MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1990

Lyapunov Exponents

27. 5. bis 2. 6. 1990

Die Tagung fand unter der Leitung von Herrn L. *Arnold* (Bremen) und Herrn J.-P. *Eckmann* (Genf) statt. Die Schwerpunkte des Interesses waren:

1. Produkte zufälliger Matrizen
2. Lineare Schiefproduktflüsse und Kontrollflüsse
3. Stochastische Flüsse
4. Entropie und große Abweichungen
5. Unendlichdimensionale Systeme
6. Intervall-Abbildungen
7. Numerische und asymptotische Aspekte
8. Ingenieurwissenschaftliche Anwendungen

Vortragsauszüge

S.T. Ariaratnam:

Lyapunov exponents in structural dynamics

The role of Lyapunov exponents in the following three problems encountered in stochastic structural dynamics is discussed:

1) Dynamic stability of elastic structures:

Equations of motion are of the form:

$$\ddot{q}_i + 2\beta_i\dot{q}_i + \omega_i^2 q_i + \xi(t)\omega_i \sum_j k_{ij}q_j = 0, \quad i,j = 1,2,\dots,n,$$

where $\xi(t)$ is a stationary, zero mean stochastic process (i.e. the loading) with smoothly varying spectral density $S(\omega) = 2\int_0^\infty \cos \omega\tau \, E[\xi(t)\xi(t+\tau)]d\tau$. β_i, $S(\omega)$ are small, $O(\varepsilon)$.

The general result for the largest Lyapunov exponent is, for $n = 2$:

$$\lambda = \tfrac{1}{2}(\lambda_1+\lambda_2) + \tfrac{1}{2}(\lambda_1-\lambda_2)\coth\left(\frac{\lambda_1-\lambda_2}{\Delta^{1/2}}\alpha\right) + \tfrac{1}{8}k_{12}k_{21}[S(\omega_1+\omega_2) - S(\omega_1-\omega_2)],\ \Delta > 0$$

where

$$\cosh\alpha = \left|\frac{1}{2k_{12}k_{21}S^+}\left\{k_{11}^2S(2\omega_1) + k_{22}^2S(2\omega_2) - 2k_{12}k_{21}S^-\right\}\right|$$

$$\Delta = \frac{1}{64}\left[\{k_{11}^2S(2\omega_1) + k_{22}^2S(2\omega_2) - 4k_{12}k_{21}S(\omega_1+\omega_2)\}\{k_{11}^2S(2\omega_1) + k_{22}^2S(2\omega_2) + 4k_{12}k_{21}S(\omega_1-\omega_2)\}\right]$$

$$S^\pm = S(\omega_1+\omega_2) \pm S(\omega_1-\omega_2);\ \lambda_i = -\beta_i + \frac{k_{ii}^2}{8}S(2\omega_i),\ i = 1,2.$$

For $\Delta < 0$, the result is obtained by suitable change of hyperbolic terms by their trigonometric counterparts, for $\Delta = 0$ by suitable limiting forms. For example, if $k_{11} = k_{22} = 0$ and $S(\omega)$ is concentrated around either $\omega_1 + \omega_2$ for $k_{12}k_{21} > 0$ or $|\omega_1 - \omega_2|$ for $k_{12}k_{21} < 0$,

$$\lambda = -\beta_{1,2} + \frac{|k_{12}k_{21}|}{8}S(\omega_1 \pm \omega_2),\ \pm \text{ according as } k_{12}k_{21} > 0, < 0, \text{ resp., } \begin{pmatrix}\beta_1 \text{ if } \beta_1 < \beta_2\\ \beta_2 \text{ if } \beta_2 < \beta_1\end{pmatrix}.$$

2) Stochastic pitchfork bifurcation:
This occurs in columns, flat plates etc. under axial stochastic loads; equation of motions is

$$\ddot{q} + 2\beta\dot{q} - (\gamma_0 + \sigma\xi(t))q + \delta q^3, \quad (\xi(t) \text{ white noise}).$$

For γ_0 near zero (i.e. the bifurcation point), the Lyapunov exponent of the linearized system $\ddot{x} + [\gamma - \sigma\xi(t)]x = 0$, $\gamma = -\gamma_0 - \beta^2$, obtained after the change of variable $q = xe^{-\beta t}$ is

$$\lambda = 0.28931\,\sigma^{2/3}\left[1 - 4.57886\,\sigma^{-4/3}\gamma + 6.10592\,\sigma^{-8/3}\gamma^2 + o\left(\frac{\gamma^2}{\sigma^{8/3}}\right)\right]$$

from which the shift in the bifurcation point due to stochastic local perturbation may be obtained by setting $\lambda = 0$.

3) Localization of wave propagation in spatially disordered structures.
In randomly disordered almost periodic structures such as long pipelines continuous over several supports or space antennae made up of several nearly identical units, the transmission coefficient $-\tau_n$ at the nth unit from the point of origin of the wave varies as $|\tau| \sim e^{-\lambda n}$, for large n, where λ is the largest Lyapunov exponent corresponding to the random wave transfer matrix of a single unit (Fürstenberg's theorem on product of random matrices). The variation of the localization length $1/\lambda$ with frequency in the pass bands is evaluated for a long continuous Euler-Bernoulli beam and compared with approximated perturbation solution for $\pm$ 10 % disorder in the span length of the beam between supports.

L. Arnold, H. Crauel:

Iterated function systems and multiplicative ergodic theory

Given a random dynamical system of affine mappings of $\mathbb{R}^d$ whose linear part is hyperbolic. We prove that it has a unique invariant measure. The proof uses multiplicative ergodic theory. The result generalizes and sheds new light on results of Barnsley and Elton on iterated function systems.

P. Baxendale:

Behaviour of invariant measures as the Lyapunov exponent crosses zero

Suppose the process $\{x_t: t \geq 0\}$ in $\mathbb{R}^d$ satisfies the stochastic differential equation

$$dx_t = V_0(x_t)dt + \sum_{i=1}^{r} V_i(x_t)dW_t^i$$

where the vector fields $V_0, V_1, \ldots, V_r$ all fix 0. Let λ be the Lyapunov exponent obtained by linearising this s.d.e. at 0. Under suitable non-degeneracy and growth conditions on the V_i, the process $\{x_t: t \geq 0\}$, when restricted to $\mathbb{R}^d - \{0\}$ is transient, or null recurrent, or positive recurrent according as $\lambda > 0$, or $\lambda = 0$, or $\lambda < 0$.

Suppose now that the coefficients in the s.d.e. are varied in a smooth manner so that $\lambda \downarrow 0$. We give estimates on the rate at which the (unique) invariant probability measure for $\{x_t: t \geq 0\}$ on $\mathbb{R}^d - \{0\}$ converges to the unit mass at 0.

P. Bougerol:

Lyapunov exponents and Kalman filtering

We consider the usual linear set up

$$\begin{cases} X_n = A_n X_{n-1} + F_n \varepsilon_n, & X_n \in \mathbb{R}^d \\ Y_n = C_n X_n + \tau_n, & Y_n \in \mathbb{R}^q \end{cases}$$

where (ε_n, τ_n) is a sequence of i.i.d. Gaussian random vectors and (A_n, C_n, F_n) is a stationary ergodic sequence independent of $\{(\varepsilon_n, \tau_n), n \in \mathbb{N}\}$, considered as known. Let

$$P_n = \mathbb{E}(\hat{X}_n - X_n)(\hat{X}_n - X_n)^*, \quad \hat{X}_n = \mathbb{E}(X_n / Y_0, \ldots, Y_n)$$

(they depend on A_n, C_n, F_n).

Let M_n be the Hamiltonian matrix: $\begin{pmatrix} A_n & S_nA_n^{*-1} \\ T_nA_n(I+T_nS_n) & A_n^{*-1} \end{pmatrix}$.

$(S_n = F_nF_n^*, \quad T_n = C_n^*C_n)$.

These matrices are in $Sp(2d,\mathbb{R})$. If the dot is the usual action of $Sp(2d,\mathbb{R})$ on the set of symmetric matrices (identified with the set of Lagrangian subspaces), then

$P_n = M_n \dots M_1 \cdot P_0$.

Let $\mathcal{P}$ be the set of positive symmetric definite matrices. Then

Theorem $\mathfrak{T} = \{M \in Sp(2d,\mathbb{R});\ M\mathcal{P} \subset \mathcal{P}\}$ is a semi group acting by *contractions* on $\mathcal{P}$, where $\mathcal{P}$ is equipped with the usual Riemannian metric.

With this theorem we prove that under weak controllability and observability assumption, there exists a unique stationary process $\overline{P}_n$ such that $d(P_n,\overline{P}_n) \to 0$ a.s. $\forall\ P_0 \in \mathcal{P}$.

We show also the exponential stability of the filters and explain the relation between this and the positivity of Lyapunov exponents of $M_n \dots M_1$, $n \geq 0$.

P. Boxler:

Lyapunov exponents as a tool in stochastic bifurcation theory

In nonlinear ordinary differential equations which depend on a parameter and which are often used as models in physics or engineering, bifurcation phenomena are frequently encountered. They are closely related to changes in the qualitative behavior of the system and in its stability properties. We are interested in understanding what happens to these features in the presence of white or real noise. A notion of a stochastic bifurcation point in terms of invariant measures is suggested. Using Lyapunov exponents a necessary condition for such a stochastic bifurcation can be derived. It allows to understand the possibility of a shift of the bifurcation point in the stochastic case. The one dimensional transcritical bifurcation is examined in detail, and the example of the saddle node bifurcation shows that noise may also destroy a bifurcation. Finally the theory of stochastic center manifolds is outlined to provide a tool to handle higher dimensional problems.

C. Bucher:

Sample stability of multi-degree-of-freedom systems

Within the analysis and design of structural systems the question of dynamic stability arises quite frequently. A typical area in civil engineering is the motion stability of long-span bridges (e.g.

suspension type) in turbulent wind. While the equations of motion can be modeled in a linear way without significant loss of accuracy it is quite essential that a relatively high number of state variables - describing both structural motion and fluid-structure-interaction - are retained in the analysis.Conventional methods used frequently in engineering analysis like stochastic averaging are less suitable for the previously mentioned cases where the interaction between different modes of vibration plays a central role. Consequently, a new concept which generalizes the 'classical' averaging to higher dimensions is presented and put for discussion. Numerical examples are provided as an additional basis for future application.

N. D. Cong:

Lyapunov exponents and central exponents of linear systems of differential equations perturbed by small random noises

We consider the spectrum of auxiliary exponents, the Lyapunov spectrum of central exponents of systems with weakly varying coefficients and close systems. Systems with weakly varying coefficients are interesting because the linear systems of almost all well known examples in theory of Lyapunov exponents, including the Perron example, are systems with weakly varying coefficients. It is proved that a system with weakly varying coefficients has exact auxiliary exponents which can be calculated by Cauchy matrix of the given system. Furthermore, central exponents of a system with weakly varying coefficients under small random nondegenerate perturbation coincide with the corresponding Lyapunov exponents with probability 1 and they are close to the corresponding auxiliary exponents of the given system.

P. Collet:

Large deviations for Lyapunov exponents and applications

Using a continuous field of uniformly contracted cones, the large deviations of the maximal exponent can be discussed using a standard thermodynamic formalism. The associated pressure function appears also in some quantitative questions. Two examples in one dimension are the length of the graph of the n^{th} iterate of the transformation, and the essential radius for the Perron-Frobenius transfer operator for expanding Markov maps.In higher dimension, this theory applies to the study of non autonomous renormalisation, where the standard critical indicies are replaced by Lyapunov exponents or their pressure at various temperatures according to the spectral nature of the noise. The pressure at inverse temperature 1 is also the dynamo rate of some suspensions of Anosov maps on the two torus, i.e. the exponential growth rate of a passive magnetic field transported by the suspended flow in the limit of infinite magnetic resistivity.

H. Crauel:

An upper bound for relative entropy

Suppose μ is an invariant measure for a smooth random dynamical system $\{\varphi(t,\omega) : M \to M \mid t \in T, \omega \in \Omega\}$ on a d-dimensional Riemannian manifold; $T = \mathbb{R}$ or $\mathbb{Z}$. Denote by μ^+ the restriction of μ to nonnegative time, i.e., to the σ-algebra $\mathcal{F}^+ = \sigma\{\varphi(\tau,\cdot) \mid \tau \geq 0\}$. The *relative entropy* associated with μ is

$$\alpha_\mu = -\int_{M\times\Omega} \log \frac{d\varphi^{-1}(1,\omega)\mu^+_{\vartheta_1\omega}}{d\mu^+_\omega}\, d\mu(x,\omega).$$

Provided some integrability condition, we show that

$$\alpha_\mu \leq d \int \max\{0, -\lambda^\mu_d(x,\omega)\}\, d\mu(x,\omega),$$

where λ^μ_d is the smallest Lyapunov exponent associated with μ. This generalizes results of Baxendale (86) and Ledrappier (84).

As a consequence, we get a criterion for 'partial hyperbolicity': If, for μ ergodic, μ is neither a functional of the future nor of the past alone, then $\lambda^\mu_d < 0 < \lambda^\mu_1$.

R.W.R. Darling:

Products of infinite-dimensional random matrices related to oriented percolation

Let A_1 be a random $\mathbb{Z}^d \times \mathbb{Z}^d$ matrix with entries in $\{0,1\}$. Let $N \subseteq \mathbb{Z}^d$ be finite, and suppose:

$$A_1(x,x+z) = 0, \text{ for } z \text{ outside } N$$

$$\{A_1(x, x+y), y \in N\} \text{ has the same law for all } x \in \mathbb{Z}^d.$$

Let $\theta = \Sigma_y \mathbb{E}[A_1(0,y)]$.

Let $A_1, A_2, \ldots$ be i.i.d. random matrices. Then the Lyapunov exponent

$$\lambda \equiv \lim_{n\to\infty} \frac{1}{n} \log\|\Sigma_y(A_1,\ldots,A_n)(0,y)\|$$

exists a.s., and

$$(*) \qquad \lambda = \begin{cases} \log\theta & \text{on } \{Y > 0\} \\ -\infty & \text{on } \bigcup_{n\geq 1}\{Y_n = 0\} \end{cases}$$

where Y is the limit of the martingale $Y_n \equiv \theta^{-n}\, \Sigma_y(A_1,\ldots,A_n)(0,y)$.

When $\dim(N) \geq 3$, then for suitable choice of the law of A_1 it is possible to prove that

$P(Y > 0) > 0$. It is conjectured that (*) gives all possible values of λ, in the sense that $P(Y > 0) = 1 - P(\bigcup_{n\geq 0}\{Y_n = 0\})$, at least when $d \geq 3$.

F. Flandoli:

Stochastic flows for stochastic PDEs of parabolic type

Some general abstract method is known to prove existence and uniqueness of solutions to ∞-dimensional stochastic equations, but the problem of existence of the associated stochastic flow is solved only in some particular case, using "ad hoc" methods. The aim of this research is to develop an abstract method for stochastic equations of parabolic type. The method is based on the fact that if the solution mapping is Hilbert-Schmidt in the square mean sense, then it is also pathwise Hilbert-Schmidt. The former property can be obtained by means of regularity results, typical of parabolic problems. One can show that this method has wide applications, to second order parabolic equations with Dirichlet, Neumann, periodic boundary conditions, to higher order parabolic equations, and to systems of parabolic equations. However, relatively to certain particular problems, the "ad hoc" methods mentioned above allow greater generality and provide more precise properties. Once the existence and regularity of the flow is established, Oseledec type theorem can be applied.

I. Goldsheid, G. Margulis:

A condition for simplicity of the spectrum of Lyapunov exponents

It is proved that Zariski closure is "responsible" for the description of the Lyapunov exponents of the product of i.i.d. random matrices. Let ν be the distribution of the matrices A_i, and F be the Zariski closure of the group generated by the support of ν. Suppose that F is a semisimple group (but $F \neq SL(m,\mathbb{R})$ in general). Denote by τ a maximal torus in F which splits over $\mathbb{R}$. Let $\mathfrak{t}$ be the Lie algebra of the group τ, and W the Weyl group of F with respect to τ. Choose some Weyl chamber $Y \subset \mathfrak{t}$ with respect to the action of W on $\mathfrak{t}$ and let R be a maximal compact subgroup of F such that $g = \kappa_1(g) \exp y(g)\, \kappa_2(g)$ and this decomposition is unique for $g \in F$; $\kappa_1(g), \kappa_2(g) \in K$, $y(g) \in Y$. It follows from Oseledec's theorem that w.p.1, there exist $\alpha(\nu) = \lim_{n\to\infty} n^{-1}\, y(A_n A_{n-1} \cdots A_1)$.

Theorem If F is a semisimple group and A_n are independent then $\alpha(\nu)$ is an in interior point of the Weyl chamber Y.

H. Herzel:

Estimation of Lyapunov exponents from experimental data

Wolf's method for the estimation of the maximum Lyapunov exponent is applied to various experimental time series. The algorithm is based on monitoring the separation of nearby trajectories in a pseudo phase space spanned by delay coordinates. Some problems of the method are discussed which are related to effects of noise and nonuniformity of the dynamics.
The algorithm is used for the characterization of several measured time series:

- solar radiation
- climatic data (EL Nino-Southern Oscillations)
- data from heterogeneous catalysis
- newborn infant cries

F. Hofbauer:

The Hausdorff dimension of an ergodic invariant measure for piecewise monotonic maps of the interval

A map $T: [0,1] \to [0,1]$ is called piecewise monotonic, if there is a finite partition $\mathcal{Z}$ of $[0,1]$ into intervals such that $T|Z$ is continuous and monotone for all $Z \in \mathcal{Z}$. If μ is a T-invariant measure on $[0,1]$, let h_μ be its entropy and λ_μ its Lyapunov exponent. The Hausdorff dimension of μ, denoted $HD(\mu)$, is defined as the infimum of the Hausdorff dimensions $HD(X)$, the infinum taken over all $X \subset [0,1]$ with $\mu(X) = 1$. The p-variation of a function $\varphi: [0,1] \to \mathbb{R}$ is defined as

$$\mathrm{var}^p\,\varphi = \sup\left\{ \sum_{i=1}^{m} |\varphi(x_{i-1}) - \varphi(x_i)|^p : m \geq 1, 0 \leq x_0 < x_1 < \ldots < x_m \leq 1 \right\}.$$

We have the following theorem:

Theorem: Let T be piecewise monotonic such that $\mathrm{var}^p\,|T'| < \infty$ for some $p > 0$. Let μ be an ergodic T-invariant measure satisfying $\lambda_\mu > 0$. Then $HD(\mu) = \frac{h_\mu}{\lambda_\mu}$.

J. Holzfuß:

Lyapunov exponents from experimental data

The spectrum of Lyapunov exponents has been calculated from a time series of an observable of a chaotic attractor in phase space. The numerical method, consisting of the construction of a trajectory in phase space and of the approximation of the matrix of the linearized flow, is investigated in detail. It is shown, that in principle all Lyapunov exponents, including negative ones, can be extracted from the data. Furthermore a new method based on the interpolation of the local flow by radial basis functions is introduced.

The analyzed systems include a numerical simulation of a Duffing oscillator as well as a data set of experimentally obtained acoustic cavitation noise.

R. Johnson:

Rotation number for higher-dimensional Schrödinger operators

We introduce a notion of oscillation for the Schrödinger operator

$$h = -\Delta + q(x), \quad x \in D \subset \mathbb{R}^n$$

when the dimension n is odd. Here D is a bounded domain with C^∞ smooth boundary, and h is defined using Dirichlet boundary conditions. It is further assumed that h has simple spectrum $\{e_i \mid i \geq 1\}$ with corresponding normalized eigenfunctions $\{\psi_i \mid i \geq 1\}$. We define a map

$$\xi: D\times(-\infty,\infty) \to \mathbb{P}^n : \xi(x,e) = \left[g(x,e), \frac{\partial g}{\partial x_1}(x,e), \ldots, \frac{\partial g}{\partial x_n}(x,e) \right]$$

where $\mathbb{P}^n$ = projective space of lines in $\mathbb{R}^{n+1}$ and g is the Green's function of h. Precisely, choose $y_0 \in D$ so that $\psi_i(y_0) \neq 0$ $(i \geq 1)$, and define $g(x,e) = (h-e)^{-1}(x,y_0)$.
It turns out that ξ can be smoothly interpolated through an eigenvalue e_i via $\xi(x,e_i) = [\psi_i(x), \nabla\psi_i(x)]$. Let ω be the volume form on $\mathbb{P}^n$, normalized so $\int_{\mathbb{P}^n} \omega = \omega_n/2$, ω_n = volume of n-sphere. Define the oscillation

$$\sigma(e) = \int_\Sigma \xi^*\omega, \quad \Sigma = (\partial D \times [e_0,e]) \cup D\times\{e\}$$

where $e_0 < e_1$ is fixed. We show that

$$(1) \qquad \lim_{|D|\to\infty} \sigma(e) = \omega_n \kappa(e)$$

where $\kappa(e)$ is the integrated density of states and D ranges over a sequence of domains increasing to $\mathbb{R}^n$. Thus we obtain a relation between the rotation number (the left hand side of (1)) and the density of states analogous to the relation which holds in dimension $n = 1$.

G. Keller:

Lyapunov exponents for interval maps

For unimodal maps $T: [0,1]\circlearrowleft$ with negative Schwarzian derivative as e.g. $Tx = ax(1-x)$

$(0 < a \le 4)$ we study the Lyapunov exponent $\lambda(x) := \lim_{n\to\infty} \frac{1}{n} \log|(T^n)'(x)|$. $\overline{\lambda}(x)$ and $\underline{\lambda}(x)$ denote the limsup and liminf, respectively. The main result is:

Thm. (to appear in Ergod. Th. & Dynam. Syst.):

If the set $\{x: \overline{\lambda}(x) > 0\}$ has positive Legesgue-measure, then T has a unique absolutely continuous invariant probability measure μ and $h_\mu(T) > 0$.

Cor.: For each T as above there is $\lambda_T \in \mathbb{R}$ such that $\overline{\lambda}(x) = \lambda_T$ for Lebesgue-a.a. x.

Furthermore: $\lambda_T < 0$ iff T has a stable periodic orbit

$\lambda_T > 0$ iff T has an invariant measure as above.

In the case $\lambda_T = 0$ a great variety of different types of asymptotic behaviours is possible. In CMPh (127) F. Hofbauer and the author constructed examples of maps $Tx = ax(1-x)$ with $\lambda_T = 0$ and various additional properties:

Let $\varpi_T(\delta_x) := \left\{ \text{weak accum. points of } \left(\frac{1}{n} \sum_{k=1}^{n} \delta_{T^k x} \right)_{n>0} \right\}$. Then

- $\exists\, T : \varpi_T(\delta_x) = \{\delta_z\}$ where $Tz = z$, $|T'(z)| > 1.9$
- $\forall\, 0 \le h_1 < h_2 < \log \frac{1+\sqrt{5}}{2}\ \exists\, T : \{h_\nu(T) : \nu \in \varpi_T(\delta_x)\} = [h_1, h_2]$ for Lebesgue-a.e. x.

Y. Kifer:

Remarks on large deviations for random transformations

Employing a general theorem on large deviations together with results and methods from Walter's paper on uniqueness of equilibrium states for some mappings which expand distances I derive full and relativized large deviations bounds for occupational measures of iterations of random expanding maps which involve full and relative entropies similarly to large deviations for dynamical systems. Relativized large deviations mean that bounds hold for almost all ω i.e. pathwise.

W. Kliemann

The Lyapunov spectrum of control systems

Many problems in control theory, like stabilization, the existence of suitable feedback controls, robustness concepts..., can be formulated using Lyapunov exponents, and by considering control systems as dynamical systems. For a control system of the form $\dot{x} = X_0(x) + \Sigma u_i X_i(x)$

on a smooth manifold, where $X_0,\dots,X_m$ are smooth vector fields and $u = (u_i) \in \mathfrak{U} = \{u: \mathbb{R} \to U, \text{integrable}\}$, $U \subset \mathbb{R}^m$ compact and convex, we formulate the corresponding control flow $\Phi_t: \mathfrak{U}\times M \to \mathfrak{U}\times M$, $\Phi = (\Theta,\varphi)$, with Θ the shift on $\mathfrak{U}$, φ the solution of the control equation. For this flow, and the associated linearized flow $T\Phi$ on $\mathfrak{U}\times TM$ and the projected flow $\mathbb{P}\Phi$ on $\mathfrak{U}\times\mathbb{P}M$, we define the concepts of control sets and chain control sets and show that these sets correspond to the maximal topologically mixing and chain recurrent components of the flows. The chain control sets lead to a subbundle decomposition of $\mathfrak{U}\times TM$, which is suitable for the analysis of ergodic properties of Lyapunov exponents and the characterization of the spectral intervals.

V.B. Kolmanovskii:

Periodic solutions of stochastic functional differential equations

Various problems related to the theory of the periodic solutions of stochastic equations with delay are considered. First the problem is stated and basic notions and definitions of the theory are given. Further general conditions for the existence of periodic solutions are established. The relationship between stability and periodicity is considered and the possibility of using the second method of Lyapunov for the proof of conditions for the existence of periodic solutions is established. Conditions for the existence of periodic solutions of concrete systems are studied and formulated immediately in terms of the coefficients of the considered equations. Periodic solutions of Ito stochastic functional differential equations are investigated.

O. Knill:

Lyapunov exponents of $Sl(2,\mathbb{R})$-cocycles

We prove, that if T is an aperiodic automorphism of a probability space (X,m), the set $\left\{A\in L^\infty(X,Sl(2,\mathbb{R})) \mid \lim_{n\to\infty} n^{-1} \log \|A(T^{n-1}(x))\dots A(x)\| > 0 \text{ a.e.}\right\}$ is dense in $L^\infty(X,Sl(2,\mathbb{R}))$.

Y. Latushkin:

Lyapunov exponents and weighted composition operators

Let T be a homeomorphism of a metric compact X. p – a Borel quasi-invariant measure on X, $a: X \to L(H)$ – continuous, $L(H)$ – an algebra of linear bounded operators, acting in Hilbert space H, $A(x,n) = a(T^{n-1}x)\cdot\ldots\cdot a(x)$ – a cocycle, $(S_a f)(x) = (dpT^{-1} / dp)^{1/2}\, a(T^{-1}x)f(T^{-1}x)$

be an acting in $L_2(X,p; H)$ *weighted composition operator*. 1) The Sacker-Sell spectrum Σ connects with the operator spectrum $\sigma(S_a)$ by the rule: $\Sigma = \{\ln|\zeta| : \zeta \in \sigma(S_a)\}$, $S = S(\hat{\alpha})$, $\hat{\alpha}: X\times H \to X\times H: (x,v) \to (\alpha x, a(x)v)$. The Riesz projectors of S_a are the operators of the multiplication on continuous operator-functions, whose images define the spectral decomposition $X\times H$. Let now $a(x), x \in X$, be compact operators, Erg – the set of T-ergodic invariant measures. 2) The spectrum Σ consists of the finite or countable numbers of the closed intervals $[\tau_\kappa^-, \tau_\kappa^+]$, $\kappa = 1,\dots, (< N+1)$, $N \le \infty$. It may be calculated over the Lyapunov exponents by $\tau_\kappa^- = \inf\{\lambda_\nu^j : \lambda_\nu^j \in [\tau_\kappa^-, \tau_\kappa^+], \nu \in \mathrm{Erg}\}$, $\tau_\kappa^+ = \sup\{\lambda_\nu^j : \lambda_\nu^j \in [\tau_\kappa^-, \tau_\kappa^+], \nu \in \mathrm{Erg}\}$. 3) The spectral radius of weighted composition operator over the endomorphism are calculated through entropy and Lyapunov exponents.

Y. Le Jan:

Asymptotic curvature for isotropic Brownian flows

The law of a measure preserving isotropic Brownian flow in $\mathbb{R}^d$ is completely defined by a spectral measure $F(d\rho)$ appearing in the representation of its covariance function.

Lyapunov exponents and the associated fluctuation constants can be explicitly computed from the dimension d and the 2^d moment of F.

The asymptotic law of the curvature induced by the flow can also be obtained from the 4th moment of F. If we reverse time this quantity converges a.s. towards what should be interpreted as the curvature of the unstable manifold the law of which is hence explicitly determined. Its tail depends only on the dimension d and one may wonder whether this result extends to a rather general class of flows.

Note this result does not really require the measure preserving property. The associated longitudinal Laplacian should then be self adjoint with respect to the random measure μ_ω obtained from the Lebesgue measure at infinity by the inverse flow.

F. Ledrappier, L-S. Young:

Stability of Lyapunov exponents

We consider small random perturbations of matrix cocycles over homeomorphisms of compact metric spaces. Lyapunov exponents are shown to be stable provided that our perturbations satisfy certain regularity conditions. In particular we require that the stationary measure is the same in the perturbed system as in the unperturbed system. These results are applicable to dynamical systems and stochastic flows.

A. Leizarowitz:

Eigenvalue representations for the Lyapunov exponents of certain Markov processes

An approach to the study of Lyapunov exponents of certain Markov processes is developed. Its utility is demonstrated by an application to two-dimensional random evolution equations, which enables the representation of p-moment Lyapunov exponents as eigenvalues of certain first order operators on the unit sphere.

We consider the relation between the Lyapunov exponent and the p-moment Lyapunov exponents, which was first observed and studied by Arnold [1]. We use this relation to represent the Lyapunov exponent as an additive eigenvalue of a certain operator on the unit sphere. This is given a simple geometrical meaning for a subclass of equations which, in particular, contains the random evolution harmonic oscillator.

X. Mao:

Lyapunov functions and Lyapunov exponents

Although there are a lot of papers on Lyapunov exponents for stochastic differential systems, there is few papers using Lyapunov functions for studying almost sure exponential stability for stochastic systems and this talk closes the gap. In this paper we first use Lyapunov functions to investigate the almost sure exponential stability for stochastic differential equation with respect to semimartingale.The result is then extended to more general Kunita's equation so that we can use Lyapunov functions to study the bound for stochastic flows of homomorphism.

M. Misiurewicz, C. Simó:

Why do we see repelling periodic orbits?

Suppose that we have a map of an interval into itself, for which there exists a probabilistic invariant measure, absolutely continuous with respect to the Lebesgue measure. When we plot a large piece of a generic trajectory then usually we see that there is considerably less points of this trajectory close to repelling periodic orbits of small periods. This cannot be explained by looking at the density of the invariant measure, since this density is not smaller there than at other points. Careful examination of the simplest model: the map $x \to 2x \pmod{1}$, shows that in this case the probability that there will be no points plotted in a neighbourhood of the fixed point of a given size, is much higher than for any other point. Other considerations show that the size of the expected "window" grows as the exponent at the fixed point decreases.

S. E.A. Mohammed, M. Scheutzow:

Lyapunov exponents of linear stochastic functional differential equations

We consider a class of linear stochastic f.d.e.'s driven by jump semimartingales Y:

$$dx(t) = dY(t)\int_{-r}^{0} \mu(t)(ds)x(t+s) \quad t \geq 0. \qquad (*)$$

In the above system, $\mu(t)$ is a measure-valued stationary ergodic process describing the memory of the system. The driving noise Y is matrix-valued and has stationary (possibly ergodic) increments. A classification of systems (*) is introduced: The *singular* ones which do not admit continuous linear stochastic (semi-) flows on the Hilbert space $M_2 := \mathbb{R}^n \times \mathbb{L}^2([-r,0),\mathbb{R}^n)$, and the *regular* ones that do. Within (*) we define a large class of regular systems whose stochastic flows are a.s. compact linear on M_2 for times t greater than the delay r. Using this result and Ruelle-Oseledec ergodic theorem in Hilbert space, we show that for the regular class of systems the Lyapunov exponents

$$\lim_{t\to\infty} \frac{1}{t} \log\|(x(t), x_t)\|_{M_2}, \quad x_t(s) := x(t+s), \quad -r < s < 0$$

forms a discrete non-random spectrum $-\infty \leq \ldots < \lambda_{i+1} < \lambda_i < \ldots < \lambda_1 < \infty$.
An exponential dichotomy for the stochastic flow is given in the hyperbolic case $\lambda_i \neq 0 \;\forall\; i \geq 1$. We also give estimates on λ_1 for some one-dimensional examples.

N.S. Namachchivaya:

Stochastic approach to small disturbance stability in power systems

This work examines the almost-sure asymptotic stability of coupled synchronous machines encountered in electrical power systems, under the effect of fluctuations in the interconnection system due to varying network conditions. A linearized multimachine model is assumed with one of the machines having negligible damping weakly coupled to the other machines with positive damping. Furthermore, the fluctuations are assumed to contain both harmonically varying and stochastically varying components. For small intensity excitations, the physical processes are approximated by a diffusive Markov process defined by a set of Ito equations. Results pertaining to the asymptotic almost-sure stability are derived using the maximal Lyapunov exponent obtainet for the Ito equations. Assumptions made for the modeling and analysis are consistent with possible operating conditions in an electrical power system.

K. Palmer:

The numerical computation of orbits of dynamical systems

A computed orbit of a chaotic dynamical system will typically diverge very quickly from the true orbit with the same initial condition. However, Hammel, Yorke and Grebogi have given a procedure which enables one to find a true orbit with slightly different initial condition which stays near the computed orbit for a large number of iterates. Shui-Nee Chow and I use the ideas of the shadowing lemma to give a different approach to this problem. We reduce the problem to the choice of an appropriate right inverse for a finite dimensional linear operator. Then we use the hyperbolicity (which need not be uniform) of the dynamical system to find a "good" (i.e. of small norm) right inverse.

Y. Peres, R. Kenyon:

Computing Hausdorff dimension via Lyapunov exponents

Motivated by studying tilted versions of the Minkowski measure and their Hausdorff dimension, the following is proved:
Theorem: Let $\lambda(p)$ be the top Lyapunov exponent for an independent random product of two 2×2 positive matrices A,B, where each element of the product is A with probability p, B with probability (1−p). Then $\lambda(p)$ is a *real-analytic* function of p which, moreover, may be continued analytically to the domain $\{z \in \mathbb{C}: |z| < 1, |1-z| < 1\}$.

This is extendable to any finite collection of matrices, with independence replaced by finite memory markov dependence.

In a different direction, extending a result of Hawkes, we show that if Λ_1, Λ_2 are Cantor sets on the circle $[0,1)$ defined by admissible digits in base b, then for a.e. $t \in [0,1)$ with respect to Lebesgue measure,

$$\dim[(\Lambda_1 + t) \cap \Lambda_2] = \frac{\lambda}{\log b}$$

with λ the top Lyapunov exponent for b easily specified 2×2 integer nonnegative matrices, taken independent with equal probabilities.

M. A. Pinsky:

Lyapunov exponents for linear systems with white noise / real noise

We consider the linear SDE $\dot{X} = AX + \varepsilon BXF(\xi_t)$ $(\varepsilon > 0)$, $X \in \mathbb{R}^d$ where the noise term may be a centered function of an ergodic reversible Markov process on a compact space *or* a white noise with mean zero, variance $\sigma^2 dt$. In case A is skew symmetric, it is shown that the small noise Lyapunov exponent $\lambda(\varepsilon)$ of the real-noise driven system is strictly less than the small-noise Lyapunov exponent of the *corresponding* white noise driven system with $\sigma^2 = -2(G^{-1}F,F)$, G = generator of real-noise process ξ_t, inner product w.r.t. invariant measure of (ξ_t). In case A is nilpotent, we obtain an expansion of $\lambda(\varepsilon)$ in fractional powers of the small parameter ε, when the leading coefficient is the same for both white noise or real noise. These results are applied to the damped oscillator equation $\ddot{y} + 2\beta\dot{y} + (k+\varepsilon F(\xi_t))y = 0$ $(\beta > 0, k > 0)$.

C. Pugh:

Critical sets

Given a C^∞ proper function $f: \mathbb{R}^m \to \mathbb{R}$, its critical set is $cp(f) = \{x \in \mathbb{R}^m : (Df)_x = 0\}$. Which compact sets $C \subset \mathbb{R}^m$ are $cp(f)$ for such an f? Equivalently, which compact subsets of $\mathbb{R}^m$ are chain recurrent sets for smooth flows on $\mathbb{R}^m \cup \infty = S^m$, having a sink at ∞? (Conley-Wilson Theorem) Answers - m = 1 - "Yes" for every compact $C \subset \mathbb{R}$. m =2 (joint with Alec Norton) "Yes" iff $\mathbb{R}^2 \setminus C$ has no simply connected components. m = 3 (joint with Matt Grayson) "Yes" for any cellular set or finite disjoint union of cellular sets. "Yes" for the Hopf link, unlink, or link of ≥ 3 components. "Yes" for Antoine's Necklace. "No" for all single knots in $\mathbb{R}^3$, "no" for all links of two components except Hopf & unlink (Corollary no smooth flow on $\mathbb{R}^3 \cup \infty = S^3$ with a sink at ∞ has chain recurrent set consisting of two closed orbits, one of which is knotted. No non-degeneracy assumptions are made.)

K.-U. Schaumlöffel:

A multiplicative ergodic theorem with applications to hyperbolic stochastic partial differential equations

We present a version of an infinite-dimensional MET which applies to cocycles in a Hilbert space X consisting of invertible operators. If, asymptotically, the cocycle has a dominating compact part, this theorem guarantees the existence of Lyapunov exponents and an associated decomposition of X. This theorem applies, e.g., to first order SPDE of the form

$$dx(t,r) = (b^0(r)\cdot\nabla x(t,r) + c^0(r)x(t,r))dt + \sum_{i=1}^{m} (b^i(r)\cdot\nabla x(t,r) + c^i(r)x(t,r)) \circ dw^i.$$

Finally, the 'asymptotic compactness' condition for this type of equations is discussed.

L. Stoyanov:

Perturbations of smooth surfaces and geodesic flows with (some) non-zero rotation numbers

Given a smooth compact (m-1)-dimensional submanifold M of $\mathbb{R}^m$, consider the geodesic flow on T*M with respect to the standard Riemannian metric g on M, inherited from $\mathbb{R}^m$. A closed geodesic c on T*M is called *non-degenerate* if the spectrum of the Poincaré map P_c related to c does not contain roots of 1. We prove that given a closed geodesic c on M there exists an arbitrarily close to id perturbation $f \in \mathcal{C}^\infty(M,\mathbb{R}^m)$ so that f = id along $\gamma = \pi_0 c$, supp f is contained in a small neighbourhood of a point $\gamma(t_0)$, γ is a closed geodesic on M' = f(M) and γ is non-degenerate with respect ot the standard metric on M. There exists a residual subset $\mathcal{R}$ of $\mathcal{C}^\infty_{einb}(M,\mathbb{R}^m)$ so that for $f \in \mathcal{R}$ every closed geodesic on M' = f(M) is non-degenerate. Similar results hold, considering the space of all smooth Riemannian metrics on M instead of $\mathcal{C}^\infty_{einb}(M,\mathbb{R}^m)$ (R. Abraham 1968, W. Klingenberg & F. Takens 1972, Anosov 1982). If m = 3 and c is an elliptic closed geodesic on T*M, there exists a small perturbation of M along $\pi_0 c = \gamma$, so that γ satisfies the assumptions of the Birkhoff-Lewis Fixed Point Theorem and therefore the closure of the periodic orbits in a neighbourhood of γ on M has positive measure.

P. Thieullen:

Pesin's formula for the α-entropy

Let M be a compact manifold of dimension d, $\phi: M \to M$ a $\mathcal{C}^2$-transformation and m a Lebesgue measure preserved by ϕ. In 1983 Brin and Katok found an equivalent definition of the usual entropy of ϕ.
If d(x,y) denotes the usual geodesic distance on M, for every n they define

$$d_n(x,y) = \max_{0\le i\le n-1} d(\phi^i(x),\phi^i(y))$$

and they proved the following theorem

$$h_m(x,\phi) = \lim_{\varepsilon\to 0} \limsup_{n\to+\infty} -\frac{1}{n-\log\varepsilon} \log m[B_n(x,\varepsilon)].\quad h_m(\phi) = \int h_m(x,\phi)m(dx).$$

When we permute these two limits we obtain the usual fractal dimension

$$\dim_F(x,\phi) = \lim_{n\to+\infty} \limsup_{\varepsilon\to 0} \frac{-1}{n-\log\varepsilon} \log m[B_n(x,\varepsilon)] = \limsup_{\varepsilon\to 0} \frac{\log m[B_n(x,\varepsilon)]}{\log\varepsilon}$$

In that particular case that notion of dimension is too crude since it is equal to d. The idea is to mix these two definitions into one. Define the new metric

$$d_n(x,y) = \max_{0 \le i \le n-1} d(\phi^i(x),\phi^i(y))e^{i\alpha} \text{ where } \alpha \text{ is a parameter} > 0,$$

and $$h_m^\alpha(x,\phi) = \lim_{\varepsilon\to 0} \limsup_{n\to+\infty} -\frac{1}{n} \log m[B_n^\alpha(x,\varepsilon)]$$

a local definition of entropy with a parameter α. Then we can generalize Pesin's formula (77):

$$h_m^\alpha(x,\phi) = \sum_{i=1}^{d} (\lambda_i(x) + \alpha)^+$$

where $\lambda_1(x) \ge \dots \ge \lambda_d(x)$ is the sequence of Lyapunov exponents. One of the interests of that formula is that it enables us to compute the whole sequence of Lyapunov exponents without knowing the tangent cocycle.

C.E. Wayne, J.-P. Eckmann:

Lyapunov exponents for infinite dimensional dynamical systems

Consider the random symplectic matrices

$$S(\omega) = \begin{pmatrix} \mathbb{1} & \Omega \\ \mathbb{1} & \mathbb{1}+\Omega \end{pmatrix},$$

where Ω is a random matrix with elements

$$\Omega_{\underline{i}\underline{j}} = \begin{cases} \sum_{|i-i'|=1} \omega_{\underline{i}\underline{i}'} & \text{if } |\underline{i}-\underline{j}| = 0 \\ \omega_{\underline{i}\underline{j}} & \text{if } |\underline{i}-\underline{j}| = 1 \\ 0 & \text{otherwise} \end{cases}.$$

The $\omega_{\underline{i}\underline{j}}$ are independent, identically distributed random variables with $\omega_{\underline{i}\underline{j}} > 0$. $\underline{i}$ and $\underline{j}$ are elements of some hypercube in $\mathbb{Z}^d$. Let $\sigma^2 = \mathrm{var}(\omega_{\underline{i}\underline{j}})$ and $\varpi = \mathrm{mean}(\omega_{\underline{i}\underline{j}})$. We are interested in the limit of the largest Lyapunov exponent as the number of points, N_1 in the hypercube goes to ∞. Denote this exponent by $\lambda_1(N)$. We prove

Theorem: If $d \ge 3$, and σ/ϖ is sufficiently small

$$\lim_{N\to\infty} \lambda_1(N) = \log\left(1+2\varpi d + 2\sqrt{\varpi d+\varpi^2 d^2}\right).$$

For any d,σ and ϖ, $\lambda_1(N) \le \log\left(1+2\varpi d + 2\sqrt{\varpi d+\varpi^2 d^2}\right)$.

The proof uses a representation of the Lyapunov exponent as a directed polymer in a random environment and then applies methods developed by Imbrie and Spencer to treat these polymer problems.

W. Wedig:

Invariant measures and Lyapunov exponents of limit cycles and stochastic systems

The paper emphasizes numerical methods in the stability analysis of dynamical systems. They are applied to calculate periodic orbits, invariant measures and associated Lyapunov exponents. To illustrate this basic concept, three typical examples are considered:

a) Linear oscillator with multiplicative white noise. Introducing polar coordinates, the Fokker-Planck equation of the decoupled angle can be solved like a dynamic system by means of a backward difference scheme.
b) Mathieu equation with harmonic coefficients. The angles of the parametric excitation and of the polar coordinate transformation define a typical limit cycle equation and its invariant measure. Both are evaluated to determine associated Lyapunov exponents.
c) Van der Pol equation with periodic orbits. To obtain an invariant measure, the geometrical stability concept of Poincaré is introduced. It allows to calculate associated orbital Lyapunov exponents.

V. Wihstutz:

Boundedness of Lyapunov exponents for linear stochastic systems with large diffusion term

The top Lyapunov exponent of a linear system perturbed by white (or real) noise with one or several independent noise sources is considered, i.e. the top Lyapunov exponent of

$$dx = A_0 x dt + \sigma \sum_{i=1}^{m} A_i x \circ dW^i,$$

where $A_0, A_1, \ldots, A_m$ are constant $d \times d$-matrices, $W^1, \ldots, W^m$ independent Wiener processes and σ a large real parameter.

For 2×2 systems boundedness versus unboundedness of the top Lyapunov exponent λ_σ, $\sigma \to \infty$, is systematically investigated. Essentially, boundedness holds only for skew-symmetric matrices $A_1, \ldots, A_m$.

Stabilizability of the control system $\dot{x} = Ax + Bu$ is investigated in case of rank $B = r < d$, using large white noise feedback control $u = \sigma \sum_{i=1}^{m} BC_i x \circ W^i$ by computing $\lim_{\sigma \to \infty} \lambda_\sigma = \lambda_\infty$.

M.P. Wojtkowski:

Systems at interacting (classical) particles with nonvanishing Lyapunov exponents

Consider a system of n point particles in the half line $q \geq 0$ with masses $m_1,\dots,m_n$. They collide elastically with each other and the bottom particle collides with the floor $q = 0$. They are all under the influence of an external potential field with the potential $V(q)$ such that $V'(q) > 0$ and $V''(q) < 0$. This system has all Lyapunov exponents different from zero if $m_1 \geq \dots \geq m_n$ and not all masses are equal. This is established by the introduction of a suitable sector bundle in the phase space. A sector C between two transversal Lagrangian subspaces L_1, L_2 of a linear symplectic space (W,ω) is defined as

$C = C(L_1,L_2) = \{w \in W \mid w = v_1 + v_2,\ v_i \in L_i,\ i = 1,2,\ \omega(v_1,v_2) \geq 0\}$.

For the model of falling particles $L_1 = \{dp_i = 0\}$, $L_2 = \{dh_i = 0\}$, where p_i's are momenta and $h_i = \frac{p_i^2}{2m_i} + m_i V$ individual energies of the particles. The same method can be applied to gas at hard balls where $L_1 = \{dp_i = 0\}$ and $L_2 = \{dq_i = 0\}$. But for technical reasons the nonvanishing of only some exponents can be established rigorously in the case of arbitrary number of particles.

L.S. Young:

Statistical properties of certain one-dimensional maps with positive Lyapunov exponents

We consider maps $f_a: [-1,1]\circlearrowleft$ defined by $f_a(x) = 1 - ax^2$ for $a \in [0,2]$. Conditions are given that guarantee 1) the existence of a unique invariant probability measure μ that is absolutely continuous w.r.t. Lebesgue (this implies positive Lyapunov exponents); 2) stability of μ under small random perturbations; and 3) exponential decay of correlations for certain classes of functions. These conditions are satisfied on a positive measure set of parameters.

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