems, it is necessary to exploit the parallelism of advanced computers. This talk will deal with new parallel algorithms for large-scale linear and non-linear networks and computational experience with these methods on multiprocessors.

C. L. Monma: *A polyhedral approach to network survivability*

In joint work with M. Grötschel and M. Stoer of University of Augsburg, we study a network survivability problem which arises in fiber communication network design. We develop classes of valid inequalities, provide conditions under which these define facets, address the separation problems, and describe a cutting plane approach for network survivability. We present computational results for several real-world problems, all of which we are able to solve to optimality on a desktop workstation.

K. Neumann: *Min-sum and min-max scheduling problems with stochastic tree-like precedence constraints*

Stochastic min-sum and min-max single-machine scheduling problems are considered where the precedence constraints are given by a so-called OR network. An OR network is a special stochastic activity network (GERT network) which may contain cycles and has some tree-structure property. It turns out that min-max-problems are harder than min-sum problems in contrast to deterministic scheduling problems with precedence constraints. If the objective function is the expected weighted flow time, an optimal scheduling policy can be computed in polynomial time. The min-max problem with unit-time activities, maximum expected completion time of the so-called operations as objective function, and precedence constraints given by a cyclic OR network is shown to be NP-hard. However, if we restrict ourselves to priority lists of operations instead of general scheduling policies, there is a polynomial algorithm for the scheduling problem where the activity durations are generally distributed and the objective function is the maximum expected lateness.

M. W. Padberg: *An analytic symmetrization of max flow - min cut*

Using affine transformations we derive a symmetrized version of the maximum flow - minimum cut problem over directed graphs. The symmetrization is obtained explicitly and turns out to be the problem studied by Lehman (1963) and Johnson (1974).

D. Pallaschke: *Higher order derivatives for quasi-differentiable functions*

Let $\mathcal{D}(\mathbb{R}^n) := \{\varphi := p - q \mid p, q : \mathbb{R}^n \to \mathbb{R}\}$ denote the lattice of differences of two sublinear functions. According to V. Demganov and A. Rubinov, a continuous function $f : U \to \mathbb{R}$ is said to be quasi-differentiable if $(\eta \mapsto \frac{df}{d\eta}\Big|_{x_0}) \in \mathcal{D}(\mathbb{R}^n)$, where $x_0 \in U \subseteq \mathbb{R}^n$, U open. Now let $\Omega \subseteq \mathbb{R}^n$ be a regular set and $\mathcal{A} \subseteq C_0(\Omega)$ a function algebra with unit $1 \in \mathcal{A}$. For $x_0 \in \Omega$ we call a linear map $d : \mathcal{A} \to \mathbb{R}$ a derivative if the Leibnitz-Rule holds, i.e. $d(fg) = f(x_0)dg + g(x_0)df$. Put $\mathcal{A}_0 := \{f \in \mathcal{A} \mid f(x_0) = 0\}$ and $\mathcal{A}_0^2 := \{\omega = \sum_{i=1}^{m} f_i g_i \mid f_i, g_i \in \mathcal{A}_0, m \in \mathbb{N}\}$. Then $d : \mathcal{A} \to \mathbb{R}$ is a derivative if and only if there exists a functional $\psi \in \left(\frac{\mathcal{A}_0}{\mathcal{A}_0^2}\right)^*$ such that $df = \psi(f - f(x_0) \cdot 1)$. If $\mathcal{A} := C^k(\Omega)$ for $k \geq 1$, then $\mathcal{H}_1\left(\frac{\mathcal{A}_0}{\mathcal{A}_0^2}\right) = L(\mathbb{R}^n, \mathbb{R})$, where $\mathcal{H}_1\left(\frac{\mathcal{A}_0}{\mathcal{A}_0^2}\right)$ denotes the set of all cosets having a positively homogeneous representant. If $\mathcal{A}$ denotes the algebra of quasi-differentiable functions over Ω, then $\mathcal{H}_1\left(\frac{\mathcal{A}_0}{\mathcal{A}_0^2}\right) = \mathcal{D}(\mathbb{R}^n)$. This result can be easily extended to higher order derivatives, i.e. a derivative of order k is a linear functional on $\frac{\mathcal{A}_0}{\mathcal{A}_0^{k+1}}$.

G. Di Pillo: *A smooth method for the discrete minimax problem*

We consider the discrete minimax problem

$$\min_{x\in\mathbb{R}^n} \max_{i\in I} f_i(x), \qquad I = \{1,2,...,m\}$$

which, due to its practical interest, has motivated much research in Nondifferentiable Optimization, both from the theoretic and algorithmic point of view. We prove that it is possible to build a smooth function whose minimizers are the same as of the max function, and which goes to $+\infty$ outside of a compact level set of the max function. The minimization of this smooth function by means of the usual unconstrained minimization methods allow the definition of an implementable algorithm which can be shown to be globally convergent at a superlinear convergence rate.

H. J. Prömel: *The Kleitman-Rothschild method, excluding weak subgraphs and a characterization*

The so-called Kleitman-Rothschild method was invented by D. Kleitman and B. Rothschild in 1975 to derive an asymptotic formula for the number of partially ordered sets on an n–element set. This proof-technique was adopted by Erdős, Kleitman and Rothschild (1976) for graphs to prove that almost every K_3–free graph is already 2-colorable (bipartite) where K_3 denotes the complete graph on 3 vertices. A generalization of this result was obtained by Kolaitis, Rothschild and the author (1987) by showing that for every fixed $l \geq 2$ almost every K_{l+1}–free graph is l–colorable. Building up on this result we describe an algorithm which colors, if possible, every K_{l+1}–free graph G with l colors, or which decides that G is not l–colorable and which has expected running time $O(n^2)$, assuming equal distribution on the K_{l+1}–free graphs. Moreover, a complete characterization of all graphs G with chromatic number $l+1$ is given such that almost all graphs which do not contain G as a weak subgraph are l–colorable. Both results are joint work with A. Steger.

W. R. Pulleyblank: *Mixed integer matching problem*

We discuss two forms of mixed integer matching problems. In the first, we have a subset I of edges of a graph $G = (V, E)$ and we wish to assign values x_j to the edges j of G such that $0 \leq x_j$, $\sum(x_j : j$ incident with $v) = 1$ for all $v \in V$, and x_j is integer for all $j \in I$. We show that the problem of determining whether such a mixed integer matching x for which $\sum x_j = |V|/2$ exists, is NP-complete. In the second, we consider which families of subsets of the blossom inequalities of the matching polytope have the property that when added to the constraints $x \geq 0$, $\sum(x_j : j$ incident with $v) \leq 1$, they result in a polytope all of whose vertices have components in $\{0, \frac{1}{2}, 1\}$ and the edges with $1/2$ assigned form the edgeset of disjoint odd cycles. We characterize such families, and describe two applications. This is joint work with Bruce Gamble, formerly of Waterloo and now at Purdue.

B. Reed: *Planarity, parity, perfection and packing paths*

We discuss a number of optimization problems related to the problem of recognizing perfect graphs. For example, we discuss the complexity of determining if two vertices lie on an induced cycle where the parity of the cycle may be specified or not. Bienstock showed that this problem is NP-complete in general. With McDiarmid, Shepherd and Schrijver we showed that determining if two vertices of a planar graph lie on an induced cycle can be done in polynomial time. We also discuss a number of results on "even points". This is a pair of vertices between which there is no odd induced path.

S.M. Robinson: *Analytical methods in nonsmooth and large-scale optimization*

Many of the mathematical objects of interest in nonlinear optimization (such as optimality conditions) can be simply and conveniently reformulated as equations involving nonsmooth functions. In addition, problems of large-scale optimization frequently have to be solved by decomposition procedures that introduce nonsmoothness into the resulting problems.

In this lecture we present several analytical tools for handling these nonsmooth functions. The tools are designed to take advantage of the special structure formed in these applications.

E. Sachs: *Numerical results for optimal control problems*

The gradient projection method has been analyzed with regard to the rate of convergence quite extensively. Also it has been investigated what consequences this analysis implies for optimal control problems. Here we consider another aspect of this method: It is the property of identifying the set of active indices after finitely many iterations under nondegeneracy assumptions. It is clear that this property is restricted to the finite dimensional case. We look at a sequence of discretized optimal control problems and observe that the number of steps to identify all active indices increases with the refinement of the discretization. If one imposes a different criterion then we show that the number of necessary steps for termination is indeed mesh independent. This can be observed also numerically for various examples from optimal control.

S. Schäffler: *Classification of critical stationary points in unconstrained optimization*

Stationary points of an unconstrained minimization problem with positive semidefinite Hessian matrix are critical, because they can't be classified by optimality conditions up to second order. It is proven, that such a critical stationary point $\bar{x}$ can be classified by classifying a stationary point $\hat{x}$ of the objective function defined on a manifold of $\mathbb{R}^n$. The Hessian matrix at $\hat{x}$ is zero, so higher order optimality conditions are applicable. If $\bar{x}$ is a saddlepoint, we discuss methods, which allow to descent from this saddlepoint along a curve.

A. Schrijver: *Matrix cuts and stable sets*

This talk reports on a new method for 0,1 optimization, developed with László Lovász. The method is based on lifting the problem to the space of $n \times n$ matrices. It yields a polynomial-time computable upper bound for any 0,1 optimization problem. The power of the method is studied by applying it to the stable set problem for undirected graphs. For perfect graphs and for t-perfect graphs, the method calculates the exact value of the stability number. More generally, it finds the exact stability number for any graph for which the stable set polytope is determined by the rank inequalities for those subgraphs H with the following property: for each vertex v of H, the graph obtained from H by deleting all neighbours of v is bipartite.

A. Sebö: *On multiflows in matroids*

We are studying the following problem: What are the undirected graphs and matroids for which the existence of a fractional multicommodity flow implies the existence of an integer one, for arbitrary, or for some restricted input data (capacities and demands)? This graph-property is an extension – and in some case the most general extension – of the property studied in the Seymour-paper "Matroids and Multicommodity flows", and roughly means the characterization of graphs in which the existence of multiflows can be nicely "well-characterized". Although this property does not behave well with respect to the composition operations of Seymour K-sum we worked out a way of characterizing it for the closure of a graph-class (with respect to the "1-" and "2-sum" operations), provided we know it for the class itself. This enables us to prove good characterizations for the existence of multiflows for general classes of graphs (matroids), namely for classes that contain both Seymour's class and his excluded minors. We would also like to show connections to recent complexity results and counterexamples of Bonner students Middendorf and Pfeiffer, and show the characterization of graphs (matroids) for which the "0-1-2-generalization" of the "cut condition" is necessary and sufficient, a common result with Werner Schwärzler.

D. Shanno: *Interior point methods - state of the art*

The talk discussed the primal-dual interior point method, derived from the logarithmic barrier method, for linear programming. The equivalence to Newton's method on the first order conditions was shown. Extensive computational experience was documented. It was shown that the method appears most efficient when the initial centering vector is of approximately the same magnitude as the initial optimality / feasibility vector. It was also demonstrated how to remove a numerical instability in Schur complements for dense vectors, and how to implement a column generation decomposition scheme. Experimental evidence was given for the complexity to be $O(\log n)$.

D. Shmoys: *A polynomial approximation scheme for a precedence-constrained scheduling problem*

We consider the following scheduling problem: Each of n jobs is to be scheduled on a single machine. Job j requires processing time p_j, which must begin at some time σ_j no less than its specified release time r_j. This job completes processing at time $\sigma_j + p_j$ and then must be delivered for q_j time and is delivered at time $\sigma_j + p_j + q_j = D_j$. The objective is to minimize the maximum D_j value over all schedules. This problem is equivalent to minimizing the maximum lateness problem $1 \mid r_j \mid L_{max}$. Finally, we have precedence constraints given by a partial order $<$. We give a family of algorithms $\{A_\epsilon\}$ where A_ϵ is a polynomial time algorithm that delivers a schedule of length at most $(1+\epsilon)D^*_{max}$, where D^*_{max} denotes the optimal length. In fact, for any fixed ϵ, the runnung time is $O(m + n\log n)$, where n is the number of jobs, and m is the number of edges in the precedence graph.

B. Simeone: *Sharp bound for a quadratic transportation problem arising in statistics*

Our research is motivated by the problem of finding a discrete joint frequency distribution with given marginals which is "as far as possible" from the independent distribution with the same marginals. If the distance between two distributions is measured by Pearson's chi-square index, the problem can be formulated as maximizing a quadratic separable function subject to the transportation constraints. We present three heuristics for this problem; one of them also yields an upper bound on the optimum, and thus one can estimate the relative error E of any given heuristic. Numerical experiments on 600 randomly generated test problems with up to 50 rows and 100 columns show that the above heuristics provide sharp bounds on the optimum ($E < 0.01$). Even more interestingly, these bounds become sharper and sharper as the problem size increases.

A. Steger: *The Kleitman-Rothschild method, excluding induced subgraphs and a general asymptotic*

In this talk we consider asymptotic properties of the class of graphs not containing a fixed graph H as an induced subgraph. Applying the Kleitman-Rothschild method we show for the class of graphs not containing an induced quadrilateral that it asymptotically corresponds to the class of split graphs. For general graphs H we generalize the notion of "extremal graph" to induced subgraphs and introduce a new parameter $\tau(H)$ which may be viewed as a common generalization of the chromatic number $\chi(H)$ and the clique covering number $\sigma(H)$. We then show that this parameter basically determines the number of edges in an extremal graph as well as the number of graphs without induced H–subgraph, i.e. we prove asymptotics corresponding to the results of Erdõs, Stone, Simonovits [1946,66] and of Erdõs, Frankl, Rõdl [1986] for weak subgraphs. (Joint work with H.J. Prõmel).

J. Stoer: *Complexity bounds for interior point methods to solve linear programs*

Consider the class $\mathcal{LP}$ of all linear programs
(LP) $\min\{c^Tx \mid x \in P\}$, $\quad P := \{x \in \mathbb{R}^n \mid a_i^Tx \le b_i,\ i = 1,...,n\}$
with P having a nonempty bounded interior P^0. To (LP) belongs a path

$x(r), r \downarrow 0, x(r) \in P^0$, of solutions of $E(x,r) \equiv \frac{c}{r} + \sum_{i=1}^{n} \frac{a_i}{b_i - a_i^T x} = 0$. Many interior point methods are path following methods, which compute a sequence $x(r_i)$ for parameters $R = r_0 > r_1 > \ldots \downarrow 0$, and their complexity is measured by the minimal number $N = N(R, \delta, LP)$ with $R = r_0 > \ldots > \delta \geq r_N$. It is pointed out that for several types of first order methods, N can be estimated by an integral of the form

$$N(R,\delta,LP) \leq c \int_{\delta}^{R} ||x''(r)||_{H(x(r))}^{\frac{1}{2}} dr, \quad H(x) := D_x E(x,r)$$

holding for all $LP \in \mathcal{LP}$, $R > \delta > 0$. By finding upper bounds for this integral one can derive complexity estimates for various classes $\mathcal{K} \subset \mathcal{LP}$ of linear programs of the type

$$N(R,\delta,LP) \leq c(\mathcal{K})[m(LP)]^{\alpha(\mathcal{K})} \log \frac{R}{\delta}, \quad \forall\, LP \in \mathcal{K}, R > \delta > 0,$$

which for some classes $\mathcal{K}$ have better exponents $\alpha(\mathcal{K}) < \frac{1}{2}$, than the known bound $\alpha(\mathcal{LP}) = \frac{1}{2}$.

R. Tapia: *Accelerating interior point methods*

To each iterate X_k generated by an interior point method we associate an indicator g_k obtained as the k-th diagonal element of a particular projection matrix. We show that as X_k converges to X^* the g_k sequence converges the the $0-1$ vector sign (X^*). Moreover, the convergence of the indicator sequence is quadratically faster than the convergence of the X_k sequence. The $0-1$ vector allows us to obtain an optimal vertex solution early on in the iteration sequence.

E. Tardos: *Using separating algorithm in fixed dimension*

The main result of the talk is the following: given a strongly polynomial separation algorithm for a convex body in fixed dimension one can optimize over the convex body in strongly polynomial time. As an application of this we extend the class of linear programs solvable in strongly polynomial time. If a linear program can be solved in strongly polynomial time, then one can also solve it if a constant number of additional variables and side constraints are added. As a special case we obtain a strongly polynomial algorithm for concurrent multicommodity flow problems (A concurrent multi-commodity flow problem is one where the same percentage is satisfied of each commodity, the problem is to maximize this precentage). The results presented are joint work with C. Haibt Norton and S.A. Plotkin.

M. J. Todd: *Anticipated behaviour of interior point algorithms for linear programming*

We discuss recent results obtained with C. Gonzaga, S. Mizuno and Y. Ye that attempt to explain the difference between observed practical behaviour of interior-point methods and their worst-case bounds. To do so we introduce the notion of the anticipated number of iterations, which is the number required if certain desirable behaviour (which occurs with high probability under a certain distribution on relevant quantities arising at each iteration) actually takes place each iteration. In particular, we describe algorithms with an $O(Ln^{\frac{1}{2}})$ worst-case and $O(Ln^{\frac{1}{4}})$ anticipated bound, and others with an $O(Ln)$ worst-case and $O(L \ln n)$ anticipated bound (including Karmarkar's algorithm). Here L is the input size and n the number of inequalities.

L. E. Trotter: *$\alpha-$critical graphs and stable set polytopes*

An edge of a simple graph $G = (V, E)$ is critical if its removal increrases the stability number $\alpha(G)$ and G is called $\alpha-$critical, when each of its edges is critical. The number $\delta(G) = |V| - 2\alpha(G)$ can be used to classify connected, $\alpha-$critical graphs. In particular, it is known that any connected, $\alpha-$critical graph with $\delta(G) = 2$ must be an even subdivision of K_4 (an even subdivision is obtained by insertion of an even number of nodes into the edges of K_4). We show that any graph for which $\delta(G) \geq 2$ must contain as a subgraph an even subdivision of K_4. This leads to a polynomial time algorithm for determining a largest stable set in a graph containing no even subdivision of K_4. It also establishes that constraints due to odd cycles and edges constitute the only "rank" facets of the polytope $P(G) =$ conv (incidence vectors of stable sets in G). This material is joint work with E. C. Sewell.

H. Warsitz: *A trajectory-following method in unconstrained optimization*

A trajectory-following method with interesting properties is considered for solving unconstrained nonlinear programming problems. The trajectory is defined by a special system of ordinary differential equations. This system uses only the gradient of the objective function. Numerical examples are given.

D. de Werra: *Chromatic scheduling with resource constraints*

An edge coloring model for a preemptive open shop problem with a resource constraint is presented. Given a sequence $(h_1, h_2, ..., h_q)$ of positive integers, does there exist an edge coloring $(M_1, M_2, ..., M_q)$ of a bipartite multigraph $G = (V, E)$ such that for $i = 1, ..., q$ $\quad \sum_{s=1}^{i} |M_s| \leq \sum_{s=1}^{i} h_s$?

This problem is shown to be NP-complete; some solvable cases are presented. These include the following situations:

a) $G = K_{m,n}$

b) G regular

c) the sequence $h_1, h_2, ..., h_q$ satisfies

$$h_p - 1 \leq h_1 \leq h_2 \leq ... \leq h_p$$

$$h_q - 1 \leq h_{p+1} \leq h_{p+2} \leq ... \leq h_q \text{ for some } p \leq q$$

A simple necessary condition for the existence of a feasible schedule in q time units is shown to be sufficient for a bipartite graph G (with maximum degree $\Delta(G) = 3$) and for all its partial subgraphs if and only if G does not contain a forbidden tree on 10 nodes as a partial subgraph. (Joint work with J. Blazewicz and W. Kubiak, Polytechnical School of Poznán).

L. A. Wolsey: *Valid inequalities for uncapacitated fixed charge networks*

Multicommodity reformulations of uncapacitated fixed charge network flow problems have significantly sharper linear programming reformulations than the standard flow formulations. Here we introduce a family of dicut inequalities that describe the projection of the multicommodity formulation onto the original variables. For the economic lot-sizing problems with start-ups we observe that a single subclass of the dicut inequalities suffices to describe the convex hull of solutions, and we examine the separation problem.

U. Zimmermann: *Minimum ratio cycles for flows and submodular flows*

Goldberg and Tarjan (87) showed that the Minimum Mean Cycle selection rule turns the negative cycle method of Klein for solving the minimum cost flow problem into a strongly polynomial procedure. By selecting lexicographical Minimum Mean Cycles, Cui and Fujishige (38) proved finiteness of such a method for solving the minimum cost submodular flow problem. We propose to select certain minimum ratio cycles which results in a quasipolynomial bound ($O(m \cdot n_{max})$). Furthermore, we discuss the use of certain different minimum ratio cycles in an interior point method for solving the minimum cost flow problem. For known optimal value, the method determines an optimal solution in polynomial time. In particular, the construction of feasible flows can be achieved in polynomial time (joint work with C. Wallader, Braunschweig).

J. Zowe: *The BT-algorithm and some applications to real life problems*

We study the minimization of a nonsmooth function f under the restrictive but realistic assumption that, at each x, we know only $f(x)$ and the subgradient $g \in \partial f(x)$. Examples for such situations are: the computation of lower bounds for TSP's via Lagrangian relaxation, the minimax eigenvalue problem, the maximization of the area of contact for the deflection of a clamped beam. By combining the attractive features of the Bundle idea with the Trustregion concept we develop an algorithm (BT-algorithm), which can deal successfully with nonsmoothness. The algorithm requires no line search (which often leads to a breakdown in other bundle implementations). Test runs show that the algorithm compares favourably with other Bundle variants. In particular, the above mentioned "hard" problems were solved successfully by our BT-algorithm.

Berichterstatter: Angelika Steger

Tagungsteilnehmer

Prof. Dr. A. Bachem
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Prof. Dr. E. Balas
Graduate School of Industrial
Administration
Carnegie-Mellon University
Schenley Park

Pittsburgh , PA 15213
USA

Prof. Dr. M.L. Balinski
Laboratoire d'Econometric de
l'Ecole Polytechnique
1, rue Descartes

F-75005 Paris

Prof. Dr. F. Barahona
Department of Combinatorics and
Optimization
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof. Dr. M.J. Best
Department of Combinatorics and
Optimization
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof. Dr. R.E. Burkard
Institut für Mathematik B
der Technischen Universität
Kopernikusgasse 24

A-8010 Graz

Prof. Dr. V. Chvatal
Institut für Diskrete Mathematik
Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof. Dr. A.R. Conn
Department of Combinatorics and
Optimization
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof. Dr. W. J. Cook
Discrete Mathematics Group
Room 2A-351
Bell Communications Research Center
445 South Street

Morristown , NJ 07960-1910
USA

Prof. Dr. W. H. Cunningham
Dept. of Mathematics and Statistics
Carleton University

Ottawa, Ontario , K1S 5B6
CANADA

Prof. Dr. J.E. Dennis
Dept. of Mathematical Sciences
Rice University
P. O. Box 1892

Houston , TX 77251-1892
USA

Prof. Dr. D. Goldfarb
Dept. of Industrial Engineering and
Operations Research
Columbia University
Seeley W. Mudd Building

New York , NY 10027
USA

Prof. Dr. G. Di Pillo
Dipartimento di Informatica e
Sistemistica
Universita di Roma "La Sapienza"
Via Eudossiana, 18

I-00184 Roma

Prof. Dr. M.D. Grigoriadis
Department of Computer Science
Rutgers University
Hill Center, Busch Campus

New Brunswick , NJ 08903
USA

Prof. Dr. R. Fletcher
Dept. of Mathematical Sciences
University of Dundee

GB- Dundee , DD1 4HN

Prof. Dr. M. Grötschel
Institut für
Angewandte Mathematik II
der Universität
Universitätsstr. 8

8900 Augsburg

Prof. Dr. J. Fonlupt
Institut IMAG
Universite scientifique, techno-
logique et medicine de Grenoble
Boite Postale 68

F-38402 St. Martin d'Heres Cedex

Prof. Dr. P.L. Hammer
RUTCOR, Centre of Operations
Research, Mathematical Sciences
Rutgers University
Hill Centre

New Brunswick , NJ 08903
USA

Prof. Dr. A. Frank
Institut für Ökonometrie und
Operations Research
Ökonometrische Abteilung
Adenauerallee 24-26

5300 Bonn 1

Prof. Dr. J. B. Hiriart-Urruty
Mathematiques
Universite Paul Sabatier
118, route de Narbonne

F-31062 Toulouse Cedex

Prof. Dr. M. Iri
Dept. of Mathematical Engineering and
Instrumentation Physics, Fac. of
Engineering, University of Tokyo
Bunkyo-ku

Tokyo 113
JAPAN

Prof. Dr. W. Krabs
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof. Dr. E. L. Johnson
Industrial and Systems Engineering
Georgia Institute of Technology

Atlanta , GA 30332-0205
USA

Dr. Ch. Kredler
Institut für Angewandte Mathematik
und Statistik
der TU München
Arcisstr. 21

8000 München 2

Prof. Dr. P. Kall
Institut für Operations Research
Universität Zürich
Moussonstr. 15

CH-8044 Zürich

Prof. Dr. E.L. Lawler
Computer Science Division
University of California
at Berkeley
591 Evans Hall

Berkeley , CA 94720
USA

Prof. Dr. B. Korte
Institut für Ökonometrie und
Operations Research
der Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof. Dr. C. Lemarechal
INRIA
Bat. 12
B. P. 105

F-78153 Le Chesnay Cedex

Prof. Dr. V. Kovacevic-Vujcic
Milovana Marinkovica 5

YU-11040 Beograd

Prof. Dr. J. K. Lenstra
Dept. of Mathematics and
Computer Science
Technical University Eindhoven
Postbus 513

NL-5600 MB Eindhoven

Prof. Dr. T. M. Liebling
Dept. de Mathematiques
Ecole Polytechnique Federale
de Lausanne

CH-1015 Lausanne

Prof. Ph.D. K. Mehlhorn
Fachbereich 10 - Informatik
Universität des Saarlandes
Im Stadtwald 15

6600 Saarbrücken 11

Prof. Dr. L. Lovasz
Department of Computer Science
Eötvös University
ELTE TTK
Muzeum krt. 6 - 8

H-1088 Budapest VIII

Prof. Dr. R. R. Meyer
Computer Science Department
University of Wisconsin-Madison
1210 W. Dayton St.

Madison , WI 53706
USA

Prof. Dr. M. Lucertini
Dipartimento di Ingeneria Elettron.
Universita di Roma
"Tor Vergata"
Via Orazio Raimondo

I-00173 Roma

Prof. Dr. C.L. Monma
Bell Communications Research Center
435, South Street

Morristown , NJ 07960-1910
USA

Prof. Dr. T. L. Magnanti
Sloan School of Management
Massachusetts Institute of
Technology
77, Massachusetts Avenue

Cambridge , MA 02139
USA

Prof. Dr. G. L. Nemhauser
Industrial and Systems Engineering
Georgia Institute of Technology

Atlanta , GA 30332-0205
USA

Prof. Dr. L. McLinden
Dept. of Mathematics, University of
Illinois at Urbana Champaign
273 Altgeld Hall -
1409 West Green Street

Urbana , IL 61801
USA

Prof. Dr. K. Neumann
Institut für Wirtschaftstheorie
und Operations Research
der Universität Karlsruhe
Kaiserstraße 12

7500 Karlsruhe 1

Prof. Dr. W. Oettli
Fakultät für Mathematik und Informatik
Universität Mannheim
Schloß

6800 Mannheim 1

Prof. Dr. B. Reed
Department of Combinatorics and Optimization
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof. Dr. M. W. Padberg
Laboratoire d'Econometric de l'Ecole Polytechnique
1, rue Descartes

F-75005 Paris

Prof. Dr. K. Ritter
Institut für Angewandte Mathematik und Statistik
der TU München
Arcisstr. 21

8000 München 2

Prof. Dr. D. Pallaschke
Institut für Statistik und Mathematische Wirtschaftstheorie
Universität Karlsruhe
Postfach 6980

7500 Karlsruhe 1

Prof. Dr. S.M. Robinson
Dept. of Industrial Engineering
The University of Wisconsin-Madison
1513, University Avenue

Madison , WI 53706
USA

Dr. H. J. Prömel
Institut für ökonometrie und Operations Research
der Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof. Dr. E. Sachs
Fachbereich IV
Mathematik
der Universität Trier
Postfach 3825

5500 Trier

Prof. Dr. W. R. Pulleyblank
Department of Combinatorics and Optimization
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Dr. S. Schäffler
Institut für Angewandte Mathematik und Statistik
der TU München
Arcisstr. 21

8000 München 2

Prof. Dr. A. Schrijver
Center for Mathematics and
Computer Science
P. O. Box 4079

NL-1009 AB Amsterdam

Dr. A. Steger
Institut für Ökonometrie und
Operations Research
der Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof. Dr. A. Sebö
Laboratoire ARTEMIS (IMAG)
Universite de Grenoble
BP 68

F-38402 St. Martin d'Heres Cedex

Prof. Dr. J. Stoer
Institut für Angewandte Mathematik
und Statistik
der Universität Würzburg
Am Hubland

8700 Würzburg

Prof. Dr. D.F. Shanno
RUTCOR, Centre of Operations
Research, Mathematical Sciences
Rutgers University
Hill Centre

New Brunswick , NJ 08903
USA

Prof. Dr. R. A. Tapia
Dept. of Mathematical Sciences
Rice University
P. O. Box 1892

Houston , TX 77251
USA

Prof. Dr. D. Shmoys
Laboratory for Computer Science
Massachusetts Institute of
Technology
545 Technology Square

Cambridge, MA 02139
USA

Prof. Dr. E. Tardos
School of Operations Research and
Industrial Engineering
Cornell University
Upson Hall

Ithaca , NY 14853-7901
USA

Prof. Dr. B. Simeone
Via Ferdinando Pinelli 12

I-00136 Roma

Prof. Dr. M.J. Todd
School of Operations Research and
Industrial Engineering
Cornell University
Upson Hall

Ithaca , NY 14853-7901
USA

Prof. Dr. L.E. Trotter
School of Operations Research and
Industrial Engineering
Cornell University
Upson Hall

Ithaca , NY 14853-7901
USA

Prof. Dr. L.A. Wolsey
Center for Operations Research and
Econometrics
34 Voie du Roman Pays

B-1348 Louvain-la-Neuve

H. Warsitz
Institut für Angewandte Mathematik
und Statistik
der TU München
Arcisstr. 21

8000 München 2

Prof. Dr. U. Zimmermann
Abteilung für Mathematische
Optimierung der TU Braunschweig
Pockelsstr. 14

3300 Braunschweig

Prof. Dr. D. de Werra
Dept. de Mathematiques
Ecole Polytechnique
Federale de Lausanne
Ecublens

CH-1015 Lausanne

Prof. Dr. J. Zowe
Fakultät für Mathematik und Physik
der Universität Bayreuth
Postfach 10 12 51

8580 Bayreuth