MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 19/1990

Gruppentheorie
-proendliche Gruppen-
29.4. bis 5.5.1990

Die Tagung fand unter Leitung von A.Lubotzky (Jerusalem) und O.H.Kegel (Freiburg) statt. Einige thematische Schwerpunkte waren: die Theorie der endlich erzeugten diskreten (residuell endlichen) Gruppen und ihrer proendlichen Vervollständigungen, die Struktur von pro-p-Gruppen und p-adischlinearen Gruppen, die Theorie der Körper unter Berücksichtigung ihrer Galoisgruppen, und kombinatorische pro-endliche Gruppentheorie. Zu all diesen Themenkreisen wurden in den Vorträgen die Theorien entwickelt und die aktuellsten Forschungsergebnisse vorgestellt. Hier wurde auch auf die Zusammenhänge zur Modelltheorie eingegangen. Da die Zeit für die Vielzahl der Vorträge knapp bemessen war, wurden zu den Vormittags- und Nachmittagssitzungen noch einige abendliche Zusammenkünfte angesetzt, so auch am Donnerstagabend, an dem offene Probleme gestellt und diskutiert wurden.

Vortragsauszüge

Zoé Chatzidakis

Profinite groups with the Iwasawa Property

We study profinite groups having the Iwasawa Property (IP), using for our study some homogeneity properties of the dual inverse system of finite quotients of such a group. We show that every profinite group H has an IP-

Gefördert durch DFG Deutsche Forschungsgemeinschaft

cover, which is unique up to isomorphism over H. We also describe some lifting properties of automorphism.

Ido Efrat

Lifting of generators in subgroups structures.

We show that Gaschütz' lemma on lifting of generators and other known results of the same nature are special cases of a general theorem in the category of, so called, subgroups structures. As another special case we obtain: If $\varphi:G \to H$ is an epimorphism of profinite groups such that $G=\langle G_1 \dots G_n \rangle$, $H=\langle H_1 \dots H_n \rangle$ and such that $\varphi(G_i)$ is conjugate to H_i, $i=1\dots n$, then there exists $g_1,\dots g_n \in G$ such that $\varphi(G_i^{g_i})=H_i, i=1\dots n$, and $G=\langle G_1^{g_1} \dots G_n^{g_n} \rangle$.

Mike Fried

Serre's lifting invariant

<u>(A) Jacobians and rough outline of the obstructions:</u>

Suppose that $\mathfrak{X} \to \mathbb{P}^1$ is a Galois cover with group, given by Riemann's existence theorem through $\sigma \in G^r$ where the corresponding conjugacy classes are C. Consider $\mathrm{Pic}^0(\mathfrak{X})_p^k = \mathrm{Pic}_{p^k}$, the collection of points of p^kth-power order on the Jacobian of $\mathfrak{X}$. The projective completion of this with respect to k is the Tate module, T_p, and because of the functoriality of it all, this gives

(1) $\quad G(M_p^0/\mathbb{C}(\mathfrak{X})) = T_p \to G(M_p^0/\mathbb{C}(x)) = {}_pG^{au} \to G(\mathfrak{X}/\mathbb{C}(x))$.

Here M_p^0 is the maximal abelian unramified extension of $\mathfrak{X}$. This should be contrasted with the group $G(M_p/\mathbb{C}(\mathfrak{X}))$ which is the maximal unramified extension of $\mathbb{C}(\mathfrak{X})$. What we are interested in is whether ${}_p\tilde{G}$ is a quotient of $G(M_p/\mathbb{C}(x))$, or whether ${}_p\tilde{G}[P_p,P_p]$ is a quotient of $G(M_p^0/\mathbb{C}(x))$. At this stage our reliance is on the "easiest" Frattini cover: ${}_pR \to G$, the "universal p-central extension." That is, is ${}_pR$ a quotient of ${}_pG^{au}$?

<u>(B) 1st Obstruction:</u> In each of the cases there must be lifts $\tilde{\sigma}$ of the entries of σ to the respective groups ${}_p\tilde{G}, {}_pG^{au}$ and ${}_pR$ with the following properties:

(2) $\quad \tilde{\sigma}_i$ has order equal to σ_i, $i=1\dots r$, and $\tilde{\sigma}_1 \dots \tilde{\sigma}_r = 1$.

Since neither ${}_p\tilde{G}$ nor ${}_pG^{au}$ have any p-power elements of finite order, this implies that $(\mathrm{ord}(\sigma_i), p) = 1$, $1=1\dots r$. If this, however, is satisfied, we can certainly lift them. But can we do so with the product equal to 1? This is the main topic of the talk.

R.I.Grigorchuk

On the growth of residually p-finite groups

Let G be a finitely generated residually p-finite group, $\gamma(n)$ be the growth function of G.

Theorem 1 : If $\gamma(n)$ grows slower than the function $e^{\sqrt{n}}$ then G is virtually nilpotent and so has a polynomial growth.

The proof uses the Lazard criterium for analyticity of pro-p-groups in terms of the Hilbert-Poincaré series $f_{G,p}(t)$ of the graded algebra

$$gr(G) = \bigoplus_{n=0}^{\infty} \Delta^n/\Delta^{n+1}$$

($\Delta \leq F_p[G]$ the augmentation ideal), associated with the group.

Theorem 2 : For any prime p there exists a finitely generated residually finite p-group G such that $a_n(G) \sim e^{\sqrt{n}}$, where

$$a_n(G) = \dim_{F_p} \Delta^n/\Delta^{n+1}$$

are the coefficients of the series $f_{G,p}(t)$.

Theo Grundhöfer

Sharply transitive linear groups

$G \leq GL(V)$ is a sharply transitive linear group on the vector space V if G acts sharply transitively on $V \setminus \{0\}$. Zassenhaus(1936) and Kalscheuer(1940) have classified these groups if V is finite, or if $V = \mathbb{R}^n$ and G is closed. In joint work with Cherlin, Nesin and Völklein, all groups G are determined if V is a finite-dimensional vector space over an algebraically closed field. If V is a finite-dimensional vector space over $\mathbb{Q}_p$, then every closed sharply transitive group G is of Dickson type, i.e. G is contained in a one-dimensional semi-linear group $\Gamma L_1 D$ for a suitable skew field D.

Fritz Grunewald

Some combinatorics relating to the local definability of ε-factors

Let k be a finite field, $\chi: k^* \to \mathbb{C}^*$, $\psi: k^+ \to \mathbb{C}^*$ two characters with ψ nontrivial. The gaussian sum for χ , ψ is defined as

$$\tau(\chi,\psi) = \sum_{x \in k \setminus \{0\}} \chi(x)\, \psi(x) .$$

I presented elementary proofs for the Hasse-Davenport relation and other relations between gaussian sums originating from ideas of Stepanov.

C.K.Gupta

Automorphism group of certain relatively-free groups

In this talk I gave a brief survey on the automorphism groups of certain relatively free groups. Highlight of the survey was recent joint work with Bryant, Levin, Mochizuki, in which, among other things, I mentioned a criterium of invertibility of certain $n \times n$ matrices over the free group ring $\mathbb{Z}F$. This yields, in particular, non-tameness of certain automorphisms of free nilpotent groups.

N.D.Gupta

Primitivity in relatively free groups

$w \in F = \langle x_1, \ldots, x_n \rangle$ is primitive if w can be included in some basis of F. If $V \leq F$ is fully invariant and if w is primitive mod V then w lifts to a primitive element of F if wv is primitive in F for some $v \in V$. We extend this notion to primitive lifting of a system $\underset{\sim}{w}=(w_1,\ldots,w_m)$, $m \leq n$, mod V.

Theorem (Gupta,Gupta, Roman'kov) If $\underset{\sim}{w}=(w_1,\ldots,w_m)$, $m \leq n-2$, is primitive mod $\gamma_{c+1}(F)F''$ then $\underset{\sim}{w}$ lifts to a primitive system of F.

Remark: There is a system $\underset{\sim}{w}=(w_1,\ldots,w_{n-1})$ which is primitive mod $\gamma_{c+1}(F)F''$ but cannot be lifted to a primitive system of F.

Dan Haran

Cohomology theory for profinite groups with involutions

An *Artin-Schreier structure* is a system $\mathbf{G}=(G, G', X)$, where G is a profinite group, G' is a subgroup of index 1 or 2, X is a boolean space on which G acts continuously such that:

(a) the stabilizer of every $x \in G$ is a group of order 2, say $G_x = \{1, d(x)\}$, where $d(x) \notin G'$;

(b) The so defined map $d: X \to G$ is continuous.

An *epimorphism* of Artin-Schreier structures is the quotient map $\mathbf{G} \to \mathbf{G/N}$, where $N \leq G'$ is a closed normal subgroup of G. One can define *projective* Artin-Schreier structures as satisfying a certain embedding property.

We define the cohomology groups of **G** *as the homology groups of the* complex $C^*(\mathbf{G},A)$, where

$$C^n(\mathbf{G},A) = \{ f: G^{n-1} \times (G \cup X) \to A \mid f \text{ continuous}\}, \quad n = 1,2,\ldots$$

with the obvious coboundary map. With this definition it is posssible to prove:

Proposition 1. An Artin-Schreier structure is projective if and only if $H^n(\mathbf{G},A)=0$.

Theorem 2 . The cohomological dimension of the absolute Artin-Schreier structure $\mathbf{G}=\mathbf{G}(K)$ of a number field K is ≤ 2.

Using the cohomology we can show that following:

Theorem 3. Let G be an Artin-Screier structure, then cdG=cdG'. In particular, **G** is projective if and only if the group G' is.

The proof of this (which uses the category of profinite modules) is an analogue of the following result:

Theorem (Serre): Let G be a torsion free pro-p-group containing an open free subgroup. Then G is free.

Karl H.Hofmann

Free compact groups, Lie groups

For any conpact pointed space X there is a compact group FX containing X (base point = identity) such that any base point preserving continuous function f: X→ G into a compact group extends to a continuous homomorphism FX→ G.

In the profinite situation, as in the abelian situation, the structure and the circumstances are fairly well understood. The talk gives a survey of the status of the general situation. Whereas the profinite case has an arithmetic and algebraic flavor, the connected case has a topological one. Lie theory and algebraic topology is used.

Moshe Jarden

Hilbertian fields and Free profinite groups*

There is an interesting analogy between theorems about extensions of Hilbertian fields and theorems about closed subgroups of the free profinite group $\hat{F}_\omega$ on $\aleph_0$ generators. The analogy becomes even stronger for a special family of Hilbertian fields, namely the fields K which are PAC and ω-free. Recall that K is PAC if each absolutely irreducible variety defined over K has a K-rational point. The field is -free if its absolute Galois group G(K) is isomorphic to $\hat{F}_\omega$. A theorem of Roquette states that each PAC ω-free field K of characteristic zero is Hilbertian.

To state the analogy, consider a property PG of subgroups of profinite groups. We say that an extension N of K has the property PF if the subgroup G(N) of G(K) has the property PG. For example, let PG be the property of a subgroup H of a profinite group G to be accessible, i.e. H is the intersection of a descending sequence of closed subgroups H such that H_0=G and $H_{i+1} \triangleleft H_i$. Then N is accessible if N= $\bigcup_{i=1}^{\infty} N_i$, N_0=K and N_{i+1} is a Galois extension of N_i.

The twinning principle: Consider the following two statements:

(G) If a subgroup H of $\hat{F}_\omega$ has the property PG , then $H \cong \hat{F}_\omega$.

(F) If a separable algebraic extension N of an ω-free PAC field has the property PF, then N is Hilbertian.

Then: 1. (G) implies (F)

2. (F) implies (G) for accessible extensions N.

As an application we transfer a theorem of Haran-Jarden and a theorem of Uchida about fields into theorems about groups:

Theorem: (a) The intersection of two closed normal subgroups of $\hat{F}_\omega$ none of them is contained in the other is isomorphic to $\hat{F}_\omega$.

(b) Let M be a closed subgroup of $\hat{F}_\omega$ whose index is divisible by two distinct primes. Suppose that M contains a closed subgroup N of $\hat{F}_\omega$ such that $\hat{F}_\omega/N$ is pronilpotent. Then M is isomorphic to $\hat{F}_\omega$.

* A report on a joint work with Alexander Lubotzky.

E.I.Khukhro

Torsion pro-p-groups.

There are two well-known conjectures:

(a) every torsion pro-p-group is locally finite;

(b) every torsion pro-p-group is of bounded exponent.

In 1989 E.I.Zel'manov proved (a) on the basis of the analysis of his positive solution of the Restricted Burnside Problem for groups of all exponents p^k. (Together with J.S.Wilson's reduction - analog of Hall-Higman's reduction - this implies that every torsion compact group is locally finite.)

But it is still an open problem whether (b) is true. Towards its solution we prove the following

Theorem: If a torsion compact group contains an open subset of elements of prime order, then it is of bounded exponent and contains a subgroup of finite index from some locally nilpotent variety.

Note that every compact group contains an open subset of elements of some fxed order.

P.H.Krobholler

Poincaré Duality groups and the Torus Theorem

I discussed the conjecture that every PD^{3+}-group is the fundamental group of a (closed aspherical) 3-manifold. This conjecture can be reduced to some special problems about PD^3-groups by using the theorem:

Every PD^{3+}-group G satisfying Max-c (max. on centralizers) acts on a tree T so that G\T is finite, each $G_e \cong \mathbb{Z}^2$, and each pair (G_v,ster(v)) is either an atoroidal or Seifert type PD^{3+}-pair.

This is an algebraic analogue of the Torus Decomposition Theorem for 3-manifolds.

C.R.Leedham-Green

Pro-p-groups and Lie algebras

If P is a group of order p^n and class c , the coclass of P is defined to be n-c. A critical step in the classification of pro-p-groups by coclass, due to S.McKay, M.F.Newman, W.Plesken, S.Donkin, A.Mann and myself, is proving that pro-p-groups of finite coclass are soluble.

As a further step in this direction, we ask what just infinite, insoluble pro-p-groups p exist with $|\gamma_i(P)/\gamma_{i+1}(P)|$ uniformly bounded. Known examples are open subgroups of Sylow pro-p-subgroups of homogeneous semi-simple classical Lie groups over local fields (these are p-adic analytic in characteristic 0), and open subgroups of the Sylow pro-p subgroups of the automorphism group of the ring $\mathbb{F}[[t]]$, where $\mathbb{F}$ is a finite field of characteristic p. This group has been studied by D.Johnson and I.York.

Alexander Lubotzky

On groups of polynomial subgroup growth.

Let Γ be a finitely generated group, $a_n(\Gamma)$ the number of subgroups of Γ of index n. The following theorem is the accumulation of results of D.Segal, A.Mann and A.Lubotzky.

Theorem: Assume Γ is a residually (finite-soluble) group. Then $a_n(\Gamma)$ grows polynoially if and only if Γ is soluble of finite rank.

The proof uses the theory of p-adic Lie groups, algebraic and arithmetic groups and the prime number theorem.

Angus Macintyre

P-adic Poincare'series: uniformity in p.

P-adic integrals $\int_A |f(x)|^s |dx|$ are rational functions of p^{-s} under very general assumptions on definability of A and f. This was proved by Denef (plus van den Dries in the greatest known generality). There are applications to nilpotent and polycyclic groups. The proof is explained, and its defects for getting precise information. Uniformity of rationality in p is obtained by working in many-sorted languages.

It is suggested that results of the above type, including the underlying elimination theory, be done directly inside p-adic analytic groups.

Avinoam Mann

Group theoretical applications of p-adic analytic groups.

Residually finite groups can be studied by their profinite completions. This is particularly useful for res.-p groups, whose pro-p-completions are p-adic analytic, because the analytic groups are linear. Using deep theorems about finite groups, we can sometimes reduce from res-finite to res-p. Examples are:

(a) A.Lubotzky : Linearity Criterium (J.Alg.113).

(b) Residually finite groups of finite rank are virtually soluble (Lubotzky-Mann, Math.Proc.Cambridge Phil.Soc.,1989).

(c) Groups with polynomial subgroup growth, described in A.Lubotzky's talk.

(d) An alternative proof for Gromov's theorem about polynomial growth for res.-finite groups (see also Prof. Grigorchuk's talk).

Avinoam Mann

Subgroup growth and probability.

A profinite group G is positively finitely generated (PFG), if for some k, the set of k-tuples that generate G has a positive measure. Free profinite groups are not PFG (Kantor-Lubotzky), but all finitely generated pro-nilpotent groups are. A group in which the number of maximal subgroups of index n is bounded by a power of n is PFG. For pro-soluble groups, and maybe for all profinite groups, the inverse implication holds. It also seems possible that all pro-soluble groups are PFG. We also remarked on an application to groups with polynomial subgroup growth, etc.

B.H.Matzat

Profinite Hurwitzian Braid Groups

Structure of the (Artinian and) Hurwitzian braid groups - Structure of the profinite completion of the Hurwitzian braid groups - The Hurwitzian braid group as Galois group - Rationality criteria for intermediate fields - Rational places of their fields of definition.

O.V.Melnikov

Combinatorial profinite group theory: a homological approach.

The purpose of the talk is to survey recent results on profinite groups acting on profinite trees. A profinite graph Γ is a tree if its naturally defined augmented chain complex $C(\Gamma,\hat{\mathbb{Z}})$ has trivial homology groups. Let a profinite group G act on a tree Γ . Then $C(\Gamma,\hat{\mathbb{Z}})$ is a short exact sequence of $\hat{\mathbb{Z}}[[G]]$ -

modules and gives an exact sequence of Mayer-Vietoris type connecting the homology groups of G and the stabilizers of vertices and edges of Γ. The introduction of such notions as segments between couples of vertices and minimal G-invariant subtrees in Γ permits to use geometrical intuition in the investigation of the structure of G. In particular a description of such groups G without non-abelian free pro-p-groups is obtained. The results are applied to profinite fundamental groups of graphs of groups.

Thomas Müller

Counting free subgroups of finite index in virtually free groups.

We investigate the growth behaviour and the asymptotics of the number of free subgroups of given finite index in a finitely generated virtually free group G. Stallings' structure theorem is used to represent G as fundamental group of a finite graph of finite groups in the sense of Bass and Serre and the asymptotic behavior of the number of free subgroups of finite index is described in terms of such a decomposition of G. Furthermore we consider the question as to what kind of information on the structure of G is contained in the number of free subgroups of finite index and as to what extent this combinatorial information determines the group G. Most of our main results depend on properties of a new combinatorial structure, an infinite triangle of rational numbers generalizing Pascal's triangle, which is associated with certain valuations on a finite connected graph.

References:

(1) Th.Müller, Combinatorial aspects of finitely generated virtually free groups, to appear in : J.London Math.Society.

(2)Th.Müller, A group-theoretical generalization of Pascal's triangle, submitted to: European J. of combinatorics.

Francis Oger

Finite images, cancellation properties and elementary equivalence of groups.

For several classes of groups, we compare equivalence relations like this: 1) G and H have the same finite images; 2) G and H belong to the same genus; 3) $G \times \mathbb{Z}$ and $H \times \mathbb{Z}$ are isomorphic; 4) G and H are elemantarily equivalent. We show the following results:

Theorem 1. Any finitely generated abelian-by-finite group is an elementarily subgroup of its profinite completion.

Corollary. Two finitely generated abelian-by-finite groups are elementarily equivalent if and only if they have the same finite images.

Theorem 2. If G and H are finitely generated finite by nilpotent groups, then G and H are elementarily equivalent if and only if $G \times \mathbb{Z}$ and $H \times \mathbb{Z}$ are isomorphic.

W.Plesken

Remarks on finite relative coclass

(Report on work in progress with D.Holt)

Let G be a finite group and $1 \to K \to X \to G \to 1$ be an extension with $K = O_p(X)$ finite. Then $\mathrm{cocl}_G(X) :=$ # G-composition factors of K - nilpotency class (K) is the G-relative coclass of X. If K is the maximal pro-p-normal subgroup of X, then $\mathrm{cocl}_G(X) := \lim \mathrm{cocl}(X/N_i)$ for a suitable chain of normal subgroups of X. The existence of extensions of pro-p-groups by a given group G with prescribed finite coclass is investigated. It turns out that almost all normal subgroups of X are of the form $\gamma_i(K)$ and for sufficiently big i G acts irreducibly on $\gamma_i(K)/\gamma_{i+1}(K)$. If K is soluble it follows that X is a subdirect product of an irreducible (even p-universal) p-adic space group and a finite group. Examples show behaviour different from the p-group case. If K is not soluble, Chevalley groups of most simple split Lie algebras over discrete complete valuation rings provide examples under mild assumptions on the residue class field.

Florian Pop

Classically projective groups and pseudo-classically closed fields.

The following generalisation of results on PRC, P_pC fields and correspondingly, real projective , p-adically projective groups was discussed:

A. Let K be an arbitrary field. We say that K is pseudo-classically closed if there exist finitely many classical fields $\mathbb{k}_k$ (k=1...n) such that: any absolute irreducible variety V/K has K-rational points if it has regular Λ-rational points for all $\Lambda \leq \tilde{K}$, $\Lambda \equiv \mathbb{k}_k$ for some k.

B. Let G be an arbitrary profinite group. We say that G is classically projective if there exist finitely many classical Galois groups $\mathbb{G}_k \cong G_{\mathbb{k}_k}$ (k=1...n) and closed subspaces $\mathfrak{G}_k \leq \mathrm{Subg}(G)$ consisting of subgroups $\Gamma \cong \mathbb{G}_k$ such that: any group extension of a finite group H by G : $1 \to H \to F \xrightarrow{\pi} G \to 1$ is split, iff all "local extensions" $1 \to H \to \pi^{-1}(\Gamma) \to \Gamma \to 1$ ($\Gamma \in \mathfrak{G}$) are split.

Theorem: A profinite group G is classically projective iff there exists a pseudo classically closed field K such that $G_K \cong G$.

Alexander Prestel

Fields elementarily characterized by their absolute Galois group.

Let K be a field of characteristic zero. We call K elementarily characterized by its absolute Galois group $G(K)$, if every field L of characteristic zero with $G(K)=G(L)$ and $L=L\tilde{\mathbb{Q}}$ is elementarily equivalent to K (in the language of fields). From the work of Tarski and recently Pop it is known that the local fields $\mathbb{C}, \mathbb{R}$ and $\mathbb{Q}_p$ are elementarily characterized by their absolute Galois group. The same applies to their algebraic parts $\mathbb{C}\cap\tilde{\mathbb{Q}}, \mathbb{R}\cap\tilde{\mathbb{Q}}$ and $\mathbb{Q}_p\cap\tilde{\mathbb{Q}}$. We proved:

Theorem: Let K be elementarily characterized by $G(K)$. Then K is $\tilde{\mathbb{Q}}$ or $\mathbb{R}\cap\tilde{\mathbb{Q}}$ or some (algebraic) extension of $\mathbb{Q}_p\cap\tilde{\mathbb{Q}}$ for some prime p.

V.N.Remeslennikov

On matrix pro-p-groups (On Zulkov's result)

1.Representation of pro-p-groups by matrices.

Let $R=\mathbb{Z}_p[[x_{ij}(k), 1\leq i,j\leq n, k=1,2...]]$ be a pro-p-ring of formal power series with commuting variables $x_{ij}(k)$ over the ring of p-adic integers.

The group $H_n(p) = \overline{\langle A_k+(x_{i,j}(k)), k=1,2...\rangle}$ in $GL_n(R)$ is called a pro-p-group of generic $n\times n$-matrices.

Thm.1: If $p\neq 2$ then $H_2(p)$ is not free.

Corollary: If $p\neq 2$ then there is a standard pro-p-identity which holds in any pro-p-subgroup of $GL_2(K)$, where K is any profinite ring.

Problem: Let p=2. Is the group $H_2(2)$ free or not? This is unknown even for the pro-2-subgroup of $GL_2(\mathbb{Z}_2[[t]])$ generated by matrices $\begin{pmatrix}1&0\\t&1\end{pmatrix}, \begin{pmatrix}1&t\\0&1\end{pmatrix}$.

2. Varieties of matrix groups.

Let V be a variety of pro-p-groups, generated by $H_2(p)$, $p\neq 2$. Let V_m be the variety generated by the group $H_2(p,m)$ (in the definition of $H_2(p)$ the ring $\mathbb{Z}_p$ is substituted by the ring $\mathbb{Z}/p^m\mathbb{Z}$).

Thm.2: Any proper subvariety of V is contained in N_cA or V_mN_cA . Here N_c is the variety of all nilpotent pro-p-groups of nilpotency class $\leq c$, and A is the variety af all abelian pro-p-groups.

Thm.3: Any subvariety V which does not contain the subvariety V_1 is locally soluble.

Luis Ribes

Almost Free Factors of Pro-p Groups

Marshall Hall has proved that if F is a free abstract group and H a finitely generated subgroup, then any basis of the free group H, can be extended to a basis of some subgroup N of finite index in F. In other words, N=H * K, for some subgroup K of N, where denotes the free product of H and K.

In this paper we consider the M.Hall property in the context of pro-p groups. First we study free pro-p groups in connection with that property, and we prove that every (topologically) finitely generated closed subgroup H of a free pro-p group F of arbitrary rank, is a free factor of some open subgroup of F, i.e., there is an open subgroup U of F such that U = H∐K, where K is a closed subgroup of U, and ∐ denotes the free pro-p product of pro-p groups, i.e. the coproduct in the category of pro-p groups. This extends a result of A.Lubotzky, who proved this for pro-p groups of finite rank. Our proof is based on a characterzation of those finite subsets of a free pro-p group F, that can be extended to a basis of F converging to 1.

Our main result deals with pro-p products of pro-p groups, in connection with the M.Hall property. Consider (topologically) finitely generated pro-p groups G_i (i=1...n), with the property that for every (topologically) finitely generated, closed subgroup H of G_i, there exists an open subgroup U of G_i such that U = H ∐ K, for some closed subgroup K; then we show that their free pro-p product $G = G_1 \amalg \ldots \amalg G_n$, satisfies the same property. This theorem generalizes a result of W.Herfort and the author.

J.Ritter

The Frattini subgroup of an absolute Galois group.

This talk which reports on joint work with Moshe Jarden is about the Frattini subgroup of an absolute Galois group of a number field or a local field. Let firstly K be local; for simplicity $K/\mathbb{Q}_p$ finite. Define T to be the maximal tamely ramified extension of K and T_p be the maximal p-elementary abelian extension of T. The Galois group $V = Gal(T_p/T)$ is a $\varprojlim \mathbb{F}_p[Gal(L/K)]$-module with L ranging over all finite tamely ramified extensions of K. Define j(V) to be the intersection of all maximal submodules of V (the Jacobson radical) and let J be the corresponding field.

<u>Theorem:</u> J is the fixed field of the Frattini subgroup of G_K.

In the global situation the picture is quite different.

<u>Theorem:</u> The Frattini subgroup of the absolute Galois group as well as its maximal soluble factor of a number field is trivial.

N.S.Romanovskii

Pro-p-groups, which have a small number of defining relations.

We say that a generalized Freiheitssatz holds in a variety $\mathfrak{M}$ of (profinite) groups, if for each group G, defined in $\mathfrak{M}$ by means of n generators $x_1,\dots,x_n$ and m defining relations $\tau_1,\dots,\tau_m$ (n >m), from these generators we can choose such n-m elements $x_{i_1},\dots x_{i_{n-m}}$, that these elements freely generate a free $\mathfrak{M}$-group.

Theorem 1: A generalized Freiheitssatz holds in the class of all pro-p-groups in each variety $\mathfrak{A}^k$ of soluble pro-p-groups and in each variety $\mathfrak{N}_k$ of nilpotent pro-p-groups.

Theorem 2: Let F be a free pro-p-group with a base X, $r \in F$, $G = \langle X \mid r \rangle$ is a pro-p-group with one relation. Then

(1) the factors $G^{(i)}/G^{(i+1)}$ are torsion free groups if and only if there isn't any element s from F, such that $r \equiv s^p \bmod F^{(k+1)}$, where $r \in F^{(k)} \setminus F^{(k+1)}$,

(2) if the latter condition is satisfied then the group algebra Z_pG is a domain and $cd(G) \leq 2$.

Marcus du Sautoy

Finitely Generated Groups, p-Adic Analytic Groups and Poincaré Series.

Let Γ be a group and denote by $a_n(\Gamma)$ the number of subgroups of index n in Γ. If Γ is finitely generated then $a_n(\Gamma)$ is finite for each n. We consider the following Poincaré series we can associate with Γ, for each prime p:

$$\zeta_{\Gamma,p}(s) = \sum_{n=0}^{\infty} a_{p^n}(\Gamma)\, p^{-ns}.$$

Question 1.

For which finitely generated groups Γ is $\zeta_{\Gamma,p}(s)$ a rational function in p^{-s}?

Segal, Grunewald and Smith proved that $\zeta_{\Gamma,p}(s)$ is rational if Γ is a finitely generated torsion-free nilpotent group.

Exploiting the special architectural features of pro-p groups discovered by Lazard, together with recent logical results due to Denef and van den Dries concerning the analytic theory of p-adic integers, we prove:

Theorem 2. Let G be a compact p-adic analytic group and denote by $A_n(G)$ the number of (open) subgroups of index n in G. Then

$$\zeta_{G,p}(s) = \sum_{n=0}^{\infty} A_{p^n}(G)\, p^{-ns}.$$

is rational in p^{-s}.

G.Schlichting

Structure of groups and graphs.

There are two remarkable theorems concerning the structure of locally finite, connected graphs $\mathfrak{g}$=(V,E) with transitive groups of automorphisms due to V.I.Trofimov.

Thm A ('83): There exists a transitive subgroup G of B($\mathfrak{g}$) (bounded automorphisms) if and only if there exists a B($\mathfrak{g}$)-invariant (even Aut($\mathfrak{g}$)-invariant) partition γ of V into finite parts such that $B(\mathfrak{g})|_\gamma$ acts freely on γ as a finitely generated free abelian group of finite rank d≥0.

Thm B ('84): $\gamma_n:=(\mathfrak{g}_n(x))$ is of polynomial growth if and only if there exists a Aut($\mathfrak{g}$)-invariant partition γ of V into finite parts such that Aut(γ) is finitely generated of polynomial growth and stabilizers of points are finite.

We relate these results to the structure theory of locally compact groups with relatively compact conjugacy classes. This is done with the aid of

Thm C : Let H be a locally compact group, Γ a compact open subgroup and H compactly generated such that $H=H_{FC}\cdot\Gamma$. ($H_{FC}:=\{g\in H|\ [g]^H$ relatively compact$\}$)
Then H_{FC} is closed, $[H:H_{FC}]<\infty$ and there exists a compact-open subgroup $K\trianglelefteq H$.

Application: For G finitely generated virtually nilpotent, torsionfree and G=<E>, E finite , the set {g: G→G bij. ; g(xE)=g(x)E , g(1)=1} is finite.

D.Segal

Quick and easy analytic pro-p groups

A survey of the theory of pro-p groups of finite rank, based on the theory of powerful pro-p groups due to Lubotzky and Mann. Main results: for a pro-p group G,

G has finite rank ⇔ G is f.g. and virtually powerful
⇔ G is f.g. and virtually uniform
⇔ (M.Lazard) G has a p-adic analytic structure

If G_0 is uniform, one defines an intrinsic structure of $\mathbb{Z}_p$-Lie-algebra on G_0. As a consequence, if G has finite rank then G is abelian - by - (linear over $\mathbb{Z}_p$).
The approach is taken from forthcoming book "Analytic pro-p groups" by J.D.Dixon, A.Mann, M. du Sautoy and D.Segal.

D.Segal

Subgroup counting problems, zeta functions and Poincaré series.

For a finitely generated group G , define $a_n = a_n(G) = \#\{H\leq G\ |\ |G:H|=n\}$.

$\zeta_G(s) = \sum_{n=1}^{\infty} a_n n^{-s}$. If G is a $\mathfrak{J}$-group (torsion-free, finitely generated nilpotent),

then $\zeta_G(s) = \prod_{p \text{ a prime}} \zeta_{G,p}(s)$ where $\zeta_{G,p}(s) = \sum_{n=0}^{\infty} a_{p^n} p^{-ns}$.

Thm: $\zeta_{G,p}(s) = F_p(p^{-s})$ where $F_p(x) \in \mathbb{Q}(x)$.

I sketched the proof, and stated a number of open problems, such as (1) how does the rational function $F_p(x)$ vary with the prime p? (2) What is the abscissa of convergence of $\zeta_G(s)$?

Reference: "Subgroups of finite index in the nilpotent groups", by F.J.Grunewald, D.Segal and G.C.Smith, Invent. Math. (1988).

Aner Shalev

Pro-p-groups and p-adic analytic groups

Recently, Zelmanov gave an affirmative solution to the longstanding restricted Burnside problem. I will describe some applications of this important development to the theory of finitely generated pro-p groups and p-adic analytic groups.

In particular it will be shown that f.g. pro-p groups which are not p-analytic must involve arbitrary large wreath products. This gives rise to some new characterizations of p-adic analytic groups.

Christian Siebeneicher

Witt-vectors for pro-finite Groups

Theorem: Let G be a profinite group, O(G) the set of open subgroups of G. There exists a unique functor W_G : com.rings → com.rings such that:

- $W_G(A) = A^{\underline{O(G)}} = \{ \alpha: O(G) \to A \mid$ constant on conjugazy classes$\}$
- $W_G(f)(\alpha) = f \circ \alpha$ for $f: A \to B$, $\alpha \in W_G(A)$
- For each $U \in O(G)$ one has a natural transformation $\Phi_U : W_G \to$ identity such that for every $\alpha \in W_G(A)$ one has:

$$\Phi_U(\alpha) = \sum_{U \lesssim V \in O(G)}{}' \varphi_U(G/V)\, \alpha(V)^{(V:U)}$$

($U \lesssim V$ means : U is subconjugate to V , $\varphi_U(G/V) = (G/V)^V =$ number of U-invariant elements in the coset G/V , summation is taken over conjugacy classes)

- $W_G(\mathbb{Z}) \cong \Omega(G)$, the Burnside - Grothendieck ring of those discrete G-spaces X , for which $\varphi_U(X) := \# X^U < \infty$ for every $U \in O(G)$.

Remarks:

- W_G = universal Witt vectors (p Witt vectors) if G is the pro-finite (pro-p) completion of the infinite cyclic group.

• Restriction and induction induce natural transformations, spezializing to Frobenius and Verschiebung; Frobenius reciprocity and Mackey's formula provide identities, wellknown for Frobenius and Verschiebung.
(A.Dress-Ch.Siebeneicher , The Burnside Ring of Profinite Groups and the Witt Vector Construction, Adv. in Math., vol.70 (1988))

Helmut Völklein

The inverse Galois problem and rational points on moduli spaces.

This is an account of joint work with M.Fried. For any finite group G that has a self-normalizing subgroup U with $\bigcap_{g\in G} U^g = 1$, and for any $r \geq 3$, we construct a (usually reducible) variety $\mathcal{H} = \mathcal{H}_r(G)$ defined over $\mathbb{Q}$, with the following property:

For any field k of characteristic 0, G is the Galois group of a regular extension of k(X) with r branchpoints if and only if the variety $\mathcal{H}$ has a k-rational point.

Further we show that for each finite group H there is a finite group G with quotient H that has a subgroup U as above, and that has the following property:

For suitably large r , the space $\mathcal{H}_r(G)$ has an absolute irreducible component defined over $\mathbb{Q}$. This component has a point rational over P for any given PAC-field P of characteristic 0 , hence every finite group is the Galois group of a regular extension of P(X).

Thomas Weigel

Residual Properties of Free Groups

W.Magnus formulated the following problem: Let be X an infinite set of finite non-abelian simple groups. Is it true that any non-abelian free group of finite rank is residually X ?

Although this is still open, there is much known in the case that X contains special classes of finite simple groups, e.g. if X contains an infinite set of alternating groups or of classical groups, then F_n is residually X.

Using easy arguments of algebraic geometry, e.g. the Lang-Weil Theorem and intersection theory on a low level, and the knowledge of the subgroup structure of finite simple groups the following is proved:

Theorem: Let X be a class containing an infinite set of exceptional groups of Lie-type not of type ${}^2B_2, {}^2G_2, {}^2F_4$ then F_n is residually X.

Kay Winberg

Decomposition of a pro-p group as a free pro-p product.

An analogue of Bieri's and Eckmann's theorem concerning the decomposition of a finitely presented discrete group of cohomological dimension 2 in a free product of duality groups is unknown in the pro-p case. However, for the Galois group G_S of the maximal p-extension $k_S(p)$ of a number field k which is unramified outside a finite set S of primes of k including the primes p and ∞ one can prove:

G_S is either a duality group or a free pro-p-product of decomposition groups and a free pro-p-group. In particular, G_S is always a free pro-p product of duality groups.

Furthermore the proof of the following result was explained:

Let G be a free pro-p-group with an action of a finite abelian group Δ of exponent p-1. Then G has a decomposition in a free pro-p-product

$G = \ast_{\chi \in \Delta^*} U^\chi$, where for each character $\chi \in \Delta^*$ the closed subgroups U^χ is Δ-invariant and $(U^\chi)^{ab} = (G^{ab})^\chi$.

P.A.Zalesskii

Combinatorial Theory of Profinite Groups: The Homotopical Approach.

The concepts of Galois-covering and fundamental group of profinite graphs are introduced. A simply connected profinite graph is defined as a graph with trivial fundamental group. Furthermore, the profinite variants of a graph of groups and its fundamental group are given. We present criteria for the representation of a profinite group G as the fundamental group of a graph of profinite groups, in terms of the connectivity and the simple connectivity of a certain standard graph S(G) . We show how these criteria can be used to prove a Kurosh subgroup theorem for open subgroups of Mel'nikov's and Haran's free profinite products.

Berichterstatter: Th.Weigel

Tagungsteilnehmer

Prof.Dr. Robert Bieri
Fachbereich Mathematik
der Universität Frankfurt
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Prof.Dr. Wulf-Dieter Geyer
Mathematisches Institut
der Universität Erlangen
Bismarckstr. 1 1/2

8520 Erlangen

Prof.Dr. Zoe Chatzidakis
U. E. R. de Mathematiques
T. 45-55, 5etage
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Rostislav Ivan. Grigorchuk
Dept. of Higher Mathematics
Moscow Institute of Railway
Transportation Engineers
Obraszowa 15

Moscow
USSR

Dr. Peter John Cossey
Department of Mathematics
Faculty of Science
Australian National University
GPO Box 4

Canberra ACT 2601
AUSTRALIA

Prof.Dr. Karl Gruenberg
School of Mathematical Sciences
Queen Mary College
University of London
Mile End Road

GB- London , E1 4NS

Ido Efrat
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Dr. Theo Grundhöfer
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. Mike D. Fried
Dept. of Mathematics
University of California
at Irvine
Irvine , CA 92717
USA

Prof.Dr. Fritz Grunewald
Mathematisches Institut
der Universität Bonn
Wegelerstr. 3

5300 Bonn 1

Prof.Dr. Chander K. Gupta
Dept. of Mathematics
University of Manitoba

Winnipeg, Manitoba R3T 2N2
CANADA

Prof.Dr. Otto H. Kegel
Mathematisches Institut
der Universität Freiburg
Albertstr. 23b

7800 Freiburg

Prof.Dr. Narain Gupta
Dept. of Mathematics
University of Manitoba

Winnipeg, Manitoba R3T 2N2
CANADA

Prof.Dr. Eugene Khukhro
Institute of Mathematics
Siberian Academy of Sciences of the
USSR/SO AN SSSR
Universitetskii pr. 4

Novosibirsk 630090
USSR

Dr. Dan Haran
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof.Dr. Norbert Klingen
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Prof.Dr. Karl Heinrich Hofmann
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Dr. Peter H. Kropholler
School of Mathematical Sciences
Queen Mary College
University of London
Mile End Road

GB- London , E1 4NS

Prof.Dr. Moshe Jarden
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof.Dr. Charles R. Leedham-Green
School of Mathematical Sciences
Queen Mary College
University of London
Mile End Road

GB- London , E1 4NS

Prof.Dr. Alex Lubotzky
Institute of Mathematics and
Computer Science
The Hebrew University
Givat-Ram

91904 Jerusalem
ISRAEL

Dr. Thomas Wolfgang Müller
Fachbereich Mathematik
der Universität Frankfurt
Postfach 11 19 32
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Prof.Dr. Angus John Macintyre
Mathematical Institute
Oxford University
24 - 29, St. Giles

GB- Oxford , OX1 3LB

Prof.Dr. Francis Oger
U. E. R. de Mathematiques
T. 45-55, 5etage
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Avinoam Mann
Institute of Mathematics and
Computer Science
The Hebrew University
Givat-Ram

91904 Jerusalem
ISRAEL

Prof.Dr. Richard E. Phillips
Dept. of Mathematics
Michigan State University

East Lansing , MI 48824-1027
USA

Prof.Dr. B.Heinrich Matzat
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1

Prof.Dr. Wilhelm Plesken
Lehrstuhl B für Mathematik
der RWTH Aachen
Templergraben 64

5100 Aachen

Prof.Dr. Oleg V. Mel'nikov
Institute of Mathematics
Academy of Sciences BSSR
ul. Surganova 11

Minsk 220604
USSR

Dr. Florian Pop
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1

Prof.Dr. Alexander Prestel
Fakultät für Mathematik
der Universität Konstanz
Postfach 5560

7750 Konstanz 1

Prof.Dr. Gerhard Rosenberger
Fachbereich Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof.Dr. Vladimir Remeslennikov
Computer Center
Academy of Sciences
Prospect Mira, 19"A"

Omsk 644050
USSR

Dr. Marcus du Sautoy
School of Mathematical Sciences
Queen Mary College
University of London
Mile End Road

GB- London , E1 4NS

Prof.Dr. Luis Ribes
Dept. of Mathematics and Statistics
Carleton University

Ottawa, Ontario , K1S 5B6
CANADA

Prof.Dr. Günter Schlichting
Mathematisches Institut
der TU München
PF 20 24 20, Arcisstr. 21

8000 München 2

Prof.Dr. Jürgen Ritter
Mathematisches Institut
der Universität Augsburg
Universitätsstr. 8

8900 Augsburg

Dr. Dan Segal
All Souls College
Oxford University

GB- Oxford OX1 4AL

Prof.Dr. Nikolai Romanovskii
Institute of Mathematics
Siberian Academy of Sciences of the
USSR/SO AN SSSR
Universitetskii pr. 4

Novosibirsk 630090
USSR

Prof.Dr. Aner Shalev
Mathematical Institute
University of Oxford
24 - 29 St. Giles

GB- Oxford OX1 3LB

Dr. Christian Siebeneicher
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Thomas Weigel
Mathematisches Institut
der Universität Freiburg
Albertstraße 23b

7800 Freiburg

Dr. Ralph Stöhr
Akademie der Wissenschaften
Institut für Mathematik
Postfach 1304
Mohrenstr. 39

DDR-1086 Berlin

Prof.Dr. Alfred Reinhold Weiss
Dept. of Mathematics
University of Alberta
632 Central Academic Building

Edmonton, Alberta T6G 2G1
CANADA

Carsten Thiel
Mathematisches Institut
der Universität Bonn
Beringstr. 6

5300 Bonn 1

Prof.Dr. John S. Wilson
Christ's College

GB- Cambridge CB2 3BU

Prof.Dr. Helmut Völklein
Dept. of Mathematics
University of Florida
201 Walker Hall

Gainesville , FL 32611
USA

Prof.Dr. Kay Wingberg
Mathematisches Institut
der Universität Heidelberg
Im Neuenheimer Feld 288

6900 Heidelberg 1

Prof.Dr. Bertram A. F. Wehrfritz
School of Mathematical Sciences
Queen Mary College
University of London
Mile End Road

GB- London , E1 4NS

Prof.Dr. Zdzislaw Wojtkowiak
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

5300 Bonn 3

Dr. Tilmann Würfel
Informatik
Institut für Mathematik
Universität der Bundeswehr
Werner-Heisenberg-Weg 39

8014 Neubiberg

Prof.Dr. Pavel Alexander Zalesskii
Institute of Technique and
Cybernetics of AN BSSR
ul. Surganova 6

220605 Minsk
USSR

Gefördert durch
DFG Deutsche Forschungsgemeinschaft