MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 14/1990

Kontinuumsmechanik der festen Körper

25.3. bis 31.3.1990

Die Tagung stand unter der Leitung der Herren G. Herrmann (Stanford) und H. Lippmann (München). Bewährt hat sich wie schon bei den vorausgegangenen gleichnamigen Treffen in Oberwolfach die Zusammensetzung des Teilnehmerkreises aus "Neulingen" und "Wiederholern", wodurch sich sofort der für das Institut typische freundschaftliche Kontakt einstellte und dies den Austausch sowie die Erörterung von wissenschaftlichen Fragestellungen und Ergebnissen anregte und intensivierte. Auch das bewußt nicht zu eng gefaßte Tagungsthema trug durch die Mannigfaltigkeit der angesprochenen Gebiete wesentlich zur gegenseitigen Anregung bei. Der Themenbogen spannte sich z.B. von funktionentheoretischen Betrachtungen bei Problemen elascher Körper mit Schädigung bis hin zur Anwendung der Wahrscheinlichkeitsrechnung und numerischer Methoden bei der Schädigung von Werkstoffen und Konstruktionsteilen. Bei der derzeitigen Entwicklung neuer Werkstoffe und dem Zwang größtmöglicher Zuverlässigkeit von Konstruktionen ist die Aktualität mathematischer und ingenieurmäßiger Behandlung von Schädigung erklärlich. Eine Reihe von Vorträgen zeigte aber auch, daß bei der Behandlung schon mehr klassischer Fragen zu konstitutiven Gesetzen und ihrer Anwendung auf verschiedene statische und dynamische Ingenieuraufgaben - etwa der numerischen und physikalischen Stabilität - weiterhin neue Erkenntnisse erzielt werden.

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Vortragsauszüge

H. ALTENBACH:

Das Kontinuumsmodell der Gebrüder Cosserat und seine Anwendung auf die Modellierung von Flächentragwerken

In Ergänzung des klassischen Cauchy-Boltzmann-Kontinuums treten beim Cosserat-Kontinuum drei zusätzliche Rotationsfreiheitsgrade auf. Entsprechend diesem kinematischen Modell können die Wechselwirkungen zwischen benachbarten Punkten des Kontinuums zusätzlich durch Momentenspannungen beschrieben werden. Diese Überlegeungen sind entsprechend dem mathematischen Kontinuumsbegriff unabhängig von der Dimension des Raumes. Neben allgemeinen Aspekten werden daher im Beitrag Fragen der Formulierung einer Flächentragwerkstheorie, die für Flächentragwerke bei Berücksichtigung spezieller Materialeigenschaften (elastisch, nichtelastisch, inhomogen in Dickenrichtung) gültig ist, diskutiert. Dabei wird insbesondere auf die Methodik der Ermittlung der effektiven Tragwerkseigenschaften eingegangen.

H. BARGMANN:

Erosion by liquid or solid impact - A random process

In constructing a general theory of erosion under repetitive impact loading one would define the "history up to time t of an attack process" by a sequence of space/time neighbourhoods $U_s^N x U_t^N$ on the surface of the target, N=1,... To each attack process an erosion process is then defined by the motion of that portion of the target which, at time t, still remains intact. In general both processes are coupled. In important cases of the decoupled problem, the stochastic space-dependence of the attack process can be brought down to the question: "What is the probability $P_{rep\,i}^N$ to hit, at the Nth attack, as an i-th repetition the same neigbourhood?" - Restricting ourselves to attacks over neigbourhoods U_s of the same relative size $p<1$ with respect to the target surface, admitting that in a 1st approximation the transition from the (N-1)th to the N-th attack configuration is Markov, and counting neighbourhoods U_s as either completely overlapping or not at all, we can show that

$$P^N_{rep\,i} := \binom{N-1}{i} p^i (1-p)^{N-1-i} \quad B(i,N-1,p) \qquad (1)$$

For the specific loss of material $\underset{\sim}{v}$ as a random variable the expected value, at the Nth attack, may then be given by

$$\mu(N) \equiv E\{\underset{\sim}{v};N\} \equiv \int_0^\infty v p(v;N)\,dv \sim \sum v_i p_i(N) \qquad (2)$$

with $p_i(N) \equiv P^N_{rep\,i}$; the determination of the particular values v_i of specific loss of material is left as an exercise in continuum mechanics. Eq. (2) agrees qualitatively well with experimentally obtained erosion curves in the literature and should allow fairly good quantitative predictions of the erosion rate.

H. BEDNARCZYK:

Mechanical influencing of absorption spectra

There exist very close relations between artificial birefringence, i.e. the stress-optical effect and the resonance peaks and frequencies of ultraviolet and infrared absorption spectra. By that means a n easy explanation of change of normal and anormalous dispersion of artificial birefringence in photoplasticity, as it was observed by many experimentalists, is possible. The absorption spectra can be influenced by mechanical loading in twofold way: Either by detuning of molecular oscillators, or by alignment of dipole-type oscillators parallel to a certain preferred direction. Using experimental data of the lierature it is shown that the effect of alignment is dominant especially for the infrared absorption spectrum when the specimen is overstained into the plastic range. The definition of an appropriate factor of anisotropy in order to describe the amount of alignment of dipole-type oscillators during mechanical loading of the material leads to a better understanding of the observed sudden rise of anormalous dispersion when overstraining the specimen into the plastic range.

J.F. BESSELING:

Pitfalls in the discretization of the nonlinear elastic deformation problem

Equations of equilibrium relate the "observable" stresses to the externally applied forces. These stresses are determined by the deformation in a neighbourhood of a material point in a continuum model, and by the deformation in a finite element in a discretized model. In both models the stresses are the duals of the deformation rates, determined by the velocities. in the discretized model the coefficients in the equilibrium equations are the derivatives of the deformation functions with respect to the kinematic nodal variables. If the dimension of the deformation space is larger than the dimension of the velocity space of the element minus the degrees of freedom as a rigid body, than the element has a statical redundancy. On the element level the redundancy is of the same order as the number of equations that express compatibility. However on the element level full compatibility usually is by no means the optimum choice for the strain distribution.

In an elasic geometrically nonlinear problem the stresses are expressed in terms of deformations. These equations can be solved by a step by step procedure with iteration after each step. However since in each step the increments of stress are expressed linearly into the increments of displacement, the geometrical nonlinearity will cause a continuity error in the dual strains, that will in part correspond to statically indeterminate stresses. These spurious stresses will be built up in the solution process and cannot be removed by iteration. the spurious stresses can only be avoided if adjoining elements have the deformation of their interface in common. They are kept small by adopting a Lagrangian updating technique in which strains are obtained by incrementation such that the compatibility conditions for the strain rates are sufficient to ensure compatibility of strains and displacements.

J.-P. BOEHLER:

Theoretical and experimental study of anisotropic hardening of rolled sheet-steel at large deformations: Influence of morphologic texture and loading path

In order to separate effects of crystallographic and morhologic texture on the anisotropic hardening at large deformations an experimental program has been developed on soft rolled steel exhibiting the same crystallographic, but different morphologic textures: one steel with equaxed grains, the other with elongated grains. The experimental results show that the morphologic texture plays a significant role in the development of anisotropic hardening. Moreover, large plastic strains applied in a direction different from the rolling and transverse direction gradually change the initial orthotropy in a new orthotropy, the privileged directions of which coincide with the principal directions of straining.

Within the theory of representation of tensor functions a model for plastic materials, including initial orthotropy and anisotropic hardening effects, has been developed. The model is able to describe correctly the experimental observations.

Finally, important loading path effects are disclosed in the anisotropic hardening behavior of both steels.

G. DEL PIERO:

Mechanics of fractured continua

For a body undergoing fracture the deformations can be assumed to be C^1 only in those regions which do not undergo any fracture. Just as in Continuum Mechanics the set of C^1 deformations is completed with respect to a Sobolev norm, the set of piecewise C^1 deformations is to be completed with respect to a norm which takes into account the discontinuities. It comes out that the elements of the completion are not functions, but triples (g,G,ψ) of L^2 functions, the domain of g and G being the region occupied by the body and that of ψ being the fractured zone.

G and ψ are interpreted as the fields of elastic deformations and microfractures, resp. If it happens that $G=Dg$ and $\psi=Jg$, the jump associated with g, then (g,G,ψ) is a deformation in the classical sense. Otherwise $Dg-G$ and $Jg-\psi$ are measures of diffused and concentrated inelastic deformations. Depending upon the constitutive equations they represent microfractures, plastic slip, damage etc.

A first advantage of this approach is that of giving a kinematical meaning to quantities which are usually considered as state variables. A second advantage is that in this space it is possible to write a Gauss-Green formula in which the deformations (g,G,ψ) are coupled with L^2 stresses with L^2 divergence. This is an essential property for the weak formulation of the equilibrium problem.

A first example is the equilibrium problem of the material which does not support tension or masonry-like material. With the use of the Gauss-Green formula the problem is transformed into a variational inequality in a convex set or into the minimum problem of the total potential energy in the same convex set.

G. FICHERA:

Analytical methods and physical reality: a difficult relationship

Two theories of Mathematical Physics are considered.
1) Viscoelasticity. It is shown how the so-called principle of fading memory strongly depends on the topology which has been introduced in the space of the admissible functions. In fact the physical results present completely different features when this topology is changed. On the other hand it is shown that this topology cannot be introduced on the basis of physical experiments, but it is a purely mathematical choice.
2) Heat diffusion. Fourier's theory of heat diffusion has been criticized by several authors on the basis that, according to it, disturbances of temperature propagate with infinite speed. It is shown that this crticism is unfounded when the theory is properly interpreted. Using correctly quantitative analysis the theories which have been proposed as alternatives to the classical one are much more open to criticism than the Fourier theory.

F.D. FISCHER:

A micromechanical study on martensitic transformation in metals

The physical aspects of martensitic transformation are discussed. The process can be explained as a sequence of a stretching/reduction (Bain strain), an invariant shear and a lattice rotation leading to a free transformation tensor $\underline{\underline{\varepsilon}}'_c$ described with respect to the local coordinate system with x', y' in the habit plane; $\underline{\underline{\varepsilon}}'_c \rightarrow \underline{\underline{\varepsilon}}'_c \Rightarrow \varepsilon_{c,ij}$, $\varepsilon_{c,33}=\delta$, $\varepsilon_{c,13}=\varepsilon_{c,31}=\gamma/2$ (δ: transformation volume change, γ: invariant shear being geometrically reversible (twinning) or not (plastification). Additionally an interaction occurs between the different transformed microregions as well as the parent phase. This interaction can be performed elastically or elasto-plastically.

Now the relation between a global load stress tensor $\underline{\underline{\sigma}}$ and the global strain tensor $\underline{\underline{\varepsilon}}$ for a transforming material is studied under the assumptions:

- $\underline{\underline{\sigma}}$ remains constant during the transformation
- each transformed microregion shows one and the same final strain tensor $\underline{\underline{\varepsilon}}$ (Taylor-Lin-hypothesis).

Two cases are studied:

- a transformation induced plasicity-steel with geometrically irreversible shear and plastic interaction
- a shape memory alloy with geometrically reversible shear and elastic interaction.

Good agreement with experiments is demonstrated.

R.D. GREGORY:

The general form of the elastic field inside an isotropic plate with free faces - an rigorous expansion theorem

a homogeneous, isotropic plate has free faces and is stretched by tractions around its edge which are symmetrical about the mi-plane, but are otherwise generally distributed. We give a rigorous proof that the most general state of stress τ_{ij} which can be generated in the plate can be decomposed in the form $\tau_{ij}=\tau_{ij}^{PS}+\tau_{ij}^{S}+\tau_{ij}^{PF}$, where τ_{ij}^{PS} is an exact plane stress state, τ_{ij}^{S} is a shear state, and τ_{ij}^{PF} is a Papkovich-Fadle state which is a 3-dimensional generalisation of the Papkovoch-Fadle eigenfunctions for the elastic strip.

Furthermore we proof that, as the plate thickness $h \to 0$, τ_{ij}^{S} and τ_{ij}^{PF} are exponentially small at points inside the plate and represent edge effcts of thickness $O(h)$.

Corresponding results are also given for the bending case.

G. GRIOLI:

St.Venat´s problem: Comparison with the exact theory

St.Venant´s theory of equilibrium of a cylindrical elastic body loaded on its ends has attracted the interest of mathematicians scince it appeared.

It is interesting to compare the conclusions of St.Venant´s theory which is an approximate theory with those of the exact theory. To do so, several authors have studied St.Venant's problem from various points of view investigating the behavior of the stored energy, the analytic characterisation of the solutions, the generalization to micropolar continua, the possible measures of overall strain. I consider this last point of view, defining for the exact solution an overall strain and showing that in three important instances the value of the overall strain for St.Venant's solution is the asymptotic limit of that for the exact theory when the length of the cylinder approaches infinity.

P. HAUPT:

On the thermodynamics of rate-independent materials

Rate-independent material behavior is characterized by hysteresis properties which do not depend on the rate of cyclic processes. Therefore, the idea of reversible processes as an asymptotic limit of slow processes is of no use in this case. Nevertheless, it is possible to model rate-independent hysteresis behavior within the classical thermodynamic theory of internal variables, based on the Clausius-Duhem-inequality. On basic idea is to interpret a plastic deformation as a sequence of equilibrium states with periodically changing stability properties. Another possibility is the application of an arc length description. Starting from the concept of dual variables and derivatives the intermediate configuration is utilized as a material reference configuration to formulate equations of state, to define thermodynamic potentials via Legendre transforms and, finally, to represent constitutive assumptions of rate-independent themoplasticity.

G. HERRMANN:

Conservation laws for systems without a Lagragian

Consider a system described by n dependent variables and m independent variables x and governed by a set of partial differential equations DE[u]=0, (1). For a variety of reasons it is most desirable to find an m-component vector P, such that $\mathrm{div}_x P=0$, (2), but only when (1) is satisfied which is called a non-trivial conservation law. Provided (1) is a set of Euler-Lagrange equations derived by a variational procedure on the basis of a Lagrangian function, Noether's first theorem supplies a complete methodology on how to obtain (2). In case a Lagrangian cannot be readily written down (dissipative systems), Noether's theorem is no longer of any help. It is shown, however, that a procedure with an integrating function leads to desired results in many cases, both for linear and nonlinear systems. To hit, one sets $fDE[u]=\mathrm{div}_x P$ and determines both f and the components of P from the resulting set of PDE's. This procedure is illustrated with several examples including the diffusion equation and linear viscoelasticity. In the latter case, (2) supplies among several other possible statements the energy balance relation which usually would be obtainable only through the application of the first and first part of the second law of thermodynamics.

KIENZLER, R:

Cracks under creep conditions

Energy-balance based concepts in fracture mechanics are derived from translational invariance of the energy-balance in terms of conservation laws and path-independent integrals. It is shown experimentally that creep crack growth is correlated by the so-called C^* integral independent of specimen size and geometry. Furthermore, some numerical aspects of finite element simulation are discussed, like proper medelling of the crack tip region, explicit and implicit time integration scheme, the influence of materail constants in the constitutive equations, 2-D and 3-D simulations and simulation of crack growth. The agreement between experimental and numerical results are rather satisfactorily.

R.J. KNOPS:

Spatial decay in elastodynamical problems

An examination is undertaken of the vibrating elastic cylinder excited by a time-dependent end displacement. The method establishes a differential inequality for the quantity ana-analoguous to the total entropy flux across a cross-section of the cylinder. Integration along the forward and backward characteristic curves enables some limited information to be

obtained on the behavior of the solution which confirms the intuitive expectation that the excitation propagates along the cylinder with a finite speed. Of interest, however, is the determination of the solution within the excited portion. In order to make progress, recourse is made to the corresponding linear theory under sinusoidal end-displacements. applied to avoid shock propagation and resonance effects. By suitable decomposition of the displacement it may be concluded that the solution in the disturbed region consists of a permanently deformed part and a part that decays spatially with axial distance. The latter component is analogous to the behavior in the corresponding equilibrium problem of the end-loaded cylinder (St.Venant´s principle).
The work described was joint with J.N. Savin and L.E. Payne.

F.G. KOLLMANN:

Two-point tensors and kinematics of rate dependent elastic solids

In this lecture the manifold theoretical approach of Marsden and Hughes is taken. Let $\mathcal{B}$ and $\mathcal{T}$ be two differentable n-manifolds and assume a bijective map ϕ: $\mathcal{B} \to \mathcal{T}$. Then 4 associated two-point tensors of rank 2 can be defined as linear maps between tangent and cotangent spaces defined on $\mathcal{B}$ and $\mathcal{T}$. If further the manifolds $\mathcal{B}$ and $\mathcal{T}$ are equipped with Riemannian metrics inner products can be defined. Next an abstract definition of transposed two-point tensors is given. Be U a linear vector space defined on $\mathcal{B}$. Then the two-point tensor $\underline{T}$ is a linear map $\underline{T}$: $U \to V$. V is a linear vector space defined on $\mathcal{T}$. The transposed two-point tensor is defined as $\underline{T}^T$: $V \to U^*$ where U^* is the dual of the space U, and equality of inner products on $\mathcal{T}$ and $\mathcal{B}$ is required. Some remarks on orthogonal two-point tensors and symmetric ordinary tensors follow.

Next the deformation gradient $\underline{F}$ which is a two-point tensor is decomposed multiplicatively into an elastic part $\underline{F}_e$ and a plastic part $\underline{F}_p$ introducing the notion of an intermediate configuration. Exploiting the concepts of push-forwards and pull-backs of metric tensors 3 families of deformation tensors are derived. Starting from the definition of Green´s strain measure on the reference configuration strain tensors and their additive decomposition on all three configurations are obtained. Next two families of covariant velocity gradients are obtained where again push-forwards and pull-backs are applied Finally two families of rate of deformation tensors are introduced. The first family defines the covariant rate of deformation tensor as symmetric part of the covariant velocity gradient. In the second approach the rate of deformation tensor is obtained as the Lie derivative of Almansi's strain tensor. By pulling back to the intermediate and reference configuration representations and additive decompositions are obtained.

M. KORZEN:

A viscoelastic-plastic model of material behavior

The constitutive model is characterized by the equations

i) $\sigma(t) = \sigma_\infty(t) + \sigma_{mem}(t)$ with

$\sigma_{mem}(t) = \underset{\tau \le t}{\mathfrak{F}}(\sigma_0(\tau), \sigma_\infty(\tau))$, $\mathfrak{F}$: fading memory

- slow: $\mathfrak{F} \equiv 0$, - fast: $\mathfrak{F} \equiv \sigma_0 - \sigma_\infty$

ii) $\sigma_\infty(t) = \underset{\tau \le t}{R_\infty}(\varepsilon(\tau))$, R_0: rate independent

iii) $\sigma_0(t) = \underset{\tau \le t}{R_0}(\varepsilon(\tau))$, R_0: rate independent

For static continuations, i.e. $\varepsilon(t) = \varepsilon(t^*) = \text{const.}$ $(t \ge t^*)$, the stress σ relaxes to its equilibrium value $\sigma_\infty(t^*)$ which is identical to the response of i)-iii) in the asymptotic limiting case of slow processes. This property characterizes the main difference to those constitutive models of viscoplastic type which are not purely viscoelastic. Representing $\mathfrak{F}$ by

$$\int_0^{z} \bar{G}(z-\bar{z})[\sigma_0'(\bar{z}) - \sigma_\infty'(\bar{z})]\,d\bar{z}, \quad dz = dt/k(\sigma_{mem})$$

one obtains an analytical solution for monotonic loading at constant strain rate $\dot{\varepsilon}_0$ in the limit $t \to \infty$. Assuming a positive sum of exponentials for $k(\sigma_{mem})$ this solution is used to identify the corresponding material parameters by a self-starting algorithm of Gustafson which guarantees convergence close to the global minimum.

J. KRATOCHVIL:

Stability analysis of non-linear continuum with microstructure

From micromechanical point of view (scale μm) any plasically deformed solid can be treated as a non-liear continuum with microstructure. The microstructure is represented by the dislocation population and the main non-linearity comes from field equations for stress and strain of a prestressed body. The stability analysis of the system of equations describing such continuum (infinite size) provides a valuable key to answers of basic problems of plasticity and damage. Two types of instability occur:

1) Structural instability leads to a self-organization of stored dislocations into characteristic patterns of low and high dislocation density regions. Piling up and subsequent annihilation of stored dislocations in the high density regions cause work-hardening which can be understood as a transient stage approaching a plastic steady state. The steady state is reached when all dislocations produced by plastic deformation are annihilated.

2) Geometrical instability caused by non-linear stress and strain effects leads to a misorientation of elementary volumes (cells) and localization of strain. Localization may result in failure or damage of the solid and thus terminate the approach to the plastic steady state.

E. KREMPL:

Viscoplasticity theory based on overstress

After a short introduction of the methods of modern plasticity theory research the viscoplasticity theory based on overstress for finite deformation was introduced. No yield surface, no loading/unloading conditions are used. The current configuration is the reference conf. and the rate of deformation tensor is the sum of its elastic and inelastic part. Since applications are restricted to metals and their alloys a rate form of the isotropic Hooke law is used for the elastic rate of deformation tensor. The inelastic rate is solely a function of the overstress, the difference between the current stress and the equilibrium stress a state variable of the theory. The rest of the talk was devoted to explain why viscoplasticity was used instead of rate independent plasticity, why F^eF^p was not used at all. Indeed arguments were presented that the unloading condition has to be represented by the constitutive equation rather than by kinematics. The talj concluded with a definition of history dependence in the sense of plasticity which was proposed as the discriminating feature for identification of "plasticity".

TH. LEHMANN:

Balance of energy and entropy in inelastic deformations including damage processes

In most cases during inelastic deformations solid bodies undergo certain changes of the internal structure. Examples are hardening of the material due to rearrangement of lattice defects, solid phase transformation, but also damage processes by arising micro-defects. Based on the assumption that the body can be considered as a classical continuum even if it is damaged by such microdefects a certain thermodynamical frame for the description of the interaction between external energy supply and changes of the internal structure is developed. This frame and some resulting consequences for the formulation of constitutive laws particularly with respect to the evolution of damage is discussed in some details.

H. LIPPMANN:

Plane rigid/plastic bending of a slender beam

A one-dimensional theory using the local bending moment $M(\zeta)$, the normal force $N(\zeta)$ to the cross-section, and the transverse force $Q(\zeta)$ as generalized stresses is presented where ζ denotes a material parameter along the axis of the beam. Large deformation is allowed, and the possible shapes of the bent beam are searched under conditions when the external loads or angular deflections are prescribed at the ends of the beam only. The governing differential equations are deduced for an arbitrarily strain-hardening or rate-sensitive material. For the special case of an ideally plastic material showing a constant generalized yield limit $|M|=L$ the bending process develops as a consequence of stationary or moving yield hinges. This leads either to local folding or to a smooth bending curve the end rotations of which, i.e. the motion of the supporting tools may easily be obtained by elementary geometric considerations.

O. MAHRENHOLTZ:

Constitutive equations of ice

Ice is considered as a building material in arctic regions. While sea ice is highly anisotropic artificial ice may be mo-modelled as isotropic and homogeneous. Hence, an isotropic flow potential or creep potential is assumed for creep of ice. The main parameters of the model are identified in an uniaxial compression test. The progress achieved lies in the success-full application to a combined state of stress, namely combi-ned compression and torsion of a cylindrical specimen.

Additionally, the model is extended to so-called tertiary creep by including a Kachanov approach. But no verification has been carried out so far.

A. MIELKE:

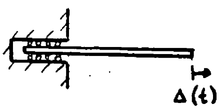
On Saint-Venant´s problem and Saint-Venant´s principle for nonlinearly elastic beams

According to ideas of Ericksen /1983/ we consider deformations of long beams having bounded strains. It can be shown that these solutions can be compared to so-called Saint-Venant so-lutions which are defined to be deformations of the infinite-ly ling beam. The difference of these solutions is found to be uniformly exponential with respect to the distance from the nearer end. Moreover it turns out that St. Venant solutions are actually describable by rod equations of Kirchhoff-Antman type. In particular we are able to derive the constitutive law of the rod in a mathematically rigorous way from the material properties of the beam.

I. MÜLLER:

Pull-out of an elastic rod glued into a rigid wall

A problem is considered as shown in the Fig. in which an ela-stic rod is glued into the hole of a rigid wall. The glue is modelled by a series of elastic springs which may break at a given extension. If the end of the rod is subject



to a time-dependent displacement $\Delta(t)$ we ask for the length of detachment of the rod as a function of time. It is hoped that some information from this simple one-dimensional detachment problem may be of heuristic value in more realistic dynamic fracture problems. e.g. in whisker reinforced ceramics.

W. MÜLLER:

Fracture mechanics and life-time predictions of ceramic roller bearings

The instability of surface and subsurface flaws in the raceway of a ceramic roller bearing are studied by means of a quantitative model: An elastic halfspace (raceway) which contains an arbitrarily oriented Griffith crack is subject to the normal and frictional forces of an infinite sliding cylinder (roller). The stress intensity factors of the crack are calculated by means of Erdogan's integral equation technique. The results are furthermore used for the prediction of the critical crack length in the raceway for different ceramic materials. It turns out that for Al_2O_3, SiC and HPSN the critical crack length is in the order of the average grain size, i.e. in the order of the length of microcracks which exist in the ceramic due to fabrication. In the use of ZrO_2, however, the K_{Ic} is high enough to give a critical crack length which is large enough to ensure a reasonable long life-time.

Z. OLESIAK:

Stress singularities in elastic matrix with surface layer generated by thermodiffusion fluxes

The fundamental question was whether the existence of fields of temperature and that of diffusion generate the significant stress, in particular stress concentrations or singularities.

For the case of axial symmetry it was possible to construct a model of a solid with inextensible membrane bonded to the bounding plane and to solve analytically the problem. There exist the singularities in the distribution of stresses at the lines of jump of temperature or diffusion (logarithmic type) and one over square root singularities at the lines of the closure of the bonded domain.

G.P. PARRY:

Invariants of defective crystals

The cassical theory of continuous distribution of dislocations has traditionally focussed on the Burgers vectors and the dislocation density tensor as descriptors of defectivness. We prove that, generally, there is an infinite number of tensor densities with similarly descriptive properties, and that there is a functional basis which strictly includes the Burgers vector and dislocation density. Moreover the changes of state which preserve these densities turn out to represent slip incertain surfaces associated with crystal geometry, so that the basic mechanism of plasticity theory emerges naturally from abstract ideas which do not anticipate the kinematics of particular types of crystal defects.

H. PETRYK:

Upper bounds to the onset of instability in elastic-plastic solids

Instability of a quasi-static deformation path at varying loading is distinguished from a narrower concept of instability of an equilibrium state. Energy criteria of instability are formulated under general assumptions concerning the nonlinear constitutive rate equations and the boundary conditions which ensure existence of a global potential for the incremental boundary value problem. In typical circumstances studied so far in the literature, the onset of path instability is shown to coincide with the primary bifurcation point while stability of equilibrium is usually maintained in some interval beyond that point. It is shown that path instability in the energy sense is associated with sensitivity of the solution to vanishingly small perturbing forces. This provides a theoretical basis for an upper bound technique for determining the onset of instability in incrementally nonlinear, time-independent solids. Analytic estimates are obtained for the onset of necking under uniaxial and biaxial tension, taking into account the effect of formation of a vertex on the yield surface.

J. SIVALOGANATHAN:

Stability of regular and singular equilibria

In the lecture I considered the static stability of smooth & singular solutions of the equilibrium equations of finite elasticity. For smooth solutions I demonstrated that every equilibrium has strong local minimising properties if the corresponding stored energy is polyconvex (uniformly).

The singular solutions considered correspond to the mathematical phenomenon of cavitation in which a hole forms in an initially perfect ball of isotropic elastic material held in tension under prescribed boundary displacements. It is known that the cavitating maps are minimising in the class of radial maps. I showed that the stability of these singular maps with respect to general 3-D perturbations depends strongly on the topology used.

E. STEIN:

Elastic-viscoplastic deformations at finite strains

On the basis of the multiplicative split of the deformation in sperical, isochoric elastic and isochoric inelastic parts and the maximum inelastic dissipation principle a flow rule for elastovoscoplasticity is derived. The flow rule is formulated in the current configuration an turns out in terms of a relaxation stress rate, i.e. strain space formulation.

With the help of the covariant Doyle-Ericksen formula, the tangent material tensor for elastoviscoplasticity in current configuration is calculated. So Newton´s method can be applied to solve the nonlinear finite element equations which leads to a effective numerical process.

Examples are presented for impacts of steel cylinders. The elastoviscoplastic solutions are compared with rigid plastic results and experimental data.

M. WAGROWSKA:

Some problems of elastic-inelastic periodic composites

Exact solutions to the boundary value problems for microperiodic composites are too complicated to be successfully applied in the engineering practice. There are many approximate methods for modelling of these kinds of materials. One of them is a non-standard homogenization method leading to the microlocal parameter theories of microperiodic composites. In this lecture the aforementioned method is applied to elastic-inelastic microperiodic materials. Problems are analysed within the small gradient deformation frame.

D. WEICHERT:

Combined shakedown- and crack-propagation analysis of elastic-plastic structures

The shakedown analysis of elastic-plastic structures under variable loads is not always sufficient in order to get reliable informations about the performance of a mechanical structural element: Even if plastic shakedown is predicted, the undercritical propagation of small cracks, initially present or developing during plastic adaptation, may lead to failure.

Some extensions and new developments concerning the solution of the shakedown problem are reviewed, in particular the influence of geometrical nonliearities and of the limitation of kinematic hardening, modelled by a simplified two-surface yield condition. The crack propagation problem is tackled at the time being by the assumption of a crack in a given place at the shakedown-loading situation. Its velocity is then determined by an empirical law for undercritical mode-I crack propagation for the state of stress associated to the shakedown load previously determined.

Z. WESOLOWSKI:

Nonlinear interaction between two waves in elastic materials

The second order elasticity theory (5 elastic constants) is applied. The total displacement consists of the fundamental motion, correction terms and interaction terms. If the fundamental motion is one pulse, then both the second and third order approximation may be obtained analytically. If the fundamental motion represents two colliding pulses one numerical integration is necessary to obtain the displacement field.

Dynamics of the transition region between two homogeneous materials

Between two homogeneous elastic materials there is a transition region where the propagation speed is a linear function of space. It is shown that the dynamic behavior of this structure may not be approximated by n steps of homogeneous material.

F. ZIEGLER:

Dynamics of visco-plastic structures

Nonlinear vibrations of ductile structures under periodic forces are considered. High velocity impact, resulting in elastic-plastic wave formation is treated as well.

In both problems the linear elastic virgin state is kept permanently to determine a background structure. Plasticity and ductile damage are considered in the form of distributed defects of the Hooke body. Going back to a thermodynamical reasoning, the internal variables become related macroscopically to inelastic strain rates an a damage parameter. Such distorsion results in an additional internal loading of the background material which drives the linear system into the nonlinear deformation state. The response to that inetrnal loading can be determined by means of static Green´s functions applied to the vibrational case or dynamic Green´s functions and time-convolution integrals in case of the hyperbolic wave problem. Displacement and stresses in both cases are calculated in two parts: A portion which is due to the external loading of the background structure and a portion which is due to the internal loading. A further splitting into the quasistatic and the complementary dynamic part is applied to the vibrational case for reasons of numerical stability. As a byproduct the plastic drift is determined. The integral equations are solved best by modal analysis of the dynamic portion. Damage produces a kind of plastic chaos in the modal responses, to be detected by Poincare maps.

Explicit results are discussed for visco-plastic and bilinear plastic semi-infinite rods loaded under kinematic as well as dynamic boundary conditions. Stability of the numerical solution after space-time discretization is almost unlimited.

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H. ZORSKI:

Dynamics of dipoles, a one-dimensional chain

An elementary dipole is defined as a pair of points with a prescribed distance between them. Equations of motion and conservation principles are discussed and some elementary solutions are presented. It is shown that there are discontinuities of first derivative in the dependence of the dipole moment of the external elastic field. Interactions between dipoles are inroduced and a one-dimensional chain of dipoles is examined in more detail. Various solutions are derived and the corresponding dispersion curves are presented. It is shown that for the ordinary interaction potential there are no solitons in the system. A class of interaction potentials exists leading to the sine-Gordon equation. It is shown that the replacement of an oscillator as the basic element of the theory of polarization by a rotating dipole affects the resulting polarization.

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