MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 21/1990

Abstrakte Konvexe Analysis

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Die Veranstalter waren Heinz König (Saarbrücken) und Helmut H. Schaefer (Tübingen). Es waren 46 Teilnehmer anwesend, davon 20 aus dem Ausland. Es wurden 35 Vorträge gehalten, davon 13 mit 45 Minuten und 22 mit 30 Minuten Dauer.

Die Tagung stand einerseits in der Tradition der "allgemeinen" Tagung über "Funktionalanalysis", die vor etwa drei Jahrzehnten von dem im Jahre 1989 verstorbenen Gottfried Köthe begründet worden war. Es sollte andererseits durch die Namensgebung ein entschiedener neuer Akzent gesetzt werden: In letzter Zeit wurde in weiten Teilen der Analysis und in benachbarten Disziplinen deutlich, daß ein Bereich "abstrakte konvexe Analysis" zentralen und ordnenden Charakter annimmt. Die Tagung sollte dazu beitragen,diese Entwicklung zu klären und zu fördern. So waren im Kreis der Teilnehmer neben der konkreteren und der abstrakteren konvexen Analysis und der "konventionellen" Funktionalanalysis eine Reihe von anderen Bereichen vertreten, in denen die Konvexität eine bedeutsame Rolle spielt. Demtentsprechend zeichnete sich die Palette der Vorträge durch eine erfreuliche Breite aus (freilich nicht auf Kosten der Tiefe!), und es herrschte nach dem Eindruck der Veranstalter ein Klima der besonders lebhaften Diskussion. Unter den "Anwendbarkeitsbereichen" der "abstrakten konvexen Analysis", die durch mehrere Vorträge vertreten waren, seien erwähnt: die Geometrie der Banachräume, positive und monotone Operatoren, Halbgruppen und Evolutionssysteme, Funktionenräume und punktweise Kompaktheit, Maßtheorie, mathematische Ökonomie.

Die Tagung konnte nach dem Eindruck der Teilnehmer zu einer im Fluß befindlichen Entwicklung einen wertvollen Beitrag leisten. Es erscheint hiernach geboten, in Oberwolfach immer wieder Tagungen von solchem "integrierenden" Charakter abzuhalten.

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Vortragsauszüge

R. BECKER :

Cones in Banach spaces and integral representation

I will develop 5 main points :

1) Cones, contained in a Banach space or its dual, which carry only localizable conical measures

2) Characterization of non-reflexive Banach spaces with the help of cones contained in their dual

3) Embeddings of weakly complete cones, contained in a Banach space, into a reflexive space

4) Characterization of operators by properties involving straitening of cones

5) Some remarks concerning integral representations (dealing with examples in the frame of the theory of Edgar's theorem)

J. BONET :

Projective tensor product of distinguished Fréchet spaces

The following question is treated : Let E and F be Fréchet spaces which are distinguished, is the complete projective tensor product $E \hat{\otimes}_\pi F$ also distinguished ? The problem was already considered by Grothendieck in 1954. A negative solution was obtained by S. Dierolf. In joint work with K.D. Bierstedt we proved that $\lambda_1(A) \hat{\otimes}_\pi F$ is distinguished if and only if (i) $\lambda_1(A)$ is Montel and F is distinguished or (ii) $\lambda_1(A)$ is distinguished and $l_1 \hat{\otimes}_\pi F$ is distinguished (i.e. F has Heinrich's density condition). Taskinen found a Fréchet Montel space F_0 such that $F_0 \hat{\otimes}_\pi F_0$ is not distinguished. On the other hand Bierstedt and I proved that if E and F have the density condition and the problem of topologies of Grothendieck is satisfied by $E \hat{\otimes}_\pi F$, then $E \hat{\otimes}_\pi F$ also has the density condition. Jointly with J. Taskinen we constructed a quojection F such that $F \hat{\otimes}_\pi F$ does not satisfy the problem of topologies of Grothendieck.

B. CARL :

Large projections in l_∞^N

Let E be a Banach space and L(E,E) the Banach space of all (bounded linear) operators from E into E. The relative projection constant $\lambda(M,E)$ of a subspace $M \subset E$ is defined by

$$\lambda(M,E) := \inf \left\{ \| P \| : P \in L(E,E) \text{ projection with } P(E) = M \right\} .$$

and the k-th projection constant $\lambda_k(E)$ of the Banach space E by

$$\lambda_k(E) := \sup \left\{ \lambda(M,E) : M \subset E, \dim M = k \right\}.$$

The value λ_k , roughly speaking, describes the smallest possible norm of a projection onto a "badly" complemented k-dimensional subspace of E. We show that the k-th projection constant of an n-dimensional subspace E of l_∞^N satisfies the estimate

$$\lambda_k(E) \geq C \min\{ \sqrt{k}, 1 + \sqrt{n-k} \} \log^{-\frac{1}{2}}\left(\frac{N}{k} + 1\right),$$

$1 \leq k \leq n \leq N$, $N = 1,2,\ldots$, where $C > 0$ is a numerical constant. The result is implicitely contained in a joint paper with A. Pajor : Gelfand numbers of operators with values in Hilbert spaces. Inventiones Math. 94 (1988), 459 - 504.

V. CASELLES :

Approximations of positive operators and continuity of the spectral radius

We prove convergence results for the peripheral spectrum and peripheral eigenvectors when a sequence T_n of positive operators in a Banach lattice E approximates a positive irreducible operator T on E such that the spectral radius r(T) of T is a Riesz point of the spectrum of T (i.e. pole of the resolvent of T with a finite rank residuum) under some conditions on the kind of approximation of T_n to T. The Banach lattice E is a weakly sequentially complete Banach lattice. We prove it first for dual Banach lattices with order continuous norm. The step to a general weakly sequentially complete Banach lattice is done through interpolation theory.

J. CONRADIE :

Generalized inductive limit topologies on Riesz spaces

The characterization of weakly compact subsets of L_1 in the Dunford-Pettis Theorem can be used to show that the Mackey topology $\tau(L_\infty, L_1)$ is the finest locally convex topology on L_∞ which coincides on the norm-bounded sets of L_∞ with the topology τ_m of convergence in measure; $\tau(L_\infty, L_1)$ is therefore an example of a generalized inductive limit topology. The following questions are therefore of interest : If L is a Banach function space, is there a finest locally convex topology coinciding on (a) the order bounded sets (b) the norm bounded sets in L with τ_m ? If so, can its equicontinuous sets be characterized ? Suitably reformulated, these questions can be asked in the context of Riesz spaces. We give partial answers to these questions and indicate the important role played by the Lebesgue dominated convergence theorem and its generalizations in this regard.

A. DEFANT :

Tensor products of operators between L_p - spaces

For k = 1,2 let $T_k \in \mathcal{L}(L_q(\mu_k), L_p(\nu_k))$. When is

$$T_1 \otimes T_2 : L_q(\mu_1) \otimes_{\Delta_q} L_q(\mu_2) \longrightarrow L_p(\nu_1) \otimes_{\Delta_p} L_p(\nu_2)$$

continuous ? Here Δ_q stands for the natural norm on $L_q(\mu_1) \otimes L_q(\mu_2)$ induced by $L_q(\mu_1, L_q(\mu_2))$. It is well-known that, if one of the operators is positive, then $\|T_1 \otimes T_2\| = \|T_1\| \, \|T_2\|$. W. Beckner proved the same norm estimate if the T_k are arbitrary and $1 \le q \le p \le \infty$. For $1 \le p < q \le \infty$ one gets continuity of $T_1 \otimes T_2$ under the assumption that $p \le 2 \le q$. However, this is false if $p < q < 2$ or $2 < q < p$, as observed by G. Bennett.
In the present talk several extensions, supplements and applications of these results are given, for example : If $1 \le p < q < 2$ then

$$\sup\left\{ \left\| T_1 \otimes T_2 : l_q^n \otimes_{\Delta_q} l_q^n \longrightarrow l_p^n \otimes_{\Delta_p} l_p^n \right\| : \|T_1\| \le 1, \|T_2\| \le 1 \right\} \asymp (\log n)^{\frac{1}{q}} ,$$

which answers a conjecture of Rosenthal and Szarek in the positive. The results are joint work with Bernd Carl.

S. DIEROLF :

Some remarks on distinguished spaces

We present an example of a distinguished Fréchet space whose bidual is not distinguished. This gives a negative answer to a question of A. Grothendieck (1954). We achieve such examples by proving the following characterization :
Proposition Let Y, X be Banach spaces and $f : Y \to X$ a continuous linear map. Then for the Fréchet space

$$F := \{ (x_k)_{k \in \mathbb{N}} \in Y^{\mathbb{N}} : (f(x_k))_{k \in \mathbb{N}} \in l^{\infty}(X) \}$$

the following assertions are equivalent :

a) F is distinguished ;

b) f is open onto its range ;

c) F is quasinormable ;

d) F is a quojection .

Consequently, whenever Y,X are Banach spaces and $f : Y \to X$ is a continuous linear map with proper dense range, the Fréchet space

$$E := \{ (x_k)_{k \in \mathbb{N}} \in Y^{\mathbb{N}} : (f(x_k))_{k \in \mathbb{N}} \in c_o(X) \}$$

is distinguished, and its bidual

$$E'' := \{ (x_k)_{k \in \mathbb{N}} \in (Y'')^{\mathbb{N}} : (f^{tt}(x_k))_{k \in \mathbb{N}} \in l^{\infty}(X'') \}$$

is not distinguished. The results are joint work with J. Bonet and C. Fernández.

G. GODEFROY :

An application of Baire's theorem to multipliers

In this joint work with F. Lust-Piquard, we show that the set of fixed points of an isometric bijection on the dual of any Banach space is stable under weak-*-limits of weak Cauchy sequences: the proof relies on a Baire category argument. An application is : if G is an abelian compact metrizable group and Λ is a subset of the dual group Γ such that the space $\mathcal{M}_{\Lambda^c}$ is stable under the Radon-Nikodym projection, then any multiplier from L^{∞}_{Λ} to $\mathcal{C}_{\Lambda}$ is the restriction to L^{∞}_{Λ} of the convolution with a function of L^1.

B. GRAMSCH :

On the structure of the set of idempotent elements in topological algebras.

The set of idempotent elements of a topological algebra with an open group of invertible elements and continuous inversion is a discrete union of locally rational homogeneous manifolds. Besides $C^{\infty}(\Omega,\mathcal{L}(\mathbb{C}^n))$, Ω compact, and the Hörmander classes of pseudodifferential operators there are many other Fréchet algebras which fulfill these requirements. Porta and Recht (1987) and Salinas (1988) introduced a covariant differentiation for the manifold $\mathcal{P}$ of idempotent elements in C^*- algebras ; along these lines they got an explicit form for the geodesics in $\mathcal{P}$. From the point of view of Fréchet algebras of pseudodifferential operators this gives the possibility for a "dynamic" definition of the geodesics which fits also submanifolds of Fréchet algebras where a Riemannian structure is missing. A complete description of the periodic geodesics in $\mathcal{P}(\Psi)$ is given. This characterization might be new also for finite dimensional C^*- algebras. The methods provide applications of relative inverses, homotopy theory and the Oka-principle for complex Fréchet-Lie- groups. The results have been obtained partly in collaboration with K. Lorentz and J. Scheiba.

R. GRZASLEWICZ :

Extreme norms on $\mathbb{R}^n$

Let N^1, N^∞ be the l^1 - and l^∞- norms on $\mathbb{R}^n$. We denote by $\mathcal{N}$ the set of all norms N on $\mathbb{R}^n$ such that $N^\infty \leq N \leq N^1$. The aim of my talk is to present the characterization of the extreme points of $\mathcal{N}$. We do this by describing the corresponding unit balls.

J. GUILLERME :

Convergential space and compacting relation

A set X will be said semi-convergential if to every point x of X is associated a fixed family $\Theta(x)$ of semi-filters on X. We introduce an ordered class of (six) notions of "compacting relation at a point" between two semi-convergential spaces. We prove that the weakest of the previous notions is sufficient to obtain some results as "the semi-continuity of the marginal function". Examples are explained, including the classical "sequential convergences".

W. HACKENBROCH :

Barycentric decomposition of measure extensions

Given an arbitrary family μ_t , $t \in T$, of probabilities on σ-algebras $\mathcal{A}_t$ on some fixed set Ω, the convex set $\mathcal{M}$ of all common measure extensions of the μ_t to some σ-algebra $\mathcal{A}$ containing all $\mathcal{A}_t$ is studied. As in the case of extensions of one measure, the extremal points of $\mathcal{M}$ are described by a criterion of the type of "subspace density" (R.G. Douglas). If T is countable, all σ-algebras $\mathcal{A}_t$, and $\mathcal{A}$, are countably generated sub-σ-algebras of some σ-algebra $\mathcal{S}$ which makes $(\Omega,\mathcal{S})$ a Blackwell space, each $\mu \in \mathcal{M}$ admits a barycentric decomposition

$$\mu = \int_{\mathrm{ex}\,\mathcal{M}} \nu \, \rho(d\nu)$$

in the sense of v. Weizsäcker - Winkler. Assuming that, for some fixed sub-σ-algebra $\mathcal{B} \subset \bigcap_{t\in T} \mathcal{A}_t$, all $\mu_t|\mathcal{B} := \mu_o$ agree and $(\mathcal{A}_t)_{t\in T}$ is $\mathcal{B}$-conditionally independent (i.e. $A_t \in \mathcal{A}_t$, $\bigcap A_t = \emptyset \Rightarrow \prod(E^{\mathcal{B}}_{\mu_t} \chi_{A_t}) = 0$ μ_o-a.e.), by $\mu(\bigcap A_t) = \int \prod(E^{\mathcal{B}}_{\mu_t} \chi_{A_t}) \, d\mu_o$ an additive common extension is given, whose image $\mu \circ \tau^{-1}$ under the canonical measurable imbedding τ of $(\Omega; \bigvee_{t\in T} \mathcal{A}_t)$ into the product space $(\Omega^T; \otimes_{t\in T} \mathcal{A}_t)$ is even σ-additive if all but one of the conditional expectations $E^{\mathcal{B}}_{\mu_t}$ admit regular versions. μ is generally not extremal.

H. HUDZIK :

Extreme points, exposed points and smooth points of Orlicz spaces equipped with Luxemburg and Orlicz norms

Some criteria for extreme points, exposed points and smooth points of the unit sphere of Orlicz spaces (or of their subspaces of order continuous elements) equipped with Luxemburg norm as well as equipped with Orlicz norm are given in terms of the generating Orlicz function.

K. JANSSEN :

Decomposition of polysupermedian measures

Denote by $\mathcal{PS}$ the class of measures which are polysupermedian for an at most countable family $\mathcal{P}$ of 1-parameter-semigroups of kernels on a nice measurable space. Existence and "unicity" of a Choquet type integral representation by minimal elements is shown to hold in $\mathcal{PS}$. For the larger class of measures which are only supermedian for each semigroup in $\mathcal{P}$ the corresponding compactness properties hold but unicity fails. The particular example of representing completely monotone measures by eigenmeasures (joint work with H. Ben Saad (Tunis)) is discussed.

I. KLUVÁNEK :

Convexity and Integration

Given are: a space, Ω : a family (a quasiring) $\mathcal{Q}$ of subsets of Ω : a family, $\mathcal{N}$, of exceptional (null) sets; a non-negative additive set function, λ, on $\mathcal{Q}$ such that $\lambda(Z) = 0$, for every $Z \in \mathcal{Q} \cap \mathcal{N}$: a locally convex space, E, continuously imbedded into a locally convex space F; and a property, (P), which an additive set function, $\nu : \mathcal{Q} \to E$, may or may not have (such as being σ-additive, having bounded variation, being λ-continuous, etc.).
A function, $f : \Omega \to F$, is said to be $(E,\mathcal{Q},\mathcal{N},(P))$-integrable with respect to λ, if there exists a unique additive set function, $\nu : \mathcal{Q} \to E$, with the property (P) such that

$$\nu(X) \in \lambda(X) \bigcap_{Y \in \mathcal{N}} \overline{\mathrm{co}}\{f(\omega) : \omega \in X \setminus Y\},$$

for every $X \in \mathcal{Q}$. By the choice of E, F, $\mathcal{Q}$, $\mathcal{N}$ and (P), one can obtain different old and new classes of integrable functions (Riemann, Lebesgue, Perron, Bochner, Pettis,...).

Heinz KÖNIG :

The Embedding of Convex and Superconvex Spaces

This is joint work with Gerd Wittstock. The problem is whether and when a convex resp.superconvex space X, I (in the sense of the author's report in "Aspects of Positivity in Functional Analysis", North Holland 1986, pp. 79-118) can be embedded (in the obvious sense) as a convex subset into a real vector space resp. as a σ-convex subset into a Hausdorff topological vector space. In the convex case this is possible, by a standard algebraic procedure, iff X, I satisfies the so-called cancellation law. Thus a superconvex space X, I with the cancellation law can be embedded as a so-called superconvex subset $Y \subset E$ into a real vector space E, defined to mean that the natural convex structure of Y can be extended to some (and hence to a unique) superconvex structure. One can of course assume that Lin(Y) = E. The main result then says that the Minkowski functional Θ for $\mathrm{conv}(Y \cup (-Y))$ is a norm on E in which E is complete. Thus Y becomes a bounded superconvex and hence σ-convex subset of the resultant Banach space (E,Θ).

Hermann KÖNIG :

Entropy numbers and weak type p spaces

Let X be a type p space. By a result of B.Carl, the entropy numbers of any map $S : l_1^n \to X$ satisfy

$$(1) \qquad e_k(S) \le c\, k^{-1/p'} \log(1 + \tfrac{n}{k})^{1/p'} \|S\| \qquad (k \le n)$$

and in particular

$$(2) \qquad e_n(S) \le c\, n^{-1/p'} \|S\| \qquad (n \in \mathbb{N})\ .$$

Here $1/p + 1/p' = 1$. For $p < 2$, it is shown that actually (1) and (2) are equivalent; both are equivalent to X being of weak type p, i.e. the type p inequality holding for vectors of equal norm only.

S. KREMP :

A short proof of the angelic theorem for continuous functions on web-compact spaces

For a Hausdorff topological space X, let $\mathbb{C}(X)$ denote the space of all real-valued continuous functions on X, endowed with the pointwise topology. In 1987 J. Orihuela established the angelic character of the space $\mathbb{C}(X)$ for a wide class of topological spaces X, which he called web-compact spaces. In our talk we present a short proof of this result which is based on a new characterization of the web-compact spaces as stated below.

A Hausdorff topological space X is web-compact if and only if there exists a dense subset U of X and a semimetric $d : U \times U \to [0,1]$ such that the following two properties are fulfilled:

(i) Every d-convergent sequence in U is relatively countably compact in X;

(ii) U is separable with respect to the semimetric d, that is there exists a countable subset D of U such that $\overline{D}^d = U$.

G. LUMER :

Averaging, integrated semigroups, and generalized solutions for linear evolutionary systems

X is a Banach space. We consider the general evolution problem

(1) $u' = Au+F(t)$, $u(0) = f$ $(F \in L^1_{loc}([0,+\infty[,X))$,

where A is a linear operator in X, closed and such that 0 is the only solution of "$u' = Au$, $u(0) = 0$" . For $n \geq 1$, v_n is called an n-strong generalized solution (n-s.g.s.) of (1), and v_n an (n-1)-mild generalized solution ((n-1)-m.g.s.) of (1), iff it is a classical solution of:

$v_n'(t) = A\,v_n(t) + (t^{n-1}/(n-1)!)\,f + F_n(t)$, $v_n(0) = 0$,

where $F_n(t) = \int_0^t ((t-s)^{n-1}/(n-1)!)\,F(s)ds$; "0-s.g.s." $\Leftrightarrow$ "classical solution".

The generalized average solution of order $n \geq 1$ is $\sigma_n(t,f) = (n/t^n)\,\rho_n(t)f$; $\sigma_i(t,f) = \rho_o(t)f$. $Z_n := \{ f \in X : \exists$ a n-s.g.s. , $v_n = v_n(t,f)$, of (1) with F=0 and $u(0) = f \}$. For $f \in Z_n$, set $\rho_n(t)f = v_n'(t,f)$. Under rather mild assumptions on F(t) (Z_{n+1} - valued), one shows that for $f \in Z_{n+1}$ (n=0,1,2,...),

(2) $w_n(t) = \rho_n(t)f + \int_0^t \rho_n(t-s)F(s)ds$

gives an n-m.g.s. of (1). One shows that $\rho_n(t) : Z_{n+1} \to Z_{n+1}$, and on Z_{n+1} :
$\rho_n(t)\rho_n(s) = \rho_{2n}(t+s) - \sum_{k=0}^{n-1}(1/k!)\left(s^k\,\rho_{2n-k}(t) + t^k\,\rho_{2n-k}(t)\right)$ for $t,s \geq 0$,
(which reduces for n=0 to the usual $\rho_0(t)\rho_0(s) = \rho_0(t+s)$). A full theory is developed for generalized solutions and evolution operators $\rho_n(t)$ (which give n-times integrated semigroups when $Z_{n+1} = X$, while in general $Z_0 \subset Z_1 \subset \ldots \subset Z_n \subset \ldots$ and all Z_n may well be $\neq X$). There are many applications : to perturbation problems, problems in age-structured population dynamics, diffusion problems with discontinuous boundary behavior (leading to very explicit formulas in the case in which we deal with bounded domains in $\mathbb{R}^3$ "very regular" in the sense of Lions).

By such methods - and with additional very recent results of this kind - one can on one hand deal very nicely with problems in which the solutions are not exponentially bounded, and on the other hand clarify considerably the situation for dissipative operators by showing :

Theorem If A is dissipative (closed) then either : (i) the Cauchy problem (1) (F=0) is not solvable on any dense subset of X in whatever n-s.g.s. (i.e. no matter how large n is chosen , or else : (ii) A generates at worst a locally lipschitz integrated semigroup on X (the latter if D(A) is not dense) or an ordinary semigroup (if D(A) is dense).

As a consequence, a special attention is justified for locally lipschitz integrated semigroups, and explicit results concerning the latter are given and applied to several of the problems mentioned above.

H. van MAAREN :

Kakutani's fixed point theorem for hull spaces with finite exchange-number

We present the analogues of Knaster-Kuratowski-Mazurkiewicz's theorem, Brouwer's fixed point theorem and Kakutani's fixed point theorem in the context of multiply-ordered spaces and hull-spaces with finite exchange number. This is done by proving a Sperner-like combinatorial lemma for finite grids in multiply-ordered spaces. We introduce the topology and convexity in these spaces which are needed for the continuous variants of this Sperner-like lemma, ending up with generalizations of KKM's-and Brouwer's theorem. The Kakutani-like theorem is proved for a multivalued mapping, from a multiply ordered space to a hull-space, satisfying some continuity-conditions. The crucial point in this latter theorem is the equality of the exchange number of the hull-space and the number of orderings from the multiply ordered space involved.

R. NAGEL :

Operator matrices and reaction-diffusion systems

A system of reaction-diffusion equations can be written as an initial value problem

(*) $\dot{u}(t) = A\,u(t),\ u(0) = u_0$.

on the product space $L^2(\Omega) \times \ldots \times L^2(\Omega)$ and for an operator matrix $\mathcal{A} = (A_{ij})_{n \times n}$, $A_{ij} = a_{ij}\,\Delta + b_{ij}(\cdot)$. It is shown how our "matrix theory" for unbounded operator matrices yields well-posedness for (*) and detailed information on the qualitative behavior (positivity, stability, convergence) of the solutions.

M. M. NEUMANN :

Applications of Convex Analysis to Flows in Networks

We first extend the classical feasibility theorem on flows in finite networks due to Gale and Hoffman to a much broader setting of a measure theoretic flavour, which allows a reasonable interpretation of flows in infinite networks. Our approach is based on the notion of a biadditive set function and is very different from the classical arguments from combinatorics and discrete mathematics. It turns out that these arguments can be replaced by the powerful machinery of convex analysis; actually our main tools are a suitable version of the Hahn-Banach theorem as well as the integration theory for submodular set functions. Our approach to the Gale-Hoffman type theorem is close in spirit to corresponding work of Fuchssteiner-Lusky and König-Neumann, but the technicalities are somewhat simpler and extend to a more general context, where the capacity constraints are given by certain submodular set functions. We thus obtain a generalization of a related result due to Lawler even in the classical setting of finitely many nodes and arcs. We finally discuss some applications to dynamic flows in a standard $L^{\infty}([0,T])$-setting, which include and improve some recent results on τ-maximal flows due to Ogier. It appears that the present approach from convex analysis works under considerably less restrictive assumptions than the methods from optimal control. Further applications to measure theory and to mathematical economics can be found in our joint book with Heinz König on convex analysis and mathematical economics.

D. NOLL :

Differentiability of Convex Functions

We discuss differentiability properties of convex functions defined on small sets. Here a convex set C in a Banach space E is called a small set if it contains no (algebraic) interior points. We pose particular emphasis on phenomena of such functions indicating that their behaviour may be quite different from that of their finite-dimensional counterparts.

J. ORIHUELA :

Resolutions of identity and pointwise compactness in function spaces

This is joint work with W. Schachermayer and M Valdivia. We solve three problems posed by Namioka showing that an Eberlein compact is characterized by being a Radon-Nikodym and Corson compact. The Banach space version of this result says the following: A Banach space E is weakly compactly generated

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if and only if the unit dual ball $B(E^*)$ is Corson compact and E is GSG. i.e. there is an Asplund space X and a continuous linear operator $T:X \to E$ with dense range. - Our method is based on a preliminary study of Talagrand's example of a Talagrand compact space which is not Eberlein, together with a careful study of the long sequences of projections, P.R.I. we can construct in Banach spaces with Corson-compact unit dual ball. - This method lead us to answer a question of J.E.Jayne on the existence of selectors for upper-semi-continuous multivalued maps $\Phi : M \to 2^{E^*}$, where M is metric and E^* is a dual Banach space with the weak-*-topology and the RNP. The following result is a joint work with A. Pallares and G. Vera: If M is metric, E^* has RNP and $B(E^*)$ is angelic, then any Φ as above has a first Baire class selector in the norm. If E does not have the c-property of Corson, there exists a Φ without such a selector. - Our results are strongly related with the σ-fragmentability property studied by J.E. Jayne, I. Namioka and C.A. Rogers, and a recent work by Ghoussoub, Maurey and Schachermayer on selection theorems.

M. PANNENBERG :

Positive Operators on Tensor Products which are the Tensor Product of their Restrictions

A positive measure on a product space is in general not uniquely determined by its restrictions to each of the factors - however, if one of the marginal distributions is a Dirac measure, the given measure on the product space is necessarily the tensor product of its marginal distributions. The talk will mainly be concerned with operator versions of this simple observation, dealing with positive operators on tensor products of Banach lattices resp. Banach algebras with involution. It turns out that e.g. a positive operator on a tensor product of Banach lattices with quasi-interior positive elements is the tensor product of its restrictions to the factors if one of them is a lattice homomorphism. In the Banach algebra setting one observes that a positive operator on a tensor product of unital C^*-algebras resp. unital commutative Banach algebras with symmetric involution is the tensor product of its restrictions if one of them is a pure completely positive operator resp. a unital algebra homomorphism. The results allow unified proofs of several Korovkin type approximation theorems.

R. R. PHELPS :

Bounded approximants to monotone operators on Banach spaces

If f is a proper convex lower semicontinuous real function on a Banach space E, then the convex, Lipschitzian everywhere defined functions $f_n(x) = \inf\{f(y) + n\,\|x-y\| : y \in E\}$ $(x \in E)$ have classically yielded a very useful approximating sequence to f. With B^* denoting the closed unit ball of E^*, one has $\partial f(x) \cap nB^* \subset \partial f_n(x) \subset nB^*$ and $\{\partial f_n\}$ converges to the subdifferential ∂f of f in the sense that $\partial f_n(x) = \partial f(x) \cap nB^*$ for all $x \in dom(f)$. I will describe an analogue of this latter result for an arbitrary maximal monotone operator in place of ∂f.

W. SCHACHERMAYER :

Jensen measures and plurisubharmonic martingales

The notion of Jensen measure is a counterpart to the concept of Choquet measure, where the convex functions - which apply to the real case - are replaced by plurisubharmonic functions.
The main theorem, obtained by Shangnan Bu and the author (T.A.M.S. 1990) states that Jensen measures on a complex Banach space X (or on a domain of $\mathbb{C}^n$) may be approximated by the image of Lebesgue measure on $\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$ under analytic functions $F : \mathbb{C} \to X$.
This allows a characterisation of the analytic Radon-Nikodym property of a Banach space X in terms of convergence of plurisubharmonic martingales.

H. H. SCHAEFER :

On Convex Hulls

It is shown that if A, B are convex subsets of a t.v.s. E, with A compact and B bounded and closed, then the convex hull of $A \cup B$ is closed. If, in addition, E is locally convex and $A \not\subset B$, then there exists an extreme point $a \in \partial_e A$ which is extreme in co $(A \cup B)$.
The preceding is applied to the closed unit ball W of the interpolation space $L^p(0,\infty) + L^\infty(0,\infty)$, which turns out to be $W = co\ (U \cup V)$ where $U = \{f : \|f\|_p \le 1\}$ and $V = \{g : \|g\|_\infty \le 1\}$. In particular, we have $\partial_e W = \partial_e U \cup \partial_e V$ $(1 \le p \le \infty)$.

G. SCHLÜCHTERMANN :

The Dual Mackey Topology of a Banach Space

For a Banach space X topological properties are investigated of the dual X^* endowed with the Mackey topology τ, the topology of uniform convergence on weakly compact subsets. With the notion of metrizability, k-space and countable tightness, relations to the structure of the Banach space X are derived.

S. SIMONS :

Minimax Theorems

In this talk we discuss some minimax theorems using the concepts of "staircase" and "pseudoconnectedness" that unify Terkelson's minimax theorem and König's minimax theorem.

H. G. TILLMANN :

The Need of Impatience
(Pareto-Optimum and Equilibrium in Mathematical Economic Systems)

We generalize results of Brown-Lewis, Econometrica **49** (1981) and Araujo, Econometrica **53** (1985) from $X = \mathbb{R}$ to general infinite dimensional Riesz spaces. Let X be a loc. convex, solid Riesz space, $Y = X'(\tau_o)$ its dual and τ_o the Mackey topology: $\tau_o = \tau_{MA}(X,Y)$. Our global commodity space is $H = l^\infty(X)$. A topology τ on $l^\infty(X)$ is called regular, if the injections $j_n : X \to l^\infty(X)$, $x \to (0,\dots,0,x,0\dots)$ are continuous. τ is called myopic, if all continuous preferences are myopic (Brown-Lewis).

Thm 1: There exists a strongest regular and myopic topology τ_{SM} on $l^\infty(X)$.

The dual is $l^\infty(X)'_{\tau_{SM}} = l^1_b(Y)$,

the bidual is $l^\infty_e(X'') = \{\bar{x} = (x_n) : x_n \in X'', \text{equicontinuous}\}$

and τ_{SM} is the Mackey topology: $\tau_{SM} = \tau_{MA} = \tau_{MA}(l^\infty(X), l^1_b(Y))$.

If X is a **reflexive** loc. conv. **solid Riesz space**, we have:

Thm 2: If τ is a topology on $l^\infty(X)$, $\tau_\infty \supset \tau \supset \sigma(l^\infty(X), l^1_b(Y))$ then: A general existence theorem for Pareto-optima exists **iff** $\tau_{MA} = \tau_{SM} \supset \tau$. ($\Leftrightarrow$ all preferences are myopic).

Thm 3: A general existence theorem for equilibria exists **iff** $\tau_{MA} = \tau_{SM} \supset \tau$.

Remark: Susanne Dierolf noticed, that the condition "X (τ_o) barrelled", I had in Thm.1 could be eliminated.

M. VALDIVIA :

Fréchet spaces without subspaces isomorphic to l_1

If E is a Fréchet space, $\mathcal{M}$ is the topology on E of the uniform convergence on every bounded, closed, absolutely convex and metrizable subset of E' $[\sigma(E',E)]$. The following results are proved:

1) If E is a Fréchet space which does not contain a subspace isomorphic to l_1 then every closed separable subspace of $E[\mathcal{M}]$ is complete.
2) If E is a separable Fréchet space which does not contain a subspace isomorphic to l_1, then $E'[\mu(E',E'')]$ is barrelled.
3) A Fréchet space is reflexive if and only if every $\mu(E',E)$-null sequence of E' is $\beta(E',E)$-null and every $\sigma(E',E)$-null sequence of E' is $\sigma(E',E'')$-null.

J. VOIGT :

On the convex compactness property for the strong operator topology

Let X, Y be Banach spaces.
Theorem. Let (Ω,μ) be a measure space with $\mu(\Omega) < \infty$. Let $U : \Omega \to K(X,Y)$ (compact operators) be bounded and strongly measurable (i.e., $U(\cdot)x$ measurable for all $x \in X$). Then the "strong integral" $\int_\Omega U(\omega)d\mu(\omega)$ belongs to $K(X,Y)$.
This property (found by L.Weis, 1988) implies that $(K(X,Y), \mathcal{T}_s)$, where $\mathcal{T}_s$ denotes the strong operator topology, has the convex compactness property, i.e., the closed convex hull of any compact subset of $(K(X,Y),\mathcal{T}_s)$ is again compact.
The corresponding property holds also for some other closed subspaces of $L(X,Y)$.

L. WAELBROECK :

Quotient bornological spaces

I speak of quotient bornological spaces, they are important in Functional Analysis, but they are not part of the subject at present. Several quotient spaces are used: The hyperfunctions, the continuous germs (of class C^∞, of distributions) near to a compact subset of a manifold, the singularities of distributions. J.-F.Colombeau's New Generalised Functions are quotient spaces. I have "quotient results": one cannot prove them without introducing a quotient space that turns out to be "trivial".
My Thèse d'Agrégation (Habilitationschrift 1960) was "quotient", but I did not know that it is (I found the category in 1962). It is only these 10 years that I have really developed Functional Analysis in the category **q**, when I learned that Colombeau's New Generalised Functions exist and is a "quotient" space.
Quotient Fréchet spaces, a category of quotient locally convex spaces probably, but no suitable category of complete or quasi-complete locally convex spaces exists. I shall describe some properties of the category **q**.

D. YOST :

Reducible Polytopes and Pseudolinear mappings

A finite-dimensional compact convex set P, symmetric about the origin, is reducible if there is a nonsymmetric compact convex Q with $P = Q - Q$. We give a necessary and sufficient condition for a polytope to be reducible, and use it to show that many polytopes, in dimension three or more, are irreducible.

A function $T : X \to Y$ between two Banach spaces is pseudolinear if it is homogeneous and satisfies the inequality

$$\|T(x) + T(y) - T(x+y)\| \leq \|x\| + \|y\| - \|x+y\|.$$

The following two questions remain open. Is every pseudolinear function between complex Banach spaces automatically linear? Is every pseudolinear function between real Banach spaces the sum of a linear function and a continuous function?

We indicate the relationship between these topics and M-ideals in Banach spaces.

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