MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3/1991

Spectral Theory of Singular Ordinary Differential Operators

20.1 - 26.1.1991

The conference was held under the leadership of Professor H.-D. Nießen (Essen), A. Schneider (Dortmund) and J. Weidmann (Frankfurt).

In the 29 lectures given and in the discussion of the 43 participants the main subjects were localization and stability properties of the absolutely continuous and essential spectrum of ordinary and partial differential operators, the Titchmarsh-Weyl m-function, orthogonal polynomials and operators in spaces with an indefinite scalar product.

Abstracts

R.R. Ashurov: On spectral resolutions of the Schrödinger operator with singular potential.

Let $\Omega \subset \mathbb{R}^N$. In Ω we consider $H = -\Delta + q$, $q(x) \in Q_\alpha$, $\alpha > 0$, where Q_α is the Stummel-space. Let $\lambda_1 \leq \lambda_2 \leq \cdots$ be the eigenvalues, $\{u_n(x)\}$ the system of eigenfunctions (complete in $L_2(\Omega)$) of H. Consider the Riesz means of the spectral resolutions

$$E_\lambda^s f(x) = \sum_{\lambda_n < \lambda} \left(1 - \frac{\lambda_n}{\lambda}\right)^s u_n(x) f_n \quad , \quad f_n = (f, u_n).$$

Theorem 1. Let $s > \frac{N-1}{2}$ and $f \in L_2(\Omega)$ be continuous in $D \subset \Omega$. Then uniformly in every compact set $K \subset D$ $\lim_{\lambda \to \infty} E_\lambda^s f(x) = f(x)$.

If q has stronger singularities, Theorem 1, in general, does not hold.

F.V.Atkinson: Schrödinger equations with oscillating potentials.

We discuss equations typified by

$$y'' + (k^2 + At^{-1}\sin mt)y = 0, \quad t \geq t_0 > 0,\ k > 0,\ m > 0,$$

which involves a "Wigner - von Neumann" type potential. It has long been known that the nature of the solutions changes as k passes through a "resonance value" $m/2$. Of recent interest is the question of the behaviour of a solution with fixed initial data $y(t_0), y'(t_0)$ and $k = m/2 + \epsilon$, as $\epsilon \to 0$. The paper discusses the phase-amplitude or Prüfer-transformation approach to this problem. It calls for matching asymptotics over intervals (t_0, t_1),, (t_1, t_2), (t_2, ∞), where $t_1 \to \infty$, $\epsilon t_1 \to 0$, $|\epsilon t_2| \to \infty$. Extensions to more general potentials, involving several frequencies or integral-type criteria are possible.

H. Behncke: Absolute continuity of the spectrum of Dirac- and Schrödinger operators with Wigner- von Neumann potentials.

We study the spectra of Dirac- and Schrödinger Hamiltonians in $[-m, m]^c$ resp. $\mathbb{R}_+$. The potentials are assumed to consist of a short range part $S \in L^1$, a long range smooth part P, whose decaying improves with differentiation and an oscillating decaying part $W = \sum f_i \sin g_i$. From $P(\infty)$ and $g_i'(\infty)$ one can easily compute the set of resonances R. Then the following holds

i) The eigenfunctions for $\lambda \notin R$ behave like plane waves at ∞.
ii) $[-m, m]^c \backslash R$ resp. $\mathbb{R}_+ \backslash R$ belongs to the a.c. spectrum.
iii) For these sets a limiting absorbtion principle holds.
iv) The m-function is continuous in the upper half plane off R.

For the proof of (ii) we use the following result

Theorem: Let τ be a Dirac or Schrödinger differential operator so that for $\lambda \in (\lambda_1, \lambda_2)$ all solutions u resp. u and u' to $\tau u = \lambda u$ are bounded. Then (λ_1, λ_2) belongs to the spectrum of τ and $\sigma(\tau)$ is abs. continuous there. If for Schrödinger operators boundedness of u only is assumed, the result holds if q_- is bounded.

R.C. Brown: Embeddings of weighted Sobolev spaces into spaces of continuous functions.

We give sufficient conditions and necessary conditions (which in some cases are both necessary and sufficient) for continuous and compact embeddings of the weighted Sobolev space $W^{1,p}(\Omega; v_0, v_1)$ where the weights v_0, v_1 have singularities only on $\partial\Omega$ or at ∞ into spaces of weighted continuous and Hölder continuous functions. The theory is illustrated by several examples, including one having to do with analysis of the spectrum of a second order symmetric differential operator on $[1,\infty]$.

Joint work with B. Opic, Mathematics Institute, Czechoslovak Academy of Science, Prague CSSR.

W. Eberhard: Regularity of indefinite eigenvalue problems of second order and the distribution of the eigenvalues.

We consider the indefinite eigenvalue problem of second order

$$(1) \quad u'' + g(x)u' + \rho^2 r(x)u = 0$$

with $g \in L(0,1)$ bounded and $r \in C^2[0,1]$ a real valued function with an arbitrary number of turning points. Using estimates of an appropriate fundamental system of solutions (cf. the abstract of Freiling) we derive explicit regularity conditions for the coupled boundary conditions at 0 and 1 generalizing those of Birkhoff in the definite case. For the corresponding regular eigenvalue problems we derive the asymptotic distribution of the eigenvalues λ_k. We get two sequences $\lambda_k^\pm$ of eigenvalues $\lambda_k^\pm = \frac{k^2\pi^2}{R_\pm^2}\left[1 + O\left(\frac{1}{k^{\sigma_0}}\right)\right]$, $k \in \mathbb{N}$, where $R_\pm = \int_0^1 \sqrt{f_\pm(t)}dt$, $f_\pm(t) = \max\{0, \pm f(t)\}$ and $\sigma_0 \in]0,1[$ is a number depending on the types of turning points. The investigations can be carried over to problems with singularities at the coefficients g and r and defined at compact intervals or at the half-axis $\mathbb{R}_0^+$ or at the complete axis $\mathbb{R}$.

W.D. Evans: Discrete inequalities and the m-function for a second-order difference equation.

In a recent paper, B.M. Brown and the speaker studied inequalities of the form

$$\left\{\sum_{n=-1}^{\infty} p_n|\Delta x_n|^2 + \sum_{n=0}^{\infty} q_n|x_n|^2\right\}^2 \le K \sum_{n=-1}^{\infty} |x_n|^2\omega_n \quad \sum_{n=0}^{\infty} \omega_n \left|\frac{1}{\omega_n} M x_n\right|^2,$$

where $Mx_n := -\Delta(p_{n-1}\Delta x_{n-1}) + q_n x_n$ ($n \in \mathbb{N}_0$), $\Delta x_n := x_{n+1} - x_n$, $\Delta^2 x_n := \Delta(\Delta x_n)$ and $p_n \neq 0$, $w_n > 0$ and q_n are real. The validity of the inequality, the value of the best constant and the existence, or otherwise, of equalizing functions, were found to depend on the behavior of the m-function for the difference equation $Mx_n = \lambda w_n x_n$ ($n \in \mathbb{N}_0$). Joint work with B.M Brown and L. Littlejohn, still in progress, on properties of the m-function will be discussed. Included will be a global representation for $m(\lambda)$ when $\lambda \in \mathbb{C}_+$

in the limit-point case, a full asymptotic formula as $|\lambda| \to \infty$ in $\mathbb{C}_+$ and connections with orthogonalizing weights for associated polynomials.

W.N. Everitt: Spectral theory and orthogonal polynomials.

The spectral theory of the classical Legendre polynomials is extended to the Legendre type orthogonal polynomials first discussed by H.L. Krall in 1938 and 1940. There are essentially five distinct Legendre type polynomials; each set is generated by a symmetric (formally self-adjoint) differential equation; one set has the equation of second order (classical Legendre), three are of the fourth-order, and one is of the sixth-order. For each set there is a right-definite and left-definite spectral theory in an appropriate Lebesgue-Stieltjes integrable-square space (right-definite) or a Sobolev integrable-square space (left-definite); in both cases each one of the five Legendre type orthogonal polynomials is represented by a self-adjoint operator.
(Joint work with L.L. Littlejohn and Susan Loveland.)

G. Freiling: Differential equations with an arbitrary number of turning points.

We consider the differential equation

$$-u'' + g(x)u = \lambda r(x)u \qquad (1)$$

on a finite or infinite interval I, where I contains m turning points x_j, $1 \leq j \leq m$, that is here, zeros of r. Using asymptotic estimates proved by R.E. Langer for solutions of (1) for intervals containing only one turning point, we derive asymptotic estimates for $\sqrt{\lambda} \to \infty$ for a special fundamental system of solutions of (1) in I.

The results obtained are fundamental for the investigation of eigenvalue problems defined by (1) and suitable boundary conditions (cf. the abstract of W. Eberhard).

S.G. Halvorsen: Sharp stability criteria for ordinary linear second order differential equations.

A method is established to find asymptotically sharp upper and lower bounds for solutions or for extremal values of solutions e.g. of equations $(*)$ $x'' + f(x)x = 0$ $(f \in L_{loc}[c, \infty))$, yielding absolute constants in limit circle criteria, (un)boundedness criteria in various norms etc.

Example: In $(*)$ let $f(t) \equiv \lambda - q(t)$.
If $n > 0$, $k > 0$ and $\alpha k t^n \leq q + k^2 t^{2n+2} \leq \beta k t^n$, then if $\beta - \alpha < n\pi$, every solution of $(*)$ is in $L^2[c, \infty)$ (limit circle case), whereas if $\beta - \alpha = n\pi$ this is not generally true for any choice of $n > 0$, $k > 0$.

B.J. Harris: The Titchmarsh-Weyl m(λ)-function.

We consider the problem of computing the Titchmarsh-Weyl m-function for $-y'' + q(x)y = \lambda y$ on $[0, \infty)$ subject to $\cos\alpha\, y(0) + \sin\alpha\, y'(0) = 0$ for $\alpha \in [0, \pi)$.
If q is such that $\int_0^\infty t|q(t)|\, dt < \frac{1}{4}\log 5$ and $q \in L^1[0, \infty)$ we show that

$$m(\lambda) = i\lambda^{1/2} + \sum_{n=1}^{\infty} p_n(0, \lambda) \quad \text{for} \quad \lambda \in \mathbb{C}^+ \backslash [-\lambda_0, 0] \quad \text{for some} \quad \lambda_0 > 0 \quad \text{where}$$

$$p_1(x, \lambda) = -\int_x^\infty e^{2i\lambda^{1/2}(t-x)} q(t)\, dt \quad \text{and}$$

$$p_{j+1}(x, \lambda) = \int_x^\infty e^{\displaystyle 2\int_x^t i\lambda^{1/2} + \sum_{n=1}^{j} p_n(s, \lambda)\, ds} p_j(t, \lambda)^2\, dt.$$

R. Hempel: The essential spectrum of Neumann Laplacians on some bounded singular domains.

Neumann Laplacians on singular domains exhibit surprising and fascinating properties. On one hand, there is the recent work of Davies, Simon and others on the spectrum of Neumann Laplacians on (unbounded) *horns*, while, on the other hand, Hempel, Seco and Simon have analysed the essential spectrum of Neumann Laplacians on some singular domains of the type *rooms and passages* and *combs*. Examples of this kind have been known at least since Courant and Hilbert in connection with Rellich's embedding theorem. It turns out that, given any closed subset S of the intervall $[0, \infty)$, we can construct a comb-like domain Ω with the property that the Neumann Laplacian on Ω has precisely the set S as its essential spectrum.

A typical rooms and passages domain consists af a sequence of rectangles (the rooms), which are symmetric with respect to the x-axis, joined by narrow passages. The main step in the analysis consists in a *decoupling* along the intervals where rooms and passages meet, by means of certain *natural* boundary conditions, which turn out to be Neumann from the side of the rooms and Dirichlet from the side of the passages (the *Organ-pipe Lemma*).

References.

E.B Davis, B. Simon: *Spectral properties of the Neumann Laplacian of horns.* Duke. Math. J., to appear.

R. Hempel, L. Seco, B. Simon: *The essential spectrum of Neumann Laplacians on some bounded singular domains.* J. Funct. Anal., to appear.

Ch. Amick: *Some remarks on Rellich's theorem and the Poincaré inequality.* J. London Math. Soc. (2) 18 (1978), 81-93.

W.D. Evans, D.J. Harris: *On the approximation numbers of Sobolev imbeddings for irregular domains.* Quart. J. Math. Oxford (2) 40 (1989) 13-42.

D. Hinton: Interpolation inequalities and L^p nonoscillation criteria.

We discuss interpolation inequalities of the form

$$\left(\int_I \left|x^\beta u^{(j)}\right|^p dx\right)^{1/p} \leq K\left(\int_I \left|x^\gamma u\right|^q dx\right)^{(1-\lambda)/q}\left(\int_I \left|x^\alpha u^{(n)}\right|^r dx\right)^{\lambda/r},$$

where the constant K may depend on n, α, β, γ, p, q, r, λ and I, but not on the function u. The function u is required to have $u, u', \cdots, u^{(n-1)}$ vanish at the endpoints of the interval I. Application of these inequalities yields L^p integral criteria on the coefficients of a linear differential operator which imply that the operator is nonoscillatory.

A.M. Hinz: The spectrum of Schrödinger operators with spherically symmetric potentials.

The concepts of a bound state, i.e. an eigenfunction for a discrete eigenvalue of the Schrödinger operator $-\Delta + V$ in $L_2(\mathbb{R}^n)$, and of strongly locallized eigensolutions, i.e. a weak solution of the eigenvalue equation $-\Delta u + Vu = \lambda u$ which decays exponentially at infinity, are closely related. This is supported by standard examples such as the harmonic oscillator, the H-atom or the von Neumann-Wigner operator. A one-dimensional example of Halvorsen, however, indicates that a bound state is not necessarily strongly locallized for potentials with a severe negative singularity at infinity. On the other hand, rather well-behaved spherically symmetric potentials in higher dimensions lead to examples of exponentially decaying eigenfunctions for eigenvalues embedded in the essential spectrum. They come from the observation that the spectra of operators like $-\Delta + \cos(|x|)$ contain intervals densly filled with eigenvalues. This fact is established by comparing these spectra with those of related singular ordinary oprators.

R.M. Kauffman: Continuous spectrum eigenfunction expansions for second-order operators.

Let $\tau = -\frac{\partial^2}{\partial x^2} + p$, $p \in C^\infty[1,\infty)$, p real and bounded below. Let H be the unique self-adjoint extension of the restriction of τ to $\{\Phi| \quad \Phi \in C^\infty[1,\infty)$, Φ has bounded support and $\Phi(1) = 0\}$, the extension is taken in $L_2[1,\infty)$. We study the existence of Banach spaces Y and Z and Borel sets Δ, such that $Z \subset Y'$ and $Y \subset L_2[1,\infty)$, and such that the following is true:
For every $\epsilon > 0 \quad \exists\{\lambda_j\}_1^k \in \Delta$ such that

$$\|\sum_{j=1}^{k} e^{i\lambda_j t} F_{\lambda_j}(\Phi) F_{\lambda_j} - P(\Delta)\, e^{iHt}\, \Phi\|_Z \le \epsilon \|\Phi\|_Y$$

for all $\Phi \in Y$, where $\tau F_{\lambda_j} = \lambda_j F_{\lambda_j}, \quad F_{\lambda_j}(1) = 0$.

M. Klaus: Spectral asymptotics near resonance points for Wigner- von Neumann type potentials.

We consider half-line Schrödinger operators $-y'' + q(r)y = \lambda y$ where $q(r) = q_1(r) + q_2(r)$ with $q_1(r) = (1+r)^{-1}p(r)$, $p(r)$ periodic and $q_2(r) = O(r^{-2})$ at infinity. Points of the continuous spectrum where the asymptotic form of solutions is not a linear combination of $e^{\pm ikr}$ $(k = \sqrt{\lambda})$ are called resonance points. In particular, such resonance points may be embedded eigenvalues. We study the asymptotic behaviour of the Jost function and of related quantities like the Titchmarsh-Weyl coefficient and spectral function near resonance points. Extensions of these results to systems, including Dirac systems, will also be discussed.

A.M. Krall: Differential operators in left-definite Hilbert spaces.

There has been considerable difficulty with dealing with differential operators in left definite settings. While Green's function (inverse) operators are bounded and cause no problem, their inverses remain enigmatic. I believe I can show how to overcome the problems.

K.H. Kwon: Spectral analysis of Bessel polynomials in Krein space and it's real orthogonalizing weight.

Bessel polynomials are polynomial solutions of

$$x^2y'' + 2(x+1)y' = n(n+1)y, \quad n \geq 0.$$

We construct a Krein space generated by Bessel polynomials, in which the Bessel operator $x^2D^2 + 2(x+1)D$ is self-adjoint and has a discrete spectrum $\{n(n+1)|n \geq 0\}$ all of which are eigenvalues of multiplicity 1.

Then, we construct a real weight function of bounded variation with support on $[0, \infty]$ with respect to which Bessel polynomials are orthogonal.

H. Langer: Spectral properties of quasileftdefinite pencils $F - \lambda G$.

Let $F - \lambda G$ be an S-hermitian linear pencil in the sense of F.W. Schäfke \ A. Schneider (Math. Ann. 162 (1965)). Recall that with the linear mappings F and G there are associated hermitian sesquilinear forms $\mathbb{F}$ and $\mathbb{G}$, respectively, and by the above named authors the pencil was studied under the assumption that at least one of these forms is positive definite. We weaken this assumption considering the case where the form $\mathbb{F}$ has a finite number of negative squares. As in the positive definite case, a theorem about expansions in eigen and associated functions is proved. The main tool is a generalization of H. Wielandt's characterization of compact operators in Hilbert spaces to Pontrjagin spaces. An application to a Sturm-Liouville system with an indefinite weight function and λ-depending boundary conditions is given. (Joint work with A. Schneider)

B.M. Levitan: The asymptotic form of the Weyl-Titchmarsh m-function.

Consider the Sturm-Liouville problem

$$-y'' + q(x)y = zy, \quad 0 \leq x < \infty, y'(0) = 0 \quad (1)$$

where q(x) is a real locally integrable function. Let $m(z)$ be the Weyl-Titchmarsh function of the problem (1). In 1984 Professor Harris published a paper [1] in which he proved the existence of a full asymptotic expansion of the m-function for z from the angle $0 < \epsilon < \arg z < \pi - \epsilon$. This expansion has the form:

$$m(z) = \frac{i}{\sqrt{z}} + \sum_{k=1}^{n} \beta_k(-z)^{-\frac{k+2}{2}} + O\left(|z|^{-\frac{k+3}{2}}\right) \quad (2).$$

The number n of the exact terms in the expansion (2) depends on the order of smoothness of the function $q(x)$ in some neighbourhood of the point $x = 0$. In the report we give a new proof of Harris' theorem using the following special Tauberian theorem for Fourier integrals, which I published in 1953 [2]:

Let $\tau(\mu)$, $-\infty < \mu < \infty$ be an odd function that satisfies the following conditions:

1) For $|a| \to \infty$ $\quad \mathrm{Var}_a^{a+1}\{\tau(\mu)\} \leq C < \infty$
2) The Fourier transform of the function $\tau(\mu)$ (in the sense of distributions) is equal to zero in some neighbourhood of zero.

Then the following asymptotic has place

$$\int_0^\mu (\mu^2 - t^2)^s \, d\tau(t) = O(\mu^s), \quad s \geq 0, \mu \to \infty.$$

References: 1. Harris, J.B. The asymtotic form of the Titchmarsh-Weyl m-function, J. London Math. Soc. 1984, v.30, pp 110-118.

2. Levitan, B.M. On a special Tauberian theorem, Jzv. Acad. Sc. USSR, serie mathemat. 1953, v.17, n.2, pp 269-282.

L.L. Littlejohn: On formally symmetric differential expressions. (Jointly with W.D. Evans.)

Suppose we are given a differential expression $M[\cdot]$ defined by

$$(*) \quad M[y](x) = \sum_{k=0}^{n} a_k(x) y^{(k)}(x) \qquad (x \in I)$$

where $I \subset \mathbb{R}$ is an interval, $a_k \in C^k(I, \mathbb{C})$ $\quad (k = 0, \cdots, n)$ and $a_n(x) \neq 0 \quad (x \in I)$.
We shall address the following two questions:
(1) When do we know that there exists $f : I \to \mathbb{R}$ such that $f \cdot M[\cdot]$ is formally symmetric?
(2) If $f \cdot M[\cdot]$ is formally symmetric so that

$$f(x)M[y](x) = \sum_{k=0}^{[\frac{n}{2}]} \left(p_k(x)y^{(k)}(x)\right)^{(k)} + i \sum_{k=0}^{[\frac{n-1}{2}]} \left\{ \left(q_k(x)y^{(k)}(x)\right)^{(k+1)} + \left(q_k(x)y^{(k+1)}(x)\right)^{(k)} \right\},$$

what are the p_k's and q_k's in terms of the a_k's ?

Clearly, efficient answers to these questions must be known before an effective spectral analysis of the expression $(*)$ can be realized.

The answer to question (1) is well understood: see L.L. Littlejohn and D. Race, "Symmetric and symmetrizable differential expressions", Proc. London Math. Society (3), 60, (1990) p 334-364. Basically, there exists a function $f : I \to \mathbb{R}$ (called a symmetry factor) such

Gefördert durch DFG Deutsche Forschungsgemeinschaft

that $f \cdot M[\cdot]$ is symmetric if and only if f is the simultaneous solution to a system of n homogeneous differential equations (called the symmetry equations) associated with $M[\cdot]$.

In answering (2), we restrict ourselves to real-valued coefficients for the sake of simplicity. We derive two formulas for expressing each p_k in terms of the a_k's. The latter of these formulas is optimal in the sense that p_k is written as a linear combination of $(n-k+1)$ a_k's.

Applications to orthogonal polynomials will be discussed.

F. Mantlik: Stability of the absolutely continuous spectrum of Sturm-Liouville operators.

We consider the Sturm-Liouville differential expression $ly = -y'' + q(x)y$ on (a,b), where the limit point case occurs at the endpoint b. Assume that for some $c \in (a,b)$ there exists a s.a. realization L_b of l in $L^2(c,b)$ which has purely absolutely continuous spectrum in an open interval $J \subset \mathbb{R}$. If additionally the spectral function ρ_b of L_b satisfies some mild growth conditions in J we show that each s.a. realization L of l in $L^2(a,b)$ has purely absolutely continuous spectrum in J, too. Our result implies that essentially the absolutely continuous spectrum of L only depends on the potential q at one singular endpoint. This confirms a conjecture of J. Weidmann. Similar results had been proved by del Rio Castillo under stronger hypotheses.

R.R. del Rio Castillo: Embedded eigenvalues of Sturm-Liouville operators.

The behaviour of embedded eigenvalues of Sturm-Liouville problems in the half axis under local perturbations is studied. When the derivative of the spectral function is strictly positive, it is proven that the embedded eigenvalues either disappear or remain fixed. In this case it is shown that local perturbations cannot add eigenvalues in the continuous spectrum. If the condition on the spectral function is removed then a local perturbation can add infinitely many eigenvalues.

B. Schultze: On the spectrum of a class of formally self-adjoint differential operators with odd order terms.

For the formally self-adjoint differential expressions

$$My = \sum_{j=0}^{[\frac{n}{2}]}(-1)^j(p_j y^{(j)})^{(j)} + \frac{i}{2}\sum_{j=0}^{[\frac{n-1}{2}]}(-1)^j\{(q_j y^{(j+1)})^{(j)} + (q_j y^{(j)})^{(j+1)}\}$$

on $I = [a,\infty), a \in \mathbb{R}$ with $p_j \in C^j(I,\mathbb{R})$ $(j = 0,\cdots,[\frac{n}{2}])$, $q_j \in C^{j+1}(I,\mathbb{R})$ $(j = 0,\cdots,[\frac{n-1}{2}])$ and $p_{\frac{n}{2}} > 0$ if n is even and $q_{\frac{n-1}{2}} > 0$ if n is odd, the following result for the essential spectrum $\sigma_e(M) = \{\lambda \in \mathbb{C}|$ range $T_0(M-\lambda)$ is not closed $\}$, ($T_0(M)$ denotes the minimal operator generated by M) is shown:

Theorem: Let

$$M_0 y = \frac{i}{2}\sum_{\sigma=0}^{k} a_\sigma \left\{\left(t^{\beta(2\sigma+1)}y^{(\sigma)}\right)^{(\sigma+1)} + \left(t^{\beta(2\sigma+1)}y^{(\sigma+1)}\right)^{(\sigma)}\right\} + \sum_{\sigma=0}^{k} b_\sigma \left(t^{2\beta\sigma}y^{(\sigma)}\right)^{(\sigma)}$$

with $\beta \le 1$, $a_\sigma, b_\sigma \in \mathbb{R}$, $a_k \neq 0$ and $My = \sum_{\mu=0}^{m} r_\mu(t)y^\mu$ be a formally selfadjoint expression, $r_\mu \in C^\mu(I,\mathbb{C})$, $\mu = 0,\cdots,m$ with $r_\mu^{(\nu)}(t) = O\left(t^{(\mu-\nu)\gamma}\right)$ for $\nu \le \mu = 0,\cdots,m$ and $\gamma < \beta$.

Then $\sigma_e(M_0 + M) = \mathbb{R}$.

The proof follows with a result on the essential spectrum of not necessarily self-adjoint expressions.

J.K. Shaw: Inverse scattering on the line for a Dirac operator.

The whole-line version of the Gelfand-Levitan-Marchenko equation for a Dirac system is studied. A new derivation of the G-L-M equation is given, independent of Frolov's earlier treatment, and the complete inversion is carried out in some explicit cases in which a spectral gap is present. Previous calculations of this type restrict either to scalar potential or degenerate gap. Applications are discussed in connection with modelling of optical couplers.

S.Stepin: Spectral theory of operator pencils and boundary eigenvalue problems of mathematical physics.

The investigation of atmospheric oscillations leads to a non-selfadjoint boundary eigenvalue problem considered on the half-line $[0,\infty)$. The spectral parameter enters both in the equation and the boundary conditions at zero. This problem can be reduced to an eigenvalue problem for an operator pencil. The spectrum of the problem turns out to be discrete and besides all eigenvalues are real. It is proved with the help of the Krein-Langer theorem that there exist two Riesz bases composed of eigenfunctions.

G. Stolz: Bounded solutions and absolute continuity of Sturm-Liouville operators.

Let $\tau u = -u'' + qu$ be a Sturm-Liouville expresssion in $(a, \infty), -\infty \leq a < \infty$ and A be an arbitrary selfadjoint realization of τ. Using the *method of subordinacy* of Gilbert and Pearson we prove the

Theorem: Let $\sup\limits_{x \geq c} \int\limits_{x}^{x+1} |q(t)|\, dt < \infty$. Assume that all solutions of $\tau u = \lambda u$, $\lambda \in (\lambda_1, \lambda_2)$ are *bounded.* Then

(i) $[\lambda_1, \lambda_2] \subset \sigma(A)$,

(ii) $\sigma(A)$ is purely absolutely continuous in (λ_1, λ_2).

Applications of this theorem range to almost all the known results on absolute continuity of SL-operators. In particular we treat *perturbed periodic* and *quasiperiodic potentials,* where results of Hinton and Shaw resp. Dinaburg and Sinai are considerably generalized.

A further application treats potentials with $q(t) \to -\infty$ as $t \to \infty$. Here the method of subordinacy is used directly to generalize results of Walter.

It is also noted that the method of subordinacy implies that the absolutely continuous spectrum is *transient* in the sense of Avron and Simon.

R. Weder: Spectral and scattering theory in perturbed stratified media.

In this talk I discuss the spectral and scattering theory for acoustic and electromagnetic waves in perturbed stratified media. In particular the spectral theory for the perturbed propagator is studied by means of the limiting absorbtion principle.

This is a problem in partial differential equations where methods of ordinary differential equations play a fundamental role.

I will present a number of open questions on the threshold behaviour of a family of Sturm-Liouville operators that are posed by the spectral theory in perturbed stratified media.

References.

R. Weder. Spectral and Scattering Theory for Wave Propagation in Perturbed Stratified Media. Applied Mathematical Science. Volumen 87, Springer Verlag, December 1990.

P. Werner: Spectral properties of periodic differential operators and applications to resonance phenomena in periodic media.

We discuss the solution $u(x,t)$ of the initial value problem

$$u_{tt} - c^2 [\rho (\frac{1}{\rho} u_x)_x - qu] = f(x)e^{-i\omega t} \quad \text{for} \quad (x,t) \in \mathbb{R} \times \mathbb{R}^+, \quad u(x,0) = u_t(x,0) = 0$$

for $\omega > 0$, $f \in C_0^\infty(\mathbb{R})$ and smooth p-periodic coefficients $c > 0$, $\rho > 0$ and $q \geq 0$. It is well known that the spectrum $\sigma(A)$ of the self-adjoint extension A of the spatial part consists of a finite or countable set of disjoint closed intervalls. In particular, $\partial\sigma(A)$ is finite or countable. We show in the case $q \neq 0$ that

$$(1) \quad u(x,t) = \frac{1-\epsilon i}{\sqrt{\pi\omega}} \sqrt{t}\, e^{i\omega t}\, V(x) \int_{-\infty}^{\infty} \frac{Vf}{\rho c^2} d\xi + W(x)e^{-i\omega t} + o(1)$$

as $t \to \infty$ if $\omega^2 \in \partial\sigma(A)$, where V is a non-trivial periodic or semi-periodic solution of

$$(2) \quad c^2 \rho (\frac{1}{\rho} V')' - qV + \omega^2 V = 0$$

and W satisfies

$$(3) \quad c^2 \rho (\frac{1}{\rho} W')' - qW + \omega^2 W = -f;$$

ϵ is a sign factor depending on the distribution of the eigenvalues of two related eigenvalue problems for eqn. (2). If $\omega^2 \notin \partial\sigma(A)$, then

$$(4) \quad u(x,t) = W(x)e^{-i\omega t} + o(1) \quad \text{as} \quad t \to \infty$$

("limit amplitude principle"), where W satisfies (3). If $q = 0$, then the static term $c \int \frac{f}{\rho c^2} d\xi$ with suitable $c \neq 0$ has to be added to the righthand sides in (1) and (4). Both estimates (1) and (4) hold uniformly with respect to x in bounded subsets of $\mathbb{R}$. Extensions to cylindrical configurations with periodic material distribution are indicated.

A. Zettl: Differential operators with an infinite number of interior singularities.

The Glazman-Krein-Naimark characterization of all self-adjoint realizations of a symmetric differential expression M on an interval I with no singular interior points is extended to the case of a finite or infinite number of interior singularities. When the defiency indices $d^+ = d^- = d = \infty$ an additional condition is required in order to place the self-adjoint domains exactly "in the middle" between the minimal and the maximal domains. This condition is automatically satisfied when $d < \infty$.
This is joint work with W.N. Everitt.

Berichterstatter: P. Furlan

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Prof.Dr. Ravshan Ashurov
Department of Mathematics
Tashkent State University

Tashkent 700095
USSR

Prof.Dr. Frederick V. Atkinson
Department of Mathematics
University of Toronto

Toronto, Ontario M5S 1A1
CANADA

Prof.Dr. Horst Behncke
Fachbereich Mathematik/Informatik
Universität Osnabrück
PF 4469, Albrechtstr. 28

4500 Osnabrück

Prof.Dr. Richard C. Brown
Department of Mathematics
The University of Alabama
345 Gordon Palmer Hall
P.O. Box 870350

Tuscaloosa , AL 35487-0350
USA

Prof.Dr. Aad Dijksma
Mathematisch Instituut
Rijksuniversiteit te Groningen
Postbus 800

NL-9700 AV Groningen

Prof.Dr. Jörg Donig
Fachbereich Mathematik
Universität-GH Duisburg
Postfach 10 16 29
Lotharstr. 65

4100 Duisburg 1

Prof.Dr. Walter Eberhard
Fachbereich Mathematik
Universität-GH Duisburg
Postfach 10 16 29
Lotharstr. 65

4100 Duisburg 1

Prof.Dr. William Desmond Evans
School of Mathematics
University of Wales
College of Cardiff
Senghennyd Road

GB- Cardiff CF2 4AG

Prof.Dr. William Norrie Everitt
Dept. of Mathematics and Statistics
The University of Birmingham
P. O. Box 363

GB- Birmingham , B15 2TT

Prof.Dr. Gerhard Freiling
Lehrstuhl II für Mathematik
(für Ingenieure)
RWTH Aachen
Templergraben 55

5100 Aachen

Dr. Hilbert Frentzen
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 10 37 64

4300 Essen 1

Prof.Dr. Don B. Hinton
Dept. of Mathematics
University of Tennessee at
Knoxville
121 Ayres Hall

Knoxville , TN 37996-1300
USA

Dr. Peter Furlan
Fachbereich Mathematik
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Dr. Andreas M. Hinz
Mathematisches Institut
Universität München
Theresienstr. 39

8000 München 2

Prof.Dr. Sigbjorn Halvorsen
Institute for Energy Technology
Division KRS
P. O. Box 40

N-2007 Kjeller

Prof.Dr. Robert Kauffman
Dept. of Mathematics
University of Alabama at
Birmingham
UAB Station

Birmingham , AL 35294
USA

Prof.Dr. Bernard Harris
Dept. of Mathematics
Northern Illinois University

DeKalb , IL 60115
USA

Martin Klaus
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-4097
USA

Dr. Rainer Hempel
Mathematisches Institut
Universität München
Theresienstr. 39

8000 München 2

Prof.Dr. Allan M. Krall
Department of Mathematics
Pennsylvania State University
218 McAllister Building

University Park , PA 16802
USA

Prof.Dr. Kil Hyun Kwon
Dept. of Mathematics, KAIST
P.O. Box 150
Cheongryang

Seoul
SOUTH KOREA

Prof.Dr. Heinz Langer
Fachbereich Mathematik
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof.Dr. Boris Levitan
Dept. of Mathematics
M. V. Lomonosov State University
Moskovskii University
Mehmat

119899 Moscow
USSR

Prof.Dr. Lance L. Littlejohn
Dept. of Mathematics and Statistics
Utah State University

Logan , UT 84322-3900
USA

Dr. Frank Mantlik
Fachbereich Mathematik
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof.Dr. Reinhard Mennicken
Fakultät für Mathematik
Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Prof.Dr. Erich Müller-Pfeiffer
Pädagogische Hochschule
Erfurt/Mühlhausen
Postfach 307
Nordhäuser Str. 63

O-5064 Erfurt

Prof.Dr. Heinz-Dieter Nießen
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 10 37 64

4300 Essen 1

Dr. Markus Poppenberg
Fachbereich Mathematik
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof.Dr. Rafael del Rio Castillo
Instituo de Investigacion en
Matematicas Aplicadas y Syste[illegible]
Univ. Nacional Autonoma de Me[illegible]o
Apartado Postal 20-726

Mexico D. F. 01000
MEXICO

Prof.Dr. Dieter Schmidt
FB 6 - Mathematik
Universität-GH Essen
Universitätsstr. 1-3
Postfach 10 37 64

4300 Essen 1

Prof.Dr. Albert Schneider
Fachbereich Mathematik
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Dr. Bernd Schultze
Department of Mathematics
University of the Philippines
Diliman

1101 Quezon City , Metro Manila
PHILIPPINES

Prof.Dr. John Shaw
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-4097
USA

Prof.Dr. H.S.V. de Snoo
Mathematisch Instituut
Rijksuniversiteit te Groningen
Postbus 800

NL-9700 AV Groningen

Prof.Dr. Stanislav Stepin
Dept. of Mathematics
Moscow State University

Moscow 119899
USSR

Dr. Günter Stolz
Fachbereich Mathematik
Universität Frankfurt
Postfach 11 19 32
Robert-Mayer-Str. 6-10

6000 Frankfurt 1

Prof.Dr. Ricardo Weder
Instituo de Investigacion en
Matematicas Aplicadas y Systemas
Univ. Nacional Autonoma de Mexico
Apartado Postal 20-726

Mexico D. F. 01000
MEXICO

Prof.Dr. Joachim Weidmann
Mathematisches Seminar
Fachbereich Mathematik
Universität Frankfurt
Postfach 11 19 32

6000 Frankfurt 1

Prof.Dr. Peter Werner
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57
Postfach 80 11 40

7000 Stuttgart 80

Prof.Dr. Anton Zettl
Dept. of Mathematics
Northern Illinois University

DeKalb , IL 60115
USA

Gefördert durch
DFG Deutsche Forschungsgemeinschaft