

## MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 4/1990

Modelltheorie

21.01. bis 27.01.1990

The main part of this conference on model theory, under the direction of L. van den Dries (Urbana), A. Prestel (Konstanz) and P. Roquette (Heidelberg), was devoted to the model theory of the ring  $\tilde{\mathbf{z}}$  of all algebraic integers. The conference started with several talks explaining Rumely's local-global principle, which plays an important role in the axiomatization of the elementary theory of  $\tilde{\mathbf{z}}$ . The connection of this principle and capacity theory was also described. In the following talks two different approaches to the model theory of  $\tilde{\mathbf{z}}$  were presented. The main results are: an elimination of quantifier theorem for  $\tilde{\mathbf{z}}$ , an effective axiomatization of  $\operatorname{Th}(\widetilde{\mathbf{Z}})$  and the decidibility of  $\widetilde{\mathbf{Z}}$ , of the ring  $\tilde{\boldsymbol{z}} \, \cap \, \boldsymbol{\mathbb{R}}$  of real algebraic integers and of the ring  $\tilde{\mathbf{Z}} \, \cap \, \boldsymbol{Q}_{_{\mathbf{D}}}$  of p-adic algebraic integers. In addition there were talks covering recent results in model theory with special emphasis on applications to algebra. The topics ranged from field arithmetic, combinatorial geometry, model theory of certain classes of fields and groups, exponentiation in  $\mathbf{Z}_{n}$  to stability theory.



Abstracts

PART A: Conference on the model theory of  $\tilde{\mathbf{z}}$ 

B. W. GREEN/F.-V. KUHLMANN/F. POP/P. ROQUETTE:

### Rumely's Local-Global Principle

As part of a prepared program on the 'model theory of the ring  $\tilde{\mathbf{z}}$  of all algebraic integers' an exposition of Rumely's Local-Global Principle was presented.

The situation considered:

 $K = \tilde{Q}$  the field of all algebraic numbers, algebraic closure of Q,

 $\theta$  =  $\tilde{\mathbf{Z}}$  the ring of all algebraic integers, integral closure of  $\mathbf{Z}$  in K.

v = v(K) the space of all non-archimedean valuations (primes) v of K; each valuation written additively,

W = an irreducible affine variety defined over K, embedded into affine r-space for some natural number r,

W(K) = the set of K-rational points of W,

 $z = (z_1, ..., z_r)$  a typical point of W(K),

 $v(z) = \min(v(z_1), ..., v(z_r))$  the v-norm of z, for  $v \in v$ .

If  $v(z) \ge 0$  then the point z is called v-integral or locally integral at v. If this is so for all  $v \in v$  then z is said to be an integral point, or globally integral, which means  $z_i \in \mathcal{O}$   $(1 \le i \le r)$ .

Theorem (Local-Global Principle for integer points): It is supposed that locally everywhere, the variety W admits a v-integral point  $z_v \in W(K)$ . Then W has a globally integral point  $z \in W(K)$ .





To prove this theorem the case where W is a curve i. e. dim(W) = 1was first considered. After that, an induction procedure with respect to the dimension of W leads to the general case.

The following talks were presented:

Roquette: Introduction to Rumely's Local-Global-Principle for  $\tilde{\mathbf{z}}$ 

Kuhlmann: Prerequisites from the theory of function-fields Green:

Good reduction and regularity

Pop: On the continuity of the zeros in function fields

Pop: The Rumely existence theorem

Roquette: The Local-Global-Principle for function fields over  $\tilde{\mathbb{Q}}$ 

(the case of curves)

Green: The Unit Density Lemma

Roquette: The Local-Global-Principle for varieties over  $\tilde{\mathbb{Q}}$  (Reduction

to the case of curves)

#### R. RUMELY:

## Capacity theory and the Local-Global Principle

This lecture, in two sessions, gave a history of the development of capacity theory culminating in recent developments by Chinburg and Rumely for capacity theory on varieties. The first paper in the subject was by M. Fekete (1923), in which he defined the transfinite diameter of a compact set  $E \subset \mathbb{C}$ , and showed that if d\_(E) < 1 then E contains only finitely many complete conjugate sets of algebraic integers. In the 1930's Polya, Szegö and Erdös gave alternate definitions for  $d_m(E)$ , proving it was equal to the so-called chebyshev-constant  $\P_{\infty}(E)$  and logarithmic capacity  $\gamma_{\infty}(E)$  . In 1955 Fekete and Szegő proved the contrary result: if  $d_{\infty}(E) > 1$ then every neighborhood of E contains infinitely many conjugate





sets of algebraic integers. Raphael Robinson made an important step in 1964 when he showed that under certain conditions equivalent to the negative definiteness of a 2x2 matrix, the real interval [a,b] contains infinitely many conjugate sets of units. In 1980 David Cantor generalized to the case of sets according a finite number of points  $X \subseteq \mathbb{P}^1(\tilde{\mathbb{Q}})$ , and in 1989 Rumely published a further generalization to arbitrary curves. This can be used to prove the following "Local-Global Principle" for the ring of all algebraic integers:

<u>Theorem:</u> Let V be an absolutely irreducible variety defined over a number field K; for each finite place v of K, let  $\tilde{\mathcal{O}}_{V}$  denote the ring of integers of the algebraic closure of the completion  $K_{V}$ . Then, if  $\tilde{\mathcal{O}}$  is the ring of all algebraic integers,  $V(\tilde{\mathcal{O}})$  is nonempty  $\Leftrightarrow V(\tilde{\mathcal{O}}_{V})$  is nonempty for all finite places v.

This Theorem was given a simpler proof in the lectures of the Heidelberg team, at this conference.

Recently Chinburg defined a new kind of capacity on smooth, projective, geometrically irreducible varieties of arbitrary dimension: The sectional capacity  $S_{\gamma}(E,D)$ , where E is an adelic set and D is an effective ample divisor on V. He showed that on  $\mathbb{P}^1$ , when  $D=(\infty)$ ,  $S_{\gamma}(E,D)$  reduces to the classical capacity  $\gamma_{\infty}(E_{\infty})$ , and proved a generalization of Fekete's Theorem. Rumely showed that for curves, if  $D=\Sigma_{\mathbf{r_i}}(\mathbf{x_i})$ 

$$\log S_{\gamma}(E,D) = -tr\Gamma(E,x)r$$

where  $\Gamma(E,X)$  is the Green's matrix in the Cantor-Rumely theory. Thus, the same quadratic form underlies both the Chinburg and Cantor-Rumely theories for curves.





#### L. VAN DEN DRIES:

# Quantifier Elimination and Axiomatization for the ring of algebraic integers

It turns out that Rumely's Local-Global Principle can be extended to take into account extra "radical" conditions. This leads to the results stated in the title. This is joint work with Macintyre.

#### A. PRESTEL:

## Model theory of domains with radical relations

Let R be a domain. A binary relation  $\le$  on R is called a <u>radical</u> <u>relation</u> if for all a,b,c  $\in$  R we have: (1) a  $\le$  a, (2) a  $\le$  b, b  $\le$  c  $\Rightarrow$  a  $\le$  c, (3) a  $\le$  b, a  $\le$  c  $\Rightarrow$  a  $\le$  b + c, (4) a  $\le$  b  $\Rightarrow$  ac  $\le$  bc, (5) 1  $\le$  a, 0  $\ne$  1, (6) a  $\le$  b<sup>2</sup>  $\Rightarrow$  a  $\le$  b. If  $\le$  is a radical relation on R satisfying also 0  $\le$  a  $\Rightarrow$  a = 0, we call (R, $\le$ ) an  $r_0$ -domain.

In our talk we proved that the class of  ${\bf r_0}\text{--domains}$  admits a model companion, axiomatized by

- (i) K = Quot R is algebraically closed,
- (ii) R is a Bezout domain,
- (iii) the "lattice" of (R,≤) is dense,
- (iv) a 'Local-Global-Principle' for the existence of point on irred, varieties in R.

As a consequence, one obtains a complete axiomatization (effective), and hence the decidability of  $\tilde{\mathbf{z}}$ , the ring of all algebraic integers. Adding the condition that R be formally real and changing (i) to 'K is real closed', one obtains decidability of  $\tilde{\mathbf{z}} \cap \mathbb{R}$ , the ring of real algebraic integers. The same can be done for the ring  $\tilde{\mathbf{z}} \cap \mathbb{Q}_p$  of p-adic algebraic integers.



#### J. SCHMID:

#### Model theory of real holomorphy ring

In continuation of Prestel's talk the following was shown:

- (1) The theory of real r-domain has a model companion (in the language:  $+,\cdot,-,0,1,\le$ ).
- (2) The theory of integrally closed integral domains has no model companion (in the language:  $+,\cdot,-,0,1,|$ ).
- (3) The theory of real holomorphy ring has no model companion (in the language:  $+,\cdot,-,0,1,|$ ).

PART B: Special talks

#### Z. CHATZIDAKIS:

#### Cohomological dimension of non-standard number fields

Let  $Q^*$  be a non-standard extension of Q, let  $\mu$  denote the group of all n-th roots of unity, and let  ${Q^*}^{ab}$  be the maximal abelian extension. Then we have:

$$cd(Q^*(i)) = 2, cd(Q^{*ab}) = 1.$$

In contrast with the standard case,  $Gal(Q^*/Q^*(\mu))$  has a closed subgroup isomorphic to  $\hat{\mathbf{Z}} \times \hat{\mathbf{Z}}$  and therefore  $cd(Q^*(\mu)) = 2$ . This implies  $Q^*(\mu) \neq Q(\mu)$  and  $Q^*(\mu_p) \neq Q(\mu_p)$ .

#### M. A. DICKMANN:

# A combinatorial geometric structure on the space of orders of a field

A structure of combinatorial geometry (matroid) is defined on the set  $\chi(K)$  of orders of any formally real field K, by means of a natural closure operator  $cl_{\mathbf{v}}$ .



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In  $\chi(K)$  lines consist of 2 points (i. e. are trivial) and circuits have cardinality  $\geq 4$ . Further  $\chi(K)$  also has the following properties:

- (i) If  $P \in cl_k(x)$  ({P},  $x \subseteq \chi(K)$ ), then P is in the closure of a finite subset of x.
- (ii) The closure of a finite set is finite (indeed, the closure of n points has cardinality  $\leq 2^{n-1}$ ).

From well-known work of Bröcker we also get:

(iii)  $cl_k$  is trivial (i. e.  $cl_K(x) = x$  for any  $x \subseteq \chi(K)$ ; equivalently,  $\chi(K)$  is a free matroid) iff the field K is SAP.

Here is a sample of the results we obtain:

- (iv) All matroids  $\chi(K)$  are binary (i. e. coordinatizable over the 2-element field,  $\mathbf{F}_2$ ).
- (v) Circuits of  $\chi(K)$  have even (finite) cardinality.
- (vi) There are both examples of fields K for which
  a)  $\chi(K)$  is unimodular (i. e. coordinatizable over every field);
  for example  $K = \mathbb{R}(X)$ .
  - b)  $\chi(K)$  is <u>not</u> unimodular; e. g.,  $K = \mathbb{R}(X_1, ..., X_n)$ , for any  $n \ge 3$ .

The matroids  $\chi(K)$  decompose as follows:

- (vii)  $\chi(K) = Arch(K) \oplus \bigoplus_{i \in I} M_i$
- where Arch(K) stands for the set of all archimedean orderings of K and the M $_1$ 's are the non-trivial valuation-components of  $\chi(K)$  Note cl $_K$  is trivial on Arch(K).
- (viii) The relation " $_{c}$ " of circuit connectivity in  $\chi(K)$  turns out to be equivalent to the following special case:

 $P \sim_{m} Q \Leftrightarrow P = Q$  or there is a 4-element circuit containing P and Q.

In particular, " $\sim$ " is an equivalence relation (a result proved by Marshall, who in a different terminology considered the relation  $\sim$ " in 1978-79).





(ix) P  $_{\rm C}$  Q implies that P,Q are in the same valuation-component of  $\chi(K)$ . The converse is also true, for valuation-components of cardinality  $\neq$  2.

This matroid-theoretic analysis gives new insights and forces a complete reorganisation of the theory of ordering spaces, leading to new and interesting results.

## Y. L. ERSHOV:

## A characterization of Kochen rings of PMC-fields

Let  $M = \{P_0, \dots, P_n\}$  be a finite set of prime numbers;  $\pi = \prod_{i \le n} P_i$   $\pi_i = \pi P_i^{-1}$ ,  $i \le n$ . If F is a field of characteristic 0 and v is a valuation on F, we call v a  $\underline{\pi\text{-valuation}}$  iff  $v(\pi)$  is the minimal positive element of  $\Gamma_v$  and  $\Gamma_v$  is a prime field.

Let  $\gamma_{\pi}(x) = \sum_{i \leq n} \frac{1}{P_i} \frac{x^{P_i} - x}{\pi_{ix}(x^{P_i} - x) - 1}$ ;  $Z(\gamma_{\pi}(F))$  is the ring generated by the elements  $\gamma_{\pi}(a)$ ,  $a \in F$  and  $\pi_{i}a(a^{P_i} - a) \neq 1$ ,  $i \leq n$ . Kochen ring  $K_{\pi}(F)$  of a field F is the ring  $\{\frac{a}{1+\pi b} | a, b \in Z(\gamma_{\pi}(F)), 1 + \pi b \neq 0\}$ .

Proposition 1.  $K_{\pi}(F) = \bigcap \{R_{\mathbf{v}} | \mathbf{v} \text{ is a } \pi\text{-valuation of } F\}.$ 

Let  $R_{\pi}$  be the class of all r-rings  $\mathbb{R} = \langle R, \leq \rangle$  (r-rings are rings with radical relation [1]) such that:

- 1) R is a domain, F = Quot(R) is a field of characteristic 0.
- 2)  $0 \le \pi$ .
- 3) For every  $a \in F$   $1 + \pi a^2 \neq 0$  and  $\frac{a}{1+\pi a^2} \in R$ .
- 4) Let  $I_{\pi} = \{a \mid a \in \mathbb{R}, \pi \leq a\}$  then  $\mathbb{R}/I_{\pi} \models \forall x (\prod_{i \leq n} (x^{p_i} x) = 0)$ .

 $\mathbb{R} \in \mathbb{R}_{\pi}$  is a <u>weakely  $\mathbb{E}_r$ -closed</u> iff  $\mathbb{R} \leq \mathbb{R}_0 \in \mathbb{R}_{\pi}$  and  $\mathbb{F}_0 = \mathbb{Q}$ uot  $\mathbb{R}_0$  is a regular extension of  $\mathbb{F} = \mathbb{Q}$ uot  $\mathbb{R}$  implies  $\mathbb{R} \leq_1 \mathbb{R}_0$ .

<u>Proposition 2.</u>  $\mathbb{R}$  weakely  $\mathbf{E_r}$ -closed iff the following conditions are satisfied:



- 1.  $F = Quot R is a P_MC$ -field (see [2] for the definition).
- 2.  $R = K_{\pi}(F)$ .
- The radical relation < in IR = <R, <> is defined by the set of all maximal ideals of R.

Remarks 1. If in the definition of  $R_{\pi}$  we change conditions 3) and 4) on a condition 3\*): For every  $a \in F$   $\gamma_{\pi}(a)$  is defined and  $\gamma_{\pi}(a) \in R$ , then for the corresponding class  $R_{\pi}^*$  of the r-rings we have the same characterization (proposition 2) of the weakely  $E_r$ -closed r-rings.

2. If we define  $\mathbb{R} \in \mathbb{R}_{\pi}(\mathbb{R}_{\pi}^{*})$  to be <u>weakely closed</u> iff  $(\mathbb{R} \leq \mathbb{R}_{0} \in \mathbb{R}_{\pi}(\mathbb{R}_{\pi}^{*}))$   $\Rightarrow \mathbb{R} \leq_{1} \mathbb{R}_{0}$  then for a characterization of the weakely closed r-rings it is necessary to change condition 1 in proposition 2 to condition 1\*:  $\mathbb{R}_{\pi} = \mathbb{R}_{\pi} =$ 

#### References

- A. Prestel, J. Schmid, Existentially closed domains with radical relations, preprint, 1989.
- V. M. Künz, Corps multiplement pseudo-p-adicement clos, C.R. Acad. Sci. Paris, t. 309, Série I, (1989), 205-208.

#### M. FRIED:

# Every finite group is a Galois group over a PAC field

The reference [M. Fried and M. Jarden, Field Arithmetic, Springer-Verlag, Ergebnisse 11] is for convenience.

A field is  $\omega$ -free if each finite embedding problem over the field is solvable. Roquette was the first to note that every  $\omega$ -free PAC field is Hilbertian (e. g., [FJ; p. 392]). It is a conjecture [FJ; p. 393] that a P(seudo) A(lgebraically) C(losed) Hilbertian field is  $\omega$ -free.

For embedding problems involving just solvable groups the data is compatible with this conjecture [FJ; p. § 24.10]. A special case of the main theorem of THE INVERSE GALOIS PROBLEM AND RATIONAL POINTS ON MODULI SPACES, a recent preprint with Helmut Völklein, is that every finite group is the Galois group of an extension of P(x)





regular over any PAC field P. In particular, if P is Hilbertian, then every group is a Galois group over P. For example, this includes the PAC Hilbertian Fried-Jarden fields P that are Galois over Q with G(L/Q) equal to products of (many) copies of  $S_n$ 's [FJ; p. 224]. But we don't yet know if such fields are  $\omega$ -free.

#### M. JARDEN:

## Compositum of Galois extensions of Hilbertian fields

A report on a joint work with Dan Haran

Theorem: The compositum of two Galois extensions of a Hilbertian field, neither of which contains the other, is Hilbertian.

Corollary 1: The compositum of two nontrivial linearly disjoint Galois extensions of a Hilbertian field is Hilbertian.

Corollary 2: The separable closure of a Hilbertian field K is not the compositum of two nontrivial linearly disjoint Galois extensions of K.

Corollary 1 (resp. Corollary 2) solves problem 12.18 (resp. 12.19) of "Field Arithmetic" (written jointly with Micael Fried).

Among the toals used in the proof are wreath product of Group, and Weil's descent method. We use ideas and results of Kuyk, Weissauer and Chatzidakis.

#### F.-V. KUHLMANN:

Henselian rational function fields and the model theory of valued fields

A henselization of a valued function field is called "henselian function field"; if it is a henselization of some rational function field then it is called "henselian rational".



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An elementary class K of valued fields is called Ax-Kochen-Ershov-(AKE-) class if for every extension (L,v) | (K,v) with (K,v), (L,v)  $\in$  K. The following implication holds:  $\overline{K} \setminus_{\gamma} \overline{L} \wedge v(K) \setminus_{\gamma} v(L) \Rightarrow (K,v) \setminus_{\gamma} (L,v)$ .

#### Thm: K is an AKE-class if

- a) All (K,v) ( K are algebraically complete.
- b) For every  $(K,v) \in K$  and  $K' \subseteq K$  rel. alg. closed with  $\overline{K} | \overline{K'}$  algebraic and v(K)/v(K') torsion group,  $(K',v) \in K$  and  $\overline{K} = \overline{K'}$ , v(K) = v(K').
- c) For every (K,v)  $\in$  K, every immediate henselian function field of transcendence degree 1 over (K,v) is henselian rational.

We discuss all known AKE-classes in view of this criterion, and in particular the meaning of condition c). We discuss restrictions of the AKE-principle to separable extensions  $(L,v) \mid (K,v)$ . Finally, we give an axiomatization of the elementary classes of "semitame" fields (with fixed finite p-degree) which includes the power series fields  $\mathbf{F}_{\mathbf{p}}((t))$ . These classes satisfy c) which yields that every semitame field is existentially closed in every immediate extension of transcendence degree 1; but it is not known whether classes of semitame fields are AKE-class (if they are not classes of tame fields).

#### A. MACINTYRE:

#### On p-adic exponentiation

I consider  $\mathbf{z}_p$ , enriched by  $f(x) = (1+p)^x$  and various p-adic "trigonometric functions". By proving a constructive uniform version of Strassman's Finiteness Theorem, and adapting the technology of Weierstrass systems in the style of van den Dries, I prove the above structure is constructively model-complete. Work in progress on a constructive version of Greenberg approximation should give the decidability of the above structure, relative to the p-adic Schanuel Conjecture.





#### D. MARKER:

## Additive reducts of real closed fields

We prove that if  $X \subseteq \mathbb{R}^n$  is semialgebraic but not semilinear then the graph of multiplication on a bounded interval is definable in  $(\mathbb{R},+,<,X)$ . This is joint work with Anand Pillay and Kobi Peterzil.

#### E. A. PALJUTIN:

## On structure of models of Horn theories with few models

A description is given of transitive Horn theories with few models in terms of  $\omega_1$ -categorical transitive Horn theories and theories with a chain of equivalence relations and some permutations.

#### T. PHEIDAS:

# Decision problems for the existential theories of certain fields

The following result was presented: The analogue of Hilbert's Tenth problem for a field of rational functions  $\mathbf{F_q}(t)$  over a finite field with  $\mathbf{q}$  ( $\mathbf{q}$  odd) elements - in the language of addition, multiplication,  $\mathbf{0}$ ,  $\mathbf{1}$  and  $\mathbf{t}$  - has a negative answer. Interconnections of the problem with the similar problems in other function fields and Hilbert's Tenth problem for the rational numbers where discussed.

#### A. PILLAY:

# Superstable differential fields

(with Z. Sokolovic)

We prove that if K is a superstable differential field, then K is closed for strongly normal extensions, where



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<u>Def.</u> L  $\supset$  K (differential fields). L is a strongly normal extension of K if L is finitely generated over K (as a differential field), and L has finite transcendence degree over K and for any isomorphism  $\sigma$ L of L over K (L, $\sigma$ L  $\subset$  U some big differentially closed field),  $\sigma$ L  $\subset$  <L,U<sub>C</sub>> where U<sub>C</sub> = constants of U.

We use U-rank, together with a characterization by Kolchin of G-extensions of K (G an algebraic group).

#### F. POP:

## Some applications of the Local-Global-Principle

We gave some consequences of the following theorem, see e. g. Rumely, Contor-Roquette, Moret-Bailly:

The Strong Local-Global-Principle. Let K be a global field,  $\Sigma, \Sigma_\star \subseteq \mathbf{P}(K)$  with  $\Sigma$  finite and  $\Sigma \cup \Sigma_\star \neq \mathbf{P}(K)$ ,  $\Sigma \cap \Sigma_\star = \emptyset$ . Then for any affine abs. irred. variety  $\mathbf{V}|K$  it holds: Suppose that open nonempty sets  $\mathbf{U}_\mathbf{V} \subseteq \mathbf{V}_{\text{reg}}(K_\mathbf{V}) \ \forall \mathbf{v} \in \Sigma$ , are given and further  $\mathbf{V}_{\text{reg}}(K_\mathbf{V}) \neq \emptyset$  for every  $\mathbf{v} \in \Sigma_\star$ . Then there exist regular points  $\mathbf{x} \in \mathbf{V}_{\text{reg}}(K)$  which are totally v-adic and lie in  $\mathbf{U}_\mathbf{V}$  for  $\mathbf{v} \in \Sigma$  and are v-integral for  $\mathbf{v} \in \Sigma_\star$ .

<u>Applications:</u> Let  $K^{\Sigma}$  denote the maximal extension of K in which all  $v \in \Sigma$  totally split and  $K^{\Sigma}$  the prolongations of  $\Sigma$  to  $K^{\Sigma}$ .

Theorem.  $K^{\Sigma}$  is pseudo  $K^{\Sigma}$ -closed, i. e. if an affine abs. irred. variety  $V | K^{\Sigma}$  has  $K_{V}$ -rational simple points for all  $v \in K^{\Sigma}$ , then V has  $K^{\Sigma}$  simple points.

As a consequence it holds:

Theorem. Let  $G^{\Sigma}$  denote the subgroup of  $G_K$  generated by all decomposition groups  $Z(\tilde{\mathbf{v}}|\mathbf{v})$ ,  $\mathbf{v} \in \Sigma$ , and let  $G^{\Sigma}$  denote this family. Then  $G^{\Sigma}$  is relatively  $G^{\Sigma}$ -projective, i. e. if an extension of a finite group H by  $G^{\Sigma}$ 

$$1 + H + G + G^{\Sigma} + 1$$

locally over any  $\Gamma = Z(\tilde{v}|v) \in G^{\Sigma}$  splits, then it globally splits.





An other problem discussed was the decidability of  $\kappa^\Sigma$  and of its maximally  $A^\Sigma\text{-extension.}$ 

#### V. WEISPFENNING:

## Gröbner bases and quantifier elimination

Let K be a field, let  $S = K[U_1, \ldots, U_m, X_1, \ldots, X_n]$  and fix a term order < on the set T of power-products of the  $X_i$ . Suppose F is a finite subset of S and let I(F) denote the ideal generated by F in S. We show how to compute a comprehensive Gröbner basis G for I(F), i. e. a finite G with  $F \subseteq G \subseteq I(F)$  such that for all specializations  $\phi$  of the parameters  $U_1, \ldots, U_m$  in some extension field K' of K,  $\phi[G]$  is a Gröbner basis wrt < for  $I(\phi[F])$  in K'[X]. As one of many applications, G easily yields quantifier-free polynomial conditions  $\Psi_d(\underline{U})$  on the parameters  $(-1 \le d \le u)$  such that for every specialization  $\phi$  of  $\underline{U}$  in some algebraically closed extension field K' of K,  $\Psi_d(\phi(\underline{U}))$  holds in K' iff dim  $V_{K'}(F) = d$ . For d = -1, this specializes to an elimination method for blocks of quantifiers in algebraically closed fields that is both asymptotically and practically fast.

## C. WOOD:

# Somewhat homogeneous partially ordered sets

 $\underline{\text{Defn.}}$  A structure M is  $\underline{\text{k-homogeneous}}$  if every partial isomorphism between k-element substructures of M extends to an isomorphism of M.

Theorem (Droste & Macpheson). A partially ordered set  $(M, \leq)$  "poset" homogeneous if and only if  $(M, \leq)$  is 1-homogeneous and 4-homogeneous.

Droste and Macpheson also gave examples to show that 4 is sharp, and asked how many such examples exist.

## Theorem (Saracino and Wood).

- a) There are  $2^{*0}$  non-isomorphic countable  $*_0$ -categorical universal 1-, 2- and not 3-homogeneous posets.
- b) There are  $2^{\approx_0}$  non-isomorphic countable  $\approx_0$ -categorical universal 1-,2-,3- and not 4-homogeneous posets.



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