MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 5/1991

Constructive Methods in Complex Analysis

3.2. bis 9.2.1991

Die Tagung fand unter der Leitung der Herren Blatt (Eichstätt), Gaier (Gießen) und Varga (Kent) statt. Besonders erfreulich war die Teilnahme einer Reihe von Mathematikern aus Osteuropa, davon drei aus der Sowjetunion und drei aus Polen. Leider mußten einige US-Amerikaner wegen des Golfkrieges absagen. Trotzdem bot die Tagung mit 29 Vorträgen ein ausgewogen breites Spektrum aus der konstruktiven Funktionentheorie.

Ein Hauptthema der Tagung war die Theorie und Numerik von konformen Abbildungen. Es wurden verschiedene Verfahren vorgestellt, die Riemannsche Abbildungsfunktion zu berechnen, darunter Interpolationsmethoden mit gleichverteilten Punkten und Approximation durch Bieberbachpolynome, sowie andere Methoden, die in Softwarepaketen im Einsatz sind.

Einen weiteren Schwerpunkt bildete die Approximation von Funktionen, sowohl mit Polynomen, als auch mit rationalen Funktionen. Inbesondere bewies Herr Stahl eine Vermutung von Varga, Ruttan und Carpenter über das exakte asymptotische Verhalten des Fehlers der besten rationalen Approximationen an $|x|$ auf $[-1,1]$ mit Hilfe potentialtheoretischer Methoden. Hingegen ist das Problem von Bernstein aus dem Jahr 1913, die Bestimmung der Konstanten $\lambda = \lim_{n\to\infty} n E_n(|x|)$ bei Polynomapproximationen, immer noch ungelöst.

Unter anderem berichtete Herr Gonchar über das überraschende Ergebnis, daß e^{-x} über $[0,\infty)$ nicht optimal durch rationale Funktionen der Klassen $R_{n,n}$ approximiert wird.

Wir hörten weiterhin über extremale Punktmengen, wie Leja-, Fekete- oder Menkepunkte, die zahlreiche Anwendungen in der konstruktiven Funktionentheorie haben. Im Zusammenhang damit standen auch Vorträge über Gleichverteilung von Nullstellen von Polynomen und Extremalpunkten bester Approximationen.

Schließlich wurde über Fragen der geometrischen Funktionentheorie, sowie über Markov-Ungleichungen und ihre Anwendungen in der Approximationstheorie vorgetragen.

Vortragsauszüge

A. I. APTEKAREV:

An approximate method for conformal mapping

A computer method for conformal mapping of simply connected domains will be presented. The method was worked out at the Keldysh Inst. of Applied Mathematic (Moscow). It has been applied for more than ten years for solving concrete problems such as the flowing problems, grid generation ect. The main points of the method are: auxiliary mapping, the integral equation of second kind for the density of the double layer potential, the boundary correspondence functions, conformal mapping from a disc (by Canchy integral), conformal mapping to a disc (additional regularisation by a nonanalytic mapping). For discretisation of the problem we use the trigonometric interpolation. It allows us to prove that the method is a method without saturation, i.e. accuracy of the method effectively depends on the smoothness of the boundary. Some numerical results in connection with it will be presented.

R. W. BARNARD:

Application of Szegö polynomials to several variables

The author discusses the problem of determining the exact value of the radii of Koebe type covered discs for mappings from the unit disc in n complex variables onto convex domains in n complex variables. He then shows how R. Varga's recently obtained computational results on Szegö polynomials in one variable can be used to obtain these exact values.

H.-P. BLATT:

Sharpness of discrepancy Theorems

By modified Jacobi polynomials it is shown that Erdös-Turan Theorems are sharp. On the other hand, let $p_n(x) = \prod_{i=1}^{n}(x-x_i)$, $x_i \in [-1,1]$, be a polynomial with (a) $\|p_n\| \leq A_n(\frac{1}{2})^n$, (b) $|p_n'(x_i)| \geq \frac{1}{B_n}(\frac{1}{2})^n$. Then the author has shown that the discrepancy of the zero counting measure and the equilibrium distribution on [-1,1] is of order $O(\frac{\log C_n}{n}\log n)$ where $C_n = \max(A_n, B_n, n)$.
Hence Kadec's theorem on the distribution of alternation points is not sharp. Moreover, the author outlined a proof for the discrepancy of order

$$O\left(\frac{\log C_n}{n}\log\log n\right).$$

It is open whether this bound is sharp.

C. BREZINSKI:

Least squares orthogonal polynomials

We intend to define orthogonal polynomials in the least squares sense that is minimizing

$$\sum_{i=0}^{m-1} [c(x^i P_k(x))]^2$$

with $m \geq k$ (the degree of P_k) and c a linear functional on the space of complex polynomials given by its moments $c(x^i) = c_i$, $i \geq 0$.

We use these polynomials in the construction of quadrature rules and Padé-type approximants.

M. EIERMANN:

Strong asymptotics for certain extremal polynomials

Let Ω be a compact subset of the complex plane with simply connected complement and let Π_m^* the set of all polynomials of degree at most m which fulfill certain linear constraints, e.g., have M prescribed coefficients or are required to fulfill N Herminitian interpolatory conditions. In Π_m^*, we seek for p_m^* minimizing the supremum norm $\|\cdot\|_\Omega$ on Ω. The classical polynomial Zolotarev problems are special cases of this question.

We describe a technique based on the Carathéodory-Fejér method to construct "nearly optimal" polynomials $\hat{p}_m \in \Pi_m^*$ for this problem. We automatically obtain strong asymptotics for $\|p_m^*\|_\Omega$ generalizing Bernstein's results on the classical Zolotarev polynomials.

(Joint work with Gerhard Starke, Institute for Computational Mathematics, Kent State University, Kent(OH), U.S.A.)

D. GAIER:

On the convergence of the Bieberbach polynomials in regions with piecewise analytic boundary

Let G be a domain bounded by a Jordan curve Γ and let f_o be the conformal map of G onto $\{w: |w| < r\}$ normalized by $f_o(0) = 0$, $f_o'(0) = 1$. Let Π_n be the n-th Bieberbach polynomial minimizing

$$\iint_G |f_o'-P'|^2 \, dz$$

in the class of (normalized) polynomials P of degree $\le n$. The problem is to estimate

$$\max \{|f_o(z) - \Pi_n(z)|: z \in \bar{G}\}.$$

Theorem. If Γ is piecewise analytic, and if $\lambda\pi$ is the smallest exterior angle of Γ, $0 < \lambda < 2$, then

$$\max \{|f_o(z)-\Pi_n(z)|: z \in \bar{G}\} \le C \cdot \log n \cdot n^{-\gamma} \text{ with } \gamma = \frac{\lambda}{2-\lambda} .$$

In general, the estimate is best possible.

In the proof, $\|f_o'-P\|_{L^2(Q)}$ is estimated by $\|g-Q\|_\infty$ where $g := f_o' \cdot R$ and $R(z) = \Pi(z-z_j)$, z_j the corners of Γ.

A. A. GONCHAR:

On the rational approximation of e^{-x} on $[0,\infty)$

Let $R_{k,l}$ be the class of all rational functions of the form $r = p/q$, $\deg(p) \leq k$, $\deg(q) \leq l$, and $\rho_{k,l} = \operatorname{dist}(e^{-x}, R_{k,l})$, with respect to the sup-norm on $E = [0,\infty)$. I shall discuss the following result (of a joint work with A. Martinez and E. Rakhmanov).

For any $\theta \in [0,1]$ and any sequences $k = k(n)$, $l = l(n)$, such that

$$k/l \to \theta, \quad \frac{1}{2}(k+l) = n \to \infty,$$

the limit

$$\lim \rho_{k,l}^{1/n} = \rho(\theta) < 1$$

exists. The value $\rho(\theta)$ (for any fixed θ) can be described in terms connected with the equilibrium distribution of the charge $\mu = \mu_F - \mu_E$ on the plates of a condenser (E,F) ($F \subset \mathbb{C}\setminus E$; μ_E, μ_F are positive measures, $\operatorname{supp}(\mu_E) \subset E$, $\operatorname{supp}(\mu_F) \subset F$, $|\mu_E| = \frac{1}{2}(1+\theta)$, $|\mu_F| = 1$), under the assumption that the external field $\varphi = \frac{1}{2}\operatorname{Re}(z)$ acts on the plate F, and this plate satisfies a certain symmetry condition in the field φ. This potential-theoretic problem admits an explicit solution in terms of elliptic integrals.

Note that $\rho(0) = 1/9$ (Schönhage, 1973) and $\rho(1) = 1/9.289\ldots$ (Gonchar-Rakhmanov, 1986); see [1],[2]. The minimal value of $\rho(\theta)$ corresponds to $\theta_o = 0.295\ldots$ and $\rho(\theta_o) = 1/16.244\ldots$.

1. A.A. Gonchar and E.A. Rakhmanov, Equilibrium distributions and degree of rational approximation of analytic functions, Mat. Sbornik, 134 (176) (1987), 3; English transl.: Math. USSR Sbornik, 62 (1969), 2, pp. 305-347.

2. R.S. Varga, Scientific Computation of Mathematical Problems and Conjectures, CBMS-NSF Regional Conference Series in Applied Math. N 60, Soc. for Industrial and Applied Math., Philadelphia, 1990, 122 pp.

R. GROTHMANN:

Interpolation points and roots of polynomials

Let $E \subset \mathbb{C}$ be kompact and $z_{\nu,n}$, $\nu=0,1,\dots,n$, $n \in \mathbb{N}$, be a matrix of interpolation points in E, such that the distribution of these points weakly converges to a measure τ on E. Then one can characterize the functions, which are analytic in certain regions containing E, by the speed of convergence of the interpolation process. (Compare the book of Krylow). The talk discusses this problem and its reverse. Moreover, the question of a faster convergence for subsequences is shown to be related to overconvergence in the sense of Ostrowski.

M. H. GUTKNECHT:

A completed theory of the unsymmetric Lanczos process and related algorithms

The theory of the "unsymmetric" Lanczos biorthogonalization (BO) algorithm, which has so far been restricted to an essentially generic situation (characterized by the nonsingularity of the leading principal submatrices of the associated moment matrix) is extended to the nongeneric case. The "serious" breakdowns due to the occurrence of two orthogonal right and left iteration vectors x_n and y_n can be overcome.

We also derive the formulas for nongeneric version of the corresponding linear equation solvers BIORES (brief for BIORTHORES or Lanczos/ORTHORES), BIOMIN (the biconjugate gradient method) and BIODIR. Here too, the breakdowns of these methods can be cured.

The whole theory is developed as a consequence of corresponding results on formally orthogonal polynomials and Padé approximants, for many of which new and simpler derivations are given. Mixed recurrence formulas are derived for a pair of sequences of formally orthogonal polynomials belonging to two adjacent diagonals in a nonnormal Padé table, and a matrix interpretation of these recurrences is developed. This matrix interpretation leads directly to a completed formulation of the progressive qd algorithm, valid also in the case of a nonnormal Padé table. Finally, it is shown how the cure for exact breakdown can be extended to near-breakdown.

D. M. HOUGH:

Numerical conformal mapping

Let Ω be a given simply connected domain with piecewise analytic boundary, let f map Ω conformally onto a canonical domain with unit circle as boundary and let $g := f^{[-1]}$. I outline the methods which are currently used in the software package CONFPACK for the construction of approximations to f and g. These methods are based on the numerical solution of Symm's equation and involve the adaptive construction of Jacobi weighted variable degree piecewise polynomial approximations to the boundary correspondence functions associated with f and g. Several examples illustrate the effectiveness of the package.

K. G. IVANOV:

Behavior of the Lagrange interpolants in the roots of unity

Let A_o be the class of functions f analytic in the open disk $|z| < 1$, continuous on $|z| \leq 1$, but not analytic on $|z| \leq 1$. We investigate the behaviour of the Lagrange polynomial interpolants $L_{n-1}(f,z)$ to f in the n-th roots of unity. In contrast with the properties of the partial sums of the Maclaurin expansion, we show that for any w, $|w| > 1$, there exists a $g \in A_o$ such that $L_{n-1}(g,w) = 0$ for all n. We also analyze the size of the coefficients of $L_{n-1}(f,z)$ and the asymptotic behaviour of the zeros of the $L_{n-1}(f,z)$.

J. KOREVAAR:

Fields of electrons on the sphere and quadrature problems

For a continuous charge distribution of constant density on the unit sphere ∂B in $\mathbb{R}^3$, it is well-know that the electrostatic field E is equal to zero throughout the unit ball B (Faraday cage effect). QUESTION: How well can one approximate this phenomenon if the total charge on ∂B is made up of n point charges 1/n? In the much simpler problem for $\mathbb{R}^2$, the field E_n due to n "electrons" on ∂B can be made $O(r^n)$ on balls B(0,r) with r < 1. It is reasonable to conjecture that the corresponding order in $\mathbb{R}^3$ would be $r^{c\sqrt{n}}$. Such a lower bound for sup $|E_n|$ on B(0,r) has been established. Our tentative upper bound for inf sup $|E_n|$ involves the exponent $c\sqrt[3]{n}$.

A number of equivalent and related problems have been identified. In particular, a good estimate for inf sup $|E_n|$ on balls $B(0,r)$ is equivalent to a good equal weights quadrature result for polynomials on ∂B. A closely related problem involves equal weights quadrature results for the standard classes of functions on the interval $[-1,1]$. For example, for analytic functions, there is an n-point equal weights quadrature formula with optimal remainder estimate $O(e^{-c\sqrt{n}})$. Our work makes use of ideas of S. Bernstein, sketched in C.R. Notes of 1937.
The work has also led to a three domains theorem of Hadamard type for harmonic functions (joint work with my student J.L.H. Meyers).

R. KÜHNAU:

Das "Kaulquappenverfahren" zur numerischen konformen Abbildung

Es wird über ein neues Verfahren berichtet, die Riemannsche - Abbildungsfunktion einfach zusammenhängender Gebiete numerisch zu realisieren. Das Verfahren arbeitet für Schliztgebiete und Gebiete, die von Vollkurven berandet sind. Dabei wird bei den Schlitzen immer nur ein Teilstück betrachtet und durch konforme Abbildung in den Einheitskreis "hineingezogen" (formale Ähnlichkeit mit der Prozedur bei der Löwnerschen Differentialgleichung).

Numerische Erprobung zeigt gute Ergebnisse. In der theoretischen Begründung sind noch viele Fragen offen.

K. MENKE:

Point systems with extremal properties and conformal mapping

Für ein- und zweifach zusammenhängende Gebiete G werden Punktsysteme

$$\{z_{n\nu} \in \partial G,\ \nu = 1,\ldots,n;\ n \in \mathbb{N}\}$$

mit Hilfe von Extremaleigenschaften definiert. Diese Punktsysteme besitzen auf ∂G gewisse Gleichverteilungseigenschaften (unter Berücksichtigung der konformen Abbildung). Über diesen Zusammenhang kann man Näherungen für die konforme Abbildung erzeugen, indem man zunächst die Punkte $z_{n\nu}$, $\nu = 1,\ldots,n$ über die entsprechende Extremaleigenschaft näherungsweise numerisch bestimmt und die approximierende Funktion durch Interpolation gewinnt.

H. N. MHASKAR:

A general discrepancy theorem

Let E be a sufficiently smooth closed Jordan curve, and σ be a signed Borel measure on E with $\sigma(E) = 0$. We define an Erdös-Turán-type discrepancy of σ. Under some mild condition on σ, we estimate this discrepancy in terms of the logarithmic potential of σ on a curve enclosing E. We apply this result to get estimates for distribution of Fekete points, alternation points of polynomials of best approximation and the zeros of orthogonal polynomials on the unit circle. These estimates are sharper than the ones known before.

N. PAPAMICHAEL:

A domain decomposition method for approximating the conformal modules of long quadrilaterals

Let Ω be a simply-connected domain in the complex plane, and consider a system consisting of Ω and four distinct points z_1, z_2, z_3, z_4 in counterclockwise order on $\partial\Omega$. Such a system is said to be a quadrilateral $Q := \{\Omega; z_1, z_2, z_3, z_4\}$.

Let R_H be a rectangle of the form

$$R_H := \{(\xi,\eta) : 0 < \xi < 1, 0 < \eta < H\}.$$

Then, the conformal module $m(Q)$ of Q is the unique value of H for which Q is conformally equivalent to the rectangular quadrilateral $\{R_H; 0, 1, 1+iH, iH\}$. (By this we mean that for $H = m(Q)$ and for this value only there exists a unique conformal map $\Omega \to R_H$, which takes the four points z_1, z_2, z_3, z_4 respectively onto the four corners of the rectangle.)

This talk concerns a domain decomposition method for approximating the conformal modules of long quadrilaterals, and is a report of joint work with N. S. Stylianopoulos. The method has been studied already by us and also by D. Gaier and W. K. Hayman, but only in connection with a special class of quadrilaterals, viz. quadrilaterals $Q := \{\Omega; z_1, z_2, z_3, z_4\}$ where: (a) The domain Ω is bounded by two parallel straight lines and two Jordan arcs. (b) The points z_1, z_2, z_3, z_4 are the four corners where the arcs meet the straight lines.

Our main purpose here is to explain how the method may be extended to a much wider class of quadrilaterals than that indicated above.

W. PLESNIAK:

Markov's inequality and extension of C^∞ functions

It is known that the Bernstein characterization of analytic functions and C^∞ functions on a compact set E that is determining for $\mathcal{O}(E)$ and $C^\infty(E)$, respectively, is possible if and only if E preserves the Bernstein-Walsh and Markov's inequality, respectively. Till now, a relation between these two conditions for E is not well understood. We discuss the question for Cantor type subsets of $\mathbb{R}$.

It remains still open the question of whether the latter condition for E implies the first one which is equivalent to saying that E is regular with respect to the Dirichlet problem for the unbounded component of $\mathbb{C} \setminus E$. Actually, we do not even know whether every compact set E preserving Markov's inequality has to be of positive logarithmic capacity.

E. A. RAKHMANOV:

Connectedness of the support of the equilibrium distributions in the external fields on $\mathbb{R}$

Let us denote

$$F := \{\varphi \in C^1(\mathbb{R}\setminus\{0\}) \cap C(\mathbb{R}) : \varphi(x)/\log|x| \to \infty \text{ as } |x| \to \infty\}.$$

We consider $\varphi \in F$ as an "external field" on $\mathbb{R}$ and denote by $\lambda_t = \lambda_{t,\varphi}$ the respective family of the equilibrium distributions with parameter $t = \lambda_t(\mathbb{R}) \in (0,+\infty)$.

It is important to know, for the most part of applications of this concept in approximation theory, whether the support of $\lambda_{t,\varphi}$ is connected; we express this property by the notations $F(t) = \{\varphi \in F\colon \operatorname{supp} \lambda_{t,\varphi} \text{ is a segment}\}$; $F_o = \bigcap_{t>0} F(t)$.
It is well known that for all convex φ in $t > 0$, F belongs to F_o. The question is also cleared up for even function φ in F. We introduce a new sufficient condition for $\varphi \in F$ to belong to F_o, which is concerned essentially with the nonsymmetric case.

Theorem. Suppose $\varphi \in F$ and the functions $\sqrt{x}\,\varphi'(x)$ and $-\sqrt{x}\,\varphi'(-x)$ are both nondecreasing in $(0,+\infty)$ and both tend to 0 as $x \to 0+0$. Then, $\varphi \in F_o$.

Example. Let $0 < p < q$ and $\varphi_{p,q}(x) = x^p$ for $x \in \mathbb{R}_+$; $\varphi_{p,q}(x) = |x|^q$ for $x \in \mathbb{R}_-$. It follows from the Theorem, that $\varphi_{p,q} \in F_o$ for $p > 1/2$. We can show that this result is sharp: $\varphi_{p,q} \notin F_o$ for $p \le 1/2$, $q > 1/2$.

Nevertheless, the Theorem doesn't give the answer for A. Gonchar's question, which asks whether $\varphi_{\frac{1}{2},1} \in F(1)$; the problem is still open.

L. REICHEL:

Leja points in scientific computing

We consider the application of Leja points to polynomial interpolation of analytic functions, and to the Richardson iteration method for the solution of large linear systems of equations. The stability of the Newton form of the interpolation polynomial, and of Richardson iteration is demonstrated.

S. RUSCHEWEYH:

On cyclic Pólya-frequency-functions

The cyclic variation v_c of a piecewise continuous, 2π-periodic function f is the supremum of the sign-changes in all sequences $f(x_1), f(x_2), \dots, f(x_n), f(x_1)$, where $x_1 < x_2 < \dots < x_n < x_1 + 2\pi$, $n \in \mathbb{N}$. A cyclic Pólya Frequency Function of order 2r+1 ($g \in CPF_{2r+1}$) is defined by the property:

$$\forall_{f, v_c(f) \le 2r} \qquad v_c(g * f) \le v_c(f),$$

where $(g * f)(x) := \frac{1}{2\pi} \int_0^{2\pi} f(\varphi)\, g(x-\varphi) d\varphi$, $x \in \mathbb{R}$. The functions in CPF_{2r+1} (introduced by Schoenberg 1959) are characterized by the non-negativity of systems of determinants with elements of the form $g(x_i - y_j)$. These determinants are difficult to deal with, and our general intention is to find alternative characterizations which admit, for instance, conclusions about the smoothness of these functions. In the present lecture we give such a characterization for CPF_3. Some applications of this to geometric function theory are also discussed.
(joint work with G. Kurth and L. Salinas)

G. SCHMEISSER:

Characterization of function spaces via quadrature

A fascinating topic in Approximation Theory deals with characterization of regularity properties of functions in terms of speed of convergence of their best approximations.

We present two concepts for a corresponding characterization in terms of speed of convergence of a quadrature process. The cases of quadrature of

(i) periodic functions over a period
(ii) non-periodic functions over a compact interval
(iii) functions over the whole real line

require separate treatment.

Here we describe only the results obtained with one of the two concepts in case (i) as far as characterization of entire functions is concerned. For illustration, a numerical example is also given.

J. SICIAK:

Compact subsets of $\mathbb{C}$ admitting inequalities of Bernstein, Bernstein-Walsh or Markov type for harmonic polynomials

Let K be a compact subset of $\mathbb{C}$. Let $s_H(K)$ be the Frechet space of functions $f \in C(K)$ such that $\forall_{k\geq 1}\ d_k(f) := \sup_{n\geq 1} n^k \operatorname{dist}_K(f,H_n)$ is finite, where H_n is the set of harmonic polynomials of degree $\leq n$ and $\operatorname{dist}_K(f,H_n) := \inf\{\|f-u\|_K: u \in H_n\}$. Consider the following conditions:

(i) There exist positive numbers M,x,μ such that for every harmonic polynomial $u: |u(z)| \leq M(1+x\delta^{\mu})^{\deg u}$, if dist $(z,K) \leq \delta \leq 1$;

(ii) There exist positive numbers M,σ s.t. $\|\operatorname{grad} u\|_K \leq M(\deg u)^{\sigma} \|u\|_K$ for every harmonic polynomial u;

(iii) There exists a continuous linear operator $L: s_H(K) \to C^{\infty}(\mathbb{R}^2)$ such that 1° for every f in $s_H(K)$, $L(f) = f$ and $\Delta L(f) = 0$ on K, 2° if u is a harmonic polynomial then $L(u|K) = u$ in a neighborhood of K.

THEOREM. (i) $\Rightarrow$ (ii) $\Leftrightarrow$ (iii).

H. STAHL:

Rational Approximation of $|x|$ on $[-1,1]$

We investigate best rational approximants in the uniform norm to the function $f(x) = |x|$ on the interval $[-1,1]$. The main result is the proof of a conjecture by R.S. Varga, A. Ruttan, and A.J. Carpenter. They have recently conjectured that

$$\lim_{n\to\infty} e^{\pi\sqrt{n}}\, E_{n,n}(|x|,[-1,1]) = 8,$$

where

$$E_{n,n}(|x|,[-1,1]) = \inf_{r\in R_{n,n}} \| |x|-r(x)\|_{[-1,1]}.$$

The theorem generalizes earlier results by D.J. Newman and by N.S. Vjacheslavov. As a byproduct of the analysis we prove theorems about the location and the asymptotic distribution of poles and zeros of the best rational approximants r_n^*, $n \in N$, and the extreme points of the error on $[-1,1]$, i.e. points of maximal deviation of the approximant r_n^* from f.

L. N. TREFETHEN:

Spijker's lemma on the Riemann sphere

Let r be a rational function of order n with maximum modulus 1 on the unit circle C. M. N. Spijker has recently proved that the are length $\|r'\|_1$ is at most $2\pi n$. As a corollary spijker's result pins down exactly the constant in the Kreiss Matrix Theorem.

A possibly more natural question is: how large can the arc length f(C) be on the Riemann sphere? We conjecture that the answer is again $2\pi n$.

R. S. VARGA:

Using the Laguerre inequalities to obtain better lower bounds for the de Bruijn-Newman constant Λ, related to the Riemann Hypothesis

For any real number λ, set

$$H_\lambda(x) := \int_0^\infty e^{\lambda t^2} \Phi(t) \cos(xt)dt \quad (0 \le t < \infty).$$

Then, H_λ is an entire function. It is known that there exists a real number Λ with $-\infty < \Lambda \le 1/2$, and that H_λ has only real zeros for $\lambda \ge \Lambda$, and some nonreal zeros when $\lambda < \Lambda$. (If the Riemann Hypothesis is true, then $\Lambda \le 0$.) Here, we apply the Laguerre inequalities to H_λ, i.e.,

$$L_\lambda(x) := (H'_\lambda(x))^2 - H_\lambda(x) \cdot H''_\lambda(x) \qquad (x \in \mathbb{R}),$$

to establish that

$$-0.0991 < \Lambda,$$

which improves all known lower bounds for Λ.

H. WALLIN:

Continued fractions and dynamical systems

We study continued fractions $K(a_n/1)$ where each a_n is chosen at random from a finite set of complex numbers. We are interested in convergence properties and a certain stability of these continued fractions, and in describing the set of values of classes of convergent continued fractions. As tools we use computer experiments, generalized iteration of Möbius transformations, iterated function systems, and Kleinian groups. (joint work with J. Karlsson)

E. WEGERT:

Nonlinear Riemann-Hilbert problems for holomorphic functions

Let T denote the complex unit circle and suppose that $\{M_t\}_{t\in T}$ is a given family of curves in the complex plane. The lecture addresses the problem of finding all functions w holomorphic in the unit disk D which are continuous on $D \cup T$ and satisfy the boundary condition

$$w(t) \in M_t$$

for all $t \in T$. We summarize relevant results about the solvability of the problem and sketch some applications: generalized Nevanlinna-Pick interpolation, H^{∞} - optimization, and polynomial hulls of sets fibered over the circle.

R. WEGMANN:

An estimate for crowding in conformal mapping

Let f be an analytic function in the unit disc. Assume that the smallest rectangle R, which contains the range of f, has sides a and b with $b \leq a$. Then the maximum norm of the derivative satisfies

$$\|f'\| \geq \frac{a}{2} \psi \left(\frac{b}{a}\right),$$

with a function $\psi(\tau)$ which behaves like $\frac{\tau}{2\pi} \exp\left(\frac{\pi}{2\tau}\right)$ for small τ. Examples show that this estimate is best possible up to a factor of 2. The estimate explains the "crowding" phenomenon, which was observed in several numerical esperiments.

A. P. WÓJCIK:

Constructive characterization of differentiable functions on Jordan domains in the complex plane

If Δ is the open unit disk in the complex plane $\mathbb{C}$ then the classical theorems of Jackson and Bernstein yield

(1) $\quad f \in \mathrm{Lip}(\beta,\bar{\Delta}) \cap \mathcal{O}(\Delta) \Leftrightarrow d_N(f,\bar{\Delta}) = O(n^{-\beta}),\ 0 < \beta < 1,$

where, for any plane compact K

$$d_N(f,K) := \min\{\|f-p\|_K : p \text{ is a polynomial of degree} \le N\}.$$

Our aim is to present some results inspired by (1). Let D be a bounded Jordan domain in $\mathbb{C}$. Denote by $\Phi: \hat{\mathbb{C}} \setminus D \to \hat{C} \setminus \Delta$ the homeomorphic extension of the conformal mapping from $\mathbb{C} \setminus \bar{D}$ onto $\mathbb{C} \setminus \bar{\Delta}$ such that $\Phi(\infty) = \infty$ and $\lim\limits_{z\to\infty} \Phi(z)/z > 0$.

Put $\psi := \Phi^{-1}$. Assume that ψ satisfies Hölder condition in some ring $\{1 \le |z| \le R\}$, $R > 1$, with the exponent $\alpha \in (0,1]$.

LEMMA. Take an integer n such that $n+1 > 16/\alpha^4$. If $f \in C^{n+1}(\mathbb{R}^2) \cap \mathcal{O}(D)$ then $d_N(f,\bar{D}) = O(N^{-(n+1)\alpha^3/8})$.

COROLLARY (to Lemma and to e.g. Lebedev, Tamrazov, Izv. AN SSSR 34 (1970)).

$$f \in C^{\infty}(\mathbb{R}^2) \cap \mathcal{O}(D) \Leftrightarrow \forall\, a > 0:\ \lim_{N\to\infty} N^a\, d_N(f,\bar{D}) = 0.$$

Berichterstatter: R. Grothmann

Tagungsteilnehmer

Prof.Dr. Alexandre I. Aptekarev
Keldysh Institute of Applied
Mathematics
Academy of Science of the USSR
Miusskaya sq. 4

125047 Moscow A-47
USSR

Prof.Dr. Dieter Gaier
Mathematisches Institut
Universität Giessen
Arndtstr. 2

6300 Gießen

Prof.Dr. Roger W. Barnard
Dept. of Mathematics
Texas Tech. University
Box 4319

Lubbock , TX 79409
USA

Prof.Dr. Andrei A. Goncar
Steklov Mathematical Institute
MIAN
Academy of Sciences of the USSR
42, Vavilova str.

Moscow 117 966 GSP-1
USSR

Prof.Dr. Hans-Peter Blatt
Mathematisch-Geographische
Fakultät
Universität Eichstätt
Ostenstr. 26 - 28

8078 Eichstätt

Dr. Rene Grothmann
Fach Mathematik
Mathematisch-Geographische Fakultät
Universität Eichstätt
Ostenstr. 26-28

8078 Eichstätt

Prof.Dr. Claude Brezinski
UFR IEEA - M3
Universite de Lille 1

F-59655 Villeneuve d'Asq Cedex

Dr. Martin Gutknecht
IPS
ETH-Zentrum

CH-8092 Zürich

Dr. Michael Eiermann
IBM Deutschland GmbH
Wissenschaftliches Zentrum - IWBS
Tiergartenstraße 15

6900 Heidelberg

Prof.Dr. David M. Hough
Department of Mathematics
Coventry Polytechnic

GB- Coventry CVI 5FB

Prof.Dr. Kamen G. Ivanov
Inst. of Mathematics
Bulgarian Academy of Sciences
1113 Sofia
BULGARIA

Prof.Dr. Wilhelm Niethammer
Institut für Praktische Mathematik
Universität Karlsruhe
Postfafch 6380

7500 Karlsruhe 1

Prof.Dr. Jacob Korevaar
Fakulteit Wiskunde en Informatica
Universiteit van Amsterdam
Plantage Muidergracht 24

NL-1018 TV Amsterdam

Prof.Dr. Nicholas Papamichael
Dept. of Mathematics and Statistics
Brunel University
Kingston Lane

GB- Uxbridge, Middlesex , UB8 3PH

Prof.Dr. Reiner Kühnau
Fachbereich Mathematik
Martin-Luther-Universität Halle
Postfach

O-4010 Halle

Prof.Dr. Wieslaw Plesniak
Instytut Matematyki
Uniwersytet Jagiellonski
ul. Reymonta 4

30-059 Krakow
POLAND

Prof.Dr. Klaus Menke
Fachbereich Mathematik
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof.Dr. Eugene Rakhmanov
Steklov Mathematical Institute
MIAN
Academy of Sciences of the USSR
42, Vavilova str.

Moscow 117 966 GSP-1
USSR

Prof.Dr. Hrushikesh Mhaskar
Dept. of Mathematics
California State University

Los Angeles , CA 90032
USA

Prof.Dr. Lothar Reichel
Department of Mathematics
Naval Postgraduate School

Monterey , CA 93943
USA

Prof.Dr. Stephan Ruscheweyh
Mathematisches Institut
Universität Würzburg
Am Hubland

8700 Würzburg

Prof.Dr. Richard S. Varga
Dept. of Mathematics
Kent State University

Kent , OH 44242-0001
USA

Prof.Dr. Gerhard Schmeißer
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2

8520 Erlangen

Prof.Dr. Hans E. Wallin
Dept. of Mathematics
University of Umea

S-901 87 Umea

Prof.Dr. Jozef Siciak
Instytut Matematyki
Uniwersytet Jagiellonski
ul. Reymonta 4

30-059 Krakow
POLAND

Prof.Dr. Elias Wegert
Fachbereich Mathematik
Bergakademie Freiberg
Akademiestr. 6

0-9200 Freiberg

Prof.Dr. Herbert Stahl
Fachbereich 2
Technische Fachhochschule Berlin
Luxemburger Str. 10

1000 Berlin 65

Dr. Rudolf Wegmann
Max-Planck-Institut für
Astrophysik

8046 Garching

Prof.Dr. Lloyd N. Trefethen
Dept. of Mathematics 2-182
Massachusetts Institute of
Technology

Cambridge , MA 02139
USA

Prof.Dr. Adam Wojcik
Institute of Mathematics
University of Mining and Metallurgy
ul. Mickiewicza 30

30059 Krakow
POLAND