MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 1/1991

Automorphe Formen und Anwendungen

06.01. bis 12.01.1991

The meeting was organized by S. Kudla (College Park, Maryland) and J. Schwermer (Eichstätt). The program of 21 lectures emphasized new developments in the theory of automorphic forms, particularly those involving interactions with number theory, representation theory and topology. The topics included:

1. The construction of motives and l-adic representations attached to automorphic forms, and the existence of Galois conjugates of Maass wave forms.
2. Trace formulas, both topological and relative and their applications to the arithmetic of automorphic forms and to a characterization of distinguished automorphic forms.
3. Properties of Eisenstein series and L–functions, in particular:
   - integral representations of L–series and the relation of local Euler factors to reducibility of principal series
   - Fourier and Fourier–Jacobi coefficients of Eisenstein series
4. Aspects of the Langlands classification of representations of real and p–adic groups.
5. Recent developments in the cohomology theory of arithmetic groups, in particular:
   - calculation of the intersection numbers of special cycles and consequences for the existence of certain types of automorphic forms,
   - A proof of Borel's conjecture on the representability of cohomology of arithmetic groups by automorphic forms.

The variety of these topics indicates the vigorous activity and diversity of current research in automorphic forms, and stimulated much fruitful discussion.

Vortragsauszüge

D. BLASIUS:

Galois conjugates of automorphic forms

Let F be a totally real field. Let π be a cuspidal automorphic representation of $GL_2(\mathbb{A}_F)$ which is arithmetic, i.e. such that, for $v|\infty$, the Langlands parameter $\sigma_v(\pi): W_{\mathbb{R}} \to GL_2(\mathbb{C})$ defined by π_v satisfies

$$\sigma_v(\pi)|_{\mathbb{C}^*}(z) = Diag(z^a\bar{z}^b, z^c\bar{z}^d)$$

with $a,b,c,d \in \mathbb{Z}$. Let $\tau \in Aut(\mathbb{C})$, and let π_f^τ be the formal conjugate of the finite part π_f of π. We prove, with M. Harris and D. Ramakrishnan, the
Theorem. There exists $\pi^{(\tau)}$, a cuspidal automorphic representation of $GL_2(\mathbb{A}_f)$, such that

a) $\pi_v^{(\tau)} \xrightarrow{\sim} \pi_v^\tau$ for all finite, unramified v

b) $\sigma_v(\pi^{(\tau)})|_{\mathbb{C}^*} \xrightarrow{\sim} \sigma_{\tau v}(\pi)|_{\mathbb{C}^*}$ if $v|\infty$

c) the set of isomorphism classes of such $\pi^{(\tau)}$ $(\tau \in Aut(\mathbb{C}))$ is finite.

Let T_π be the field generated by the unramified Hecke eigenvalues of π. We deduce the
Corollary. a) $[T_\pi : \mathbb{Q}] < \infty$ b) T_π is contained in a CM field
c) π_f admits a model defined over a finite extension of $\mathbb{Q}$.

We briefly sketch the proof of this result, showing the role played by a recent result of Kudla, Rallis and Soudry about the degree 5 L-function of forms on $GSp(4)$.

A. KRIEG:

The Maaß space for Hermitian modular forms

Let K be an imaginary quadratic number field. The Maaß space for Hermitian modular forms of degree 2 and weight k with respect to K is defined in analogy with the Maaß Spezialschar. Using the Fourier Jacobi expansion the Maaß space proves to be isomorphic to a certain subspace of the space of elliptic modular forms

on $\Gamma_o(D_K)$ of weight $k-1$ and character χ_K, where $-D_K$ is the discrimimant of K and χ_K the Kronecker symbol of K. This subspace can be viewed as a generalization of Kohnen's "+"-space and can be described by the eigenspaces of certain Hecke operators.
The isomorphism is compatible with Hecke operators. Hence one can show that the Hermitian Eisenstein series (of Siegel type) always belongs to the Maaß space. Its Fourier coefficients can be calculated explicitly. Surprisingly they have a simpler form than those of the Siegel Eisenstein series.
The restriction of a modular form in the Maaß space to the Siegel half-space always belongs to the Spezialschar. For $K = \mathbb{Q}(i\sqrt{3})$ this restriction is an isomorphism.

F. SHAHIDI:

Twisted endoscopy and local period integrals

In this talk we discuss the irreducibility of representation of $Sp_{2n}(F), F$ a p-adic field, induced from irreducible super cuspidal representations of its Siegel parabolic. Consequently we relate this question to the theory of twisted endoscopy for GL_n as being developed by Kottwitz and Shelstad. Using their theory we show that the reducibility depends on vanishing of a certain period integral on $GL_n(F)$. (More precisely over the space $Z_n(F)Sp_n(F)\setminus GL_n(F)$ when n is even. For n odd and a self dual super cuspidal unitary representation of $GL_n(F)$ the unitarily induced representation is always reducible). Finally we give a formula for the local Langlands L-function attached to the representation and the exterior square L-function of $GL_n(\mathbb{C})$, the L-group of GL_n.

J. ROHLFS:

Intersection numbers of special cycles on arithmetic varieties

This is a report on joint work with J. Schwermer. Let $G/\mathbb{Q}$ be a connected semi simple algebraic group over $\mathbb{Q}$ and let $\sigma, \tau \in AutG(\mathbb{Q})$ be $\mathbb{Q}$-rational automorphisms of finite order which commute. Assume that $\Gamma \subset G(\mathbb{Q})$ is arithmetic, torsion free and σ,τ-stable. Let X be the space of maximal compact subgroups of $G(\mathbb{R}), G(\mathbb{R})$ non compact, and consider the special cycles $C(\sigma) = X^\sigma/\Gamma^\sigma$ and $C(\tau) = X^\tau/\Gamma^\tau$. Here the upper index indicates "fixed points". We assume that $C(\sigma), C(\tau)$ and X/Γ are oriented and that there is one point x_o where $C(\sigma)$ and $C(\tau)$ intersect transversally. If now X/Γ is compact the intersection number $[C(\sigma)][C(\tau)] \in \mathbb{Z}$ is defined. We assume that Γ is sufficiently small. Then it is shown that the contribution of a connected component F of $C(\sigma) \cap C(\tau)$ to the intersection number is up to sign $\chi(F)$, the Euler–Poincaré characteristic of F. If X/Γ has the structure of a complex variety and if σ and τ act analytically then $[C(\sigma)][C(\tau)] = L(\sigma, \Gamma^\tau)$ where $L(\sigma, \Gamma^\tau)$ denotes the Lefschetz number of σ acting on the cohomology of Γ^τ. It is known that $L(\sigma, \Gamma^\tau) > 0$ for Γ sufficiently small. If S is non compact the last formula also holds if $[C(\sigma)][C(\tau)]$ is interpreted correctly. Thus we generalize results of Millson and Raghunathan where $C(\sigma)$ and $C(\tau)$ have only isolated intersection points.

D. VOGAN:

Geometric Langlands parameters for real groups

Suppose G is a complex connnected reductive algebraic group endowed with an inner class of real forms. Let ${}^vG$ be the dual group, and ${}^vG^\Gamma \supset {}^vG$ the Galois form of the L–group. Fix an infinitesimal charater for G; this amounts to a semi simple ${}^vG$–orbit $\mathcal{O} \subset {}^v\mathfrak{g}$. We define a smooth algebraic variety $X(\mathcal{O}, {}^vG^\Gamma)$ with ${}^vG$ action, having the following properties:

(1) ${}^vG$ has finitely many orbits, parametrized by L–packets of infinitesimal character $\mathcal{O}$.

(2) $X(\ )$ is functorial for L–homomorphisms.

We show that the (perverse) geometry of the orbit closures corresponds nicely to irreducible character theory on real forms of G. (This is joint work with D. Barbasch and J. Adams).

J. FRANKE:

Some results on Eisenstein cohomology of arithmetic groups

Let $C^\infty_{umg}(\Gamma \setminus G)$ be the space of C^∞–functions of uniformly moderate growth and $\mathcal{A} \subset C^\infty_{umg}(\Gamma \setminus G)$ the space of automorphic forms (i.e., $Z(g)$–finite functions). It is shown that the first of the two maps

$$H^*_{(g,K)}(\mathcal{A} \otimes E) \to H^*_{(g,K)}(C^\infty_{umg} \otimes E) \overset{\text{Borel}}{\underset{\sim}{\longrightarrow}} H^*(\Gamma, E)$$

is an isomorphism, proving an old conjecture of Borel. The proof is an inductive argument which implies the vanishing of certain local cohomology groups of C^{∞}_{umg}. This can also be applied to coherent cohomology. We also construct a filtration on $\mathcal{A}$ whose quotients are given by Eisenstein series. From this filtration we get an Eisenstein spectral sequence converging to $H^*(\Gamma, E)$. If the highest weight of the G–representation E is regular, then the Eisenstein spectral sequence collapses and there is a formula for $H^*(\Gamma, E)$ in terms of cusp forms. – For GL_n, the proof of Borel's conjecture also implies the rationality of the decomposition into classes of associate parabolic subgroups $H^*(\Gamma, E) = \bigoplus_{\{P\}} H^*(\Gamma, E)_{\{P\}}$.

T. IKEDA:

On the Fourier–Jacobi coefficients of Eisenstein series

Let k be a global field and $\mathbb{A}$ be its adele ring. A Jacobi group D is a semi–direct product $V \rtimes H$. Here V is 2–step nilpotent and unipotent algebraic group defined over k and H acts on the center Z of V. A homomorphism $S : Z \to k$ is called non–degenerate if $V/KerS$ is a Heisenberg group with center $Z/KerS$. We fix a non–degenerate S.

Example: All of the following groups are subgroups of $Sp_{m+n} = G$.

$$V = \left\{ \left(\begin{array}{cc|cc} I_m & x & z & \frac{y}{2} \\ 0 & I_n & \frac{{}^t y}{2} & 0 \\ \hline & & I_m & 0 \\ & 0 & & \\ & & -{}^t x & I_n \end{array} \right) \mid x, y \in M_{m,n}(k), z - \frac{x\,{}^t y}{2} \in Sym_m(k) \right\}$$

$$Z = \left\{ \left(\begin{array}{c|cc} 1_{m+n} & z & 0 \\ & 0 & 0 \\ \hline 0_{m+n} & 1_{m+m} & \end{array} \right) \mid z \in Sym_m(k) \right\}$$

$$H = \left\{ \left(\begin{array}{cc|cc} 1_m & & & \\ & A & & B/2 \\ \hline & & 1_m & \\ & 2C & & D \end{array} \right) \mid \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp_n \right\} \simeq Sp_n$$

Any homomorphism $Z \to k$ is given by $z \mapsto tr(Sz); S \in Sym_m(k)$. S is non–degenerate if and only if $detS \neq 0$.

Let $\widetilde{H}(\mathbb{A})$ be the covering of $H(\mathbb{A})$ given by the fibre product of $H(\mathbb{A}) \to Sp_{v/z}(\mathbb{A})$ and $\widetilde{Sp_{v/z}}(\mathbb{A}) \to Sp_{v/z}(\mathbb{A})$. Put $\widetilde{D(\mathbb{A})} = V(\mathbb{A}) \times \widetilde{H(\mathbb{A})}$. Let ψ be a non–trivial additive character of $\mathbb{A}/k$. Let $C^{\infty}_S(D(k) \setminus \widetilde{D(A)})$ be the space of C^{∞}–functions on

$\widetilde{D(A)}$ such that $\varphi(\gamma g) = \varphi(g), \forall \gamma \in D(k), \varphi(zg) = \psi(S(z))\varphi(g), \forall z \in Z(A)$. Let π be any closed $\widetilde{D(A)}$–invariant subspace of $C_S^\infty(D(k)\backslash D(\mathbb{A}))$. Then π is generated by

$$\theta_1(vk) \int_{V(k)\backslash V(\mathbb{A})} \varphi(uk)\theta_2(uk)du, \ \varphi \in \pi \ , \ \theta_1, \theta_2 \text{ are some theta functions.}$$

When φ is the Fourier–Jacobi coefficient of Eisenstein series $E(g; f)$ of Siegel type on $G = Sp_{m+n}$, the integral is an Eisenstein series on $H(A)$. When m is odd, this is an Eisenstein series of half–integral weight.

M. HARRIS:

L–functions and periods of polarized regular motives

Let M be a polarized motive over $\mathbb{Q}$ of odd weight w. M is regular if the Hodge spaces $M^{p,q}$ are of dimension ≤ 1. Let K be an imaginary quadratic field and $\chi : K_{\mathbb{A}}^x / K^x \to \mathbb{C}^x$ an algebraic Hecke character with $\chi_\infty(z) = z^{-k}, k \in \mathbb{Z}$; let $M(\chi)$ be the corresponding rank 2 motive over $\mathbb{Q}$. We define Petersson inner product invariants $Q_i(M) =< w_i, F_\infty w_i >$, where $w_i \in M^{p_i, q_i}$ is arithmetic over $\mathbb{Q}, p_1 > p_2 > \cdots > p_n, n = \text{rank}(M)$. It can be shown that the Deligne period

$$C^+(M \otimes M(\chi)) = (\text{product of} CM \text{ periods}) \times \prod_{j=1}^{r'} Q_j(M), \text{ where } r' = min(r, n-r)$$

if $w - 2p_r + 1 < k < w - 2p_{r+1} - 1$.

Motives of the form $M \otimes M(\chi)$ as above are supposed to appear in the middle dimension cohomology, with twisted coefficients, of unitary Shimura varieties of signature $(1, n-1)$ (under certain local hypotheses). However, one can show that critical values of the corresponding automorphic L–functions are expressed in terms of Petersson inner products of arithmetic holomorphic forms on unitary Shimura varieties of signature $(r, n-r)$ with r as above. This reduces Deligne's conjecture for $M \otimes M(\chi)$ to period relations for automorphic forms on different unitary Shimura varieties. A program for proving these period relations by means of the theta correspondence was sketched. The main difficulty is to prove that certain L–functions don't vanish.

G. HENNIART:

An explicit Kazhdan–Langlands correspondence in the unramified case

Let F be a finite extension of $\mathbb{Q}_p$ and K an unramified extension of F, of degree n. To each regular character χ of K^x, D. Kazhdan has attached (LN 1241) a cuspidal representation $\pi(\chi)$ of $G = GL(n, F)$ which should correspond, by the (conjectural) Langlands correspondence to the representation $\sigma(\chi)$ of the Weil group of F induced from the character of the Weil group of K corresponding to χ. This yields a bijection between $Gal(K/F)$–orbits of χ's and cuspidal representations π of G satisfying $\pi \otimes (\varepsilon \circ det) = \pi$ for ε a character of $Gal(K/F)$. On the other hand Howe and Gérardin (PSPM 33) have given another map $\chi \mapsto \pi'(\chi)$ with the same properties, via induction from compact mod center subgroups. The talk showed how to achieve the comparison between those two parametrizations. It turns out that $\pi'(\chi) = \pi(\chi \cdot (-1)^{(n-1)val})$. The proof was sketched when n is prime: one has to prove that $\pi'(\chi)$ satisfies the same character identities as $\pi(\chi(-1)^{(n-1)val})$. But fortunately it is enough to verify them on elements of K^x where they are easy to get.

R. TAYLOR:

ℓ–adic representations for Siegel modular forms of low weight

If f is a Siegel modular form of weight $a_1 \geq a_2 \geq 3$ which is an eigenform of the Hecke algebra, Shimura, Deligne and Chai/Faltings associate to f an ℓ–adic representation, such that the Frobenius at a good prime p satisfies a polynomial given by the eigenvalues of the Hecke operators. Unfortunately neither the dimension nor the characteristic polynomials of Frobenius elements are known. We extend this result to weight $a_1 \geq a_2 \geq o$. This uses the theory of congruences between modular forms together with the idea of pseudo–representations. These are maps from a group G to a ring R which satisfy the polynomial identities one would expect of a trace. In char 0 one can recover a representation from a pseudo–representation. In special cases forms of weight $(a_1, 2)$ come from forms on GL_2. In these cases for general reasons we can establish control of the dimension of the representations we obtain, and calculate the characteristic polynomials of Frobenius.

U. EVERLING:

On some local intertwining integrals for SL_3 and $SU(3)$

The group $G = SL_{3|\mathbb{Q}_p}$ [or $SU(3)$, quasisplit over $\mathbb{Q}_p$, split over $\mathbb{Q}_{p^2}$] contains a copy of $H = PGL_2$, and a Borel subgroup B. A character $\varphi : B(\mathbb{Q}_p) \to \mathbb{C}^*$ varies with α, β [or γ] $\in \mathbb{C}^*$ when $\varphi_{|B(\mathbb{Z}_p)}$ is fixed, and $\chi : H(\mathbb{Q}_p) \to \{\pm 1\}$. We study the space of intertwining operators $Hom_{G(k)}(Ind_{B(k)}^{G(k)}\varphi,\ Ind_{H(k)}^{G(k)}\chi)$ $(k = \mathbb{Q}_p)$. Among the orbits $B(k)\xi H(k)$, ten [or three] are "thick", i.e. $H_\xi := \xi^{-1}B(k)\xi \cap H(k)$ is finite, and three [or one] "thin". Each thick orbit contributes a nonzero intertwining operator τ_ξ defined by $\tau_\xi(f)(g) = \int_{H_\xi \backslash H(k)} f(\xi h g)\chi(h)^{-1} dh$, provided $\forall h \in H_\xi : \varphi(\xi h \xi^{-1}) = \chi(h)$. On functions like the spherical function, these integrals depend on φ by rational functions of α, β [or γ] with at most three [or one] denominators. Each denominator belongs to a thin orbit. When a denominator vanishes, the thin orbit supports an additional intertwining operator, and some of the τ_ξ of thick orbits cease to exist. When the denominators do not vanish, the τ_ξ of thick orbits generate the whole space of intertwining operators.

D. SOUDRY:

Rankin–Selberg convolutions for $SO(2\ell+1) \times GL(n)$

Let π, τ be irreducible, automorphic, cuspidal representations of $SO_{2\ell+1}(\mathbb{A})$ and $GL_n(\mathbb{A})$. We consider integrals $\int_{SO_{2\ell+1}(k)\backslash SO_{2\ell+1}(\mathbb{A})} \varphi(g) E_{X,\psi}(\xi_{\tau,s}, g)dg$, when $\ell < n$ $\varphi \in \pi, E(\xi_{\tau,s}, g)$ Eisenstein series for $Ind^{SO_{n,n}(\mathbb{A})} \tau \otimes |det \cdot|^s$ (from Siegel parabolic) and $E_{X,\psi}$ is a Fourier coefficient with respect to a certain unipotent subgroup. $SO_{2\ell+1}$ is embedded in a natural way in $SO_{n,n}$. There is a "dual" construction when $\ell \geq n$: $\int_{SO_{n,n}(k)\backslash SO_{n,m}(\mathbb{A})} \varphi_{X,\psi}(h)E(\xi_{\tau,s}, h)dh$. We showed Euler product expansion, and the corresponding local theory. It gives the standard L-functions $L(\pi \times \tau, s)$. We introduce the corresponding local factors (L-factors and γ-factors). We prove their existence over a nonarchimedean place. Our main result up to now is that $\gamma(\pi \times \tau, s, \psi)$ is multiplicative (inductive) in π and in τ.

G. HARDER:

Topological trace formula for Hecke operators

Let $G/\mathbb{Q}$ be a reductive algebraic group, we choose a suitable subgroup $K_\infty K_f \subset G(\mathbb{A})$ and consider the locally symmetric space

$$S_K^G = G(\mathbb{Q}) \setminus G(\mathbb{A})/K_\infty K_f$$

Given a rational representation $\rho : G \times_{\mathbb{Q}} \bar{\mathbb{Q}} \to GL(\mathcal{M})$ we get a sheaf on the locally symmetric space and we may consider the cohomology $H^\cdot(S_K^G, \mathcal{M})$. These cohomology groups are modules for the Hecke algebra $\mathcal{H}(G(\mathbb{A}_f)//K_f)$ and we propose a formula for the trace of an element $h \in \mathcal{H}(G(\mathbb{A}_f)//K_f)$ on the cohomology, which should follow from the results of Bewersdorff in rank one and Goresky and MacPherson in the general case.

To get this formula we have to compactify $S_K^G \xrightarrow{i} S_K^{G,\wedge}$ which has a stratification labeled by the conjugacy classes of parabolic subgroups defined over $\mathbb{Q}$. Then the trace formula is

$$tr(h|H^\cdot(S_K^{G,\wedge}, i_*\mathcal{M})) = \text{elliptic terms coming from fixed points in the interior} +$$

$$\sum_P tr(h^{M,\mathrm{trunc}}|H^\cdot(S_{K^M}^M, H^\cdot(\mathfrak{u}, \mathcal{M}))) \cdot (-1)^{d(P)+1},$$

where the second term yields the contribution of the fixed points at infinity.

V. GRITSENKO:

Eisenstein series for $Sp(1,1)$

It was proved that the nonarchimedian part $b_m(s)$ (m is a three dimensional quaternion) of Fourier coefficients in ∞ of Eisenstein series for $Sp(1,1)$ (group over quaternion algebras over $\mathbb{Q}$) depend only on the norm $N(m)$ and the integral divisors of the quaternion m. In the case of the algebra with class number 1 the exact formulas were obtained. The integral representation of the L–function with local factor of degree 4 for $Sp(1,1)$ automorphic forms was written down. It gives us the analytical continuation of the L–function. (The kernel of this representation is an Eisenstein series for $SL_2(\mathbb{C})$. The generalization of that integral to $SO(1,n)$–groups was discussed.

U. WESELMANN:

Cohomological congruences between automorphic forms on GL_2

For a numberfield F we consider the spaces $S_{K_f} = GL_2(F) \setminus GL_2(\mathbb{A}_f)/K_\infty K_f$ and their cohomology $H^\cdot(S_{K_f}, \bar{\mathbb{Q}}) = \bigoplus_\pi H^\cdot(\hat{S}, \bar{\mathbb{Q}})(\pi)^{K_f}$, where π runs over indecomposable representations of $GL_2(\mathbb{A}_f)$. If R is a local ring with quotient field $\bar{\mathbb{Q}}$ then the lattice $H^\cdot(S_{K_f}, R)_{\text{free}}$ = image of $H^\cdot(S_{K_f}, R)$ in $H^\cdot(S_{K_f}, \bar{\mathbb{Q}})$ does not respect this decomposition. $H^\pi = H^\cdot(S_{K_f}, R)_{\text{free}} \cap H^\cdot(S_{K_f}, \bar{\mathbb{Q}})(\pi)$ lies inside H_π = projection of $H^\cdot(S_{K_f}, R)_{\text{free}}$ in $H^\cdot(S_{K_f}, \bar{\mathbb{Q}})(\pi)$. If $H^\pi \neq H_\pi$ we get congruences between π and some other π'. We vary K_P in $K_f = K^P \cdot K_P$ and want to construct nontrivial H_π/H^π in this way. The dependence of H^π from K_P can be computed in small degrees by group cohomological methods using the congruence subgroup property for S–arithmetic groups and for the Hilbert modular group. The theory of modular symbols shows that H^π is as small as possible if the algebraic part of some Dirichlet twisted L–value of π is a unit in R. H_π is determined by Poincaré duality respectively the knowledge of the cohomology of the boundary for the Eisenstein part. For totally real F one gets a lot of congruences in this way, while for general F we only obtain results in base change situations.

H. JACQUET:

Relative trace formulas

Let G be a reductive group, over a number field F and σ an automorphism of order 2. We let H be the fixator of σ and set $S = H \setminus G$. Assuming G quasi–split we let N be the unipotent radical of a Borel subgroup of G. We then consider a geometric trace formula of the form

$$\sum_\xi \int \Phi\,({}^t n \xi n)\, \Theta(n) dn$$

where Φ is a smooth function of compact support on $S(\mathbb{A})$ and Θ a generic character of $N(\mathbb{A})$ trivial on $N(F)$; the sum is over an appropriate subset of the orbits of $N(F)$ on $S(F)$. We try to match it with a trace formula:

$$\int K'(n_1'n_2')\,\Theta'(n_1')\,\Theta'(n_2')\,dn_1'dn_2'\,,\; K'(x,y) = \sum_{\xi} f'(x^{-1}\xi y)$$

where G' is another quasi-split group, N' a maximal unipotent subgroup of G', Θ' a generic character of N'. This trace formula allows us to describe the image of a functorial map from automorphic representations of G' to those of G.

Ch. DENINGER:

Local L-factors of motives and regularized determinants

Let $X/\mathbb{Q}$ be a smooth projective variety, $o \leq w \leq 2dimX$. Consider the L-function of its w-piece:

$$L(H^w(X),s) = \prod_{p\leq\infty} L_p(H^w(X),s) \quad \text{for } Re\,s > \frac{w}{2}+1.$$

Let $\Delta_p = \mathbb{L}_p[\theta_p]$ where $\mathbb{L}_p = \mathbb{C}[Y_p, Y_p^{-1}], Y_p = exp(\frac{2\pi i}{\log p}z)$ for $p < \infty\,, \mathbb{L}_\infty = \mathbb{C}[Y_\infty^{-1}], Y_\infty = exp(z), \theta_p = \frac{d}{dz}$ on $\mathbb{L}_p$. We construct additive functors for $p \leq \infty$:

$$\left\{\begin{array}{l}\text{pure absolute}\\ \text{Hodge motives } M/\mathbb{Q}_p\end{array}\right\} \longrightarrow \left\{\begin{array}{l}\Delta_p\text{–modules which}\\ \text{are free of finite rank } /\mathbb{L}_p\end{array}\right\}$$

$$M \longmapsto \mathcal{F}_p(M)$$

such that on good reduction motives $\mathcal{F}_p(M) = H^w(M/\mathbb{L}_p)$ where the right hand side is a cohohomlogy theory with Künneth–formula and Poincaré duality and the right Betti–numbers (over $\mathbb{L}_p$). We have:
<u>Thm:</u> Assume $X/\mathbb{Q}$, smooth projective, Fr_p acts semisimply on $H^w(X_{\bar{\mathbb{Q}}}, \mathbb{Q}_\ell)^{I_p}$ (e.g. $w = o, 1$). Then for $s \in \mathbb{C}$

$$L_p(H^w(X),s) = det_\infty\,(\frac{\log p}{2\pi i}(s-\theta_p)\,|\,\mathcal{F}_p)^{-1}$$

where $\mathcal{F}_p = \mathcal{F}_p(H^w(X)), det_\infty$ is a regularized $det, \log\infty = i$.

We then explained how the meromorphic continuation and functional equation of L would follow from the existence of a suitable Grothendieck site as in the function field case. This philosophy suggests the following result which is proved by explicit fomulas in number theory.

Thms: $\frac{1}{(2\pi)^2} s(s-1) 2^{-1/2} \pi^{-\frac{s}{2}} \Gamma(\frac{s}{2}) \zeta(s) = exp(-\frac{d}{dz}|_{z=0}\ a.c. \quad \sum_{\rho} \frac{1}{[\frac{1}{2\pi}(s-\rho)]^z})$
non-tri. zeros of ζ

T. ODA:

The cohomology groups of degree 3 of Siegel modular varieties of genus 2

Let $Sp(2:\mathbb{R})$ be the real symplectic group of rank 2, $Sp(2:\mathbb{Z})$ the integral symplectic group. For a natural number $\ell \geq 3$, let $\Gamma(\ell)$ be the principal congruence subgroup of $Sp(2:\mathbb{Z})$. The quotient $V = \Gamma(\ell) \setminus H$ of the Siegel upper half space H by $\Gamma(\ell)$, is a smooth algebraic variety over $\mathbb{C}$. Let $\tilde{V}$ be a toroidal smooth compactification of V.

Put $D = \tilde{V} - V = \bigcup_i D_i$, each irreducible component being smooth. The Poincaré dual of the restriction map $\varrho_{D_i} : H^1(D_i, \mathbb{Q})(-1) \to H^3(\tilde{V}, \mathbb{Q})$ has an image which is a sub Hodge structure of type $\{(2,1)+(1,2)\}$ in $H^3(\tilde{V}, \mathbb{Q})$. We define a sub Hodge structure $H^3(M_\infty, \mathbb{Q})$ of $H^3(\tilde{V}, \mathbb{Q})$ by

$$H^3(M_\infty, \mathbb{Q}) := \sum_i Im\, \rho_{D_i} \quad \text{in } H^3(\tilde{V}, \mathbb{Q}).$$

Put $H^3_!(V, \mathbb{Q}) = Im(H^3_c(V, \mathbb{Q}) \to H^3(V, \mathbb{Q}))$. Then we have a decomposition

$$H^3(\tilde{V}, \mathbb{Q}) = H^3(M_\infty, \mathbb{Q}) \oplus H^3_!(V, \mathbb{Q})$$

of polarized Hodge structures.

Satake classified the modular embedding $W \to V$. By his result, W is a Hilbert modular surface or a product of elliptic modular surfaces. Let $\tilde{W} \to \tilde{V}$ be the corresponding smooth compactification and regular maps.

Theorem. We put $F'^1_{\text{mod}} H^3(\tilde{V}, \mathbb{Q}) = \sum_{\substack{W \to V \\ \text{modular}}} Im(H^1(\tilde{W}, \mathbb{Q})(-1) \longrightarrow H^3(\tilde{V}, \mathbb{Q}))$.
Then $F'^1_{\text{mod}} H^3(\tilde{V}, \mathbb{Q}) \subset H^3(M_\infty, \mathbb{Q})$.

The proof of Theorem uses the intersection theory, and reduce the problem to a result of Yamazaki (Amer. J. Math., 98, 1976).

R. PINK:

Application of Eisenstein cohomology to the construction of unramified field extensions

Let M_n be the module of homogenous polynomials in 2 variables with coefficients in $\mathbb{Z}[\frac{1}{6}]$. One has an exact sequence $(2|n > o)$

$$0 \to \underbrace{H^1_!(SL_2(\mathbb{Z}), M_n)}_{H^1_!} \to \underbrace{H^1(SL_2(\mathbb{Z}), M_n)}_{H^1} \to \underbrace{H^1(SL_2(\mathbb{Z}) \cap \begin{pmatrix} * & * \\ 0 & * \end{pmatrix}, M_n)}_{H^1_\partial} \to 0$$

A generator of the free part of H^1_∂ lifts to a Hecke-Eigenvector only after multiplying by a "denominator".
Theorem (Harder): This denominator is equal to the numerator of $\zeta(1-(n+2))$.
Let $\mathcal{H} \subset End(H^1 \otimes \mathbb{Z}_p)$ be the $\mathbb{Z}_p$–algebra generated by all Hecke operators, $\mathcal{H}_! \subset End(H^1_! \otimes \mathbb{Z}_p)$ analogous. Let $\mathcal{M} \subset \mathcal{H}$ be the maximal ideal generated by p and $T_\ell - 1 - \ell^{n+1}$ for all primes ℓ. Put $X := (H^1_! \otimes \mathbb{Z}_p)_\mathcal{M}$ (localization). If $p^\delta \|$ "denominator", then there is a Galois–equivariant embedding $\mathbb{Z}/p^\delta\mathbb{Z}(-n-1) \hookrightarrow X/p^\delta X$, the Galois–action coming from p–adic cohomology. Analyzing this Galois representation in detail, and using a result of crystalline cohomology, gives another proof of (Mazur–Wiles').
Theorem. There exists an unramified abelian extension of $\mathbb{Q}(\mu_{p^\delta})$ of order p^δ, on which $Gal(\mathbb{Q}(\mu_{p^\delta})/\mathbb{Q})$ acts by the (cyclotomic character)$^{-n-1}$.

This result extends to the cohomology of congruence subgroups, yielding unramified extensions of other cyclotomic fields.
Under suitable conditions on the algebra $(\mathcal{H}_!)_\mathcal{M}$ and its module X (order freeness, Gorenstein, or the like) one can construct a cyclic extension of the desired p^δ. This is related to Vandiver's conjecture.

B. SPEH:

Lefschetz numbers and trace formula

Let $G_o/\mathbb{Q}$ be a connected semisimple simply connected algebraic group, k a cyclic purely imaginary extension of $\mathbb{Q}$, α a generator of the Galois group, and $G = Res_{k/\mathbb{Q}} G_o$. We construct a function $f_\mathbb{A}$ with compact support in $G(\mathbb{A})$, so that the elliptic terms in the Arthur trace formula twisted by α can be interpreted as the Lefschetz number of α on the cohomology of a locally symmetric space and a local system corresponding to an α invariant representation of G.
This is joint work with J. Rohlfs.

S.J. PATTERSON:

Cuspidal biquadratic theta functions

Let k be a global field containing the 4^{th} roots of 1. By a theorem of Flicker one can associate with any Größencharacter χ of $k_{\mathbb{A}}^{\times}$ such that $\chi_w(-1) = 1$ for every place w of k an automorphic representation $V^o[\chi]$ of the four-fold cover of $GL_2(k_{\mathbb{A}})$. Likewise there exist corresponding local representations. If ψ is an arbitrary Größencharacter then $V^o[\chi\,\psi^4] \cong V^o[\chi] \otimes (\psi \circ \det)$. If χ is itself a fourth power then $V^o[\chi]$ is a quotient of a principal series representation (in the local case) and is a residue of an Eisenstein series (in the global case). Otherwise it is cuspidal. In the former case C. Eckhardt and the author developed a conjecture describing the Fourier coefficients of the automorphic form ("biquadratic theta function"). This was based on an analysis of the local spaces of Whittaker functionals. In this talk we discuss the analogous problems in the cuspical case. (In the local case, over a field F with odd residual characteristic C. Blondel has given an explicit realization of these representations). In particular we discuss the problem of the explicit computation of the Fourier coefficients numerically.

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