MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 22/1990

The Schrödinger Equation and its Classical Counterparts

20.5. bis 26.5.1990

The meeting was organized by Volker Enss (Aachen). 43 participants from 11 countries discussed recent progress in the analysis of Schrödinger operators. Special emphasis was given to analogies as well as differences between the properties of classical and the corresponding quantum mechanical systems.

The limiting behaviour of Schrödinger operators under the change of various parameters was treated in detail, in particular the semiclassical analysis. In addition to Planck's constant h a variety of other quantities could play the role of a small parameter. The properties of eigenfunctions - both square-integrable and improper - have been investigated as well as estimates on the position and distribution of eigenvalues. Another major topic was scattering theory where the attention was centered on one hand on resonances and on the other hand on N-body systems, in particular asymptotic completeness. In addition, some more general mathematical investigations were initiated by related problems.

Further details can be found in the abstracts of the 28 lectures. We regret that it is impossible to include the equally valuable discussions.

The participants and the organizer are grateful for the hospitality at the Institute and for the support of its director, Professor Barner. This provided the stimulating environment for research and exchange of ideas.

Abstracts

Mark Ashbaugh

Proof of the Payne-Pólya-Weinberger Conjecture

In 1955 Payne, Pólya, and Weinberger proved $\lambda_2/\lambda_1 \leq 3$ for the ratio of the first two eigenvalues of any homogeneous membrane problem, $-\Delta u = \lambda u$ on a bounded domain $\Omega \subset \mathbb{R}^2$ with Dirichlet boundary conditions, and conjectured that $\lambda_2/\lambda_1 \leq \lambda_2/\lambda_1 |_{\text{disk}} \approx 2.539$ is the optimal bound of this form. This and its higher dimensional analogues were recently proved by Rafael Benguria and the speaker. In general form, the proof follows a scheme introduced by Chiti (who obtained the bound 2.586 in 1983) but uses ratios of Bessel functions as trial functions in the variational characterization of the gap $\lambda_2 - \lambda_1$ rather than simply the radial variable r. This improvement allows us to obtain the sharp constant. The proof starts from the Rayleigh-Ritz inequality for λ_2 and uses spherical rearrangements and an inequality of Chiti (1982) to produce a chain of inequalities culminating in the optimal bound. The bound is attained if and only if Ω is a disk (ball, for $\mathbb{R}^n$). Various technical results concerning Bessel functions and their zeros are key elements of the proof.

Volker Bach

Ionization Energies of Bosonic Coulomb Systems

We consider atomic and molecular systems with fixed nuclei where the electrons are assumed to be bosons. Then the ionization energy is rigorously computable in the limit of large particle numbers. It is given in leading order by the chemical potential of the corresponding Hartree model and is of order $O(Z^2)$ up to accuracy of order $O(Z^{5/2})$, Z denoting the sum of the nuclear charges. This contrasts with the fermionic case where ionization energies are expected to be $O(1)$ and thus shows the importance of the Pauli Principle in the real world. Moreover, in the atomic case, we show the excess charge of this model to be given by $(0.21) \cdot Z + O(Z^{2/3})$.

Jean Bellissard

2D Electrons on a Lattice in a Magnetic Field

An electron on a lattice in a magnetic field is Quantum mechanically described through magnetic translations T_1, T_2 which satisfy $T_1T_2 = T_2T_1 e^{2i\pi\alpha}$, $\alpha = \Phi/\Phi_0$ where Φ is the flux through a unit cell and $\Phi_0 = h/e$ is the flux quantum. α plays the role of a "Planck constant". Therefore a small field expansion can be obtained by semiclassical methods. The same expansion works near rational values of α. Any Hamiltonian in the form of a polynomial in T_1, T_2 admits spectrum which looks like a fractal set. The gap edges are continuous functions of α, with right and left derivative at rational values of α, giving a dense set of cusps. Application of this calculus concerns the properties of flux phases in high T_c superconductors.

Vladimir Buslaev

Spectral Properties of the Operators $H\psi = -\psi_{xx} + p(x)\psi + v(\varepsilon x)\psi$, p is Periodic

We describe the spectrum in $L_2(\mathbb{R})$ and the asymptotic behaviour of the eigenfunctions of the operator $H\psi = -\psi_{xx} + p(x)\psi + v(\varepsilon x)\psi$, p is periodic, as $\varepsilon \to 0$. In detail we consider three special cases:

A. $v(\xi) \to +\infty$, $\xi \to \infty$; B. $v(\xi) \to 0$, $\xi \to \infty$; C. $v(\xi) = -\xi$.

All of them have natural applications, especially in quantum solid state physics. The used approach is some generalization of the standard semi-classical approximation. The generalization introduces in the WKB-constructions new geometrical ideas. We can prove the existence of Wannier-Stark ladder of resonances for the case C and give the whole asymptotic estimates for the resonances.

Michael Demuth

On Kato's Class and Large Coupling Convergences in Spectral Theory of Schrödinger Operators

We consider the asymptotic behaviour of Schrödinger operators of the form $H(\alpha,\beta) = H_0 + V_\alpha + \beta U$, where H_0 is the Laplacian, V_α is a Kato's class potential, and U is a nonnegative potential of compact support.

Resolvent convergence is studied in the large coupling limit, i.e.

$$\lim_{\beta\to\infty} (H(\alpha,\beta) - z)^{-1} = (H(\alpha,\infty) - z)^{-1}$$

with respect to different operator norms. Sufficient and necessary conditions are given for the Hilbert-Schmidt and trace-class convergence. That holds both for regular resolvent values and in the limiting absorption case.

The V_α has a finite Kato-norm. If V_α tends to V_{α_0} with respect to this Kato-norm it yields a further operator norm convergence of the form

$$n - \lim_{\alpha\to\alpha_0} (H(\alpha,\beta) - z)^{-1} = (H(\alpha_0,\beta) - z)^{-1}.$$

Sufficient conditions on V_α are given such that this holds also in the limiting absorption case.

Both convergences imply for instance the corresponding asymptotics of wave and scattering matrices.

Jan Derezinski

Algebraic Approach to Long-Range N-body Scattering Theory

If H is a long-range 2-body Schrödinger operator then there exist $s - \lim_{t\to\pm\infty} e^{itH}\, g(\frac{x}{t})\, e^{-itH}$. One can express differential scattering cross sections by these limits. In the case of N-body long-range Schrödinger operators similar asymptotic observables can be defined. They can be used in the study of scattering theory in the case when we do not know whether asymptotic completeness is true.

Christian Gérard

Mourre Estimate for Dispersive Systems

Dispersive N-body Hamiltonians arise when one replaces the non relativistic kinetic energy for N particles $\sum_1^N -\frac{\Delta x_i}{2m_i}$ by a general kinetic term $\omega(D_x)$ for a real function $\omega(\xi)$. We describe in this work a class of dispersive Hamiltonians which we call regular for which we prove a Mourre estimate outside a closed and countable set of energies called **thresholds** which can be explicity described. The regularity condition depends only on the kinetic term $\omega(\xi)$ and on the family of coincidence planes X_a which describe the N-particle structure of the potential energy term. As a consequence of the Mourre estimate we establish abscence of singular continuous spectrum and H-smoothness of $\langle x\rangle^{-s}$, for s bigger than 1/2, which is a basic tool in scattering theory. We then prove some results on scattering theory for short-range interactions. We obtain existence of wave operators, orthogonality of channels and asymptotic completeness of wave operators in the two cluster region.

Gian Michele Graf

Asymptotic Completeness for N-body Short-Range Quantum Systems

We review some standard notation for N-body Schrödinger operators. We then give a sketch of a geometrical proof of asymptotic completeness for an arbitrary number of quantum particles interacting through short-range pair potentials. This is the statement that for large times the quantum system is asymptotically given by a superposition of breakups into one or several independently moving bound clusters.

A framework which allows to describe and obtain some of the propagation properties of the system is introduced. These properties are expressed by means of propagation estimates. In particular we focus on an estimate showing that the center of mass motion of clusters of particles concentrates asymptotically on classical trajectories.

Sandro Graffi

Quantization of the Classical Lie Algorithm in the Bargmann Representation

This talk reports some results on the classical limit obtained in collaboration with Mirko Degli Esposti and Jan Herczynski. For any polynomial perturbation of a system of nonresonant harmonic oscillators in d degrees of freedom it is proved, writing the classical Hamiltonian in complex coordinates and the corresponding Schrödinger operator in the Bargmann representation, that the classical Lie algorithm yielding the canonical perturbation expansion can be "exactly" quantized to yield the Rayleigh-Schrödinger perturbation expansion near any quantum unperturbed eigenvalue. By exact quantization it is meant the explicit computation of all quantum corrections to the principal symbol, given by the classical Lie expansion for the relevant operators in perturbation theory. A consequence of this result is the explicit construction, to any order in perturbation theory, of all corrections to the Bohr-Sommerfeld quantization formula.

Andreas M. Hinz

Dense Point Spectrum of a Spherically Symmetric Schrödinger Operator

Schrödinger operators with spherically symmetric potentials give rise to some amazing spectral phenomena. In particular they serve as counter examples to a number of general opinions about the asymptotic behavior of eigensolutions. The most surprising result is the presence of intervals of dense point spectrum for the operator $-\Delta + \cos(|x|)$. This is proved in two steps: Orthogonal separation shows that the gaps in the spectrum of the one-dimensional operator are filled up in higher dimensions, while spherical decomposition yields that the separated one-dimensional operators, all having the same essential spectrum, can only contribute to this filling up by eigenvalues.

Peter D. Hislop

Semiclassical Theory of Stark Ladder Resonances

Joint work with J.M. Combes. We prove the existence of resonances in the semiclassical regime of small h for the Stark ladder Hamiltonian $H(h,F) = -h^2 \frac{d^2}{dx^2} + v + Fx$ in one dimension. The potential v is a non-constant, real periodic potential with period τ which is the restriction to $\mathbb{R}$ of a function analytic in a strip about $\mathbb{R}$. We consider F fixed and satisfying the bounds $0 < F < \|v'\|_\infty$. For h sufficiently small, there exists an infinite family of resonances with the same negative imaginary part and whose real parts differ by multiples of $F\tau$. In general, the resonance width is bounded above by $ce^{-\beta h^{-1}}$ where β is expressible in terms of the Agmon metric and depends on F. In the special case where the distance between resonant wells is $O(F^{-1})$, we prove that there is at least one resonance z_0 for which the width satisfies an Oppenheimer formula : $|\operatorname{Im} z_0| \leq ce^{-\beta_0 (Fh)^{-1}}$, where $\beta_0 > 0$ is independent of F and h.

Maria Hoffmann-Ostenhof

Nodal Properties of Solutions of Schrödinger Equations

The following is joint work with Thomas Hoffmann-Ostenhof. Let Ψ be a real-valued solution of $(-\Delta + V)\Psi = 0$ in Ω, Ω a domain in $\mathbb{R}^n$, $n \geq 3$, where V is realvalued and $C^\infty(\Omega)$. Let D denote a nodal domain of Ψ in Ω, i.e. $|\Psi| > 0$ in D, D is connected and $\Psi = 0$ in $\partial D \backslash \partial \Omega$. It is shown that D satisfies an interior cone condition for points in $\partial D \backslash \partial \Omega$. Assume that Ψ has a zero of order M in $x_0 \in \Omega$, then the number of nodal domains D of Ψ with $x_0 \in \partial D$ is smaller than or equal to the number of nodal domains of some harmonic homogeneous polynomial of degree M. For $\Omega = R^n$ and $\Psi \in L^2(R^n)$ we obtain under suitable conditions on V upper bounds to M.

Werner Horn

Semi-classical Construction of Approximate Eigenfunctions near the Top of a Potential Barrier

We consider the operator $P(h) = -h^2 \frac{d^2}{dx^2} + V(x)$ where V is a real analytic function on $I\!R$, which has a potential barrier at the origin, i.e. $V(0) = 0$; $V'(0) = 0$; $V''(0) < 0$. A double well potential is a typical example of this situation. We construct explicit functions $V^N(x, E, h)$ which satisfy $(P(h) - E(h))V^N(x, E, h) = O(h^{N+2})$ for x and E sufficiently close to the origin. This is done by reducing the problem to the task of finding solutions of the equation $-h^2 \frac{d^2}{d\xi^2} w - \xi^2 w = cw$. We also show how the functions $V^N(x, E, h)$ can be used to construct approximate eigenfunctions in $L^2(I\!R)$. This yields an implicit condition, which can be used to find approximate eigenvalues.

Arne Jensen

Classical and Quantum Scattering for Stark Hamiltonians

For the ordinary Schrödinger operator $H = -\Delta + V$ on $L^2(I\!R^n)$ the borderline between short range and long range potentials is given by the Coulomb potential $V(x) = \frac{c}{|x|}$. This potential is also the borderline in classical mechanics.

For Stark Hamiltonians the situation is different. In the quantum case it is shown that the ordinary wave operators $W_\pm = s - \lim_{t \to \pm\infty} e^{itH} e^{-itH_0}$ do not exist for $H_0 = -\frac{1}{2} \Delta + E \cdot x$, $E \in I\!R^n, E \neq 0$ and $H = H_0 + \lambda |x|^{-\gamma}$, $\lambda \in I\!R$, $\lambda \neq 0$, $0 < \gamma \leq \frac{1}{2}$ $(\gamma < \frac{1}{2}$ if $n = 1)$. The classical mechanics case is different: We prove the following result: Let $V \in C^2(I\!R)$ realvalued with bounded derivatives, such that $|V(x)| \leq c(\log(2 + |x|))^{-\alpha}$ for some $\alpha > 1$. Let $x(t)$ solve Newton's equation $\ddot{x}(t) = -1 - V'(x(t))$. Assume that the solution is not a bound state as $t \to +\infty$, i.e. $\limsup_{t \to +\infty} (|x(t)| + |\dot{x}(t)|) = +\infty$. Then there exist $\xi, v \in I\!R$ such that

$$\lim_{t \to +\infty} |x(t) - (-\frac{1}{2} t^2 + t v + \xi)| = 0,$$

$$\lim_{t \to \infty} |\dot{x}(t) - (-t + v)| = 0.$$

Thus the solution $x(t)$ is asymptotic to a free solution.

The results presented are joint work with Tohru Ozawa, Nagoya Univ., Japan.

Hitoshi Kitada

Asymptotic Completeness of N-body Wave Operators

A new proof of the asymptotic completeness of N-body wave operators was presented. Let $H = H_0 + V = H_0 + \sum_\alpha V_\alpha(x_\alpha)$ be the N-body Hamiltonian. Then the completeness is shown under short-range decay assumption on $V_\alpha(x_\alpha)$ with some additional conditions. The method is based on the framework of Enss: We asymptotically decompose the approximate state $\Phi_n e^{-it_nH} f$ for $f \in \mathcal{H}^c(H)$ like

$$\Phi_n e^{-it_nH} f \sim \sum_{2\leq|a|\leq N} L_a(t_n) e^{-it_nH} f \quad (t_n \to \infty)$$

and prove the existence of the strong limit

$$\Omega_a f = s - \lim_{t\to\infty} e^{itH_a} L_a(t) e^{-itH} f.$$

Then one has $\Phi_n e^{-it_nH} f \sim \sum_{2\leq|a|\leq N} e^{-it_nH_a} \Omega_a f$, from which follows the asymptotic completeness. In the proof of the existence of $\Omega_a f$, Sigal-Soffer's propagation estimates for the non-threshold case are used.

Andreas Knauf

Scattering Theory for Coulombic Potentials

Joint work with Markus Klein. We consider the planar scattering of a classical or quantum mechanical particle by a potential with n Coulombic $(-z/r)$ singularities.

In the classical case, a Levinson Theorem is proven. The high energy bound states are analysed using symbolic dynamics. For $n \geq 3$ the time delay is infinite on a Cantor set of asymptotic data, whereas the differential cross section is a smooth function. For the special case of two fixed Coulomb singularities the resonances associated to the hyperbolic collision orbit are computed by separation of variables and complex WKB methods.

Michael Loss

Minimization of Functionals Arising in Mathematical Physics and Geometry

This is joint work with Eric Carlen. A new method is presented for proving sharp constants in a variety of inequalities. The method is effective in all those cases where the functional to be minimized is conformally invariant and improves under rearrangements. Examples are the Sobolev inequality, the Hardy-Littlewood-Sobolev inequality and Onofri's inequality to name a few.

One constructs a map consisting of a well chosen "conformal rotation" on function space followed by the symmetrically decreasing rearrangement. Starting from any function by iterating this map one gets an optimizing sequence which has excellent convergence properties and which drives the functional to its minimal value.

André Martinez

Born-Oppenheimer Expansions for N-body Problems

Consider an $N = (n + 1 + p)$-body problem with Coulombic interactions, where $n + 1$ is the number of nuclei and p is the number of electrons. When the mass M of the nuclei tends to infinity, Born and Oppenheimer conjectured in 1927 that the low lying eigenelements of the Hamiltonian (after removing the center of mass motion) should admit asymptotic expansion in powers of $M^{-1/4}$. We prove this result for a finite number of eigenvalues in the case of the diatomic molecule, as well as for the first one in the case of the polyatomic molecule. We also obtain WKB-type expansions for the associated eigenfunctions. The method consists first in reducing the problem to a pseudodifferential operator acting only on the nuclei-position variables x. This can be done using an x-dependent change of the electronic variables, which regularize in some sense the initial potential. This is a joint work with M. Klein, R. Seiler, and X.P. Wang.

Eric Mourre

Algebraic Character of Pseudo-Differential Calculus

I) Let $\mathcal{A}(t;\cdot)$ be an associative algebra with some norm $\|\cdot\|$ (usually not complete), and $(D_i)_{i\in\{1,\cdots,m\}}$ a family $\mathcal{D}$ of commuting derivations. It may be assumed to be complete for a natural metric topology; we define a subalgebra $\tilde{\mathcal{A}}$ such that

$$a *_{(z;D_1,D_2)} b = \sum_{\alpha\in N^n} (D_1^\alpha a)\cdot(D_2^\alpha b)\cdot\frac{z^\alpha}{\alpha!};$$

$$D_\alpha = (D_\alpha^i)_{i=1,\cdots,n} \in \mathcal{D},\ \alpha = 1, 2, \quad z\in \mathbb{C}^n$$

define on $\tilde{\mathcal{A}}$ new associative laws. The procedure may be iterated and has a group property.

II) Let $\mathcal{A}(t)$ be a vector space equipped with two associative laws $\cdot, *$. Let L_0, L_1 be derivations, repectively, on $\mathcal{A}(+;\cdot)$, $\mathcal{A}(+;*)$.

We discuss the problem of constructing a vectorial isomorphism between ker L_0 and ker L, which solves in a very special case the problem of inducing from a deformation of the algebra $\mathcal{A}(+;\cdot)$ a deformation on a subalgebra.

Mary Beth Ruskai

Bounds on the Total Excess Charge of a Stable Diatomic Molecule

Although the fixed-nucleus approximation is useful for many purposes, it is not very satisfactory for describing such phenomena as the breakup of a molecule into atomic subsystems. This talk reports some recent results about the existence of bound states for Hamiltonians describing diatomic molecules with dynamic nuclei, i.e., for which the nuclear motion is completely unrestricted and the kinetic energy of the nuclei is included. It is shown that a system with N electrons and nuclear charges Z_1 and Z_2 cannot have a bound state below the continuum if the nuclear charges are either too large (in which case the molecule splits into two atomic subsystems) or too small, relative to the number of electrons. Equivalently, there are constants $N_{\min}$ and $N_{\max}$ associated with every pair of charges (Z_1, Z_2), such that the existence of a bound state implies that N satisfies $N_{\min} \leq N \leq N_{\max}$. Asymptotic bounds on $N_{\min}$ and $N_{\max}$ are presented. The proof indicates that differences in threshold energies play an important role in bound state molecular problems.

Carol Shubin Christ

An Inverse Spectral Problem

We study the inverse spectral problem for the time independent Schrödinger operator H with a radial potential in $x \in \mathbb{R}^3$, $|x| \leq 1$, with Dirichlet boundary conditions on $|x| = 1$. By expansion in spherical harmonics H is unitarily equivalent to an infinite direct sum of Sturm-Liouville operators acting on $L^2[0,1]$. We answer the question of whether one can uniquely determine the potential given the spectra of the first two operators in this sum. The main result is that the intersection of the corresponding isospectral manifolds is locally compact.

Marianna A. Shubov

Asymptotics of the Discrete Spectrum for a Radial Schrödinger Operator with Nearly Coulomb Potential

The radial Schrödinger equation with Coulomb potential perturbed on a compact set is considered. An asymptotic formula for the discrete spectrum is obtained. It follows from this formula that the quantum defect tends to a constant when the principal quantum number tends to infinity. An explicit expression of this constant through the perturbation is obtained. The result is based on the eigenvalue selection principle proved in the previous work of the author.

Heinz Siedentop

A New Phase Space Localization Method with Application to the Sum of Negative Eigenvalues of Schrödinger Operators

Joint work with R. Weikard.
A Schrödinger operator, under suitable conditions, may be decomposed as $H = -\frac{d^2}{dr^2} - \varphi(r) = \sum_n H_n$ with $H_n = \psi_n' \otimes \psi_n' - (\varphi^{1/2}\psi_n) \otimes (\varphi^{1/2}\psi_n)$. Choosing $\psi_n(r) = (\frac{\rho}{N})^{1/2}(r) \exp\{2\pi i\, n \int_0^r \frac{\rho}{N}(t)\,dt\}$, $\int \rho = N$, $\rho > 0$, and $\rho = \frac{1}{\pi}\varphi^{1/2}$ yields the leading quasiclassical expression with sharp estimates on the remainder. Applying this to $-\Delta - \frac{Z}{|\cdot|} + \rho_{TF} * \frac{1}{|\cdot|}$ where ρ_{TF} is the minimizer of the corresponding Thomas-Fermi functional (nuclear charge Z, Z electrons), yields a simple proof of a lower bound of Scott type to the groundstate energy of

$$H_{Z,N} = \sum_{i=1}^{N}(-\Delta - \frac{Z}{|x_i|}) + \sum_{i<j=1}^{N} \frac{1}{|x_i - x_j|}, \quad N = Z,$$

in $\bigwedge_{i=1}^{N}(L^2(\mathbb{R}^3) \otimes \mathbb{C}^4)$, i.e., inf spectrum$(H_{Z,Z}) \geq E_{TF}(Z,Z) + \frac{q}{8}Z^2 - c\,Z^{2-\epsilon}$ where $E_{TF}(Z,Z)$ is the Thomas-Fermi energy.

Israel M. Sigal

Periodic Solutions of Non-Linear PDE's

Consider a linear Schrödinger or wave equation. A key feature of such an equation is that it admits time-periodic solutions, called bound states in the Schrödinger case and standing waves in the wave case. We show that such solutions are generically unstable under small non-linear perturbation for the wave equation, and could be stable or unstable depending on their nature for the Schrödinger equation. We study also life-times of such solutions. To this end we introduce the notion of resonance for non-linear equations.

Avy Soffer

N-body Long Range Scattering

Asymptotic Completeness is proved for systems of 4 particles interacting with long range potentials of the Coulomb type.

The proof requires the development of spectral and scattering theory for a class of N-body Hamiltonians with extra time dependent potentials added. This is done jointly with I.M. Sigal.

Günther Stolz

Expansions in Generalized Eigenfunctions of Schrödinger Operators with Singular Potentials

We prove the existence of an expansion in generalized eigenfunctions of Schrödinger operators $H = (\frac{1}{i}\nabla - b)^2 + V$ in $L^2(\mathbb{R}^\nu)$. To do this we use an expansion theorem of BGK-type together with some modifications.

If the electromagnetic potentials are polynomially bounded (up to local singularities), the generalized eigenfunctions lie in a polynomially weighted Sobolev space of second order. Using this result we get polynomial boundedness of the generalized eigenfunctions by methods from regularity theory.

From these results we get one part of the proof of

$$\sigma(H) = \overline{\{\lambda : \exists u \in K_s L_2 \cap W_{2,1,\,\mathrm{loc}}\ (s > 0),\, H u = \lambda u\}},$$

which is shown under general assumptions on b and V.

X.P. Wang

Semiclassical Resolvent Estimates of N-body Schrödinger Operators and Classical Trajectories

Let $P(h) = -h^2\,\Delta + \sum_{i<j} V_{ij}(x_i - x_j)$ be an N-body operator with smooth interacting potentials. We denote by $T^Q(h)$ the quantum thresholds of $P(h)$ and by T^C the classical ones of $p = \xi^2 + \sum_{i<j} V_{ij}(x_i - x_j)$.

We prove:

(a): For $h_0 > 0$ small, $\bigcup_{0<h\leq h_0} T^Q(h) \subseteq T^C$; and

(b): The resolvent estimate

$$\|\langle F_0\rangle^{-s} R(\lambda \pm i0, h) \langle F_0\rangle^{-s}\| \leq c\,h^{-1}, \quad h \in]0, h_0]$$

is true for some $s > 1/2$, $|\lambda - E| < \delta$, if and only if $E \notin T^C$. Here $F_0 = h(x \cdot D_x + D_x \cdot x)/2$.

Dmitri Yafaev

On Resonant Scattering by Perturbations Periodic in Time

Consider a quantum system described by a Hamiltonian $H(t) = H_1 + \varepsilon V(t)$ where the self-adjoint operator H_1 has the absolutely continuous spectrum $[0, \infty)$ and the negative eigenvalue λ_1 and $V(t) = V(t + 2\pi)$. For such perturbations scattering of a plane wave of energy λ is determined by a set of amplitudes $S^{(n)}(\lambda, \varepsilon)$, $n \in \mathbb{Z}$, corresponding to the "final" energy $\lambda + n$.

The limit of $S^{(n)}(\lambda, \varepsilon)$ as $\varepsilon \to 0$ is studied. In the case $\lambda - \lambda_1 \notin \mathbb{Z}$ the amplitudes satisfy $S^{(n)}(\lambda, \varepsilon) = O(\varepsilon^n)$ and $S^{(0)}(\lambda, \varepsilon)$ is convergent to the scattering matrix $S_1(\lambda)$ for the Hamiltonian H_1. At the resonant energies $\lambda - \lambda_1 = m \in \mathbb{Z}$ the perturbation theory in ε becomes singular because of the small denominator problem. In particular, if $m = 1$, $S^{(-1)}(\lambda, \varepsilon) \sim \varepsilon^{-1}$ as $\varepsilon \to 0$ and $S^{(0)}(\lambda, \varepsilon)$ has a finite limit which is different from $S_1(\lambda)$. The additional phase shift is explained by the resonant interaction between the plane wave and the bound state of H_1.

Kenji Yajima

Dirac Equations with Moving Nuclei

We consider the Cauchy problem for the Dirac equation

$$i\partial_t u = (\alpha \cdot D + m\beta + (\phi - \alpha \cdot A))u, \quad u\,|_{t=s} = u_0,$$

where (A, ϕ) is the Lienard-Wiechert potential produced by a finite number of nuclei $q_1(t), \cdots, q_N(t)$ with charges $Z_1, \cdots, Z_N$. We assume: $q_j \in C^3(\mathbb{R}, \mathbb{R}^3)$; $|\dot{q}_j(t)| \leq v_j < 1$, $|\ddot{q}_j(t)| \leq Q_j < \infty$ for $1 \leq j \leq N$, $\forall t \in \mathbb{R}$; $q_j(t) \neq q_k(t)$ if $j \neq k$.

When $|Z_j| < \sqrt{3}/2$, $j = 1, \cdots, N$, we show, by using a local pseudo-Lorentz transformation, which is the spatial part of the Lorentz transformation near each $q_j(t)$ and which freezes the singularity of the potential, that the equation generates a unique unitary propagator which preserves $H^1(\mathbb{R}^3)$, $L^2(\mathbb{R}^3)$, $H^{-1}(\mathbb{R}^3)$.

When $1 > |Z_j| \geq \sqrt{3}/2$, we can still construct a unique propagator if v_j is sufficiently small, but it preserves $H^{1/2}$, L^2, $H^{1/2}$.

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