MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 3/1990

Nonlinear Evolution Equations, Solitons and the Inverse Scattering Transform

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The conference on

Nonlinear Evolution Equations, Solitons and the Inverse Scattering Transform

was organized by Professors Mark J. Ablowitz (Boulder), Benno Fuchssteiner (Paderborn) and Martin Kruskal (Princeton).

The participants (35 mathematicians and physicists from several countries) presented their most recent work in the meeting. The lively scientific atmosphere of the conference resulted in very many stimulating discussions which certainly will influence future directions and will contribute to further progress in the field.

The lecture program consisted of 35 lectures on topics such as Inverse problems in multidimensions, notions of integrability, algebraic and geometrical aspects of nonlinear evolution equations, Solitons, Painlevé analysis, explicit solutions of special systems in 1 + 1 and 2 + 1 dimensions, direct linearization of special systems, computational and algorithmic aspects, Quantum systems, soliton equations in differential geometry and various other applications.

Vortragsauszüge

M.J. ABLOWITZ

Aspects of Solitons and Computations

There are many nonlinear evolution equations which have discrete approximations which for suitable initial values do not properly reflect the correct solution, in a numerical sense, for intermediate values of the mesh size. The prototype equation is the nonlinear Schrödinger equation (NLS) with periodic conditions. The NLS equation possesses a denumerably infinite number of homoclinic orbits which are related to soliton solutions of the defocusing NLS equation. Initial values proximate to these homoclinic orbits are difficult to resolve numerically. In particular standard numerical schemes reflect puious chaotic behavior which disappears for sufficiently refined mesh. Well known integrable numerical schemes associated with the NLS equation give good approximations which coverage uniformly as the mesh is refined. Finally homoclinic orbits are, in fact, selected to multiple eigenvalues of the underlying spectral problem.

S.J. ALBER

Associated integrable Hamiltonian systems. Regular and singular problems.

Link between different problems is established in the form of isomorphism of spaces of solutions. These problems form a complex of nonequivalent Hamiltonian structures over the same space of solutions. Shrinking process is used for the investigation of the singular integrable problems.
Then associated discrete and continuous integrable systems are introduced as the systems with the same spectrum, the same dynamical or spatial Hamiltonian and similar latticed Jacobi systems of inversion.
In particular, hierarchies of continuous integrable systems associated with Toda lattices, relativistic Toda lattices and Volterra lattices are found and investigated.

M. BLASZAK

Symmetries, Conservation Laws and Multisoliton Perturbation Theory

On the basis of action/angle variables for multisolitons new symmetries (mastersymmetries) are constructed. For a system with known hierarchies of nonhamiltonian mastersymmetries the hierarchies of hamiltonian mastersymmetries are constructed (KdV for example) and for a system with known hamiltonian mastersymmetries the hierarchies of non-hamiltonian mastersymmetries are constructed (Benjamin-Ono for example). Moreover, with the help of the action/angle variables, the N-solitons perturbation theory, on soliton submanifold (adiabatic approximation), is formulated. The explicit form of the time evolution of asymptotic data under the influence of perturbation is presented.

A.I. BOBENKO, L.A. BORDAG

Periodic multiphase solutions of the Kadomtsev-Petviashvili equation

N-Phase solutions of the Kadomtsev-Petviashvili (KP) equation, that are periodic in space variables x and y, were obtained and effectively investigated using the Schottky uniformisation, of which a short description is given. Our automorphic approach leads to a general result; namely, to a natural description of an arbitrary number of interacting phases and to an effective determination of the periodic solutions. Many wave patterns are represented graphically as contour plots and as isometric projections for different parameter values of two-, three- and four-phase solutions of the KP equation.

F. CALOGERO

Some results on C-intergrable nonlinear PDE's

(1) C-integrable and S-integrable equations
Some examples were presented, and a definiton: "A nonlinear PDE is C-integrable ("integrable by change of variables") if its solution can be obtained by solving:

(i) A finite system of nondifferential (possibly nonlinear, i.e., algebraic or transcendental) equations.

(ii) A finite system of linear PDE's (including ODE's and quadratures)".

The corresponding definition of S-integrability ("integrability by the Spectral transform , or inverse Scattering, technique") is closely analogous, except for the replacement (extension) of (ii) as follows: "(ii) a finite system of linear PDE's (including ODE's and quadratures), as well as linear integral equations (Fredholm, Volterra, Riemann)".

(2) A class of C-integrable Equations
A technique to generate C-integrable equations has been presented, and some examples given. The results reported are those published in the Addendum to the chapter by F. Calogero, "Why are certain nonlinear PDE's both widely applicable and integrable?", in the forthcoming book "What is integrability?", edited by V.E. Zakharov, Springer, 1990. Some extensions have also been mentioned.

(3) Burgers equation on the semiline with general boundary conditons at the origin
(joint work with S. De Lillo)
The problem characterized by the following equations is treated: $u \equiv u(x,t)$;
$u_t = u_{xx} + 2u_x u$, $x \geq 0$, $t \geq 0$; $u(x,0) = u_0(x)$ given , $x \geq 0$;
$H[u(0,t); u_x(0,t); t] = 0$, $t \geq 0$, $H(\alpha;\beta;\gamma)$ given.

H.W. CAPEL

Integrable lattice systems and hierarchies

Integrable equations on three-dimensional lattices are studied via the direct linearization method which is based on a linear integral equation with arbitrary measure and contour. For every measure and contour the integral equation yields the basis functions of a Lax representation and the potential satisfying the compatibility condition can be evaluated by integration over the same contour with the same measure. The integrable equations are obtained as N x N matrix equations embedded in an infinte matrix structure. As an example the N x N matrix lattice version of the isotropic Heisenberg ferromagnet in 2+1 dimensions ist discussed. Special cases include the three-dimensional N x N matrix lattice version of the KP, the modified KP and the two-dimensional Toda equation. Hierarchies of integrable systems with one or more continuous coordinates are obtained applying continuum limits with a vertex operator to the lattice equation as well as to the Lax representation. In fact, the equations of the hierarchy can be expressed in terms of the monodromy matrices for a special value of the spectral parameter. The monodromy matrices can be evaluated via an expansion in terms of the (canonical) field determined by the leading part of the expansion. The continuum limits yield a variety of hierarchies including the ones associated with the KP, the modified KP, the DS, the 2+2 dimensional Toda equations and their discrete analogues.

S. CARILLO

An application of the action-angle transform: The Liouville equation
(joint work with B. Fuchssteiner)

A method for constructing the action-angle transformation especially in the case of multi-soliton solutions has been obtained in [B. Fuchssteiner, S. Carillo:"The action-angle transformation for soliton equations" sub. Physica A].
There 1+1-dimensional nonlinear evolution equations admitting a hereditary recursion operator have been considered. Indeed, a key role, at first, seemed to be played by the nonlinear link between the two isospectral problems; however, a further analysis lead us to show how such action-angle transformation can be

obtained from the study of the symmetry structure of the nonlinear evolution equation under investigation and, specifically, of the related interacting soliton equation.
In this perspective we have been investigating various different nonlinear evolution equations such as the KdV, mKdV (thus, also sine-Gordon) and nonlinear Schrödinger equations.
Here, as an example of application of the action-angle transformation, it is shown how such transformation naturally induces the construction of the general solution of the initial value problem for the Liouville equation.

A. DEGASPERIS

Explicit solutions of the Davey-Stewartson equation

The interest in the Davey-Stewartson 1 equation, a 2+1 (space and time) nonlinear evolution equation, is well motivated: it is both applicable and integrable (via the spectral transform). Here we face the practical problem of constructing explicit solutions of this equation. In the case of (1+1) integrable evolution equations (such as KdV, NLS etc.) this problem has been solved by various techniques (f.e. inverse spectral theory, Bäcklund transformations, Hirota method, τ -function theory), which have all produced the multi-soliton class of solutions. The building-blocks of these construction methods are the exponentials $exp(px+\omega t)$, which are just particular solutions of the linearized form of the evolution equation (namely $u_t+u_{xxx}=0$ for KdV, $iu_t+u_{xx}=0$ for NLS etc.). We show that, by means of the spectral theory, solutions of the DS1 equation can be explicitly constructed by using as building-blocks any solution of the linear 1+1 Schrödinger equation $if_t+f_{xx}+v(x,t)f=0$, with a given (solvable) potential $v(x,t)$.To this purpose we discuss the algorithmic way of generating solvable potentials $v(x,t)$. In particular, we introduce a class of solutions of the DS1 equation, which describe the interaction of bumps that are localized with a gaussian (rather than exponential) profile (gaussons); each bump, however, disperses away with a characteristic dispersion time τ. In the limit in which all dispersion times go to infinity, this solution coincides with the multi-dromion solution, that turns out to be just a special subcase. Collissions of gaussons with gaussons and dromions can be easily investigated in this formalism. Work is in progress, and this includes also radiation and dromion creation processes.

J. DORFMEISTER

Banach manifolds and evolution equations

Let $H=l^2(S^1)$ and choose the natural $ONB\{f^n;n\in\mathbb{Z}\}$, $H=H_++H_-$.
Let $w:Z\rightarrow(0,\infty)$ be a weight $(w(r+s)\leq w(r)w(s),\ w(0)=f,\ w(r)=w(-r))$.

Set $B_{KP} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in B(H), \sum w(ij)|b_{ij}| < \alpha, \sum w(ij)|c_{ij}| < \infty, \right.$

and similar conditions on $a, d\}$. Set $G_{KP} = (B_{KP}{}^*)^0$, $P_{KP} \equiv \begin{pmatrix} \alpha & \alpha \\ 0 & \alpha \end{pmatrix} \in G_{KP}$. Then $X_{KP} = G_{KP}/P_{KP}$ is a "Segal-Wilson Like" manifold. We define a τ-function τ_w for $w \in X_{KP}$ as usual and set $S_{KP} = \{\sum \partial_x{}^2 ln\tau_w, \ w \in X_{KP}\}$. Theorem: $G_{KP} \to X_{KP} \to S_{KP}$ is a chain of submersions of Banach manifolds. In particular, the set S_{KP} solutions to the KP equation associated with $w \in X_{KP}$ is a Banach manifold. We consider also the natural KdV-reduction and obtain: Theorem: $G_{KdV} \to X_{KdV} \to S_{KdV}$ is a chain of submersions of Banach manifolds. We consider the natural inclusions and obtain: Theorem: The natural inclusions $G_{KdV} \to G_{KP}$, $X_{KdV} \to X_{KP}$ and $S_{KdV} \to S_{KP}$ are embeddings of Banach manifolds.

B. FUCHSSTEINER

Symplectic ideals and the structure of mastersymmetries

For completely integrable systems the hereditary algebra spanned by symmetries and mastersymmetries is considered. This algebra is made into a Lie algebra module by introduction of scalar fields. Furthermore, by use of a suitable symplectic form a natural completion of ideals in this Lie algebra module is defined. Ideals which are complete in this sence are called symplectic ideals. It turns out that multisoliton manifolds can be characterized as zero sets of symplectic prime ideals. The identities obtained by this characterization are used to determine the nonlocal part of the mastersymmetries of the systems under consideration.

J.P. FRANCOISE

Periodic orbits and perturbation theory for Calogero systems

The systems we studied are of the following type:

$$H_0(x,y) = \frac{1}{2}\sum_{i=1}^{m} y_i{}^2 + \sum_{\alpha \in R_+} \frac{y_\alpha{}^2}{< x, \alpha >^2} + \lambda^2 \sum_{i=1}^{m} x_i{}^2$$

$$H_\epsilon(x,y) = H_0(x,y) + \epsilon \sum_{i=1}^{m} h(x_i)$$

$$\omega = \sum_{i=1}^{m} dx_i \wedge dy_i$$

We proved that the systems (1) are associated to a symplectic action of the torus Π^m when the root system ist A_m, B_m, C_m, D_m and σ_2. One gets as consequences the spectrum of the quantum system and a formula of Gallavotti and Marchioro. Then we reported on works obtained in collaboration with O. Ragnisco and A. Celletti on perturbations of $H_0(2)$. The perturbed system remains integrable in the Arnold-Liouville sense if h is a polynomial of degree four. But there are computer experiments which tend to establish the non-integrability of six-order perturbations in the two-particles case. We mentioned related subjects and questions. Second-order matrix differential equations of the form $\ddot{x} + 2h(x)h'(x) = 0 \quad h(x) = \lambda x + \mu x^2$ where x is symmetric have m integrals in involution . We do not know if such an equation is integrable.

C. HOENSELAERS

Prolongation structures for nonlinear differential equations

One first rewrites a given partial differential equation in two independent variables as a set of first order equations and then as a set of differential forms. These forms are supposed to form a closed ideal and, when restricted to a solution manifold and annulled, to give back the original equations. Pseudo potentials are introduced by enlarging the dimension of the manifold of dependent and independent variables. The enlarged ideal consisting of the original set of forms and additional one-forms for the pseudo potentials is again supposed to be closed. One is thereby led to a set of vector fields and commutation relations between them; not all commutators, however, are given. The question now is, whether those vector fields generate an infinite dimensional Lie algebra and if so which one. If they generate a loop algebra one uses a matrix representation to write an AKNS pair for the pseudo potentials. Let us now assume that we are given an infinite dimensional Lie algebra, one forms dual to the generating vector fields and the Maurer-Cartan forms. By setting almost all one forms to zero one derives two sets of equations for the remaining ones. These sets are a closed ideal and yield on the integral manifold some differential equations which depends on the parametrisation. Again using a matrix representation one can derive Bäcklund transformations for the ideal, and thereby for all differential equations on its integral manifold, by purely algebraic means.

B.G. KONOPELCHENKO

Nonlinear soliton eigenfunction's equations: The IST integrability and some properties

The soliton equations are related with the linear systems of equations with variable coefficients. The corresponding eigenfunctions obey nonlinear equations. It is shown that these eigenfunction equations are integrable by the inverse spectral transform (IST) method too. The operator form of the corresponding compatibility conditions is given by the Manakovs triad operator equations.

The eigenfunction equations are the generating equations and possess other interesting properties. Eigenfunction equations for several typical 1+1 and 2+1 dimensional soliton equations are considered.

H. LANGE

Attractors of nonlinear Schrödinger systems
(joint work with G. Rünger)

We consider nonlinear time-dependent Schrödinger equations of type

$$(NLS) \quad iu_t = u_{xx} + \omega u$$

where the potential ω may depend nonlinearily on u. We investigate the asymptotical behavior of solutions to initial-boundary-value problems for (NLS) for $t \to \infty$, namely in case where

$$(i) \quad \omega(u) = f(|u|^2) + \beta h(|u|^2)_{xx} h'(|u|^2)$$

or

$$(ii) \quad \omega_{xx}(u) = 1 - |u|^2 \quad (Vlasov - NLS).$$

For (i) we prove that in some cases (e.g. $f(s) = \lambda s^p$, $h(s) = s^q$, $\lambda > 0$, $\beta < 0$; "non-focusing dispersive" case) all finite energy solutions decay to zero uniformly in space. For (ii) with appropriate periodic initial-boundary-value conditions we prove the existence of a maximal global attractor for all H^2-solutions of the associated problem for (NLS) which has finite Hausdorff and fractal dimension when a damping term $i\gamma u$ $(\gamma > 0)$ is included additionally into (NLS). Also in this case a condition is given such that a solution of the problem has a finite set of "determinating modes" (in the Fourier expansion).

D. MAISON

Gravitating solitons

The only known 3-dimensional solitons (= particle-like solutions) in nonlinear special-relativistic field theories are the non-abelian magnetic monopoles (resp. dyons) and so-called non-topological solitons of charged scalar fields. The inclusion of gravity (in the form of General Relativity) changes the situation in several respects. Although there are still no regular solitons in the Einstein-Maxwell theory and in a large class of Kaluza-Klein theories, those theories allow for finite mass black holes showing particle like behaviour. In addition to gravitating versions of the flat-space solitons there exist new smooth solitons for charged scalars (so-called Boson stars). However, what came as a surprise to many was the recent discovery of static, spherically symmetric solutions of the Einstein-Young-Mills theory by Bartnik and Mc Kinnon (through numerical integration). Although a rigorous existence proof for these solutions based on standard method seems difficult, we present a computer-aided existence proof combining the numerically obtained knowledge on approximate solutions with rigorous estimates using Lipschitz conditions. Unfortunately linear stability analysis shows that these solutions are unstable against gravitational collapse.

S.V. MANAKOV

On a Class of Integrable Nonlinear Nonlocal Evolution Equations

The solutions $\psi(\lambda, \bar{\lambda}, t)$ of the local $\bar{\partial}$ - problem $\frac{\partial\psi}{\partial\bar{\lambda}} = \psi R$, $\psi(\infty) = I$ with $R(\lambda, \bar{\lambda}, t)$ evolving in time according to the equation $R_t = [K, R]$, where $K(\lambda, \bar{\lambda})$ is an arbitrary vanishing at infinity matrix - valued function on the complex plane λ, satisfy the equation in $2+1$ dimensions

$$\frac{\partial\psi}{\partial t} + \psi K = \partial_{\bar{\lambda}}{}^{-1}(\psi \frac{\partial K}{\partial \bar{\lambda}} \psi^{-1})\psi \,.$$

This equation is Lagrangian and integrable. To solve the initial - value problem for it one has to construct $R(\lambda, \bar{\lambda}, 0) = (\psi^{-1}\frac{\partial\psi}{\partial\bar{\lambda}})|_{t=0}$ and then to solve the integral equation

$$\psi(\lambda, \bar{\lambda}, t) = I + \partial_{\bar{\lambda}}{}^{-1}(\psi R(t)), \text{ with } R(t) = e^{Kt}R(0)e^{-Kt} \,.$$

An analogous equation can be obtained via the Riemann - Hilbert problem $\psi^+ = \psi^- R$, $R_t = [K(\lambda), R]$, $\lambda \in \mathbf{R}_1$. In this case one gets a $1+1$ dimensional nonlocal evolution equation. The simplest example of this type is a *nonlinearization* of the $2+2$ AKNS spectral problem:

$$\frac{\partial}{\partial t} X + i\lambda[\sigma_3, X] = [\sigma_3, < X >]X(\lambda),$$

$$\text{where } < X >= \int_{-\infty}^{\infty} X(\lambda, t) d\lambda.$$

J. MATSUKIDAIRA

Soliton Equations expressed by trilinear form and their solutions

Most of all soliton equations which have N-soliton solutions can be expressed by bilinear form through suitable dependent variable transformation. It has been shown by various people that bilinear forms of soliton equations such as KP equation or Toda equation are nothing but the identities of Wronskian determinant or Casorati determinant. Recently, we have found Brouer-Kaup system or the classical Boussinesq equation can be expressed by trilinear form through suitable dependent variable transformation. It will be shown in my talk that the trilinear form of this system may be considered to be an identities of two-component Wronskian. Furthermore, the existence of infinite number of trilinear forms of equations connected with characteristic polynomials of General Linear Group will be discussed.

V.B. MATVEEV

Algebrogeometrical aspects of the solvable chiral Potts model

The conjecture of Perk, Baxter et al. about the structure of the normalization factor R(p,q,r) entering into their new solution of the star-triangle equation is verified. Some properties of the underlying algebraic curve are studied. Particularly the exact formula of its matrix of B-periods is obtained. As a consequence the related multidimensional theta-function may be represented as a sum of finite number of terms, each term being a product of one-dimensional theta-functions. The relation of the Picard-Fuchs equations of the curve with thermodynamic properties of the model are discussed.

F.W. NIJHOFF

Multidimensional Lattices, the Simplex Equation and Quantum Groups

The d-simplex equations are generalizations of the wellknown Quantum Young-Baxter equations (QYBE's). The first member of the hierarchy of simplex equations is matrix commutativity ($d = 1$), next we have the QYBE's ($d = 2$), the Zamolodchikov equations ($d = 3$), etc. The work in collaboration with J.M. Maillet (CERN) is motivated by the problem of arriving at a genuinely multidimensional notion of integrability. It is convenient to work on lattices (also time-discrete) because in $d = 2$ the corresponding Zakharov Shabat (ZS) equations look like

$$L_l{}^{l'} * L_{l'} = L^l{}_{l'} * L_l$$

(in which L_l, $L_{l'}$ are translation matrices of the linear problem, l, l' denoting different lattice directions, and superscript denotes translation in the corresponding directions). Thus the ZS system is very similar to a 1-simplex equation. We develop, analogous to the structure in the simplex hierarchy, a method of building in obstructions at any level, thus climbing to one higher level in the hierarchy. Thus we arrive in d=3 at an equation

$$K_{ll'} * K^{l''}_{ll''} * K_{l'l''} = K^{l}{}_{l'l''} * K_{ll''} * K^{l''}_{ll'}$$

for plaquette-objects $K_{ll'}, \ldots$, in which the couplings are exactly as in the

QYBE. We interpret this local version of the QYBE's as a classical equation, generalizing the discrete ZS equations, governing the dynamics of a d=3 dimensional lattice. Connections with quantum groups, and gauge theories on spaces of loops are also discussed.

G. OEVEL

Canonical variables for Multi-Soliton Systems
(joint work with B. Fuchssteiner and M. Blaszak)

For all integrable hamiltonian equations in (1+1) dimensions we present the geometrical structure of the manifold of N-soliton solutions. With the help of mastersymmetries we are able to derive canonical variables for the multi-soliton systems in terms of the field variable u. Furthermore we present a hereditary recursion operator for the multi-soliton system of the Benjamin-Ono equation and discuss its properties.

W. OEVEL

The Bi-Hamiltonian Structure of Fully Supersymmetric Korteweg-de Vries Equations
(joint work with Z. Popowicz)

The bi-Hamiltonian structure of supersymmetric extensions of the KdV related to the N=1 and the N=2 superconformal algebras is found. It turns out that some of these extensions admit "inverse" Hamiltonian formulations in terms of presymplectic operators rather than in terms of Poisson tensors. For one extension related to the N=2 case additional symmetries are found with bosonic parts that cannot be reduced to symmetries of the classical KdV. They can be explained by a factorization of the corresponding Lax operator. All the bi-Hamiltonian are derived in a systematic way from the Lax operators.

U. PINKALL

Soliton equations in differential geometry

Many classical topics in differential geometry (surfaces with constant mean - or Gaussian curvature, minimal surfaces in $S^n \dots$) are directly equivalent to the study of certain soliton equations. For example, all tori with constant mean curvature in R^3 ("soap-bubble tori") can be classified using soliton theory. Conversely, describing soliton theory in differential geometric terms adds new insight to the subject and reveals very naturally the connection between soliton equations and Kac-Moody algebras.

J. PÖSCHEL

Quasiperiodic Solutions of Nonlinear Schrödinger and Wave Equations

I describe an extension of the classical KAM-Theorem about the existence of quasi-periodic motion to some infinite dimensional Hamiltonian systems, such as the nonlinear wave equations $u_{tt} = u_{xx} - Q(x)u - \epsilon f(x,u)$ *on* $o \leq x \leq \pi$ with Dirichlet b.c., depending on a potential $Q \in L^2(0,m)$, with f analytic in u. The idea is to consider finite dimensional KAM-tori in infinite dim. systems and to make use of the asymptotic behaviour of the Dirichlet eigenvalue $\mu_k(Q)$ of Q. The result is: Given $n \geq 2$, for suff. small ϵ there exists "large" n-dimenional Cantor set $C_n(\epsilon) \subset E = \{Q : \mu_1(Q) > 0\}$ such that the corresponding wave equation possesses quasi - periodic solutions with basic frequences close to $\sqrt{\mu_1}, \dots \sqrt{\mu_n}$. Moreover, if $\sum f(x,y) = u^3 + h.o.t$, then C_n is in fact open and dense in E. Analogous results hold for the nonlinear Schrödinger equation $iu_t = u_{xx} - Q(x)u - \epsilon f(x,(us^2)u)$, and the idea also applies to perturbed periodic KdV-equations. This theory was also developed independently by S.B. Kuksin.

O. RAGNISCO

Integrable Mappings and stationary versions of Nonlinear Integrable Systems

It is shown that, under suitable regularity assumptions, Lagrangian mappings (i.e.: discrete-time systems coming as stationary point of a given action functional) can be viewed as simplectic mappings on a manifold. This result allows to introduce the concept of completely integrable mappings, which are the discrete counterparts of completely integrable systems in Hamiltonian mechanics. It is proven that stationary versions of integrable nonlinear discrete evolution equations provide families of completely integrable mappings. As an example, some mappings coming from an integrable hierarchy of discrete equations recently introduced by Tu Guizhang are briefly discussed. A discrete integrable version of the harmonic oscillator is also presented.

A.G. RAMM

Multidimensional inverse scattering problems and completeness of the set of products of solutions to PDE

A general method is given for proving uniqueness theorems for a wide class of inverse scattering problems. For example, it is proved that the scattering amplitude $A_q(\theta', \theta)$ given at a fixed energy $K^2 > 0$ for all θ' and θ running through solid angles (however small) determines uniquely the potential $q(x)$ in the class

$Q_a := \{q : q = \bar{q},\ q \in L^2(B_a),\ q = 0 \ in \ B_a{}'\},\ B_a = \{x : x \in \mathbb{R}^3,\ |x| \leq a\}$. A reconstruction formula for finding $q(x)$ given $A(\theta', \theta)$ for all θ', $\theta \in S^2$ is given. Stability problem is discussed. Namely, suppose $\{\delta,\ A_\delta A(\theta',\theta)\}$ are given, where $\sup_{\theta',\theta} < \delta\ |A_\delta - A| < \delta$, A_δ is not necessarily a scattering amplitude. Then $\tilde{q}_\delta(\lambda)$ is constructed such that $|\tilde{q}_\delta(\lambda) - \tilde{q}(\lambda)| \leq \eta(\delta) \to 0 \ as \ \delta \to 0$. Here $\lambda \in \mathbb{R}^3$ is an arbitrary fixed vector, $\eta(\delta)$ can be chosen the same for all $|\lambda| \leq \lambda_0$, where $\lambda_0 > 0$ in an arbitrary fixed number. Other results are mentioned. The method is based on the property C for pairs $\{\mathcal{L}_1, \mathcal{L}_2\}$ of partial differential expressions. Property C means completeness of the set of products $\{u_1, u_2\}\ \forall u_j \in N_D(\mathcal{L}_j) := \{u : \mathcal{L}_j u_j = 0 \ in \ D\}$.

S. RAUCH-WOJCIECHOWSKI

On connection between stationary flows of soliton equations and integrable, separable natural Hamiltonian systems

F. RENNER

Necessary conditions for Painlevé property

For a given system of polynomial evolution equations (SPE) the r-equivalent SPE is introduced and a theorem stated that connects Painlevé property (PP) of the first in a necessary way with PP of the second. Defining when a SPE is r-reduced respectively r-degenerate and citing some further conditions for the resonance polynomial a theorem is proved concerning the leading order of a Painlevé expansion for a given SPE.

C. ROGERS

Pinney-Ermakov Systems

A linearization of the celebrated Ermakov system is presented along with associated nonlinear superposition principles. The linearization is exploited for a particular Ermakov system which arises out of a symmetry reduction in two-layer long wave hydrodynamics . Thus reduction to Ince-type equations is presented and used to infer stability and periodicity properties for the original Ermakov system. In conclusion, it is shown how, in the case of the simplest Ermakov system, the nonlinear superposition may be used to solve a wide class of initial boundary value problems. Detailed results are presented for Heaviside and blast loading boundary conditions applied to a thin hyperelastic tube.

S.N.M. RUIJSENAARS

KP Soliton vectors in fermion Fock space

From work by the Kyoto school, Segal/Wilson and other authors it is known that the group $G_2(\mathcal{H})$ of Bogolinbov transformations of the Dirac field on the fermion Fock space $F_a(\mathcal{H})$ over $\mathcal{H} \equiv l^2(Z)$ may be viewed as a symmetry group for the KP equation and its associated higher order equations. More in detail, viewing $l^2(Z)$ as the Fouriertransform of L^2-functions on $S^1 \subset C$, one needs a Dirac decomposition $\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-$, where $\mathcal{H}_+/\mathcal{H}_-$ contains the holomorphic functions inside/outside S^1 with L^2 boundary values; then $G_2(\mathcal{H})$ is the group of operators on $\mathcal{H}$ that are bounded and boundedly invertible and have off-diagonal Hilbert-Schmidt parts w.r.t. the $\mathcal{H}_+/\mathcal{H}_-$ decomposition. Denoting the Fock space implementer of an operator $G \in G_2(\mathcal{H})$ by $\mathcal{G}$, the vacuum expectation value $\tau(x) = (\Omega,\ e^{H(x)}\mathcal{G}\Omega)$ solves the KP hierarchy of Hirota bilinear equations. Here, the KP evolution operator $e^{H(x)}$ is the implementer of the "i-particle evolution" $e^{h(x)}$. The latter belongs to $G_2(\mathcal{H})$, provided this time sequence $(x_1, x_2, \ldots)$ in the multiplication operator $h(x) \equiv \sum_{k=1}^{\infty} x_k z^{-k}$ decays fast enough. We have reported an explicit solution to the problem of finding implementers and (hence) Fock space vectors yielding the KP N-soliton τ-functions. The solution exhibits remarkable parallels with the so-called soliton vertex operators. The latter are formal maps that do not operate within Fock space, but which can and have been used by the Kyoto school to solve the problem just mentioned in a formal fashion.

P.M. SANTINI and A.S. FOKAS

Solitons and dromions, coherent structures in a nonlinear world

We linearize an initial-boundary value problem for the Davey-Stewartson I equation, showing the genericity and the spectral interpretation of the associated multidimensional coherent structures. Unlike solitons, which are nonlinear modes of integrable equations in 1+1 dimensions, these coherent structures are nonlinear distorsions of the modes of the linearized version of the Davey-Stewartson equation. Upon interaction they not only exhibit a two-dimensional phase-shift, but also a change of form and an exchange of energy; furthermore they can be driven everywhere in the plane choosing a suitable motion of the boundaries and they radiate energy if their motion is not uniform. We have called the above novel, localized, coherent structures "dromions" since they travel on the tracks (in greek "dromos") generated by the boundaries and are driven by them.

J. SATSUMA

A Representation of Solutions for the Soliton Equations and its Algebraic Structures

The determinant representation of solutions for the KP equation proposed by Nakamura is generalized for all equations of the KP hierarchy. It is shown that the algebraic structure of this determinant is parallel to that of the Wronskian. The structure of the KP hierarchy in the bilinear form is clearly seen through this representation of solutions. The relation between the determinant and the solution of the Gel'fand-Levitan-Marchenko equation is also discussed. Furthermore, the extension to the modified KP hierarchy, Toda hierarchy and the multi-component KP hierarchy are presented.

F. SCHWARZ

An Algorithm for Determining the Size of Symmetry Groups

To determine the symmetry group of a differential equation is of great theoretical and practical importance. There does not seem to exist a finite procedure for determining the symmetry group of an arbitrary differential equation. However, the more restricted problem of determining the size of the symmetry group may be solved in a finite number of steps. An algorithm is described which allows it to determine the number of parameters if the group is finite and the number of unspecified functions and its arguments if it is infinite. In many cases this algorithm allows it to determine the symmetry group completely, e.g. if it is just the identity or if the coefficients of the infinitesimal generator are algebraic of low degree.

W. STRAMPP

Symmetries of the KP and Sato's theory

We use Sato's theory for obtaining symmetries of the KP-equation. The bilinear form of higher order KP-equations is given. The connections to the recursion operator obtained by Fokas and Santini are discussed. Further recursion operators arising from Sato's theory are described.

TU GUIZHANG

A trace identity and its application to discrete and continuous integrable Systems

This is a summary on some recent common works in collaboration with Professors Fuchssteiner, Ragnisco, Oevel and my colleague Mr. Zhang. We start from a wide class of continuous and discrete isospectral problems and propose a systematic procedure for deriving the corresponding hierarchies of evolution equations and then reducing them into forms of generalized Hamiltonian equations. In this approach a trace identity - which takes exactly the same form both in continuous and in discrete case - plays an important role. This identity can be effectively used to derive both the whole hierarchy and the sequence of Hamiltonians from a single stationary zero-curvature equation. Furthermore we present a general and explicit formula for Poisson brackets which implies immediatly the Liouville integrability of the derived hierarchy of Hamiltonian equations. We emphasize that the above procedure applies to both continuous and to discrete systems. A comparison between these two cases is given to clarify their common points and to emphasize the different features. As an illustration of the procedure, a new hierarchy of discrete integrable systems is proposed. The relevant isospectral probem is given by $E\psi = U\psi$ with $U = \begin{pmatrix} \lambda + gr & g \\ r & 1 \end{pmatrix}$, where $g = g(m,t)$, $r = r(m,t)$ are two field variables depending on $t \in \mathbf{R}$ and $m \in Z$, λ is the spectral parameter, E is the translation operator defined by $(Ef)(n) = f(n+1)$. The mastersymmetries and the recursion structures are also addressed for this new hierarchy and for the Bogoyavlensky hierarchy of discrete integrable systems.

W. WIWIANKA

Algorithms to detect complete integrability in 1+1 - dimension
(joint work with B. Fuchssteiner)

Algorithms to test and detect complete integrability for nonlinear partial differential equations are given, their implementations in MAPLE are described. For some examples of solvable equations these algorithms are used to find symmetries and mastersymmetries, yielding the recursive structure of their hierarchies.

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