

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12/1990

Mathematische Stochastik

11.3. bis 17.3.1990

Die Tagung fand unter der Leitung von H. Föllmer (Bonn) und L. Rüschendorf (Münster) statt. Auch diesmal hatte sie vor allem das Ziel, Querverbindungen zwischen Wahrscheinlichkeitstheorie und Statistik zu betonen und zu verstärken. Im Vordergrund standen also Themen, bei denen Fragestellungen und Methoden aus beiden Gebieten in engem Zusammenhang stehen und sich in ihrer Entwicklung gegenseitig beeinflussen. Insbesondere ging es um

- stochastische Analysis und partial likelihood in der Statistik stochastischer Prozesse
- Zusammenhänge zwischen statistischer Inferenz, stochastischer Analysis und Differentialgeometrie
- statistische und wahrscheinlichkeitstheoretische Aspekte räumlicher stochastischer Modelle, insbesondere auch für Expertensysteme
- metrische Methoden bei der Konstruktion von Dichteschätzern und von Schätzern für stochastische Prozesse und bei der Untersuchung von Bootstrap-Verfahren
- Anwendungen der stochastischen Analysis auf Optionen und Portfoliostrategien
- große Abweichungen und ihre Anwendung in der Risikotheorie.

Insgesamt wurde deutlich, daß sich die Wechselwirkungen zwischen den verschiedenen Arbeitsrichtungen der Stochastik weiter intensivieren.

## Vortragsauszüge

S. V. Anulova:

### Functional limit theorems for semimartingales and diffusion approximations for queueing systems

We consider a multidimensional queue  $Q^n$ , where  $n = 1, 2, \dots$  is a parameter such as the number of servers, capacity of the waiting room etc. In order to study the behaviour of  $Q^n$ , appropriate approximations are investigated. Under reasonable assumptions the process  $q^n = Q^n/n$  converges to a (deterministic) solution  $q$  of a certain differential inclusion with values in a certain convex set  $\mathcal{O}$  in  $\mathbb{R}^d$ . And the deviation  $\sqrt{n}(Q^n/n - q)$  converges to a solution of a certain linear Itô equation with reflection from  $\partial\mathcal{O}$  (roughly speaking). It turns out that the latter convergence holds not for the usual Skorohod topology, but for some other one, admitting convergence of continuous functions to discontinuous.

Convergence is proved by the martingale method. To this end we establish a FLT for semimartingales under general metrics. Furthermore we provide a metric on the Skorohod space which suits the case of queueing systems.

O. E. Barndorff-Nielsen:

### Stochastic calculus, statistical inference and differential geometry

Recent developments in analysis, stochastic calculus, statistical inference and relativistic physics have turned out to have a common ground in a mathematical concept that it is proposed to term *phyla*. Phyla are geometric objects, on manifolds, of a very general kind. The concept comprises those of ordinary and covariant higher order derivatives, connections, tensors, higher order differentials, stochastic second order (differential) tangent vectors, and others. While coordinate-free formulations are possible and important, it is often useful to characterize particular phyla in terms of the transformation law they follow under a change of coordinate system. To exemplify, let  $M$  be a  $d$ -dimensional differentiable manifold and let  $\omega^i, \omega^j, \dots$  and  $\psi^a, \psi^b, \dots$  be generic coordinates of two coordinate systems  $\omega$  and  $\psi$  on  $M$ . One may define *extended Christoffel symbols*  $\Gamma_{j_1 \dots j_n}^{i_1 \dots i_m}$  by  $\Gamma_{j_1 \dots j_n}^{i_1 \dots i_m} = \sum \Gamma_{J_1}^{i_1} \dots \Gamma_{J_m}^{i_m}$  where the summation is over all *ordered partitions* of  $J = j_1 \dots j_n$  into  $m$  blocks  $J_1, \dots, J_m$  and where the single factors follow a transformation law as in  $\Gamma_{j_1 \dots j_m}^i = \sum_{\nu=1}^n \Gamma_{b_1 \dots b_\nu}^a \psi_{/j_1 \dots j_m}^{b_1 \dots b_\nu} \omega_{/a}^i$  where  $/$  indicates differentiation and  $\psi_{/j_1 \dots j_n}^{b_1 \dots b_\nu} = \sum \psi_{/J_1}^{b_1} \dots \psi_{/J_m}^{b_\nu}$ , summing again over ordered partitions. The extended Christoffel symbols give rise to tensorial higher order derivatives

that are symmetric in the indices if the  $\Gamma_{j_1 \dots j_n}^{i_1 \dots i_n}$  are. A related concept, termed *yokes*, induces higher order geometries — with extended Christoffel symbols that are symmetric — and also *invariant Taylor expansions*. A yoke is a function on  $M \times M$  that satisfies (1)  $g_{j_i}(\omega, \omega) \equiv 0$  and (2)  $g_{j_i j_i}(\omega, \omega)$  is nonsingular (here differentiation refers to coordinates in the first factor of  $M \times M$  only). Important statistical examples are the *expected likelihood yoke*  $g(\omega; \omega') = E_{\omega'} \{ \ell(\omega) - \ell(\omega') \}$  and the *observed likelihood yoke*  $g(\omega; \omega') = \ell(\omega; \omega')a - \ell(\omega'; \omega', a)$  where  $a$  denotes an ancillary statistic.

A paper giving a brief survey of these developments is in preparation, joint with Wilfrid Kendall and Peter E. Jupp.

*T. Bednarski:*

### Statistical expansions and locally uniform Fréchet differentiability

Statistical functionals which are Fréchet differentiable at a smooth model have stochastic expansions that hold locally uniformly in probability over product measures induced by shrinking neighbourhoods. This means in particular that if  $\{P_\theta\}$  is a model,  $\mathcal{F}_{\theta, n} = \{(1 - \varepsilon/\sqrt{n})P_\theta + \varepsilon/\sqrt{n}H : H \text{ probability measure}\}$  are the shrinking neighbourhoods, while a functional  $T$  estimating  $\theta$  is Fréchet differentiable at  $P_\theta$ , then for some function  $\Psi(x, \theta)$

$$\sqrt{n}(T(F_n) - \tau_n) = \frac{1}{\sqrt{n}} \sum_{i=1}^n \Psi(x_i, \tau_n) + o_{G_n}(1)$$

holds for all sequences  $\{G_n\}$  and  $\{\tau_n\}$  such that  $G_n \in \mathcal{F}_{\tau_n, n}$ ,  $\sqrt{n}|\tau_n - \theta|$  is bounded and  $F_n$  is the empirical distribution function based on the sample from  $G_n$ . This strong expansion may be very useful in studies of estimators' behaviour under small violations of the model assumptions.

In fact a converse of the statement holds. An estimator having strong expansion is asymptotically equivalent to an  $M$ -estimator whose  $M$ -functional is Fréchet differentiable. Therefore the strong expansions are typical to Fréchet differentiability only.

The results were obtained jointly with B. Clarke and W. Kolkiewicz.

*D. T. Daley:*

### Order relations applied to a general epidemic model

Consider the class of epidemics occurring in a closed homogeneously mixing finite population. The total size of any such epidemic depends only on the infection distribution. Depending on the first two moments of this distribution, the mean

total size can be bounded above and below. The requisite techniques for this study involve order relations for distributions and their transforms and of regarding branching processes in finite populations as embedded in random walks. The latter technique facilitates the numerical study of the total size distribution because, using forward Chapman-Kolmogorov equations, the only operations involved are addition and multiplication of probabilities.

*P. L. Davies:*

### Global dispersion functionals

Consider the set  $W(\mathbb{R}^k)$  of probability measures on  $\mathbb{R}^k$  with

$$\delta(Q) = \sup_{\vartheta, \|\vartheta\|=1} Q(\{x : x^T \vartheta = 0\}) < 1.$$

One problem arising in the context of linear regression is to define a functional  $T : W(\mathbb{R}^k) \rightarrow \text{PDS}(k)$  into the set of positive definite symmetric matrices which satisfies the following conditions:

- (i)  $T$  is well defined.
- (ii)  $T(Q^A) = AT(Q)A^T$  for all non-singular  $k \times k$ -matrices  $A$ .
- (iii)  $T$  has a high breakdown point at each  $Q \in W(\mathbb{R}^k)$ .
- (iv)  $T$  is smooth.

Using an idea due to Donoho and Stahel such a  $T$  may be defined by considering all projections onto one-dimensional subspaces.  $T$  satisfies (i) and (ii) and has a finite sample breakdown point of  $\frac{1}{3}(1 - \delta(Q))$  at each point  $Q$ . This is slightly less than the highest possible breakdown point of  $\frac{1}{2}(1 - \delta(Q))$ . Furthermore  $T$  satisfies locally a Lipschitz condition of order 1 with respect to a linearly invariant Prohorov type metric based on ellipsoids and parallelograms.

*H. Dinges:*

### Wiener germs and formal Edgeworth expansions

Wiener germs are the objects in a theory designed to give a new approach to asymptotic normality and (elementary aspects of) large deviation theory. They are families of distributions  $\{d\mu_\varepsilon(x) : \varepsilon \rightarrow 0\}$  tending to a  $\delta$ -measure in a peculiar way. They can be represented by "admissible exponents" in various ways. An admissible exponent is given by a uniform asymptotic expansion

$$\frac{1}{\varepsilon} K(\varepsilon, x) = \frac{1}{\varepsilon} K(x) + K_0(x) + \varepsilon K_1(x) + \dots + \varepsilon^m \cdot K_m(x) + o(\varepsilon^m)$$

where  $K''(\cdot)$  is positive definite  $2m$ -times continuously differentiable and  $K_j(\cdot)$  is  $(2m - 2j)$ -smooth. Three ways to specify a Wiener germ  $\{d\mu_\varepsilon(x) : \varepsilon \rightarrow 0\}$  are particularly nice and important:

- a) Referring to densities (or something analogous in the lattice case)
- b) Referring to cumulant generating functions
- c) Referring to uniform asymptotic expansions of "tail probabilities"

The one-to-one correspondances connecting the various admissible exponents are different for smooth and for lattice-Wiener germs. This is one of many indications that something is wrong with the philosophy of formal Edgeworth expansions, which says that refinements of asymptotic normality are best described by "approximate cumulants", i.e., by the asymptotic expansions of the cumulant generating function. The good coefficients seem to be the Taylor coefficients of the admissible exponent arising in c) ("tail probabilities"). On similar grounds there are objections against the practice in traditional large-deviation theory:  $\ln \Pr(X_e \in A)$  will not have a nice asymptotic expansion;  $\frac{1}{2} [\Phi^{-1}(\Pr(X_e \in A))]^2$ , however, is connected with an admissible exponent.

*E. Eberlein:*

#### **Almost sure approximation and an application to stock price modelling**

Two topics concerning strong approximation of continuous time stochastic processes are discussed. First we sketch sufficient conditions which allow the almost sure approximation of semimartingales by continuous processes with independent increments due to N. Besdzik. Second, let  $(X^n(t))_{t \geq 0}$ ,  $(Y^n(t))_{t \geq 0}$  be two sequences of stochastic processes. We discuss pathwise approximations in the sup-norm on finite intervals of the form

$$\|X^n(t) - Y^n(t)\|_{t_n} \ll \varepsilon_n \quad \text{a.s.}$$

As an application we show that geometric Brownian motions modelling the stock price in the Black-Scholes model of option pricing can be approximated in a pathwise sense by processes with price changes at discrete time points only.

*P. Embrechts:*

#### **Aggregate claims and generalised renewal theory**

Motivated by the following examples:

- (EX.1) The Spitzer-Baxter identity for the first ascending ladder height of a random walk on  $\mathbb{R}$ ;
- (EX.2) The Random Record Process where times (and interrecord times) till the records in a renewal process, marked point process are studied, and
- (EX.3) The aggregate claim distribution in a Pólya process over  $[0, 1]$  where the claimsize distribution satisfies Cramèr's condition,

we discuss the following result.

**Theorem.** Suppose  $\{X_n; n \in \mathbb{N}\}$  i.i.d. ( $F$ ) with mean  $\mu$  and  $F$  non-lattice. Suppose  $a: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ ,  $a(x) = x^\alpha L(x)$  with  $L$  slowly varying.

$$(i) \text{ If } \alpha > -1, \text{ then } \forall h > 0 : \sum_{n=1}^{\infty} a(n)P\{x < S_n \leq x + h\} \sim \frac{h}{\mu^{\alpha+1}} a(x) \quad (1)$$

as  $x \rightarrow \infty$ .

(ii) If  $\alpha = -1$ , and

(\*)  $L$  is monotone, and (\*\*)  $\exists K \geq 0 : 1 - F(x) \sim K a(x)$  as  $x \rightarrow \infty$ ,

then (1) holds.

(iii) If  $\alpha < -1$ , and (\*\*), then (1) holds.

*M. Falk:*

### Statistical inference of conditional curves: Poisson process approach

A Poisson approximation of a truncated, empirical point process enables us to reduce conditional statistical problems to unconditional ones. Let  $(X, Y)$  be a  $(d + m)$ -dimensional random vector and denote by  $F(\cdot|x)$  the conditional density function of  $Y$  given  $X = x$ . Applying our approach one may study the fairly general problem of evaluating a functional parameter  $T(F(\cdot|x_1), \dots, F(\cdot|x_p))$  based on independent replicas  $(X_1, Y_1), \dots, (X_n, Y_n)$  of  $(X, Y)$ . This will be exemplified in the particular cases of nonparametric estimation of regression means and regression quantiles.

*U. Gather:*

### Outlier detection

The problem of detecting the "outliers" in a univariate sample  $x_1, \dots, x_n$  is formulated as follows: identify those observations if any which lie in some outlier region  $\text{out}(\alpha_n; N(\mu, \sigma^2)) := \{x; |x - \mu| > \sigma z_{1-\alpha_n/2}\}$  say with respect to a normal target and some given  $\alpha_n \in (0, 1)$  where  $\mu$  and  $\sigma^2$  are unknown. The performance of corresponding  $\alpha_n$ -outlier identifiers is measured first by their masking breakdown point adapting a definition given by Donoho and Huber (1983) for estimators. We also consider least favourable constellations for the "outliers" and are thus able to distinguish further between outlier identifiers with the same optimal masking breakdown point: for Rousseeuw's procedure (Rousseeuw 1984) and the outward testing procedure (Rosner 1975, Simonoff 1987) we give asymptotical results about largest non identifiable outliers and the conditions under which all large outliers will or will not be identified. (This is joint work with P.L. Davies)

*H. O. Georgii:*

**Large deviations and conditional limit theorems for Gibbs measures**

An alternate approach to the large deviations principle for the empirical field of a Gibbs random field is presented. This approach has two features. On the one hand, it allows to sharpen the large deviations estimates, in that the weak topology is replaced by the finer topology of convergence of cylinder-probabilities. On the other hand, it also yields a limit theorem for conditional distributions of microcanonical type and thus sheds some light on the intimate relationship between the large deviations principle, the maximum entropy principle, and the equivalence of Gibbs ensembles.

*P. E. Greenwood:*

**Partially and fully specified semimartingale models and efficiency**

Suppose we observe two semimartingales,  $X_1$ , a process of interest, and  $X_2$  which supplies additional information. The triplet of predictable characteristics,  $T_\theta^1$ , of  $X_1$ , with respect to the filtration generated by both processes, is specified up to an unknown parameter which we wish to estimate. The stochastic structure of  $X_2$  is unknown and unmodelled. A concept of asymptotic efficiency for estimation of a functional of  $\theta$  is proposed which can be based on the partially specified likelihood as defined by Jacod. We show that the partially specified likelihood is the true likelihood of a family of measures of  $(X_1, X_2)$  obtained by changing  $T_\theta^1$ . As a result this likelihood can be used without loss of efficiency whenever the dependence of the joint law of  $(X_1, X_2)$  on  $\theta$  is only through  $T_\theta^1$ . This is joint work with Wolfgang Wefelmeyer.

*G. R. Grimmett:*

**Percolation and the contact process: local changes and global effects**

The technique of making "local changes at bounded cost" may be used to establish an attractive general condition on a pair  $\mathcal{L}, \mathcal{L}'$  of lattices [of arbitrary dimension and such that  $\mathcal{L} \subset \mathcal{L}'$ ] which ensures the strict inequality  $p_c(\mathcal{L}) > p_c(\mathcal{L}')$  of the critical probabilities of the corresponding percolation processes. A general formulation of such a remark may be used to see that the "entanglement transition" takes place at a parameter-value strictly less than the critical probability (joint work with Michael Aizenman).

Use of the same general technique allows the application of percolation technology to the contact process, yielding thus exponential-tail theorems and critical-exponent inequalities (joint work with Carol Bezuidenhart).

C. Hesse:

### Theorems and applications for processes with infinite variance

We will discuss some inferential methods for linear processes with infinite variance, especially autoregressive processes with stable innovations. It has been claimed that these processes provide a better fit to certain economic time series, such as stock price changes, than classical finite-variance models. In this talk we focus on the construction of confidence interval sequences for distribution percentiles and rates of convergence of estimators of autoregressive parameters.

C. Hipp:

### Efficient estimation under constraints

If  $X_1, X_2, \dots$  are iid with unknown distribution  $P$  lying in some unrestricted family of distributions, then the empirical distribution  $P_n$  of  $X_1, \dots, X_n$  is an efficient estimator for the underlying distribution  $P$ , i.e., for smooth functionals  $H$  the estimator  $H(P_n)$  is an efficient estimator for  $H(P)$  in the sense of Hajek's convolution theorem. Here we consider the case of families of distributions  $P$  which are restricted by a finite set of linear or nonlinear constraints. This type of families of distributions occurs e.g. when  $\mu$  is estimated while  $\sigma^2$  is known, or when  $\sigma^2$  is estimated while  $\mu$  is known. For these families, a minimum Pearson distance projection of  $P_n$  is efficient. Modifications of these projections are simpler and work under less restrictive assumptions than the minimum discriminant information adjusted estimators investigated by Haberman (1984) and Sheehy (1988). They are also simpler than the nonparametric maximum likelihood estimators for this problem (see Gill (1989)).

Most semiparametric models can be written as families of distributions that are restricted by an infinite set of nonlinear constraints. Efficient estimators under constraints can be used to construct approximately efficient estimators in these models. Consider, e.g., the classical problem to estimate the mean  $\mu$  of a distribution which is known to be symmetric. For this problem the sample mean  $X_1 + \dots + X_n$  of the iid observations is not efficient, we can improve the estimator using additional information. If the underlying distribution  $P$  has density  $h(x - \mu)$  then the following information can be used:

$$\int g(x - \mu)P(dx) = 0, \text{ where } g(x) = h'(x)/h(x).$$

This information yields an estimator which, at  $P$ , has minimal asymptotic variance

$$\left( \int g^2(x - \mu)P(dx) \right)^{-1}.$$



R. Höpfner:

### A “second LeCam lemma” for Markov step processes

Consider a Markov step process  $X = (X_t)_{t \geq 0}$  taking values in some state space  $(E, \mathcal{E})$ , starting from an arbitrary point  $x \in E$ . Assume that the generator of  $X$  is governed by an unknown parameter  $\vartheta \in \Theta$ ,  $\Theta \subset \mathbb{R}^d$  open. This induces a statistical model  $\{P_{x, \vartheta} : \vartheta \in \Theta\}$  on the canonical path space  $(\Omega_x, \mathcal{F}_\infty, \mathbb{P})$  of the process  $X$ .

Observing a trajectory of  $X$  over a long time and considering the resulting statistical model locally over shrinking neighbourhoods of some point  $\vartheta \in \Theta$ , we prove a decomposition of log-likelihood ratio processes in the local model, of type

$$h^T M_\vartheta^n + \frac{1}{2} h^T \langle M_\vartheta^n \rangle h + \text{remainder terms},$$

where  $h \in \mathbb{R}^d$  is the local parameter and  $M_\vartheta^n$  is the score function martingale, suitable normed and scaled. Our assumptions link together convergence properties of  $X$ , under  $\vartheta$ , and smoothness properties of the parametrization near  $\vartheta$ . They are sufficiently general to include

- “ergodic/non-ergodic models” in the sense of Basawa and Scott, (e.g. models for supercritical branching processes),
- models containing points  $\vartheta \in \Theta$  such that  $X$  is recurrent (positive or null) under  $\vartheta$ ,
- a non-LAMN example.

I. Ibragimov:

### About probability density estimation

The talk (joint work with R. Hasminski) deals with upper and lower bounds for the quality of density estimation on the base of iid observations  $X_1, \dots, X_n$  with common density  $f(x)$ ,  $x \in \mathbb{R}^k$ . Connections are established between these problems and the theory of approximations of functions. In particular, for any  $\nu = (\nu_1, \dots, \nu_k)$  there exists an estimator  $\tilde{f}_{n, \nu}$  such that

$$E_f \|f - \tilde{f}_{n, \nu}\|_p \leq \begin{cases} c_1(1 + \|f\|_p) \left[ \mathcal{E}_\nu(f) + \left(\frac{\nu_1 \dots \nu_k}{n}\right)^{\frac{p-1}{p}} \right], & 1 \leq p < 2 \\ c_2(1 + \|f\|_p) \left[ \mathcal{E}_\nu(f) + \sqrt{\frac{\nu_1 \dots \nu_k}{n}} \right], & 2 \leq p < \infty \\ c_3(1 + \|f\|_p) \left[ \mathcal{E}_\nu(f) + \sqrt{\frac{\nu_1 \dots \nu_k \ln(\nu_1 \dots \nu_k)}{n}} \right], & p = \infty. \end{cases}$$

Here  $\mathcal{E}_\nu(f)$  denotes the value of the best approximation of  $f$  by entire functions of the exponential type  $\nu$  in  $\mathcal{L}_p(\mathbb{R}^k)$ -norm.

*J. Jacod:*

### Regularity for likelihood and partial likelihood

Consider a statistical experiment where we observe a semimartingale  $X$ , plus possibly some covariates. The value  $\theta$  of the parameter describes the triple of characteristics of  $X$ , but nothing else, so for each  $\theta$  we have a family  $\mathcal{I}_\theta$  of probability measures.

Pick  $P_\theta$  in  $\mathcal{I}_\theta$  for each  $\theta$ . Regularity of the experiment is differentiability of  $\theta \mapsto \sqrt{Z^\theta}$  in  $\mathcal{L}^2$ , where  $Z^\theta$  is the likelihood process of  $P_\theta$  with respect to  $P_0$  and differentiability holds "locally" in time.

We then introduce the partial likelihood process  $\bar{Z}^\theta$ , which is given by the explicit form of Girsanov's theorem: hence  $\bar{Z}^\theta$  is some sort of a projection of  $Z^\theta$  on the stable subspace of martingales generated by  $X$ . We can then introduce partial regularity as differentiability of  $\theta \mapsto \sqrt{\bar{Z}^\theta}$  as above.

Then we have two theorems:

1. Regularity implies partial regularity.
2. Partial regularity is equivalent to a suitable notion of differentiability of the characteristics of  $X$ , in  $\theta$ .

Hence partial regularity is something which can be read off directly from the model, without actually knowing the true measures  $P_\theta$ .

*M. Janžura:*

### A general view of the maximum likelihood estimation

Let the model for an estimation problem be given by a measurable space  $(X, \mathcal{S})$  with some family  $\mathcal{M}$  of probability measures which are assumed to be invariant with respect to a suitable group  $\Gamma$  of transformations. The data are considered to be given by

$$\{f \circ \gamma(x)\}_{f \in \mathcal{F}, \gamma \in \Gamma_n}$$

where  $\mathcal{F}$  is a collection of real-valued observable functions and finite  $\Gamma_n \subset \Gamma$  stays for the set of repetitions of observations. We shall assume  $X$  metrizable, all  $\gamma$  continuous, and  $\mathcal{M}$  a subset of Radon invariant measures  $M_I$ . Due to the ergodic theorem and Choquet's representation theorem we conclude that it is not worth to try to distinguish between measures belonging to the same (minimal for them) faces in  $M_I$ . Therefore, instead of a family of distributions, we rather consider a family of pairwise disjoint closed faces.

Since we know that an affine upper semicontinuous (a.u.s.) function assumes its maximum on a closed face we shall use the (more special) functions of the type

$$b(\mu) + \int h d\mu$$

(where  $b$  is fixed a.u.s., and  $h$  belongs to some space  $\mathcal{H}$ ) to parametrize the faces by the functions  $h \in \mathcal{H}$ .

Thus, the estimate of unknown  $h$  is constructed in order to approach the "empirical" value to the theoretical maximum. Under some natural conditions the method provides a consistent estimate. As an example we mention the random fields. In the special case of i.i.d. we obtain the usual MLE. Some robust estimates can be re-formulated in this way as well.

*J. L. Jensen and J. Møller:*

### **Pseudolikelihood for exponential family models of spatial processes**

A number of properties for the pseudolikelihood method are investigated for general spatial processes with a density which belongs to an exponential family model. In particular, the pseudolikelihood for spatial point processes including marked point processes is derived. Moreover, consistency of the maximum pseudolikelihood estimate is established for lattice processes as well as for spatial point processes.

*I. Karatzas:*

### **Clark's formula for the representation of Wiener functionals, and its applications**

A modification of J.M.C. Clark's formula is established, for the stochastic integral representation of Brownian functionals as stochastic integrals, under an equivalent (Girsanov) change of probability measure. It is shown how this modified Clark formula leads to the representation of optimal portfolios for a variety of situations in the modern theory of financial economics.

*H. Kellerer:*

### **On a stochastic process arising in a lottery**

The stochastic model (for a "permanent jackpot") given by the recursion

$$X_0 \geq 0 \quad \text{and} \quad X_n = Y_n(X_{n-1} + 1) \quad \text{for } n \in \mathbb{N}$$

is treated under the assumptions

- (a)  $0 \leq Y_n \leq 1$  (resp.  $0 < Y_n < \infty$ ) with identical distribution  $\nu$ ,
- (b)  $X_0, Y_1, Y_2, \dots$  independent random variables.

Denote by  $\eta$  the essential infimum of  $Y_n$  and let  $(S_n)_{n \geq 0}$  be the random walk with summand  $Z_n = \log Y_n$ ; then:

- (1)  $S_n \rightarrow +\infty$  a.s.  $\iff X_n \rightarrow \infty$  a.s.,  
 (2)  $S_n$  oscillating  $\implies \liminf X_n = \eta/(1-\eta)$  and  $\limsup X_n = \infty$ ,  
 (3)  $S_n \rightarrow -\infty$  a.s.  $\iff \mathcal{L}(X_n)$  converges to some limit  $\mu$ .

In case (3) the distribution function  $F$  belonging to  $\mu$  is the solution of an integral equation which, hardly being solvable, at least yields continuity of  $F$  for all  $\nu$ . Moreover, due to special features of the kernel associated with the Markov process  $(X_n)_{n \geq 0}$ ,  $\mu$  is either absolutely continuous or singular for all  $\nu$ .

Applications concern some learning model and the original lotto problem.

*H. Künsch:*

### Long range dependence

We discuss the use of models with long range dependence in statistics. For simplicity we consider processes with second moments and define long range dependence by requiring the spectral density to behave like  $|\lambda|^{1-2H}$ ,  $\frac{1}{2} < H < 1$ , as  $\lambda \rightarrow 0$ . This implies that the covariances are not summable and the variance of the arithmetic mean is  $O(n^{2H-2})$ . We present some empirical evidence for such models in geophysical data and also in measurement processes. Results for estimating  $H$  are surveyed and the behavior of least squares in a regression model with long range dependent errors is studied. An application to testing treatment effects in randomized comparisons is also given.

*S. L. Lauritzen:*

### Probabilistic expert systems based on directed Markov fields

A rigorous approach to reasoning with uncertainty in expert systems is presented. The approach is based upon modelling the system under investigation as a directed Markov field which is a generalisation of the notion of a Markov chain to a directed acyclic graph.

The directed Markov field is then embedded into an undirected Markov field on a triangulated graph. The computation of conditional probabilities is carried out locally in a junction tree of cliques of the undirected graph by "message passing" among neighbours.

Finally extensions of the methodology involving sequential Bayesian updating and structural monitoring were touched upon. These "learning" processes were based on random probability measures of so-called hyper Markov type.

A. Le Breton:

### Linear filtering in linear models with infinite variance

The problem of linear filtering in a linear stochastic system is revisited in the case of models which generate possibly non second-order processes. At first some examples are discussed in order to motivate the need of modifying the ordinary Kalman-Bucy algorithm and to introduce a possible approach for doing that. Then a formulation of the filtering problem is proposed for some class of random systems in such a way that the optimality criterion for a linear filter extends the usual minimum variance criterion. The optimal linear filter is derived; it appears that it is recursively defined through equations which reduce to the Kalman-Bucy equations when the system is driven by gaussian white noises. Finally, some comments are made about the prediction and smoothing problems.

The talk is based on a joint work with Marek Musiela, U.N.S.W., Kensington, Australia.

F. Liese:

### Strong convergence of distributions of diffusion processes

Given a diffusion process  $X_t$  defined by  $dX_t = a(X_t) dt + dW_t$  and a non-negative function which vanishes for negative arguments an explicit formula for the Laplace transform of  $\int_0^{T_c} h(X_s) ds$  is given.  $T_c$  denotes the hitting time of level  $c$ . Combining this result with a general criterion in terms of the Hellinger processes for the variation convergence of distribution of stochastic processes, a necessary and sufficient condition on the sequence  $a_n$  is established to guarantee the variation convergence of the distribution of the corresponding diffusion processes. Explicit upper and lower bounds for the variational distance are formulated in terms of the corresponding drift coefficients. To test the hypothesis  $H_0 : W$  versus the trend alternative  $H_A : X$  where  $a(x) \geq 0$  we use the test statistic  $U_c = \int_0^{T_c} h(X_s) ds$ . The Laplace transforms of  $U_c$  both under the null hypothesis and under the alternative are calculated. A limit theorem for  $\frac{1}{c} U_c$  is established as  $c \rightarrow \infty$ .

H. Luschgy:

### Deficiencies between location models for stochastic processes

Consider location models  $Y = \vartheta + X$  for stochastic processes with noise distribution  $Q$ . The possible loss of statistical information about  $\vartheta$  when  $Q$  is replaced by some other noise distribution  $P$  is measured by the deficiency  $\delta(P, Q)$ . This

number is shown to be the supremum of the deficiencies between location models given by finite dimensional marginal distributions of the noise (arising from discrete sampling, observation of Fourier coefficients of sample paths and so forth). Moreover,  $\delta(P, Q) \leq \|P - Q\|$  (variational distance), a convolution divisibility criterion for the equation  $\delta(P, Q) = 0$  holds and in case  $\delta(P, Q) = 0$ , one obtains  $\delta(Q, P) = \|P - Q\|$  provided  $P$  and  $Q$  are Gaussian. For particular Gaussian noise processes good lower and upper bounds of  $\delta(P, Q)$  are available and sometimes precise formulas for  $\delta(P, Q)$  can be derived.

*T. J. Lyons:*

### Infinite dimensional martingales and inequalities in parametric statistics

Let  $\mu : M \rightarrow \mathcal{P}(\Omega, \mathcal{F})$ ,  $\mu_m = p(m, \omega)\nu(d\omega)$  be a smooth parametrisation of a family of probability measures. Let  $(\mathcal{F}^t) \subset \mathcal{F}$  be a (partially ordered) filtration and  $p^t = \mathbb{E}(p|\mathcal{F}^t)$  be the associated likelihood function. For  $\nu$ -a.e.  $\omega$  observe that for rich enough families  $(\mathcal{F}^t)$  the single sample path  $t \mapsto p^t(\cdot, \omega)$  of the function valued process carries full information about  $\mu$  and  $\omega$ . Incompletely specified models can be described by considering  $p^t(\cdot, \omega)$  for restricted classes of  $t$ .

Although some models have natural filtrations, it can be informative to introduce others deliberately. For example if  $\Omega = \mathbb{R}^n$  then the dyadic partition of  $\mathbb{R}^n$  can be informative and makes available classical Littlewood-Paley theory. They do not seem to have been exploited by statisticians.

This structure can be exploited by considering the score form process (and related stochastic integrals). Let  $\alpha^t$  be the differential of  $\log p^t$  on  $M$ . This process is intrinsic — and independent of the reference measure  $m$ . Moreover, it can easily be transformed into a  $\mu_m$ -martingale for any  $m$ . Stochastic integrals of these martingales can then be analyzed against their  $\langle \cdot \rangle$  processes. In continuous cases the  $\langle \cdot \rangle$  is independent of  $m$  and so the Gaussian measure with this co-variance provides an approximate pivotal distribution. The degree of this approximation can be established using Bernstein type inequalities. Examples in the literature (e.g. Heyde + Jacod) do not operate at this level of abstraction and so have tended to consider the score form process over one or two points of  $m$ . It is not clear that such restricted processes will be as powerful at discriminating.

The  $\langle \cdot \rangle$  of  $\alpha$  (or its martingale modification) provide a rich structure on  $M$  including a stochastic flow which represents the uncertainty in the likelihood function from a boot-strap perspective. But that's another story.

*E. Mammen:*

### Bootstrap and asymptotic normality

In this talk we give a simple proof for the equivalence of asymptotic normality and consistency of bootstrap for linear functionals  $T_n(G) = \int h_n(x)G(dx)$ . The proof gives a simple explanation for this equivalence which does not hold in general for arbitrary statistical functionals: Here the conditions for consistency of bootstrap and normal approximation coincide: (a) continuity of the distribution of the estimator as a functional of the underlying distribution (bootstrap) and (b) Lindeberg condition (asymptotic normality).

The presented result is a generalization of results of Athreya (1985) and Giné and Zinn (1989).

*A. Martin-Löf:*

### Entropy and risk theory

We study the standard model of Risk Theory, i.e.,  $U(t) = u + pt - S(t)$ ,  $t \geq 0$ .  $U(t)$  = surplus of the company,  $pt$  = inflow of premiums,  $S(t)$  = payments = compound Poisson process defined by

$$E(e^{zS(t)}) = \exp \lambda t \int_0^{\infty} (e^{zx} - 1)F(dx) =: \exp \lambda t g(z).$$

$F(dx)$  = distribution of the claim sizes.

We derive large deviations estimates of the time of ruin:  $T = \min\{t; S(t) > u + pt\}$ . These estimates can be expressed in terms of the entropy function  $h(\bar{x})$  associated with  $g(z)$  by the conjugate relation  $h(x) = \min_z (g(z) - zx)$ , so that  $h(x) = g(z) - zx$ , when  $x = g'(z)$ .

The well known Esscher-Cramér estimate

$$P(S(t) \geq x) \approx (\exp \lambda t h(x/\lambda t)) / z \sqrt{2\pi \lambda t g''(z)},$$

with  $x/\lambda t = g'(z)$ , valid for  $z > 0$ , i.e.,  $x/\lambda t > \mu$ , can be extended to an estimate in path space: Put  $S_\lambda(t) = \lambda^{-1}S(t)$ , and think of  $\lambda$  as large. Then we have  $P(S_\lambda(s) \approx x(s), 0 \leq s \leq t) \approx \exp \lambda \int_0^t h(x'(s)) ds$ , where  $x(\cdot)$  is an absolutely continuous path with  $x(0) = 0$ . Using this we can derive the following estimate of the ruin probabilities: (putting  $u/\lambda = a$ ,  $p/\lambda = b$  and  $R_t = \{x(\cdot); x(0) = 0, x(s) > a + bs \text{ for some } s \leq t\}$ )

$$P(T \leq t) = P(S_\lambda > a + bs \text{ for some } s \leq t) \approx \exp \lambda \max_{x(\cdot) \in R_t} \int_0^t h(x'(s)) ds.$$

For any fixed  $\tau \leq t$  the maximum over all  $x(\cdot)$  going from  $(0, 0)$  to  $(\tau, a + b\tau)$  is obtained for the straight line with slope  $x' = b + a/\tau$ , so the maximum is given by  $\max_{0 \leq r \leq t} \tau h(b + a/\tau)$ . The entropy function of the random variable  $T$  is hence  $H(\tau) = \tau h(b + a/\tau)$ , and

$$P(T \leq t) \approx \exp \lambda H(t) \text{ for } t \leq \bar{T}$$

$$P(T \leq t) \approx \exp \lambda H(\bar{T}) \text{ for } t \geq \bar{T}$$

$$P(T \geq t) \approx \exp \lambda H(t) \text{ for } t \geq \bar{T},$$

where  $\bar{T}$  is the point of maximum for  $H(t)$ :  $H(\bar{T}) = \max H(t)$ , determined by the equations  $g(R)/R = b$ ,  $g'(R) = b + a/\bar{T}$ ,  $H(\bar{T}) = -aR$ . This means that  $P(T < \infty) \approx e^{-\lambda a R}$ , and  $P(|T - \bar{T}| > \varepsilon | T < \infty) \approx \exp -\lambda(H(\bar{T}) - H(\bar{T} + \varepsilon)) \rightarrow 0$  rapidly. The above "equations of state" define a natural splitting of  $u$  and  $p$  for two independent subsystems "in equilibrium" with a common value of  $R$  and  $\bar{T}$ : If  $\lambda g = \lambda_1 g_1 + \lambda_2 g_2$  then  $p = \lambda b = \lambda g(R)/R = \lambda_1 g_1(R)/R + \lambda_2 g_2(R)/R = p_1 + p_2$  and  $u/\bar{T} = \lambda a/\bar{T} = \lambda g'(R) - p = \lambda_1 g'_1(R) - p_1 + \lambda_2 g'_2(R) - p_2 = (u_1 + u_2)/\bar{T}$ , and  $P(T < \infty) \approx e^{-Ru} = e^{-Ru_1} \cdot e^{-Ru_2} \approx P(T_1 < \infty)P(T_2 < \infty)$ . This can be used for decentralized planning of a company with several independent branches.

Reference: A. Martin-Löf: Entropy, a Useful Concept in Risk Theory. Scand. Act. J. (1986).

*J. Møller and M. Sørensen:*

### Parametric models of spatial birth-and-death processes with a view to modelling linear dune fields

Statistical inference for parametric models of spatial birth-and-death processes is discussed in detail. In particular, a flexible and statistically tractable parametric class of such processes, defined on the real line, is presented. The suggested methods are illustrated by applying them to two sets of data given in the form of air photos from the Kalahari Desert.

*M. Röckner:*

### Martin boundary on Wiener space

We study the positive parabolic functions of the Ornstein-Uhlenbeck operator on an abstract Wiener space  $E$  using the approach developed by E. B. Dynkin. This involves first proving a characterization of the entrance space of the corresponding Ornstein-Uhlenbeck semigroup and deriving an integral representation for an arbitrary entrance law in terms of extreme ones. It is shown that the Cameron-Martin



densities are extreme parabolic functions, but that if  $\dim E = \infty$ , not every positive parabolic function has an integral representation in terms of those (which is in contrast to the finite dimensional case). Furthermore, conditions for a parabolic function to be representable in terms of Cameron-Martin densities are proved.

*U. Rösler:*

### How reliable is Quicksort?

Let  $X_n$  be the number of comparisons needed by the sorting algorithm Quicksort to sort a list of  $n$  numbers into their natural ordering.

We show that  $(X_n - E(X_n))/n$  converges weakly to some random variable  $U$ . The distribution of  $U$  is characterized as the fixed point of some contraction. It satisfies a recursive equation, which is used to provide recursive relations for the moments. The random variable  $|U|$  has exponential tails. Therefore the probability that Quicksort performs badly, e.g. that  $X_n \geq 2E(X_n)$ , converges to zero polynomially fast of every order.

*H. Rost:*

### Minimal entropy production

Recent results by Guo-Papanicolaou-Varadhan, Spohn, Mürmann are interpreted as providing examples how in a possible non-equilibrium theory of thermodynamics "entropy production" at the macroscopic level can be derived from the corresponding quantity at the micro-level, in a striking analogy to what happens for entropy. (This latter example is well known since a couple of years under the name of "projection principle" in large deviation theory.) The connection of the two levels for entropy production also gives the right transport (diffusion) coefficient: Spohn's interpretation of the Green-Kato formula as solution to a variational problem.

*A. Rukhin:*

### Estimation of variance: New procedures for an old problem

The problem of estimating an unknown variance in a multivariate normal distribution is reviewed. Inadmissibility of the traditional estimator is related to the absence of solutions to the adjoint heat equation which can be approximated by positive and integrable functions. Behavior of admissible and minimax improvement due to Brewster and Zidek is discussed. An asymptotic setting of a nonparametric variance estimation problem is studied and asymptotic estimation of variance (also of the mean vector) is shown to be related to the estimation of

a positive mean on a basis of one observation. Finally, admissibility of the usual estimator of the discriminant coefficient is established.

*M. Schweizer:*

### Semimartingale methods in option hedging

We use a general semimartingale  $X \in \mathcal{S}^2$  on a probability space  $(\Omega, \mathcal{F}, \mathbb{F}, P)$  to model the price process of a stock; an option is described by a random variable  $H \in \mathcal{L}^2$ . The existence of an optimal hedging strategy for  $H$  is then equivalent to a representation

$$H = H_0 + \int_0^T \xi_s^H dX_s + L_T^H$$

of  $H$ , where the martingale  $L^H$  must be orthogonal to the martingale part  $M$  of  $X$ . We obtain such a decomposition in the case where  $H$  is a stochastic integral of  $X$  with respect to a larger filtration  $\tilde{\mathbb{F}} \supseteq \mathbb{F}$ . Furthermore, we show how the optimal strategy  $\xi^H$  can be computed directly in terms of a minimal equivalent martingale measure  $\hat{P} \approx P$  for  $X$ .

*J. Steinebach:*

### Some remarks on the convergence rates of Bahadur-Kiefer type statistics

Let  $S_n$  denote the  $n$ -th partial sum of an i.i.d. sequence  $\{X_k\}$  with  $EX_1 = \mu > 0$  and  $0 < \text{Var}(X_1) = \sigma^2 < \infty$ . Denote by  $N(t) = \min\{n \geq 0 : S_{n+1} > t\}$  the corresponding renewal process, and define a Bahadur-Kiefer type process

$$K(t) = (\mu^{-1}S_{[t]} - t) + (N(\mu t) - t) = S^*(t) + N^*(t), \quad t \geq 0.$$

Deheuvels and Mason (1990) have recently proved via strong invariance that, if  $EX_1^4 < \infty$ , then

$$(1) \quad \lim_{n \rightarrow \infty} \|K\|_n / (\|N^*\|_n \log n)^{\frac{1}{2}} = \sigma/\mu \quad \text{a.s.}$$

which also implies

$$(2) \quad \limsup_{n \rightarrow \infty} \|K\|_n / (n \log \log n)^{\frac{1}{2}} (\log n)^{\frac{1}{2}} = 2^{1/4} \sigma^{3/2} / \mu^{3/2} \quad \text{a.s.,}$$

where  $\|f\|_n = \sup_{0 \leq t \leq n} |f(t)|$ . This provides analogues of Kiefer's (1970) uniform rates in the Bahadur representation of sample quantiles. It was left as an open

problem whether an extension of (1) and (2) might be possible under a lower moment condition.

It is shown here that the fourth moment condition above is essentially sharp, i.e., it cannot be replaced by  $E|X|^r < \infty$  for some  $r < 4$ . By a more detailed discussion under Pareto-type distributions, it turns out that a possible extension of (1) and (2) under lower moments depends upon the underlying distribution. Finally, it is outlined that even more general Bahadur-Kiefer type processes can always be handled via strong invariance, provided an approximation rate of order  $o\left\{n/\log \log n\right\}^{\frac{1}{2}}(\log n)^{\frac{1}{2}}$  is given.

I. Vajda:

### Asymptotic Rényi distances

Let  $P, Q$  be probability measures on  $(\mathcal{X}, \mathcal{F})$ . Rényi distances are defined by means of Hellinger integrals of order  $a$  by  $R_a(P, Q) = (a - 1)^{-1} \ln H_a(P, Q)$  for  $a > 0$ ,  $a \neq 1$ , and by

$$R_1(P, Q) = \lim_{a \uparrow 1} R_a(P, Q) = I(P, Q)$$

for  $a = 1$ , where  $I(P, Q)$  is the Kullback-Leibler distance. Let us now consider a filtration  $(\mathcal{F}_\lambda | \lambda \in \Lambda)$  on  $(\mathcal{X}, \mathcal{F})$ , where  $\Lambda$  is a directed set. Asymptotic Rényi distance characterizes the rate at which  $R_a(P_\lambda, Q_\lambda)$  tend to  $\infty$ . It is defined by the condition

$$\bar{R}_a(P, Q) = \lim_{\Lambda} c_\lambda^{-1} R_a(P_\lambda, Q_\lambda) \quad , \quad a > 0,$$

where  $c_\lambda$  is positive, increasing to  $\infty$ , such that for at least one  $a > 0$  it holds  $0 < \bar{R}_a(P, Q) < \infty$ . This condition defines  $\bar{R}_a(P, Q)$  uniquely up to a multiplicative constant.

Let  $\alpha_\lambda, \beta_\lambda$  be probabilities of error of a test  $\varphi_\lambda$  of simple hypothesis  $P_\lambda$  against simple alternative  $Q_\lambda$ . Under mild regularity conditions on  $\bar{R}_a(P, Q)$ ,  $a > 0$ , it holds for every  $\alpha$ -level Neyman-Pearson test

$$\lim_{\Lambda} (\beta_\lambda)^{1/c_\lambda} = \exp\{-\bar{R}_1(P, Q)\}$$

and for every Bayes or minimax test

$$\lim_{\Lambda} (\alpha_\lambda + \beta_\lambda)^{1/c_\lambda} = \exp\{-(1 - a_*)\bar{R}_{a_*}(P, Q)\},$$

where  $a_*$  maximizes  $(1 - a)R_a(P, Q)$  on  $(0, 1)$ . Various examples and interesting particular cases are discussed.

A. Wakolbinger:

### Palm formula and persistence of discrete and continuous measure valued critical branching processes

Let  $N^t$  be a population of individuals on  $\mathbb{R}^d$  which has evolved from an initially homogeneous Poisson population as follows: each individual moves and after an exponentially distributed lifetime gives rise to a cluster of children (all this independently of the other individuals). The "Palm population" of relatives of an individual  $\delta_x$ , given that  $\delta_x$  belongs to  $N^t$ , can be constructed in an intuitive way from the random genealogy of  $\delta_x$ . This population of relatives stays for  $t \rightarrow \infty$  locally finite if and only if the mobility of the individuals is strong enough to spread out the branching clusters. If this is the case, then the population  $N^t$  is "persistent" in the large time limit. The results are stated for a class of monotype branching populations (Gorostiza/Wakolbinger, to appear in Ann. Probab.) and of multitype populations with mutation and monotype branching (Gorostiza/Roelly-Coppoletta/Wakolbinger 1990); in this latter case, persistence depends on the most mobile and the most clumping type.

An analytic proof for the Palm formula, using Feynman-Kac representation, is indicated. Also the corresponding results for (multitype) Dawson-Watanabe processes, which arise as limits in case of many particles, small masses, small lifetimes (and small mutation probabilities) are stated (Gorostiza/Roelly-Coppoletta/Wakolbinger, to appear in Sém. Probabilités; same authors, 1990).

W. Wefelmeyer:

### Efficient estimation in partially specified semimartingale models

Suppose we observe two stochastic processes  $X_1$  and  $X_2$  such that  $X_1$  is a semimartingale with respect to the filtration generated by  $X_1$  and  $X_2$ . Assume that the predictable characteristics of  $X_1$  are known up to a possibly infinite-dimensional parameter. Such a model is called partially specified. We describe conditions on the characteristics under which Jacod's partially specified likelihood is locally asymptotically normal, and obtain a convolution theorem for estimators of differentiable functionals of the parameter. As examples we consider a diffusion process with multiplicative drift process, a counting process with multiplicative intensity process, a Markov chain in a random environment, and a linear regression model. This is joint work with P. E. Greenwood.

*L. Younes:*

**Some remarks on synchronously simulated fields**

Existence theorem and uniqueness condition for the invariant law of a probability kernel where simulation at each site  $\in \mathbb{Z}^2$  is simultaneous. The particular case of the synchronous Ising model, i.e., where simulation at site  $ij$  is done according to

$$\pi_{ij}(x_{ij}|y) \propto \exp \left[ -kx_{ij} - \alpha x_{ij}y_{ij} - \beta \sum_{\substack{\text{nearest} \\ \text{neighbours}}} y_{i'j'} x_{ij} \right],$$

is studied; in that case, the potential can be explicated and identifiability can be shown.

Berichterstatter: Martin Schweizer

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