MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 18/1990

Einhüllende Algebren und Ringe von Differentialoperatoren

22.4. bis 28.4.1990

Leitung : W. Borho (Wuppertal)
M. Duflo (Paris)
J.C. Jantzen (Eugene)
R. Rentschler (Paris)

Die Tagung Einhüllende Algebren und Ringe von Differentialoperatoren war dazu bestimmt neue Entwicklungen auf dem Gebiet der Einhüllenden Algebren von Lie-Algebren und Wechselwirkungen mit dem Bereich der Ringe von Differentialoperatoren aufzuzeigen. Des weiteren sind neben unendlich-dimensionalen Lie-Algebren die sogenannten Quanten-Gruppen in Erscheinung getreten, die als Deformationen von Einhüllenden Algebren auftreten.

Die Tagung konzentrierte sich fast ausschließlich auf den halbeinfachen Fall. Bei einer Reihe von Vorträgen kamen geometrische Fragen zum Tragen.

Eine entscheidende Rolle spielen relle und komplexe adjungierte nilpotente Orbiten, ihre Abschlüsse und deren Überlagerungen, ihre Beziehungen zu unipotenten unitären Darstellungen, zu Weyl-Gruppen Darstellungen, zu primitiven Idealen und Fragen der "Quantisierung", und zu den sogenannten Dixmier-Algebren (endliche äquivariante Erweiterungen primitiver

Quotienten Einhüllender Algebren von halbeinfachen Lie-Algebren). Ein ganzes Programm "Quantisierung von Überlagerungen von adjungierten Orbit-Abschlüssen und Dixmier-Algebren" zeichnet sich ab. Erwähnt sei insbesondere die Möglichkeit normaler Einbettungen solcher Überlagerungen (Kostant/R.K. Brylinski) in grössere Lie-Algebren.

Weitere geometrische Behandlungen galten verallgemeinerten Schubert-Varietäten, der Bruhat-Ordnung in symmetrischen Raumen, der Beschreibung des minimalen K-typs in Harish-Chandra Moduln, sowie in einem Fall (für nilpotente Lie-Algebren) des Zusammenhangs gewisser Moduln endlicher Länge mit Lagrange-Untervarietäten bestimmter coadjungierter Orbiten.

Differentialoperatoren kamen zum Tragen über die Beilinson-Bernstein Lokalisierung, bei unitarisierbaren Darstellungen mit höchstem Gewicht, bei der Behandlung homogener Räume, bei der Bestimmung des Zentrums von Algebren invarianter Differentialoperatoren. Zwei Vorträge galten Differentialoperatoren auf singulären Kurven.

Verschiedene Fragen zu Lie-Gruppen Darstellungen haben sich nunmehr (wie schon viele in der Vergangenheit) als mit algebraischen Mitteln behandelbar erwiesen : Die Bestimmung der unitarisierbaren Darstellungen mit höchstem Gewicht und die Bestimmung der $\mathfrak{n}$-Homologie zulässiger Banachraum-Darstellungen.

Zwei Vorträge galten der Behandlung von Whittaker Moduln (mit der Bernstein-Beilinsen Lokalisierung) bezw. der Existenz verallgemeinerter Whittaker-Vektoren in Harish-Chandra Moduln.

Neu in Erscheinung getreten ist der Zusammenhang zwischen der sogenannten Kategorie $\mathcal{O}$ und Koszul-Algebren; erwähnt sei insbesondere der Beweis der Isomorphie (Soergel) des Endomorphismenrings der Summe aller direkt unzerlegbaren Projektiven mit der Ext-Algebra der Summe aller Einfachen in $\mathcal{O}$ (mit zentralem trivialen Chara kter).

Ein Vortrag befasste sich mit bestimmten kommutativen Unteralgebren der Einhüllenden Algebra von halbeinfachen Lie-Algebren, welche von einer Cartan-Algebra und bestimmten Elementen vom Grad zwei erzeut werden. Ein Vortrag galt dem Zusammenhang von Volichenko-Algebren, Lie-Superalgebren und endlichdimensionalen einfachen Lie-Algebren in Charakteristik 2.

Drei Voträge galten unendlichdimensionalen Lie-Algebren : Eigenschaften von Linienbündeln über Schubert-Varietäten von $\hat{SL}_n$; modulare Invarianz, conforme Symmetrie und Zusammenhang mit der Virasoro-Algebra. Klassifizierung (Mathieu) aller einfachen Harish-Chandra Moduln für die Virasoro-Algebra unter Verwendung von Charakteristik-p Methoden.

Besondere neue Aspekte ergaben sich durch das Erscheinen von Quanten-Gruppen, das heisst von Deformationen von Einhüllenden Algebren, und spezieller unter ihnen, den sogenannten Yangians. Im Unterschied zu den üblichen Einhüllenden Algebren sind die Quantengruppen vergleichsweise starr und die Comultiplikation ist nicht kommutativ.

Vorgetragen wurde über endlichdimensionale Darstellungen von Quanten-Schleifengruppen, über Quantenalgebren gehörig zu Schubertvarietäten, sowie über Verschwindungssätze, Induktion und Charaktere. Für Quanten-Gruppen vom Typ $U_q(\mathfrak{g})$ wurde der ad-endliche Teil und im Zusammenhang damit mit einer originellen Methode das Zentrum bestimmt. Ein Vortrag zeigte eine enge Beziehung zwischen eine narürlichen Erweiterung der Einhüllenden Algebra von $\mathfrak{gl}(\infty,\mathbb{C})$ und der Vereinigung der aufsteigenden Kette der zu den $\mathfrak{gl}(n,\mathbb{C})$ gehörigen Yangians auf.

Vortragsauszüge

D. BARBASCH : UNIPOTENT REPRESENTATIONS OF REAL REDUCTIVE GROUPS

Special unipotent representations play a central role in the unitary dual of a complex group. In particular it is important to study their character theory.

In this talk we find a formula for the number of unipotent orbits attached to a given nilpotent orbit in terms of multiplicities in induced representations from subgroups of the Weyl group.

A similar formula is described for the number of real nilpotent orbits conjugate to a given complex orbit.

Y. BENOIST : SIMPLE MODULES ON A NILPOTENT LIE ALGEBRA CONTAINING AN EIGENVECTOR FOR A SUBALGEBRA

Let $\mathfrak{g}$ be a nilpotent Lie algebra over $\mathbb{C}$, $\mathfrak{k}$ a subalgebra, f a character of $\mathfrak{k}$. Let I be a primitive ideal of the enveloping algebra $\mathcal{U}$ of $\mathfrak{g}$ associated to a coadjoint orbit Ω. We describe the $\mathfrak{g}$-module $\mathcal{U}/(I+\mathcal{U}\mathfrak{k}^f)$ when it is of finite lenght (where $\mathfrak{k}^f=\{X-f(X);X\in\mathfrak{k}\}$); in particular, it is semisimple and its simple quotients are in bijection with the irreducible components of the lagragian manifold $Z := \Omega\cap(f+\mathfrak{k}^\perp)$.

K. BROWN : RINGS OF DIFFERENTIAL OPERATORS ON AFFINE CURVES : THEIR FINITE DIMENSIONAL TOPS

Let X be an irreducible affine curve over $\mathbb{C}$ with regular functions $\mathcal{O}(X)$ and ring of (global) differential operators $\mathcal{D}(X)$. By results of S.P. Smith and J.T. Stafford, the Noetherian affine domain $\mathcal{D}(X)$ has a unique minimal non-zero ideal J(X), the factor $H(X) := \mathcal{D}(X)/J(X)$ being a finite dimensional $\mathbb{C}$-algebra, with $H(X) = \bigoplus_{x \in \mathrm{Sing}X} H(X,x)$, the factor H(X,x) associated with each singular point x of X being indecomposable. I give information on the structure of J(X) and of H(X), including a description of H(X) as a set of linear maps on $\mathcal{O}(\bar{X})/E$, where $\bar{X}$ is the unramified closure of X and E is the conductor of $\mathcal{O}(\bar{X})$ into $\mathcal{O}(X)$. These results are used to split the K theory of $\mathcal{D}(X)$ into the direct sum of the K-theory of H(X) with the K-theory of $\mathcal{O}(\tilde{X})$, where $\tilde{X}$ is the normalisation of X.

R.K. BRYLINSKI : INTEGRATION OF INVARIANT VECTOR FIELDS ON A LIE ALGEBRAS

Joint work with B. Kostant. Let G be a complex semisimple algebraic group. Let $\mathcal{O}=G.e$ be the adjoint orbit of a nilpotent $e \in \mathfrak{g}$. Let $\mathfrak{z}(\mathfrak{g}^e)$ be the center of the Lie centralier $\mathfrak{g}^e$.

Theorem: $Z_e = \mathfrak{z}(\mathfrak{g}^e) \cap \mathcal{O}$ is open dense in $\mathfrak{z}(\mathfrak{g}^e)$. So all direction vectors in $\mathfrak{z}(\mathfrak{g}^e)$ are tangent to $\mathcal{O}$ at e. So $\dim(N(G^e_o)/G^e_o) = \dim$ (invariant vector fields on $\mathcal{O}$) = $\dim(\mathfrak{z}(\mathfrak{g}^e))$.

Moreover the Lie algebra normalizer is $\mathfrak{n}(\mathfrak{g}^e) = \mathfrak{g}^e \oplus [e_-, \mathfrak{z}(\mathfrak{g}^e)]$, where (e,h,e_-) is an S-triple. Furthemore, $N(G^e_o)/G^e_o$ is solvable, and the boundary of Z_e in $\mathfrak{z}(\mathfrak{g}^e)$ is a union of hyperplanes. For e principal, we find $\dim(N(G^e)/G^e) = 1 = \mathrm{rank}\ \mathfrak{g}$. Application: $N(G^e_o)/G^e_o$=G-symplectic structures on G/G^e_o.

V. CHARI : QUANTUM LOOP ALGEBRAS

We classify irreducible representations of the quantum affine algebra associated to $\mathfrak{sl}(2)$.

Theorem 1 : There exists a bijective correspondance between irreducible finite-dimensional representations of $U_q(\hat{\mathfrak{sl}}(2))$ and polynomials with constant coefficient one.

If $\mathfrak{g}$ is of type $\mathfrak{sl}(n)$, there exists a map $\sigma_a : U_q(\hat{\mathfrak{g}}) \to U_q(\mathfrak{g})$ for every $a \in \mathbb{C}$. Thus every representation V of $U_q(\mathfrak{g})$ gives rise to a representation V(a) of $U_q(\hat{\mathfrak{g}})$.

Theorem 2 : Any finite-dimensional irreducible representation is isomorphic to $\otimes V_i(a_i)$ for some choice of V_i and a_i.

J. ENRIGHT : COVARIANT DIFFERENTIAL OPERATORS

Here we extend to the general Hermitian symmetric setting the correspondance between singular modular forms and covariant differential operators (CDOS). This connection was set down in an article by Harris and Jakobsen (Math. Ann. 1982). The results described here are part of joint works with Joseph and Davidson and Stanke.

Let G/K be a Hermitian symmetric space and H a Cartan subgroup of K. Let $\mathfrak{g}, \mathfrak{k}$ and $\mathfrak{h}$ be the corresponding complexified Lie algebras. We have decompositions $\mathfrak{g} = \mathfrak{m} \oplus \mathfrak{k} \oplus \mathfrak{m}^+ = \mathfrak{n} \oplus \mathfrak{h} \oplus \mathfrak{n}^+$ with $\mathfrak{k} \oplus \mathfrak{m}^+$ (resp. $\mathfrak{h} \oplus \mathfrak{n}^+$) a maximal (resp. minimal) parabolic subalgebra. The $\mathfrak{k}$-decomposition of the symmetric algebra $S(\mathfrak{m})$ gives an isomorphism of $S(\mathfrak{m})^{\mathfrak{n}}$ with $\mathbb{C}[T_1, \ldots, T_r]$ with r equaling the split rank. Let V_i be the $\mathfrak{k}$-module in degree i of $S(\mathfrak{m})$ with lowest weight corresponding to the generator T_i.

Suppose L_τ is a unitarizable highest weight representation of G with level (of reduction) l, $1 \le l \le r$, and defining $\mathfrak{k}$-type E_τ. Let (σ, D) be the Parthasarathy-Ranga Rao-Varadarajan component with $\mathfrak{k}$-module projection $P: V_l \otimes E_\tau \to D$.

Theorem 1 : D occurs in the generalized Verma module $\mathbb{E}_\tau$ with multiplicity one and gives an intertwining map

$$\Phi : \mathbb{E}_\sigma \to \mathbb{E}_\tau$$

The image is the maximal $\mathfrak{g}$-submodule of $\mathbb{E}_\tau$ and so taking adjoints Φ^* we find L_τ is characterized as the kernel of Φ^*.

Theorem 2 : Let X_i and X^i be basis and dual basis for V_1 (with respect to a Gaussian measure).

Then Φ^* has the form of a gradient

$$\Phi^* f = P \sum_i (X^i * f) \otimes X_i$$

H. HECHT : A COMPARISON THEOREM FOR $\mathfrak{n}$-HOMOLOGY

Let (π, M_π) be an admissible Banach space representation of a semisimple Lie group G_o, and M its Harish-Chandra module with respect to a fixed maximal compact subgroup K_o of G_o. Let $\tilde{M}$ be the (dense) G_o-submodule of analytic vectors in M_π. Our goal is to compare homology groups of M and $\tilde{M}$, with respect to a nilradical $\mathfrak{n}$ of a Borel subalgebra $\mathfrak{b}$ of $\mathfrak{g}$ ($\mathfrak{g}$ is the complexified Lie algebra of G_o). Both G_o and the comlexification K and K_o act on the flag variety X of $\mathfrak{g}$ with finitely many orbits, paired under Matsuki correspondance. Any point in the intersection of a pair of related (G_o and K) orbits is called special.

Theorem (jointly with J. TAYLOR): Let $x \in X$ be special and let $\mathfrak{n}_x$ be the nilradical of the Borel subalgebra $\mathfrak{b}_x$ corresponding to x. Then the natural map $H_p(\mathfrak{n}_x, M) \to H_p(\mathfrak{n}_x, \tilde{M})$ is an isomorhism.

The proof uses the following ingredients : 1) $M \to \tilde{M}$ is an exact functor (result of W. Schmid); 2) Localisation of $\mathfrak{b}$-modules theory (Berlinson-Bernstein, and Hecht-Taylor in global case).

M. P. HOLLAND : THE INFLUENCE OF SL_2 ON $\mathcal{D}$-MODULES ON RATIONAL CURVES

Let $\mathcal{X}$ be a singular rational projective curve over $\mathbb{C}$ such that the normalisation $\mathbb{P}^1 \to \mathcal{X}$ is bijective. The talk was about the properties of $\mathcal{D}(\mathcal{X})$, the global differential operator ring on $\mathcal{X}$. If $\mathcal{D}_{\mathcal{X}}$-mod is the category of quasi-coherent $\mathcal{D}_{\mathcal{X}}$-modules then the functor $\mathcal{D}_{\mathcal{X}} \otimes - : \mathcal{D}(\mathcal{X})\text{-mod} \to \mathcal{D}_{\mathcal{X}}\text{-mod}$ is exact and makes $\mathcal{D}_{\mathcal{X}}\text{-mod} = \mathcal{D}(\mathcal{X})\text{-mod}/\langle H^1(\mathcal{X}, \mathcal{O}_{\mathcal{X}})\rangle$. Moreover every $\mathcal{M} \in \mathcal{D}_{\mathcal{X}}$-mod is generated by sections. Further $\mathcal{D}(\mathcal{X})$ is Noetherian with a unique minimal ideal $J(\mathcal{X})$. The factor $\mathcal{D}(\mathcal{X})/J(\mathcal{X}) \cong \left(\begin{array}{c|c} M_t(\mathbb{C}) & \mathbb{C}^{(t)} \\ \hline 0 & \mathbb{C} \end{array}\right)$ where t=arithmetic genus of $\mathcal{X}$. $\mathcal{D}(\mathcal{X})$ is Morita equivalent to $\mathcal{D}(\mathcal{Y})$ whenever $\mathcal{Y}$ has injective normalisation $\mathbb{P}^1$ and is singular.

A. JOSEPH : UNITARY HIGHEST WEIGHT MODULES

Let $\mathfrak{g}$ be a complex semisimple Lie algebra. The classification of unitarizable highest weight modules associated to a non-compact form has been known for some time; but was based on much case by case analysis. In joint work with T.J. Enright I gave a new and quite intrinsic description. Any such module L is associated to a triangular decomposition $\mathfrak{g} = \mathfrak{m}^+ \oplus \mathfrak{k} \oplus \mathfrak{m}$ with $\mathfrak{m}^+$, $\mathfrak{m}$ commutative and $\mathfrak{k}$ the Levi factor of the parabolic $\mathfrak{m} \oplus \mathfrak{k}$. Our analysis involves the $\mathfrak{k}$ modules $V_i : i=1,2,\dots t$ of $S(\mathfrak{m})$ generated by certain fundamental invariants, the notion of the level of a $\mathfrak{k}$ dominant weight τ and the Parthasarathy-Ranga Rao-Varadarajan component P_i of $V(\tau) \otimes V_i$.

Further work was aimed at understanding a result of M.G. Davidson, T.J. Enright and R.J. Stanke who showed for $\mathfrak{g}$ classical that $\mathrm{Ann}_{\mathcal{U}(\mathfrak{m})} L$ is always a prime ideal! This was shown to hold in general. Furthemore it takes the form $\mathcal{U}(\mathfrak{m}) V_{l(\tau)+i-1}$ with $l(\tau)$ explicitly computed. (Conventionally $V_j = 0$ if $j > t$). When $\mathrm{Ann}_{\mathcal{U}(\mathfrak{m})} L \neq 0$, a sufficient (and conjecturally necassary) condition for $\mathrm{Ann}_{\mathcal{U}(\mathfrak{g})} L$ to be maximal is given. For $\tau = 0$ this recovers a result of T. Levasseur and J.T. Stafford (obtained for $\mathfrak{g}$ classical) that $\mathrm{Ann}_{\mathcal{U}(\mathfrak{g})} L$ is always maximal. For $\tau \neq 0$ this fails. Nor need $\mathrm{Ann}_{\mathcal{U}(\mathfrak{g})} L$ be completely prime; but the Goldie rank of the quotient algebra always divides dim $V(\tau)$.

F. KNOP : THE CENTER OF THE ALGEBRA OF INVARIANT DIFFERENTIAL OPERATORS

Let G be a connected reductive group, and X a smooth G-variety. Let $\mathcal{D}(X)$ be the algebra of linear differential operators on X.

Theorem : Let X be affine. The center Z of the algebra $\mathcal{D}(X)^G$ of invariant operators is a polynomial ring and $\mathcal{D}(X)^G$ is a free Z-module.

More specifically one can find an isomorphism between Z and the algebra $S(\mathfrak{a})^{W_X}$, where W_X is a finite reflection group acting on $\mathfrak{a}$.

Application : Let $H \subseteq G$ be a connected reductive subgroup. Then the center of the centralizer $\mathcal{U}(\mathfrak{g})^H$ is generated by $\mathcal{U}(\mathfrak{g})^{\mathfrak{g}}$ and $\mathcal{U}(\mathfrak{h})^{\mathfrak{h}}$.

B. KOSTANT : NILPOTENT ORBITS AND NORMAL EMBEDDINGS

Joint with R.K. Brylinski. Let G be a connected simply connected complex semi-simple Lie group with Lie algebra $\mathfrak{g}$. Let $\mathcal{O}=G.e$ be a nilpotent adjoint orbit. The affine variety $X=\mathrm{Spec}\,\mathbb{C}[\mathcal{O}]$ is the normalization $\eta: X \to \bar{\mathcal{O}}$ of the closure $\bar{\mathcal{O}}$ in $\mathfrak{g}$; X is an important geometric object for representation theory. Let E be the Euler field on $\mathcal{O}$.

Proposition : Let m be the G-stable complement to $\mathbb{C}$ in $\mathbb{C}[\mathcal{O}]$; so m is the maximal ideal of $\eta^{-1}(0)$. Then X embeds equivariantly into $(m/m^2)^* = T_{\eta^{-1}(0)} X$ (Zariski tangent space) and this is the minimal embedding. Any equivariant (for G and E) complement U to m^2 in m provides the embedding map.

Theorem :

(1) $U \cong \mathfrak{g}' = \mathbb{C}[\mathcal{O}]_2$ (Euler grading) is a Lie algebra under Poisson bracket on $\mathbb{C}[\mathcal{O}]$.

(2) $\mathfrak{g}'$ is semi-simple, and its action integrates to an Hamiltonien action of its Lie group G' on X. G' is the maximal Lie group with a Hamiltonian action on X. The moment map $\mu: X \to (\mathfrak{g}')^*$ realizes X as the normalization of a nilpotent orbit closure $\bar{\mathcal{O}}'$ for $\mathfrak{g}'$; $X=\mathrm{Spec}\,\mathbb{C}[\mathcal{O}']$.

(<u>Conjecture</u> : $\mathfrak{g}$ simple => $\mathfrak{g}'$ simple). Our results extend orbit covers $\tilde{O}$.

<u>Example</u> : (1) $\mathfrak{g}=\mathfrak{g}_2$, $\bar{O}$=non-normal =>$\mathfrak{g}'=\mathfrak{so}(7)$ (Levasseur-Smith)

(2) $\tilde{O}$=simply connected cover of principal nilpotent orbit for $\mathfrak{sl}(3)$=>$\mathfrak{g}'=\mathfrak{g}_2$. (3)$O=O_{(3,3,1,1)}$ for $\mathfrak{g}=\mathfrak{sp}(8)$=>$\mathfrak{g}'=E_6$ (with Vogan). Thus for, examples support U=$\mathfrak{g}'$ for orbits O.

<u>S. KUMAR</u> : <u>A GEOMETRIC REALIZATION OF THE MINIMAL K-TYPE OF HARISH-CHANDRA MODULES FOR COMPLEX SEMISIMPLE GROUPS</u>

Let G be a connected simply connected complexe semi-simple algebraic group viewed as a real Lie group with Lie algebra $\mathfrak{g}$. We fix a maximal compact subgroup K⊂G. As is known, any irreducible ($\mathfrak{g}$,K)-module M admits a unique minimal K-type. In the case when M has integral infinitesimal character, we realize the minimal K-type of M by a purely geometric construction and connect it with some special components in the tensor product of two finite dimensional irreducible holomorphic representations of G. This is a joint work with B. Kostant.

<u>V. LAKSHMIBAI</u> : <u>SCHUBERT SCHEMES</u> - <u>CLASSICAL, GENERALIZED ET QUANTUM</u>

I : "<u>Classical</u>" : Let G be a semi-simple algebraic group over an algebraically closed field k. Let B be a Borel subgroup of G and X a Schubert variety in G/B. Let L be an ample line bundle on G/B. Let $R_x=\oplus_L H^0(X,L)$. We construct explicit bases for R_x in terms of standard monomials, as a generalization of the classical Hodge-Young theory.

II : "<u>Generalized</u>" : We prove similar results (as in I) for Schubert varieties in G/B where $G=\widehat{SL}_n$.

III : "<u>Quantum</u>" : Assume k=$\mathbb{C}$ and G classical. One can associate a Hopf algebra A(G) (in a canonical way) which is a deformation of k[G]. We refer to A(G) as a quantum deformation of G. We construct a "quantum G/P" by giving a subalgebra A(G/P) of A(G) which is also a left coideal in such a way that A(G/P) is a deformation of k[G/P](=$\oplus H^0(G/P,L^m)$). For a Schubert variety X in G/P, we construct a quantum space A(X), as a deformation of k[X]. We obtain similar bases for A(G/P) as well as $A(X_w)$ (as in I). We have similar results for G/B and its Schubert varieties.

D. LEITES : NON COMMUTATIVE GEOMETRY AND VOLICHENKO ALGEBRAS

(Joint with Vera Serganova). Conjectorially, there exists a not too noncommutative geometry in which the algebras of functions are replaced by metabelian algebras (satisfying $[x,[y,z]]=0$ for any three of their elements), the role of Lie algebras there is played by nonhomogeneous subalgebras of Lie superalgebras (called Volichenko algebras in honour of I. Volichenko who described them as PI-variety). Simple Volichenko algebras (finite-dimensional over $\mathbb{C}$) constitute a discrete (and nice-looking) list modulo a hypothesis (projection of the simple Volichenko algebra to the even part of the ambiant simple Lie superalgebra is onto). Their enveloping algebras, albeit no Hopf ones, provide with an interesting invariant which measures their "nonhopfness". As an application the following conjecture is offered :

Reduction of structure constants of integer forms of simple finite-dimensional or "vectory" Lie algebras, Lie superalgebras or Volichenko algebras modulo 2 (with subsequent deformations) gives (after selection a simple subquotient) all simple finite-dimensional Lie algebras over an algebraically closed field of char 2 (perhaps, restricted in some sence).

G. LETZTER : ADJOINT ACTION AND QUANTUM ENVELOPING ALGEBRAS

(Joint work with A. Joseph). Let $\mathfrak{g}$ be a semisimple Lie algebra. There is a quantum group $U_q(\mathfrak{g})$ associated to $\mathfrak{g}$ which is a deformations of the enveloping algebra $U(\mathfrak{g})$. Using the Hopf algebra structure of $U_q(\mathfrak{g})$, we may define an adjoint action of $U_q(\mathfrak{g})$ on itself. Unlike the enveloping algebra $U(\mathfrak{g})$, the quantum group $U_q(\mathfrak{g})$ does not act locally finite on itself under the adjoint action. We find a subalgebra P of $U_q(\mathfrak{g})$ that admits a locally finite $U_q(\mathfrak{g})$ action such that $PK=U_q(\mathfrak{g})$ where K is a group of invertible elements in $U_q(\mathfrak{g})$. We apply this result to obtain a description of the center of $U_q(\mathfrak{g})$.

O. MATHIEU : CLASSIFICATION OF SIMPLE HARISH CHANDRE MODULES FOR THE VIRASORO ALGEBRA

We classify all the simple Harish Chandra modules for the Virasoro algebra $\hat{W}$ (Kac's conjecture). The proof uses modular representation theory of restricted Lie algebras.

H. MATUMOTO : WHITTAKER VECTORS FOR COMPLEX SEMISIMPLE LIE GROUPS

In this lecture, we generalize the notion of Whittaker vectors to certain parabolic subalgebras. For complex semisimple Lie groups, we give the condition for the space of $C^{-\infty}$-Whittaker vectors of an irreducible Harish-Chandra module V to be non-zero and finite dimensional in terms of the wave front set of V, under the assumption that V has an integral infinitesimal character. We also give the relation to Goldie rank polynomial representations.

W. M. McGOVERN : QUANTUM OF NILPOTENT ORBIT COVERS IN COMPLEX SEMISIMPLE GROUPS

Let $\mathfrak{g}$ be a complex semisimple Lie algebra $\mathcal{O}\subset\mathfrak{g}^*$ a coadjoint orbit (usually nilpotent), X a finite cover of $\mathcal{O}$. David Vogan has formulated a program to attach to X an algebra extension of finite type over $U(\mathfrak{g})$ (a Dixmier algebra) which quantizes the latter in the sence that both admit filtrations with the associated graded objects isomorphic. We describe a conjectural implementation of the program for trivial and universal covers X. Using an important construction of induced Dixmier algebras due to Vogan, we make substantial progress towards proving that the implementation satisfies Vogan's desired properties. It $\mathfrak{g}$ is of classical type, the implementation generalizes Arthur's construction of special unipotent representations.

D. MILICIC : TWISTED HARISH-CHANDRA SHEAVES AN WHITTAKER MODULES

We discussed the application of Beilinson-Bernstein localisation theory to the study of the category of finitely generated modules over the enveloping algebra $U(\mathfrak{g})$ of a semisimple Lie algebra $\mathfrak{g}$ which are locally finite over the center $Z(\mathfrak{g})$ of $U(\mathfrak{g})$ and over the algebra $U(\mathfrak{n})$ of the nilpotent radical $\mathfrak{n}$ of a fixed Borel subalgebra in $\mathfrak{g}$. In particular, we gave a geometric proof a Kostant's results on Whittaker modules with nondegenerate action of $\mathfrak{n}$ on the Whittaker vector. This is a joint work with Wolfgang Soergel.

G.I. OLSHANSKII : EXTENSIONS OF ENVELOPING ALGEBRAS OF INFINITE-DIMENSIONAL CLASSICAL LIE ALGEBRAS AND THE YANGIANS

Let us consider the simplest infinite-dimensional classical Lie algebra $\mathfrak{gl}(\infty,\mathbb{C})$, the union of the algebras $\mathfrak{gl}(n,\mathbb{C})$. The center of $U(\mathfrak{gl}(n,\mathbb{C}))$ has n generators but the center of $U(\mathfrak{gl}(\infty,\mathbb{C}))$ is trivial. However it is possible to define a natural extension A of $U(\mathfrak{gl}(\infty,\mathbb{C}))$. The algebra A has a rich center Z(A) and operates in highest weight $\mathfrak{gl}(\infty,\mathbb{C})$-modules.

Theorem : The algebra A is isomorphic to $Z(A)\otimes Y(\infty)$ where $Y(\infty)$ denotes the union of the ascending chain of the Yangians $Y(\mathfrak{gl}(m,\mathbb{C}))$, m=1,2,...

Thus the Yangians turn out to be closely related to representation theory of infinite-dimensional classical Lie algebras. There are some generalisations involving other Lie algebras and new "Yangian-type" algebras.

P. POLO : REPRESENTATIONS OF QUANTUM ALGEBRAS

We report on a joint work with H.H. Andersen and Wen Kexin. To each Cartan matrix Drinfeld and Jimbo have associated a so-called quantum group, which is a certain Hopf algebra U' over $\mathbb{Q}(v)$. Let $\mathcal{A}$ denote the localisation of the polynomial ring $\mathbb{Z}[v]$ at the maximal ideal generated by v-1 and p, where p is an odd prime. Following Luszting, we consider some $\mathcal{A}$-lattice U of

U' which is a Hopf algebra over $\mathcal{A}$. We consider a Borel type subalgebra U^b of U and study induction from U^b to U. We prove that it can be obtained as a composite of $|R^+|$ rank one inductions. This enables us to obtain analogues of Grothendieck and Kempf vanishing theorems for the flag variety, and of Demazure character formula. Considering a specialization of $\mathcal{A}$ into a field Γ, we get a Borel-Weil-Bott theorem and Serre duality. From these it follows that finite dimensional U_Γ-modules are completely reducible when the image of v is not a root of unity. In the root of unity case, the situation resembles modular representation theory. Namely, we obtain a linkage principle a "Jantzen type" sum formula. This allows us to compute all irreducible character in rank 2 and in type A_3. The results provide further evidence for the truth of Lusztig's conjecture.

F. RICHTER : THE ALGEBRA OF FUNDAMENTAL DIFFERENTIAL OPERATORS ON A HOMOGENEOUS SPACE AND RELATIONS TO THE RADON TRANSFORM

The classical Radon transform in Euclidean space is to integrate rapidly decreasing functions along p-dimensional planes. The range of this transformation is described by a finite system of partial differential equations coming from the universal enveloping algebra attached to the Euclidean group.
The system mentioned arises in a very natural way from a group theoretical point of view and becomes finite, since all generators of a special ideal are computed (a complete system of generators).

A. ROCHA-CARIDI : MODULAR INVARIANT REPRESENTATIONS IN CONFORMAL SYMMETRY

Conformal symmetry is the study on invariance with respect to a group of conformal transformations. In two-dimensions, the Lie algebra of the conformal group is equal to two copies of the Virasoro algebra. One of the consistency conditions in conformal symmetry is modular invariance of the partition function. Since the partition function can be expressed as a finite linear combination of products of two irreducible characters of the Virasoro algebra, conformal symmetry leads to the study of modular invariant

representations. These are the representations for which the orbits of their characters, under the modular group, are finite dimensional.
Exact formulas for the weight multiplicities of the modular invariant representations are obtained.

W. ROSSMANN : REAL NILPOTENT ORBITAL INTEGRALS AND REPRESENTATIONS OF WEYL GROUPS

Let $\mathfrak{g}_{\mathbb{R}}$ be a semisimple real Lie algebra, O a nilpotent orbit in $\mathfrak{g}_{\mathbb{R}}$. The problem is to find a formula for the invariant measure μ_O on O analogous to *Harish-Chandra's Limit Formula* for $O=\{0\}$:

$$\lim_{\lambda\to 0} \omega_\lambda \mu_\lambda = const. \mu_0$$

where μ_λ is the invariant measure of the orbit O_λ of the regular element λ in a real Cartan subalgebra $\mathfrak{h}_{\mathbb{R}}$ and $\omega_\lambda = \Pi_{\alpha\in\Delta^+}\partial\alpha$ is the product of the derivatives along the positive roots, considered as differential operator in λ. Formulas of this type for complex orbits have been obtained by Barbash and Vogan, Hotta and Kashiwara, and myself; for orbits in $\mathfrak{g}_{\mathbb{R}}=\mathfrak{u}(p,q)$ by Barbash and Vogan. The problem is solved here under the following hypothesis. We assume that there is a chamber C for the imaginary roots of $\mathfrak{h}_{\mathbb{R}}$ so that

$$O = O_{\mathbb{C}} \cap \left(\bigcap_{\lambda\in C} \overline{\mathbb{R}^{\times}_{+} O_\lambda} \right)$$

where $O_{\mathbb{C}}$ is the complex orbit containing λ. Under this hypothesis, Harish-Chandras formula holds for O when ω_λ is replaced by a differential operator that transforms under the Weyl-group representation associated to $O_{\mathbb{C}}$ in Springer' theory and the limit is taken from within the chamber C.

W. SOERGEL : COMBINATORIES OF CATEGORY O

Let $\mathfrak{g}\supset\mathfrak{b}$ be a semisimple complex Lie algebra and a Borel, $(W,\mathcal{S})$ it's Coxeter system, $e\leq W\leq w_o$.

Consider the category $O=O(\mathfrak{g},\mathfrak{b})$ of all finitely generated $\mathfrak{g}$-modules which are $\mathfrak{b}$-locally finite and semisimple over some Cartan $\mathfrak{h}$ of $\mathfrak{b}$.

Let $M_x, L_x, P_x \in O$ for $x\in W$ be the Vermas, simples and projectives with trivial central character, parametrivad in such a way that $M_e = L_e$, $M_{w_o} = P_{w_o}$, $L_{w_o} = \mathbb{C}$. We prove :

Theorem : (Beilinson-Ginsburg-Duality)
There is an isomorphism of $\mathbb{C}$-algebras

$$\mathrm{End}_{\mathcal{O}}(\oplus P_{x w_o x}) \cong \mathrm{End}^{\bullet}_{\mathcal{O}}(\oplus L_x, \oplus L_x)$$

such that the obvious idempotents 1_x correspond.

T.A. SPRINGER : THE BRUHAT ORDER OF SYMMETRIC SPACES

The talk was about joint work with R.W. Richardson. Let G be a connected reductive group over an algebraically closed field of characteristic not 2 and let θ be an automorphism of G of order 2 (in the sense of algebraic groups). Denote by K the fixed point group of θ and by B a Borel group of G. The quotient G/K is an affine variety, the symmetric variety associated to (G,θ). Il is known that B (acting by left translations) has finitely many orbits in G/K. We describe the inclusion relations between the closures of these orbits.

Our description is similar to the description (due to Chevalley) of the inclusion relations between closures of Bruhat calls in G/B by the Bruhat order of the corresponding Weyl group. As a matter of fact, the latter result is a special case.

M. VERGNE : SPRINGER REPRESENTATION AND JOSEPH POLYNOMIALS

Let $\mathfrak{g}$ be a complex semisimple Lie algebra. Let $x_o = \mathfrak{h} \oplus \mathfrak{n}$ be a fixed Borel subalgebra. Let $u \in \mathfrak{n}$ and let $\mathcal{O}_u = \overline{G.u}$ the closure of the G-orbit of u. Let C be an irreducible component of $\mathcal{O}_u \cap \mathfrak{n}$. A Joseph associated to C a polynomial J_C on $\mathfrak{h}$, the equivariant multiplicity of C. Consider now the Springer subvariety $D_u = \{x \in D;\ x = g.x_o;\ u \in x\}$ of the flag variety $D = G/B$. Let

$$D_u(C) = \{x \in D;\ x = g.x_o;\ g^{-1}u \in C\}$$

This is a cycle in D, thus determins an harmonic polynomial $S_{D(C)}(\lambda)$ on $\mathfrak{h}^*$. We prove that $S_{D(C)}(\lambda)$ is proportionnal to $J_C(\lambda)$ (when identitying $\mathfrak{h}$ with $\mathfrak{h}^*$

by the Killing form).

This result was proven earlier by Hotta, Ginsburg, Borho-Brylinski-Mac Pherson, Joseph. The proof given here relies on double integration technique of Rossmann's which is an elementary analogue of the direct image technique of Borho-Brylinski -Mac Pherson.

E. VINBERG : ON SOME COMMUTATIVE SUBALGEBRAS OF THE ENVELOPING ALGEBRAS OF SEMI-SIMPLE LIE ALGEBRAS

Let $\mathfrak{g}$ be a semi-simple Lie algebra, $\mathcal{U}$ its enveloping algebra and $\mathcal{P}=\mathrm{gr}\,\mathcal{U}$ the corresponding Poisson algebra. Some commutative subalgebras of $\mathcal{U}$ generated by a Cartan subalgebra of $\mathfrak{g}$ and some elements of degree 2 are described. It turns out that the corresponding graded algebras are contained in commutative subalgebras of $\mathcal{P}$ obtained by the method of translating invariants. The problem is posed, if the latter subalgebras of $\mathcal{P}$ can be lifted to commutative subalgebras of $\mathcal{U}$. A connection of this problem with the known construction of the Gelfand-Cetlin basis is discussed.

Berichterstatter : R. RENTSCHLER

Tagungsteilnehmer

Prof.Dr. Jacques Alev
Institut de Mathematiques Pures et Appliquees, UER 47
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Ken Brown
Dept. of Mathematics
University of Glasgow
University Gardens

GB- Glasgow , G12 8QW

Prof.Dr. Dan Barbasch
Dept. of Mathematics
Cornell University
White Hall

Ithaca , NY 14853-7901
USA

Prof.Dr.Rannee K. Brylinski
Department of Mathematics
Pennsylvania State University
McAllister Building

University Park , PA 16802
USA

Dr. Yves Benoist
U. E. R. de Mathematiques
T. 45-55, 5etage
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Jean-Yves Charbonnel
U. E. R. de Mathematiques
T. 45-55, 5etage
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Brian D. Boe
Dept. of Mathematics
University of Georgia

Athens , GA 30602
USA

Prof.Dr. Vyjayanthi Chari
School of Mathematics
Tata Institute of Fundamental Research
Homi-Bhabha Road

Bombay 400 005
INDIA

Prof.Dr. Walter Borho
Fachbereich 7: Mathematik
der Universität/Gesamthochschule
Wuppertal, Gaußstr. 20
Postfach 10 01 27

5600 Wuppertal 1

Sophie Chemla
U. E. R. de Mathematiques
T. 45-55, 5etage
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Dr. Fokko du Cloux
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Henryk Hecht
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Prof.Dr. David H. Collingwood
Dept. of Mathematics
University of Washington
C138 Padelford Hall, GN-50

Seattle , WA 98195
USA

Prof.Dr. Martin P. Holland
Dept. of Pure Mathematics
University of Sheffield

GB- Sheffield S3 7RH

Prof.Dr. Michel Duflo
U. E. R. de Mathematiques
T. 45-55, 5etage
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Jens Carsten Jantzen
Dept. of Mathematics
University of Oregon

Eugene , OR 97403-1222
USA

Prof.Dr. Aleksander G. Elashvili
Institute of Mathematics
Academy of Sciences of the Georgian
SSR
Z. Rukhadze Str. 1

380093 Tbilisi
USSR

Prof.Dr. Anthony Joseph
Dept. of Mathematics
The Weizmann Institute of Science
P. O. Box 26

Rehovot 76 100
ISRAEL

Prof.Dr. Thomas J. Enright
Dept. of Mathematics
University of California, San Diego

La Jolla , CA 92093
USA

Prof.Dr. Friedrich Knop
Mathematisches Institut
Universität Basel
Rheinsprung 21

CH-4051 Basel

Prof.Dr. Bertram Kostant
Dept. of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139
USA

Prof.Dr. Thierry Levasseur
Dept. de Mathematiques
Universite de Bretagne Occidentale
6, Avenue Victor Le Gorgeu

F-29287 Brest Cedex

Prof.Dr. Shrawan Kumar
School of Mathematics
Tata Institute of Fundamental
Research
Homi-Bhabha Road

Bombay 400 005
INDIA

Prof.Dr. Marie-Paule Malliavin
10, rue Saint Louis en l'Ile

F-75004 Paris

Prof.Dr. Venkatramani Lakshmibai
Dept. of Mathematics
Northeastern University

Boston , MA 02115
USA

Prof.Dr. Olivier Mathieu
28, bd. de l'hopital

F-75005 Paris

Prof.Dr. Dimitry Leites
Dept. of Mathematics
University of Stockholm
Box 6701

S-113 85 Stockholm

Prof.Dr. Hisayosi Matumoto
4 - 15 - 2 Shiden
Ichikawa-shi

Chiba-ken 272
JAPAN

Dr. Gail Letzter
Department of Mathematics
Wayne State University

Detroit , MI 48202
USA

Prof.Dr. William M. McGovern
Dept. of Mathematics
Yale University
Box 2155, Yale Station

New Haven , CT 06520
USA

Prof.Dr. Dragan Milicic
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Dr. Frank Richter
Sektion Mathematik
Humboldt-Universität Berlin
Postfach 1297
Unter den Linden 6

DDR-1086 Berlin

Prof.Dr. Gregori I. Olshanskii
Institute of Geography
USSR Academy of Sciences
Staromonetny 29

Moscow 109017
USSR

Prof.Dr. Alvany Rocha-Caridi
Mathematics Department
Baruch College
City University of New York
17, Lexington Avenue

New York , N. Y. 10010
USA

Prof. Dr. Patrick Polo
Laboratoire de Mathematiques
Fondamentales, U. E. R. 48
Universite Paris VI
4, Place Jussieu

F-75230 Paris , Cedex 05

Prof.Dr. Wulf T. Rossmann
Dept. of Mathematics
University of Ottawa

Ottawa, Ontario K1N 9B4
CANADA

Prof.Dr. Andrew Nicholas Pressley
Department of Mathematics
King's College London
Strand

GB- London WC2R 2LS

Prof.Dr. Brad Shelton
Dept. of Mathematics
University of Oregon

Eugene , OR 97403-1222
USA

Prof.Dr. Rudolf Rentschler
16, rue du Champ de Mars

F-75007 Paris

Dr. Wolfgang Soergel
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

5300 Bonn 3

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Prof.Dr. Tonny A. Springer
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof.Dr. Michele Vergne
Centre de Mathematiques
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05

Prof.Dr. Ernest Vinberg
Department of Mechanics and
Mathematics
Moscow State University
Lenin Hills

Moscow , 119899
USSR

Gefördert durch
DFG Deutsche Forschungsgemeinschaft