MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 7/1990

Nukleare Fréchetträume

4.2. bis 10.2.1990

The meeting was organized by E. Dubinsky (Purdue University), R. Meise (Düsseldorf) and D. Vogt (Wuppertal). As in the previous meetings the participants appreciated the pleasant atmosphere which contributed essentially to the success of the conference.
In 23 lectures the speakers presented recent results on the theory of Fréchet spaces and their duals and on the application of this theory to problems of analysis. In particular problems like structural properties of Fréchet spaces, splitting properties of exact sequences and surjectivity of convolution- and partial differential operators were treated in several lectures.
The 29 mathematicians from Austria, Bulgaria, Finland, Italy, Poland, Russia, Spain, Turkey, USA, and West Germany, which attended the meeting, enjoyed the comfortable environment and made extensive use of the opportunity to participate in fruitful discussions, which gave valuable stimulation to further research.
V. P. Kondakov of Rostov on Don, who could not take part in the conference, sent us an abstract, which is contained in the report.

Abstracts

Aytuna, A.

Analytic function spaces and linear operators

The following results are discussed.

1. Let M be a Stein manifold, then $\mathcal{O}(M)$ is tamely isomorphic to a power series space of infinite type if and only if there exists a plurisubharmonic continuous function proper and maximal off a compact set.
2. If V is a closed submanifold of a Stein manifold M for which $\mathcal{O}(M)$ is tamely isomorphic to a power series space of infinite type then there exists a l. tame extension operator if and only if there exists a plurisubharmonic function p maximal off a compact set and satisfies $A\Phi + B \le p \le C\Phi + D$ where Φ is the tameness function of M.

Bonet, J.

Remarks on some questions of Grothendieck

In the first part we report on joint work with J. C. Diaz and J. Taskinen. We introduce the class of Fréchet- and (DF)-T^*-spaces. We show that they provide a general frame in which the answer to several problems of Grothendieck on tensor products is positive. We have: (a) If E and F are Fréchet T^*-spaces then every bounded subset of $E\hat{\otimes}_\pi F$ can be lifted by bounded sets, (b) If E is a Fréchet-T^*-space and G is a (DF)-T^*-space, then $L_b(E,G)$ is (DF), (c) If G and H are (DF)-T^*-spaces, then $G\otimes_\varepsilon H$ is also (DF). In the second part we present some joint results with A. Defant and A. Galbis that emphasize the relation of the questions of Grothendieck with the local theory of Banach spaces. Some consequences in infinite holomorphy are also discussed.

Braun, R. W.

Surjectivity of convolution and partial differential operators on Roumieu classes of ultradifferentiable functions

(Joint work with R. Meise and D. Vogt)

For $\alpha>1$ let

$$\Gamma^{\{\alpha\}}(\mathbb{R}^N) = \{ f \in C^\infty(\mathbb{R}^N) \mid \forall k\ \exists m: \sup_{|x|\le k} \sup_{\beta\in\mathbb{N}_0^N} |f^{(\beta)}(x)| \exp(-m|\beta|^\alpha) < \infty \}$$

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denote the classical Gevrey class of exponent α. For $\mu \in \Gamma^{\{\alpha\}}(\mathbb{R})'$ we denote by $\hat{\mu}$ its Fourier transform and by $T_\mu : \Gamma^{\{\alpha\}}(\mathbb{R}) \to \Gamma^{\{\alpha\}}(\mathbb{R})$, $T_\mu f(x) = \langle \mu, f(x - \cdot)\rangle$ the convolution operator induced by μ.

Theorem I: T_μ is surjective if and only if

(1) T_μ has a fundamental solution

and (2) there is a partition $V(\hat{\mu}) = V_0 \cup V_1$ of $V(\hat{\mu}) = \{\hat{\mu}(z)=0\}$ such that

$$\lim_{z\in V_0} \frac{|\mathrm{Im}\, z|}{|z|^{1/\alpha}} = 0 \quad \text{and} \quad \liminf_{z\in V_1} \frac{|\mathrm{Im}\, z|}{|z|^{1/\alpha}} > 0.$$

Theorem II: A partial differential operator $P(D)$: $\Gamma^{\{\alpha\}}(\mathbb{R}^N) \to \Gamma^{\{\alpha\}}(\mathbb{R}^N)$ is surjective if and only if $\forall \mu\ \exists k\ \forall L,m\ \exists C\ \forall u \in PSH(V)$ we have that (1) and (2) imply (3) where

(1) $\forall z \in V$: $u(z) \le \mu|\mathrm{Im}\, z| + |z|^{1/\alpha}$

(2) $\forall z \in V$: $u(z) \le L|\mathrm{Im}\, z|$

(3) $\forall z \in V$: $u(z) \le k|\mathrm{Im}\, z| + \frac{1}{m}|z|^{1/\alpha} + C$.

The proof is based on Palamodov's Proj^1-functor and, in the case of Theorem II, on techniques developed by Hörmander to solve the corresponding problem in the space of real analytic functions.

Dierolf, S.

<u>A remark on distinguished spaces</u>

For the following 3-space-problem:

> Given a short exact sequence $0 \to F \to E \to E/F \to 0$ of Fréchet spaces such that F and E/F are distinguished, is then E also distinguished ?

a positive answer holds if - in addition - we assume that the quotient map $E \to E/F$ lifts bounded sets. In contrast to this we construct a non-distinguished, separable Fréchet space E containing a reflexive Fréchet subspace F such that the quotient space E/F is a Banach space.

Domański, P.

<u>Injective Fréchet spaces of continuous functions</u>

Let T be a locally compact σ-compact topological space such that the Fréchet space of continuous functions $C(T)$ equipped with the compact-open topology is injective. We prove that either $C(T)$ contains a product $\prod_{i\in\mathbb{N}} \ell_\infty(\Gamma_i)$, Γ_i

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uncountable, or T splits into a sequence of disjoint compact-open sets (T_n) and, therefore, $C(T) \cong \prod_{n\in\mathbb{N}} C(T_n)$. Other topological properties of T are also obtained. We use in the proof the method developed for another purpose and compact T by Wolfe (1974).
We give an example of a completely regular topological space T such that $C(T) \cong \ell_\infty^{\mathbb{N}}$ (i. e. , C(T) is injective) while T contains no open non-empty extremally disconnected (or locally compact) subset, which is impossible in the Banach case by the results of Amir (1962).

Floret, K.

Composition of accessible operator ideals

(Joint work with A. Defant, Oldenburg)
A quasi-Banach operator ideal $(\mathcal{A}, A)$ (in the sense of A. Pietsch) is called left-accessible, if for all Banach spaces E, N (where N is finite-dimensional), $T \in \mathcal{A}(E,N)$ and $\varepsilon > 0$ there is a factorization

$$\begin{array}{ccc} E & \xrightarrow{\quad T \quad} & N \\ & \searrow \quad \nearrow_{S} & \\ & E/L & \end{array}$$

with a finite codimensional subspace $L \subset E$ such that $A(S) \leq (1+\varepsilon)A(T)$. It is called right-accessible, if every $T \in \mathcal{A}(N,E)$ factors through $N \xrightarrow{S} M \hookrightarrow E$ with $A(S) \leq (1+\varepsilon)A(T)$. The operator ideal is called accessible, if it is both: right- and left-accessible. These notions are analogues of notions in the metric theory of tensor products: A maximal normed operator ideal is (left/right) accessible if and only if its associated tensornorm is (left/right) accessible. No non-accessible operator ideal seems to be known. A p-Banach operator ideal is accessible if and only if $\mathcal{A}^{\min} := \overline{\mathfrak{F}} \circ \mathcal{A} \circ \overline{\mathfrak{F}} = \mathcal{A} \circ \overline{\mathfrak{F}} = \overline{\mathfrak{F}} \circ \mathcal{A}$ (where $\overline{\mathfrak{F}}$ is the ideal of approximable operators). In particular: Minimal p-Banach operator ideals are accessible.
Cyclic composition theorem: Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be quasi-Banach operator ideals such that $\mathcal{A} \circ \mathcal{B} \subset \mathcal{C}$.

(a) If $\mathcal{A}$ is right-accessible then $\mathcal{C}^* \circ \mathcal{A} \subset \mathcal{B}^*$.

(b) If $\mathcal{B}$ is left-accessible then $\mathcal{B} \circ \mathcal{C}^* \subset \mathcal{A}^*$.

Applications of this result to the composition of minimal operator ideals, the Persson-Pietsch multiplication table for p-summing, p-integral, and p-nuclear operators, Grothendieck's inequality, quotient formulas $(\mathcal{A} \circ \mathcal{B})^* = \mathcal{B}^* \circ \mathcal{A}^{-1}$ and to Kwapien's characterization of subspaces of quotients of $L_p(\mu)$ were mentioned.
Details will appear in the forthcoming book (with A. Defant) on "Tensornorms and operator ideals".

Haslinger, F.

Convolution equations and related properties of conformal mappings

We consider a nuclear Fréchet algebra $\mathcal{F}_P$ of entire functions $f: \mathbb{C}^n \to \mathbb{C}$ such that

$$\int_{\mathbb{C}^n} |f(z)|^2 \exp(-2p_m(z))\, d\lambda(z) < \infty \quad \forall m \in \mathbb{N},$$

where $P = (p_m)$ is a suitable family of weight functions. Let $T \in \mathcal{F}_P'$ be a continuous linear functional, $T \neq 0$, and define a convolution operator $\mathcal{T}_T: \mathcal{F}_P \to \mathcal{F}_P$ by $\mathcal{T}_T(f) = T*f$, where $T*f(z) = T(f(z+\cdot))$. We discuss the surjectivity of convolution operators $\mathcal{T}_T$. We state a necessary condition for surjectivity in terms of the Young conjugate functions p_m^* and a sufficient condition with the help of conformal mappings. From the theory of convolution equations we get on the other hand some new results about the boundary behavior of the involved conformal mappings.

Kocatepe, M.

A note on the conditions S_2^* and DS^*

It has been shown that if $\lambda(A)$ is a Fréchet Köthe space such that $\mathrm{Ext}(\lambda(A), \lambda(A)) = 0$ or the pair $(\lambda(A), \lambda(A))$ has the property DS^* then $\lambda(A)$ has a step-space $\lambda(A)_I$ which has property DN or $\bar{\Omega}$.
It also has been shown that if $\lambda(A)$ is Schwartz and has the property $\bar{\Omega}$ then the condition $(\lambda(A), \lambda(A)) \in DS^*$ is equivalent to the condition that $\lambda(A) \cong \lambda(B)$ where (b_j^k) is defined as follows: there is a sequence of increasing functions $\psi_k: (0,1) \mapsto (0,1)$ such that $\sqrt{x} \le \psi_k(x)$, $x \in (0,1)$ and a sequence (β_j) such that $0<\beta_j<1$ and $\beta_j \downarrow 0$ and $\psi_k \circ \ldots \circ \psi_1(\beta_j) \le \psi_{k+1}(\beta_j)$ and $b_j^1 = \beta_j$, $b_j^{k+1} = \psi_k \circ \ldots \circ \psi_1(\beta_j)$.

Kondakov, V. P.

Bases in complemented subspaces of weak-mixed Köthe spaces

A Köthe space

$$\ell_1([a_r(n)], \ell_1^{k(n)}) = \{x=(x_n): x_n \in \ell_1^{k(n)}, \sum \|x_n\|_{\ell_1} a_r(n) = |x|_r < \infty \ \forall\, r \in \mathbb{N}\}$$

is called weak-mixed (block-unstable), if $k(n)<\infty$, $n \in \mathbb{N}$, and

$$\exists\, r \;\forall\, p \;\exists\, q \;\forall\, s \qquad \lim \frac{a_p(n+1)}{a_q(n+1)} \frac{a_s(n)}{a_r(n)} = 0.$$

Theorem 1. In a weak-mixed Köthe space every complemented subspace has an unconditional basis.
This extends a result of E. Dubinsky and D. Vogt for the case of a tame power series space. For any projection P in a weak-mixed Köthe space E there exists an automorphism T of E such that $T^{-1}PT$ is a diagonal projection.
Theorem 2. Assume that the F-space E is isomorphic to some weak-mixed Köthe space. Then all unconditional bases in E are quasi-equivalent.

Langenbruch, M.

Splitting of the $\bar\partial$-complex in weighted L^2-spaces

Let $\bar\partial$ be the Cauchy-Riemann-system and let $W = \{W_n \mid n \geq 1\}$ be an increasing system of weights, defined on a pseudoconvex open set $\Omega \subset \mathbb{C}^N$ (W is supposed to satisfy some mild technical conditions).

Let $L^2(W, \Omega) := \{f \in L^2_{loc}(\Omega) \mid \|f\|_n^2 := \int |f(z)|^2 e^{-2W_n(z)} dz < \infty$ for some $n \geq 1\}$ and $\mathcal{H}_2(W,\Omega) := L^2(W, \Omega) \cap \mathcal{H}(\Omega)$. Then $\mathcal{H}_2(W,\Omega)$ is complemented in $L^2(W, \Omega)$, if the following condition is satisfied:

For any $t \in \Omega$ there are plurisubharmonic functions Φ_t
and for any $n \geq 1$ there are $I(n)$, $A(n)$: (*)
$\Phi_t(t + \xi) \geq 0$ for $|\xi|_\infty \leq r(t)$
$\Phi_t(z) \leq W_{I(n)}(z) - W_n(t) + A(n).$

Explicit continuity estimates for the continuous projection in $L^2(W, \Omega)$ onto $\mathcal{H}_2(W, \Omega)$ can be proved. If $L^2(W, \mathbb{C}^N)$ is shift invariant, then the $\bar\partial$-complex on $L^2(W, \Omega)$ splits, if (*) is satisfied. In many cases (e. g. for $\Omega \subset \mathbb{C}$ or for certain weighted algebras $\mathcal{H}_2(W, \mathbb{C}^N)$), (*) is also necessary for the splitting of the $\bar\partial$-complex.

Meise, R.

Whitney's extension theorem for non-quasianalytic classes of ultradifferentiable functions

(Joint work with J. Bonet, R. W. Braun and B. A. Taylor)
Let ω: $[0, \infty[\;\rightarrow [0, \infty[$ be a continuous increasing function which satisfies

(α) $\omega(2t) = O(\omega(t))$ (β) $\int_1^\infty \frac{\omega(t)}{t^2}\,dt < \infty$

(γ) $\log t = o(\omega(t))$ (δ) $\varphi: x \longmapsto \omega(e^x)$ is convex

Define $\varphi^*: y \longmapsto \sup(yx - \varphi(x)),\ y \geq 0$ and let

$\mathcal{E}_{\{\omega\}}(\mathbb{R}^N) :=$

$$\{ f \in C^\infty(\mathbb{R}^N) \mid \forall K \subset\subset \mathbb{R}^N\ \exists m \in \mathbb{N}: \sup_{x\in K} \sup_{\alpha \in \mathbb{N}_0^N} |f^{(\alpha)}(x)|\, e^{-\frac{1}{m}\varphi^*(m|\alpha|)} < \infty \}.$$

For a compact set K in $\mathbb{R}^N$ denote by $\mathcal{E}_{\{\omega\}}(K)$ the set of all $F = (f^\alpha)_{\alpha\in\mathbb{N}_0^N}$ in $C(K)^{\mathbb{N}_0^N}$ for which there exist $m \in \mathbb{N}$ and $M>0$ so that

(1) $\sup_{x\in K} \sup_{\alpha\in\mathbb{N}_0^N} |f^\alpha(x)| \exp(-\frac{1}{m}\varphi^*(m|\alpha|)) \leq M$

(2) $\forall l \in \mathbb{N}_0,\ |\alpha| \leq l,\ x, y \in K: |(R_x^l F)_\alpha(y)| \leq M \frac{|x-y|^{l+1-|\alpha|}}{(l+1-|\alpha|)!} \exp(\frac{1}{m}\varphi^*(m(l+1))),$

where $(R_x^l F)_\alpha(y) := f^\alpha(y) - \sum_{|\alpha+\beta|\leq l} \frac{1}{\beta!} f^{\alpha+\beta}(x)(y-x)^\beta.$

Then $\rho_K: f \longmapsto (f^{(\alpha)}|_K)_{\alpha\in\mathbb{N}_0^N}$ maps $\mathcal{E}_{\{\omega\}}(\mathbb{R}^N)$ into $\mathcal{E}_{\{\omega\}}(K)$.

The following theorem extends a result of Bruna from 1980.

Theorem. For ω as above, the following conditions are equivalent:

(1) $\rho_K: \mathcal{E}_{\{\omega\}}(\mathbb{R}^N) \to \mathcal{E}_{\{\omega\}}(K)$ is surjective for each compact set K in $\mathbb{R}^N$.

(2) $\rho_{\{0\}}: \mathcal{E}_{\{\omega\}}(\mathbb{R}^N) \to \mathcal{E}_{\{\omega\}}(\{0\})$ is surjective.

(3) $\exists C \geq 1\ \forall y > 0: \int_1^\infty \frac{\omega(yt)}{t^2}\,dt \leq C\omega(y) + C.$

Melichov, S. N.

On absolutely representing systems of quasipolynomials in spaces of analytic functions

Let $\mathfrak{G}$ be a bounded convex region in $\mathbb{C}$ with support function $h(-\varphi)$, let $\mathfrak{L}$ be an exponential function with indicator $h(\varphi)$, and let Λ be a set of zeros λ of $\mathfrak{L}$ with multiplicity $p(\lambda)$. In the Fréchet space $A(\mathfrak{G})$ of functions analytic in $\mathfrak{G}$ we investigate absolutely representing systems (ARS) of quasipolynomials with exponent in Λ. In particular we prove

1. Theorem. The following are equivalent:

(i) $\mathfrak{L}$ is a function of completely regular growth.

(ii) For some system of disks $S_j := \{z \in \mathbb{C} \mid |z-\mu_j| < r_j\}$, $j \in \mathbb{N}$, of zero linear density, for some sequence of nonintersecting nonempty finite subsets $\Lambda_j \subset \Lambda$, $j \in \mathbb{N}$, and for some system of $(m(\lambda))_{\lambda \in \Lambda} \subset \mathbb{N}^{\Lambda}$ such that $m(\lambda) \leq p(\lambda)$ $\forall \lambda \in \Lambda$, $\Lambda_j \subset S_j$ $\forall j \in \mathbb{N}$, $\lim_{j \to \infty} \frac{m_j}{|\mu_j|} = 0$, $m_j := \sum_{\lambda \in \Lambda_j} m(\lambda)$, $j \in \mathbb{N}$, in $A(\mathfrak{G})$ there exists an ARS of quasipolynomials $(w_{j,q})_{j=1,\ q=0}^{\infty \quad m_j-1}$ such that $w_{q,j}(z) \in \operatorname{span}\{z^p \exp \lambda z \mid 0 \leq p \leq m(\lambda)-1,\ \lambda \in \Lambda_j\}$, $0 \leq q \leq m_j-1$, $j \in \mathbb{N}$.

Theorem 1 strengthens and extends some results of Leont'ev and Ju. Korobeinik on representation of analytic functions by series of exponentials and quasipolynomials.

Metafune, G.

A special class of quojections

A first order quojection is a Fréchet space E isomorphic to a quotient of a countable product of Banach spaces by a Banach subspace. Most of known quojections are of this type. A quojection is of the first order if and only if there is a continuous seminorm on it whose kernel is a product. A first order quojection $E = P/B$ is a product if and only if there exists $Z \supset B$, Z Banach and complemented in P or, equivalently, if there exists an automorphism T of P such that $T(B) \subset \prod_{i=1}^{k} X_i$, for a suitable k, where $P = \prod_i X_i$. New examples of twisted quojections are given. A twisted first order quojection is not complemented in any product. If F is a (general) twisted quojection and X is a Banach subspace of F, then F/X is twisted. Applications to the case of prequojections associated to first order quojections are given.

Napalkov. V.

Some questions in the theory of convolution equations

Let U be a convex domain in $\mathbb{C}^n$, H(U) is the space of analytic functions in U with the topology of compact convergence. If F is some analytic functional in $\mathbb{C}^k$ and K its convex support then F generates the following convolution equation:

$$F * y = g(t), \qquad (1)$$

where $y \in H(U+K)$, $g \in H(U)$. In the talk problems of surjectivity for the operator (1) are discussed.

Palamodov, V. P.

Splitness of complexes of differential operators with constant coefficients and related topics

Let A be the algebra of differential operators with constant coefficients in $\mathbb{R}^n$, M be an A-module of a finite type, F be either $\mathcal{E}(\Omega)$ or $\mathcal{D}'(\Omega)$ where $\Omega \subset \mathbb{R}^n$ is open and convex. Then the complex $Hom_A(R, F)$ is exact where R is a free A-resolvent of M and the problem is when does this complex split ? The special case when M corresponds to a scalar differential operator is known as L. Schwartz's problem and was investigated by R. Meise, B. A. Taylor and D. Vogt. A criterion of splitness of $Hom_A(R, F)$ for general M is given in the lecture in terms of complex analysis on the algebraic set supp(M) (to be exact on the set of algebraic varieties associated with M). Namely a PL-condition is used which is a variant of Phragmen - Lindelöf property but adapted to a given algebraic variety. In the trivial case of the variety $V = \mathbb{C}^n$ this condition is a corollary of well-kown S. N. Berenstein inequality. The class of modules M which satisfy this criterion for example for $\Omega = \mathbb{R}^n$ is much more rich than the class of strictly hyperbolic operators which have explicitely written splitting operators.

Pełczyński, A.

On ellipsoids of maximal volume inscribed into a convex compact symmetric body in $\mathbb{R}^n$ and Dvoretzky-Rogers-factorizations

$\mathcal{C}_n$ - the class of convex compact bodies in $\mathbb{R}^n$ symmetric with respect to the origin;

$B_n = \{x=(x(j)) \in \mathbb{R}^n : |x|_2 := (\Sigma x(j)^2)^{1/2} \leq 1\}$; $Q_n = \{x \in \mathbb{R}^n : \max |x(j)| \leq 1\}$.
Given $C \in \mathcal{C}_n$, E_C is (the unique) ellipsoid of maximal volume inscribed into C.

Theorem (F. John 1949). $\forall C \in \mathcal{C}_n$ E_C satisfies: $\exists m$ with $n \leq m \leq \frac{1}{2}n(n+1)$ such that $\exists x_1, x_2, \ldots, x_m \in \mathbb{R}^n$ and $\lambda_1, \lambda_2, \ldots, \lambda_m$ such that

(i) $|x_j|_C = |x_j|_{E_C} = 1$ $(j=1, 2, \ldots, m)$ (x_j are contact points)

(ii) $x = \sum_{j=1}^{m} \lambda_j \langle x, x_j \rangle_{E_C} x_j$ for $x \in \mathbb{R}^n$.

The converse is also true.

Theorem 1 (Volklor). Let $C \in \mathcal{C}_n$, $E \in C$ an ellipsoid. Assume that (i) and (ii) are satisfied with E_C replaced by E for some $x_1, \ldots, x_m \in \mathbb{R}^n$,

$\lambda_1, \ldots, \lambda_m > 0$ and some integer m (not necessary $\le \frac{1}{2}n(n+1)$). Then E is uniquely determined and $E = E_C$.

Terminology. A set (x_j) is called an essential set of contact points if it satisfies (i) and (ii) with some (λ_j); it is called minimal if m=n; it is called maximal equidistributed if $m=\frac{1}{2}n(n+1)$ and $\lambda_1=\lambda_2=\ldots=\lambda_m=\frac{2}{n+1}$.

A Dvoretzky-Rogers-factorization through a Banach space X with dim X = n is a pair of linear operators $u: \ell_n^2 \to X$, $v: X \to \ell_n^\infty$ such that $vu = i_{2\infty}^n: \ell_n^2 \to \ell_n^\infty$ is the formal identity.

$$dr(X) = \inf \|u\| \, \|v\|,$$

the infimum extends over all Dvoretzky-Rogers-factorizations through X.

Proposition 1 . The following are equivalent:

1) $dr(X) = 1$.

2) X is isometrically isomorphic to $(\mathbb{R}^n, C)$ for some $C \in \mathcal{C}_n$ such that $B_n \subset C \subset Q_n$.

3) X is isometrically isomorphic to $C \in \mathcal{C}_n$ satisfying (a) $B_n \subset C$
(b) $\exists$ orthonormal basis $(g_j)_{1\le j\le n}$ with $|g_j|_2 = |g_j|_C = 1$ for $j = 1, 2, \ldots, n$.

4) X is isometrically isomorphic to $(\mathbb{R}^n, C)$ with E_C having the minimal number of essential contact points.

Proposition 2. Given X with dim X = n, $\exists$ Y with $dr(Y) = 1$, $d(X,Y) \le dr(X)$ ($d(X,Y)$ the Banach Mazur distance)

Theorem Szarek (1987) $\forall n \; \exists X_n$ with $\dim X_n = n$ such that $\lim_n dr(X_n) = +\infty$.

Some positive results.

If dim X = n then $\exists$ $Z \subset X$ with $\dim Z = \alpha(n)$ and $dr(Z) \le 2$

$\alpha(n) \ge n^{1/2}$ Dvoretzky-Rogers (1950): $\alpha(n) \ge cn$ Bourgain - Szarek (1988)

Theorem 2. Let $X = (\mathbb{R}^n, C)$. Assume that E_C has a set of m essential contact points. Then $\exists Z$, dim Z = m, $dr(Z) = 1$ and X is isometrically isomorphic to a norm one complemented subspace.

Theorem 3. $\forall X$ with dim X = n, $dr(X) = 1$ $\exists$ $D \in \mathcal{C}_n$ such that $d(X, (\mathbb{R}^n, D)) \le 4$ and E_D has a maximal number of equidistributed essential contact points.

Poppenberg, M.

Subspaces and quotient spaces of (s) in the tame category

Let E and F be graded Fréchet spaces, i. e. Fréchet spaces equipped with fixed increasing sequences of seminorms $\| \; \|_n$ defining their topologies. A linear map $L: E \longrightarrow F$ is called tame if L satisfies continuity estimates $\|Lx\|_n \leq C_n \|x\|_{n+b}$ for some fixed b. There is given a complete characterization of all graded Fréchet spaces that are tamely isomorphic to a graded subspace or to a graded quotient space of (s), the space of rapidly decreasing sequences. To do this, the two tamely invariant properties (DNDZ) and (ΩDZ) are introduced corresponding to the topological properties (DN) of Vogt and (Ω) of Vogt and Wagner. Properties (DNDZ) and (ΩDZ) are also shown to be sufficient and necessary for two tame splitting theorems for certain tamely exact sequences of graded Fréchet spaces.

Taskinen, J.

A Fréchet-Schwartz space with basis having a complemented subspace without basis

Pełczyński showed in 1971 that for each Fréchet space E with finite dimensional decomposition we can find a Fréchet space F having a basis and a complemented subspace isomorphic to E. We show that if E is chosen to be a suitable nuclear Fréchet space without basis, the preceding construction leads to a Fréchet-Schwartz space F. This yields the result mentioned in the title.

Taylor, B. A.

Equivalence of Kaneko's Phragmen-Lindelöf principle for an operator and its principal symbol

(Joint work with R. Meise, D. Vogt)
A. Kaneko has shown that the following Phragmen-Lindelöf type inequality is the necessary and sufficient condition that every real analytic solution, f, of a constant coefficient partial differential equation $P(D)f = 0$ in an open set $U \setminus K$ extends across the obstacle, K. Here, K is an obstacle open to one side, $K = \overline{K} \cap \{x_n < 0\}$ and U is an open neighborhood of K, $U = \Omega \cap \{x_n < 0\}$, where $\overline{K} \subset \Omega$, such that $U \setminus K$ is connected. The condition is:

$V = \{P(z) = 0\}$.

For all $A>0$, $a>0$, there exists $B \geq A$, $b < b' \leq a$, and $\delta > 0$ such that all functions $u = \log|F|$ with F entire that satisfy for $z = (z', z_n) \in V$,

(KPL)

i) $u(z) \leq A\,|\mathrm{Im}\, z'| + a \max(-\mathrm{Im}\, z_n, 0) + o(|z'|)$

and ii) $u(z) \leq A\,|\mathrm{Im}\, z'| + b\,|\mathrm{Im}\, z_n| + o(|z|)$, $\quad |\mathrm{Im}\, z| < \delta(1+|\mathrm{Re}\, z|)$

also satisfy on V

iii) $u(z) \leq B\,|\mathrm{Im}\, z'| + b'\,|\mathrm{Im}\, z_n| + o(|z|)$.

We prove that an algebraic variety V satisfies (KPL) if and only if its tangent cone at infinity does also.

Terzioglu, T.

Unbounded operators and nuclear Köthe quotients

In a joint paper with S. Önal we had shown that a Fréchet space has a nuclear Köthe quotient if and only if it is not a prequojection.
In the proof the existence of an unbounded continuous linear operator between certain Fréchet spaces was exploited. Recently we have obtained that any two Fréchet spaces which are not prequojections have a common nuclear Köthe quotient.

Valdivia, M.

On totally reflexive Fréchet spaces

In this lecture we give the following results:

a) A Fréchet space E does not contain a copy of l_1 if and only if every sequence $\mu(E', E)$-null in E' is $\beta(E', E)$-null.

b) A Fréchet space E is totally reflexive if and only if it has the following properties:

(1) Every sequence $\mu(E', E)$-null in E' is $\beta(E', E)$-null.

(2) Every sequence $\sigma(E', E)$-null in E' is weakly locally null in $E'[\sigma(E', E)]$.

Vogt, D.

On the structure theory of power series spaces

For power series spaces

$$\Lambda_\infty(\alpha) = \{x=(x_0, x_1, \ldots): |x|_t^2 = \sum_j |x_j|^2 e^{2t\alpha_j} < \infty \text{ for all } t\}$$

where $\alpha_0 \leq \alpha_1 \leq \ldots \uparrow +\infty$, $\limsup \frac{\alpha_{2n}}{\alpha_n} < \infty$ a complete characterization of subspaces, quotient spaces and complemented subspaces is given in terms of linear topological invariants (DN), (Ω) and $\Lambda_N(\alpha)$-nuclearity. This extends earlier work of M. J. Wagner and the speaker to the nonnuclear case.

Zaharjuta, V. P.

Linear topological invariants, isomorphic classification, spaces of analytic functions

We consider the following problems: (a) isomorphic classification, (b) existence of basis, (c) description of the structure of Köthe matrices for some classes of spaces of analytic or infinitely differentiable functions, connected in some sense with sets having a thin edge.
A basis is constructed in the space of all infinitely differentiable functions, defined in a domain, whose boundary is smooth except for a unique edge-like point and with all derivatives vanishing at the point of this edge; it is given a complete isomorphic classification ; also we investigate non-trivial structure of Köthe matrix for such spaces.

Zobin, N.

Jordan representation of linear operators and related spaces

Let H be a Banach space, A: H $\to$ H a bounded linear operator. We define a locally convex space $\mathfrak{A}$ connected with H, A such that there exist dense immersions $\mathfrak{A} \hookrightarrow H \hookrightarrow \mathfrak{A}^*$, $\mathfrak{A} \hookrightarrow H^* \hookrightarrow \mathfrak{A}^*$ and A acts in $\mathfrak{A}$ and $\mathfrak{A}^*$. It turns to be possible to define generalized root vectors of A, lying in $\mathfrak{A}^*$. We give conditions on A, H necessary and sufficient for completeness of the root vectors and give a method of summation. So we obtain the Jordan representation of A. The crucial point is the construction of $\mathfrak{A}$, which cannot be a Banach space. We study nuclearity-type conditions on $\mathfrak{A}$ and their relations with the spectral nature of A.

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