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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 12/1991

Mathematische Stochastik

10.03. bis 16.03.1991

Die Tagung fand unter der Leitung von P. L. Davies (Essen) und B. W. Silverman (Bath) statt. Im Vordergrund standen die engen Verbindungen zwischen Wahrscheinlichkeitstheorie auf der einen und Statistik auf der anderern Seite. Verdeutlicht wurden diese Verbindungen insbesondere durch zwei Schwerpunktthemen der Tagung,

Anwendung der Theorie empirischer Prozesse

und

Inversprobleme
(Computertomographie und Bildverarbeitung).

Wie vielfältig solche Verbindungen inzwischen sind, wurde durch Vorträge aus den Gebieten

- Epidemilogie
- robuste Statistik
- Zeitreihenanalyse
- semi–Markov–Prozesse
- Versuchsplanung
- nicht–parametrische Regression
- große Abweichungen
- Schätztheorie
- starke Gesetze
- Warteschlangentheorie und Computernetze

eindrucksvoll belegt. Während der ganzen Woche fand eine rege Diskussion und ein intensiver Gedankenaustausch statt, was einen wesentlichen Beitrag zum Erfolg der Tagung lieferte.

An der Tagung nahmen insgesamt 45 Wissenschaftlerinnen und Wissenschaftler aus 8 Ländern teil.

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Vortragsauszüge

Valerie Isham (London)

Stochastic epidemic models

The AIDS epidemic has motivated much recent progress in this area, so the focus of the talk is on models for the transmission dynamics of HIV/AIDS, although the results have wider implications and applications. Properties of a simple model and extensions to a more realistic model for AIDS are described and discussed. An outline is given of some areas of current interest and activity, and the particular problem of determining the variability of realisations of an epidemic is considered in more detail.

Jürgen Eichenauer–Herrmann (Darmstadt)

Pseudorandom numbers generated by inversions

A classical method of generating uniform pseudorandom numbers in the interval [0,1] is the linear congruential method. However, linear congruential pseudorandom numbers show several undesirable regularities, like the relatively coarse lattice structure, which are due to the linearity of the underlying recursion.

A particularly attractive alternative is the inversive congruential method. Inversive congruential pseudorandom numbers do not show any unfavourable lattice structure; they even "avoid the planes". Moreover, bounds for the discrepancy of tupels of consecutive numbers indicate that the inversive congruential method has nice statistical independence properties.

Claudia Klüppelberg (Zürich)

Spectral estimates and stable processes

Let $X_t = \sum_{j=-\infty}^{\infty} \psi_t Z_{t-j}$ be a discrete time MA process based on i.i.d. symmetric random variables Z_t with a common distribution function from the normal domain of attraction of a p–stable law $(0<p<2)$. We derive the limit distribution of the normalized periodogram

$$I_{n,X}(\lambda) = \left| n^{-\frac{1}{p}} \sum_{t=1}^{n} X_t e^{-it\lambda} \right|^2, \quad -\pi \leq \lambda \leq \pi .$$

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For fixed $\lambda = 2\pi\omega$, ω irrational, we show that

$$I_{n,X}(\lambda) \Rightarrow |\psi(e^{-i\lambda})|^2 S(N_1^2+N_2^2) \qquad (n \to \infty)$$

where S is a positive $\frac{p}{2}$–stable random variable $(0<p<2)$ and N_1, N_2 are i.i.d. normal and independent of S .

Alois Kneip (Bonn)

Ordered Linear smoothers

Let Y be a random vector satisfying $Y_i = \mu_i+\epsilon_i$, where $\epsilon_1,\ldots,\epsilon_n$ are i.i.d. zero mean random variables with variance $\sigma^2<\infty$. The following approach for estimating $\mu = (\mu_1,\ldots,\mu_n)'$ is considered: first, a family Ω of symmetric $n \times n$ matrices is specified. Then, an element $\hat{S} \in \Omega$ is selected by Mallow's C_L and $\hat{\mu} := \hat{S} \cdot Y$. It is assumed that Ω is an "ordered linear smoother" according to some easily interpretable qualitative conditions. Examples include the James–Stein estimator, as well as estimators corresponding to smoothing procedures in nonparametric regression (as e.g. kernel estimators or smoothing splines).

The estimator $\hat{\mu}$ is compared to the "optimal" estimator $S_\mu \cdot Y$ minimizing $\mathbb{E}\, \frac{1}{n} \|\mu - S\cdot Y\|_2^2$ for $S \in \Omega$. Under some conditions on ϵ, the difference

$$\left|\frac{1}{n} \|\mu - \hat{S}\cdot Y\|_2^2 - \frac{1}{n} \|\mu - S_\mu \cdot Y\|_2^2\right|$$

is bounded by exponential probability inequalities which hold for all $n \in \mathbb{N}$, all $\mu \in \mathbb{R}^n$, and each ordered linear smoother. The result yields detailed qualitative insight into the properties of estimators constructed in this way.

Laurie Davies (Essen)

Mythical urns and approximate densities

It is argued that classical statistics (Bayes, Neyman–Pearson) is the statistics of the mythical urn. A mythical urn is one which cannot be opened or does not exist and may be characterized by the phrase "the true but unknown parameter θ_0". The statistics of the mythical urn should be replaced by the statistics of the approximate urn, one which cannot be distinguished from the mythical one. The ideas were demonstrated using "approximate density functions".

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Theo Gasser (Mannheim)

Analysis of samples of curves

More often, data are collected in forms of curves for a sample of subjects (or experimental units as Y_{ij} (i=1,...,m_j, j=1,...,n_i)) . A nonparametric regression model for the Y_{ij} is as follows:

$$Y_{ij} = f_i(t_{ij}) + \epsilon_{ij}$$

where f_i are the individual regression functions and ϵ_{ij} are the residuals. Usually the f_i are differing both with respect to intensity and with respect to the dynamic of the development.

An approach for the analysis of such data is presented, starting with estimating an average curve. The essential steps are as follows:

(i) define features which are common to all curves and important such as extrema
(ii) estimate these features by nonparametric regression and differentiation
(iii) estimate shift functions to synchronize the raw curves to the average development
(iv) apply the shift functions to the interpolated raw data
(v) average the shifted curves and smooth if neccessary.

Rates of convergence have been derived. We have applied these methods to several biomedical problems, such as characterizing the development of children or the processing of stimuli by the brain.

Iris Pigeot (Dortmund)

Asymptotics of the jackknifed Mantel–Haenszel estimator

In recent years resampling techniques, especially the jackknife and the bootstrap procedure have gained in importance e.g. when estimating the bias or variance of a given estimator. The jackknife principle has originally been proposed by Quenouille (1949, 1956) for bias reduction. This procedure is applied to the classical Mantel–Haenszel estimator (Mantel, Haenszel, 1959) which is widely used in the analysis of epidemiological studies especially in the analysis of stratified 2×2 tables. It estimates the so–called odds ratio which can be interpreted as the relative risk of developing a certain disease when a person is exposed to some risk factor.

The asymptotic properties of this jackknife estimator are derived in the case that the sample sizes within each stratum tend to infinity while the number of strata remains fixed. It can be shown that the jackknifed Mantel–Haenszel estimator is strongly consistent and asymptotically normal with the same asymptotic variance as the Mantel–Haenszel estimator itself (Pigeot, 1989).

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Frank Ball (Nottingham)

Stochastic models of ion channel gating mechanisms

A single ion channel is usually modelled by a continuous time Markov chain with a finite state space. The state space is partitioned into open and closed states and it is only possible to observe whether the process is in an open or a closed state. A further complication is that short sojourns in either the open or closed class of states fail to be detected, a phenomenon known as time interval omission.

The aim is to make inferences concerning the structure and parameters of the underlying process from the observed aggregated process. We show that an embedded semi–Markov process provides a flexible framework for analysing the dynamic properties of aggregated Markov processes, that is invariant to time interval omission. We use this framework to examine the robustness of structural inferences to time interval omission. Parameter estimation and identifiability problems are discussed briefly.

Dieter Rasch (Rostock)

Experimental design – the unloved step–child of mathematical statistics

Experimental Design is the second–born of two brothers. The crown prince "Statistical Analysis" or first–born is in a very powerful position with many more followers and proud of its mother "Probability Theory" (we use the term crown prince even if the father, Sir Ronald, may not be a king but is at least a Lord). Experimental Designs suffers from all disadvantages of a second–born son and step–child and a non–mathematical mother, variety testing.

In the talk statistics is defined decision–theoretically as a unity of experimental design and analysis. A lemma gives conditions under which both parts of statistics may be treated independently. Sequential analysis is a branch in which such a separation is senseless. Two theorems for designing experiments in nonlinear regression are presented. It is mentioned that in the case of selection procedures the analysis is trivial i.e. the problems lay mainly in the design part and it is conjectured that this is the reason that it is seldom applied.

Gabriele Laue (Leipzig)

Self–reciprocal functions in probability theory

Teugels (1971) studied the class of probability density functions p_F satisfying the relation $p_F(x) = 1 / \sqrt{2\pi} \cdot f(x)$. These are functions coinciding with their own

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cosine–transforms. Obviously, densities which coincide with their own sine–transforms must be densities of non–negative random variables, and they must satisfy the relation

$$(1) \qquad p_F(x) = \sqrt{\tfrac{2}{\pi}} \cdot \operatorname{Im} f(x),\ x \geq 0\ .$$

For instance, the density of the Rayleigh–distribution, $p_F(x) = x \cdot e^{-x^2/2}$, $x \geq 0$, has this property. We remark that Teugels' problem can be reformulated for densities of non–negative random variables

$$(2) \qquad p_F(x) = \sqrt{\tfrac{2}{\pi}} \cdot \operatorname{Re} f(x),\ x \geq 0.$$

To solve the problems (1) and (2) we use the theory of self–reciprocal functions. Especially, the connection between such functions lead to new solutions of (2) . We obtain further solutions using the theory of characteristic funtions of non–negative random variables and the theory of Mellin–transforms.

Hermann Dinges (Frankfurt)

Cumulatives of Wiener germs

<u>Theorem</u> (rigorously proved for d=1)

For every m–smooth family $f_\epsilon(x)dx = \frac{1}{\sqrt{2\pi\ \epsilon}} \exp(-\frac{1}{\epsilon}(K(\epsilon,x)+o(\epsilon^m))dx$ there exist constants $c(\epsilon) = 1 + o(\epsilon^m)$, an m–smooth family $\frac{1}{\epsilon} H(\epsilon,x)$, $R(\epsilon,x) \to 0$ uniformly on compacts, such that

$$c(\epsilon) \int_x^y f_\epsilon(u)du = \Phi(\pm \sqrt{2} \sqrt{\tfrac{1}{\epsilon}\ H(\epsilon,y)+\epsilon^m \cdot R(\epsilon,y)})) - \Phi(\pm \sqrt{2} \sqrt{\tfrac{1}{\epsilon}\ H(\epsilon,x)+\epsilon^m \cdot R(\epsilon,x)})$$

Roughly speaking: With $\Lambda(p) := \frac{1}{2} [\Phi^{-1}(p)]^2$ ("probabilistic logarithsm")

$$\Lambda \circ \Pr(X_\epsilon \leq x) = \frac{1}{\epsilon} H(\epsilon,x) + o(\epsilon^m)$$

uniformly on compacts.

In the second half on the talk the character of the "WG–approximation"

$$F_\epsilon(x) \approx \Phi(\frac{1}{\sqrt{\epsilon}} A(x) + \sqrt{\epsilon}\ A_0(x))$$

was confronted with traditional approximation formulas for the cumulative distribution functions of asymptotically normal distribution and distributions satisfying the principle of large deviation (in the regular case in finite dimension) of Edgeworth–Expansion, Cramér's theory of large deviations, the formula of Lugannami and Rice.

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Sara van de Geer (Leiden)

Applications of empirical process theory

We briefly summarize some concepts used in empirical process theory, with as illustration the empirical process ν_n indexed by monotone functions. It can be shown that a class of uniformly bounded monotone functions $\mathcal{F}$ is a VC–hull class, and hence is universal Donsker. This an be applied in various statistical problems. We mention differentiability of the Kaplan Mayer estimator as function of $\{\nu_n(f): f \in \mathcal{F}\}$. Also, we present a simple proof of asymptotic normality of the nonparametric maximum likelihood estimator of the mean, in a model with interval censored observations, using the asymptotic equicontinuity of ν_n.

We discuss 'manageable' (in a sense a little more general than in Kim & Pollard (1989)) classes, which are useful in some parametric statistical problems, where estimators do not converge at $n^{-1/2}$–rate, as well as in nonparametric problems. For instance, we show that manageability of $\mathcal{F}$ leads to an easy derivation of the rate of convergence of the Grenander estimator of the uniform density.

Finally, we look at increments of ν_n, with as application rates for least squares and maximum likelihood estimators. As example, we reprove the rate $n^{-1/3}$ for the least squares estimator of a monotone regression function.

Grace Wahba (Madison)

Analysis of variance in function spaces

We consider the model

$$y_i = f(t_1(i),\dots,t_\alpha(i)) + \epsilon_i, \quad i=1,\dots,n$$

where $t_\alpha \in T^{(\alpha)}$, some index set (which we will take as $E^{d(\alpha)}$), $\epsilon = (\epsilon_1,\dots,\epsilon_n) \sim \mathfrak{N}(0,\sigma^2 I)$, σ^2 unknown, $f \in \mathfrak{H}$, some reproducing kernel Hilbert space (rkhs). Let $t = (t_1,\dots,t_\alpha)$, $t \in E^{\Sigma_\alpha d(\alpha)}$. We consider (ANOVA) decompositions of f of the form

$$f(t) = \mu + \sum_\alpha f_\alpha(t_\alpha) + \sum_{\alpha<\beta} f_{\alpha\beta}(t_\alpha, t_\beta) + \dots \qquad (*)$$

By considering $\mathfrak{H} = \prod_{\alpha=1} (\alpha \cdot Y \oplus \mathfrak{H}^{(\alpha)})$ where $\mathfrak{H}^{(\alpha)}$ is an rkhs of functions on $T^{(\alpha)}$, we can obtain the decomposition of (*) as an ortholgonal decomposition in $\mathfrak{H}$. Once the model is chosen (i.e. the number of terms in (*) to be included is chosen) then f can be estimated

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as the minimizer of

$$\frac{1}{n}\sum_{i=1}^{n}(y_i - f(t(i)))^2 + \lambda \sum_{\alpha\in M} \theta_\alpha^{-1} J_\alpha(f_\alpha) + \sum_{\alpha,\beta\in M} \theta_{\alpha\beta}^{-1} J_{\alpha\beta}(f_{\alpha,\beta}) + \dots$$

where M indexes the set of subspaces included in the model and the J_α, $J_{\alpha\beta}$ etc. are spline penalty functionals. If $d(\alpha) > 1$ then a thin–plate penalty functional will be involved. The minimization can be carried out via the publicly available code RKPACK (write netl.b). λ and the θ's are obtained by GCV .

Nina Gantert (Bonn)

Large deviations of self–similarity of Brownian motion

Using the Lévy–Cisielsi construction of Brownian motion, we identify $\mathscr{C}[0,1]$ with a subset of $\mathbb{R}^{\mathcal{I}}$, where $\mathcal{I}$ is an index set with "tree structure": Wiener measure is a product measure on $\mathbb{R}^{\mathcal{I}}$ with marginal distribution $\mathscr{N}(0,1)$. We define shifts T_0 , T_1 : $\mathbb{R}^{\mathcal{I}} \to \mathbb{R}^{\mathcal{I}}$ and call a probability measure on $\mathbb{R}^{\mathcal{I}}$ "self–similar", if it is invariant unter T_0, T_1. With these shifts we construct a sequence of random measures which converges $\mathbb{P}$–a.s. to $\mathbb{P}$, and we investigate large deviations of this convergence. This leads to the class of self–similar probability measures on $\mathbb{R}^{\mathcal{I}}$ (N. Gantert, Einige große Abweichungen vom Wienermaß, Dissertation, Bonner Schriften, to appear).

Luis Cruz–Orive (Bern)

Stereology: new trends for old problems

Smoothing methods are continuously being refined to obtain stable numerical solutions of ill–posed inverse problems. In stereology, a standard illustration is Wicksell's approach to estimate the size distribution of 3D particles embedded in a solid, from information solely in 2D sections of the solid. The general problem is indeterminate, and to make progess it has commonly been assumed that the particles are spheres, as Wicksell suggested in 1925.

Over the last five years, however, an entirely different approach has been adopted in stereology, leading to unbiased estimation of particle size irrespective of particle shape. Essential ingredients are new equal–probability sampling methods on the one hand, and intercept–based volume estimators on the other. Since estimation is linear and direct, no inversion problems are involved in the stereology of particles any more. It is therefore suggested that, in future, the notable advances produced in smoothing estimation are put to good use in stereology for alternative inverse problems.

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Jim Kay (Glasgow)

Bootstrapping blurred and noisy data

The use of the bootstrap was introduced and discussed within the context of image restoration. In such problems the observed image is a noisy convolution of the underlying true image, so that the restoration process involves deconvolution as well as smoothing. The bootstrap was used to

(i) choose an appropriate amount of smoothing,
(ii) select the amount of smoothing adaptively and
(iii) enable the construction of simultaneous interval estimates at a number of points of the true image.

The methods were illustrated using some synthetic one–dimensional examples and the strengths and weakness of the various methods were discussed.

I. A. Ibragimov (Leningrad) and R. Z. Khasminskii (Moscow)

Estimation under restrictions

The estimation problem of the parameter $\theta\epsilon\Theta$ in the set $P_\theta^{(2)}$ with the LAN property is considered for the case where the statistician knows additionally that

$$\theta \in T = \{\theta\epsilon\Theta,\ F(\theta)=0\}\ .$$

Here $F = (F_1,...,F_r)^T$ is a differentiable function. The asymptotically effective (a.e.) estimator with restrictions is found as a correction to the a.e. one without restrictions. The case of infinite dimensional Θ is considered too.

Siegfried Heiler (Konstanz)

Bounded influence regression based on rank statistics and on linear combinations of order statistics

In the seventies M–estimators (based on a "generalized" maximum likelihood principle), L–estimators (as linear combinations of order statistics) and R–estimators (using rank statistics) were suggested as robust alternatives to least squares in regression. The idea of bounding the influence of points in the factor space, that led to the development of generalized M–estimates, is carried over to L– and R–type estimators. This is achieved by using an unified approach for all three types, based on the maximum likelihood principle.

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Ursula Gather (Dortmund)

Estimating a scale parameter under censorship

The problem of estimating a power of the scale parameter under normalized squared error loss is studied. It is shown that under the presence of Type I–censoring a trivial estimator not depending on the sample can be both minimax and admissible. This contrasts with the case of a Type II–censoring model where this anomaly does not occur. Some possible treatments of the problem are discussed.

Jack Cuzick (London)

Strong laws for weighted sums

Conditions for a strong law for partial sums are well known, requiring $\mathbb{E}|X|<\infty$ in the i.i.d. case. For triangular arrays, the situation is different as shown by Hsu–Robbins (1947) where for $S_n = \sum_{i=1}^{n} X_{in}$, $\{X_{in}$, i.i.d.$\}$,

$$S_n/n \overset{a.s.}{\to} 0 \quad \Leftrightarrow \quad \mathbb{E}\, X^2 < \infty \ .$$

The intermediate case of weighted partial sums $S_n = \Sigma\, a_{in} X_i$ of a mean zero i.i.d. sequence is important in many applications. The best general result here is due to Stout (1968) who showed e.g. if $|a_{in}| \le K$ and $\mathbb{E}\, X^2 < \infty$ the $S_n/n \overset{a.s.}{\to} 0$. Here the result is improved to replace the moment condition by

$$\mathbb{E}|X|\ (\log^+|X|)^{1/2} < \infty$$

where $\log^+ x = \max(1, \log x)$. When $\frac{1}{n}\Sigma|a_{in}|^{2p} \le K$ for $p>1$, a necessary condition is

$$\mathbb{E}(X^2 \log^+|X|)^{\frac{1}{2-1/p}} < \infty \ .$$

Many other generalizations can be given. This contrasts with the weak law which can be established for $\mathbb{E}|X| < \infty$ and $\{a_{in}\}$ uniformly square integrable.

Evarist Giné (Storrs, Connecticut)

Bootstrapping of M–estimators and other statistical functions

M–estimators, in the generality of Huber and of Pollard, are shown to to be differentiable in a restricted sense (along sequences $n^{-1/2} x_n$ with $x_n \to x_0$ uniformly continuous and bounded) and therefore they can be bootstrapped in probability as a consequence of a bootstrap CLT for empirical processes of Zinn and me. However, the proof that, with some

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mild restrictions, they can also be a.s. bootstrapped is more difficult and requires an oscillation result for empirical processes indexed by VC–graph classes which is a consequence of Alexander's exponentional bounds.

As a consequence, the CLT for the spectral median can be a.s. bootstrapped just as in the case of $\mathbb{R}$; k–means can also, and so can Huber's estimator of location even for non–smooth functions L . This is joint work with M. Arcones.

Manfred Riedel (Leipzig)

Bias–robust estimators in parametric models

Equivariant functionals are used to describe robust estimators of parameters of models with some neighbourhood. The models are generated by groups which are generalizations of location models. A representation of the minimax bias, the minimum of the maximal asymptotic bias over all equivariant functionals, is derived; it plays an essential role for determining the minimax bias as a functional of the model and its neighbourhood.

Moreover, we will construct a most bias–robust functional. In particular, there are considered the case of gross–error neighbourhoods and neighbourhoods induced by metrics. Finally, we discuss applications to location and scale families.

Jutta Steffens (Düsseldorf)

Balayage and a Skorohod stopping theorem

Given a transient (standard) Markov process on a state space $(E,\mathfrak{E})$ with potential kernel U , a classical theorem by H. Rost (1971) states that whenever μ , ν are finite measures on E with $\mu U \leq \nu U$ then μ is obtained by randomly stopping the process started according to ν .

In 1988 Fitzsimmons generalized this theorem to right processes and excessive measures involving a balayage operation in terms of the associated stationary process.

Within the analytic (or measure theoretic) approach (in the sense of Dellacherie/Meyer) one starts out with a substochastic resolvent $(U_\alpha)_{\alpha>0}$ on $(E,\mathfrak{E})$. Defining a balayage operation on excessive measures w.r.t subordinated resolvents one obtains a generalization of Rost's stopping theorem:

Given an excessive measure m dominated by a measure potential μU, there exists a family

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of subordinated resolvents $(V^u_\alpha)_{\alpha>0}$ such that

$$m = \int_0^1 R_{V^u}(\mu U)du \quad \text{where} \quad R_{V^u}(\mu U) := \mu(U-V^n) .$$

As a consequence one obtains another characterization for the existence of right processes given a resolvent.

Guido Dietel (Essen)

Global location and dispersion functionals

The term "global" means here, that the functionals which were introduced, are defined on the whole space of distribution functions in $\mathbb{R}^k$. Further, these functionals are uniquely defined and affin equivariant with high breakdownpoint. Properties like Fréchet differentiablity and Lipschitz boundedness were considered. Recently an application for linear regression was introduced.

Deborah Nolan (Berkeley)

Assessing sequential forecasts in the continuous case: an application of empirical process theory

We extend binary scoring rules to the case where observations are $\mathbb{R}^d$–rational and the one–step forecast, conditional on $X_0, X_1, \ldots, X_{i-1}$, is a distribution P^i on $\mathbb{R}^d$. The scoring rule $S(P^i(E), \{X_i \in E\})$ is a function of the forecastor's probability that event E occurs and the indicator of the event that $X_i \in E$. The total score $\frac{1}{n}\sum_i S(P^i(E), \{X_i \in E\})$ is viewed as a stochastic process indexed by $\mathfrak{C}$ a collection of sets in $\mathbb{R}^d$. Real–valued functions of this process are of interest here: examples include

$$\sup_{\mathfrak{C}} \frac{1}{n}\sum_i [P^i(E) - \{X_i \in E\}]$$

and in one dimension

$$\int [P^i((-\infty,t]) - \{X_i \leq t\}]^2 \, d\,G(t) .$$

Conditions are found for the weak convergence of these processes to mixtures of Gaussian processes.

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Richard Liu (Cornell)

Geometry in Nonparametrics and Robustness

In this talk, we adopt a gemometric view point to clarify efficiency–phenomena, especially for singular estimation problems, in quite a general way. Our central tool is a geometric measure called "modulus of continuity". For example, by this approach,

- we can determine "optimal rates of convergence" as well as "optimal constant" in many interesting problems simply (e.g. density estimation, nonparametric regression etc.). Furthermore, we can determine nearly optimal procedures for many of such problems intuitively, which can be within 25% to the best among all measurable procedures.
- we can easily explain why optimal rates have to be slower than $n^{-1/2}$ in all singular problems.
- we can also clarify a long suspicion of why all existing good estimators in singular problems are biased, namely the phenomenon of "non–existence of good unbiased estimators in singular problems". Consequently, bias–variance trade off becomes an essential component for singular problems while it is not absolutely necessary in the classical literature.

Even though our focus in this talk is on singular problems, this method applies also generally to non–singular problems. For example, we can now immediately see certain phenomena of efficiency in many useful stochastic processes (e.g. empirical process).

Hans–Joachim Roßberg (Leipzig)

Characterising the exponential distribution by properties of the differences of order statistics: a problem posed by Ursula Gather

Let $X_{1:n} \leq \dots \leq X_{n:n}$ be order statistics with basic distribution function F. We put

$$d_{\ell k} = X_{k+\ell:n} - X_{k:n}.$$

Our problem: From which properties of $d_{\ell k}$ does it follow that F is exponential on $(0,\infty)$?

We showed in 1972 that either of the conditions

(1) $P(d_{1k} \geq x) = e^{-x(n-k)}$, $x \geq 0$, $F(0)=0$ (conjecture of Gnedenko)

(2) $P(d_{1k} \geq x) = P(X_{1:n-k} \geq x)$, $x \geq 0$,

implies the desired conclusion; the means of the proofs were very different methods of analytic function theory; for (1) an unpleasant additional assumption was shown to be necessary.

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Here we present an elementary method permitting to solve the analogue two problems in a unified way where d_{1k} occuring in (1), (2) is replaced by $d_{\ell k}$, $\ell \geq 2$. One of them was posed by U. Gather. The method demands additional assumptions, but in view of the above remarks concerning (1), (2) it is not to be expected that a unified approach solves either problem in a satisfactory way.

We also present an analytic function method for the solution of the Gather problem.

Finbarr O'Sullivan (Seattle)

Inverting the attenuated Radon transform

The talk describes an iterative approach to regularized least squares and maximum likelihood inversion of the attenuated Radon transformation arising in positron emission tomography. The approach is based on a splitting of the attenuated transform into a piece which is porportional to the unattenuated transform plus a residual. This splitting leads to iterative application of the filtered backprojection procedure, with an operator count per iteration equivalent to ART CRV EM algorithms.

Convergence of the scheme is established and the operating characteristics examined relative to ART and EM. Much of the improvement associated with maximum likelihood reconstruction seems to be associated with the imposition of positivity. A positively constrained least squares procedure is developed whose numerical efficiency is substantially better than maximum likelihood implemented with the EM algorithm. Asymptotic RMS error characteristics of these procedures are described and some numerical illustrations with the Hoffman phantom presented.

Wolfgang Hārdle (Louvain–la–Neuve)

How many terms should be added into an additive model?

Smoothing in high dimensions faces the problem of data sparseness. Additive regression models alleviate this problem by fitting a sum of one–dimensional smooth functions. Given a set of predictor variables, some of these functions could actually be zero, so that a further simplication of high dimensional smoothing occurs. A two–stage procedure is proposed here to decide how many and which components should be added into such an additive model. After a first step determining the number and sequence of components the model is fit by the kernel method. The asymptotic distribution of this regression estimate is given. A resampling procedure based on wild bootstrapping is proposed for computing p–values.

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Bernard Silverman (Bath)

How much information is lost in continuous and discretised statistical inverse problems?

In a statistical inverse problem – such as Position Emission Tomography – one observes data from a density $g = \mathfrak{P}f$, where $\mathfrak{P}$ is a linear operator and f is the density of real interest. A helpful way of quantifying the information loss due to $\mathfrak{P}$ is to calculate the equivalent sample size of observations from f itself that would yield the same minimax risk. For smoothness classes based on the SVD of $\mathfrak{P}$, corresponding to bounds on square integrals of p^{th} derivatives, and for linear estimates, expressions – and numerical values – for equivalent sample sizes in the PET problems were found.

Usually, the observations obtained are discretised, for example because of the finiteness of the number of PET detectors. Under a matching SVD assumption, which is satisfied in a surprising range of contexts, much of the work on equivalent sample sizes and minimax risks can be partially extended. Upper and lower bounds on the minimax risks are obtained. A general conclusion is that if the discretisation is sufficiently fine relative to the sample size, it is not felt and the accuracy obtained is the same, asymptotically, as for exact data. At the critical rate of discretisation, the order of magnitude of the minimax risk is unaffected, but the constant of proportionality is increased. For PET, the critical behaviour is obtained if the number of detectors is of order $n^{1/(2+p)}$, where n is the sample size and the smoothness class corresponds to bounds on square integrals of p^{th} derivatives.

Erwin Bolthausen (Zürich)

Convergence of path measures in models with polaron like interaction

We consider path measures $\hat{P}_T^\lambda$ of the form

$$d\,\hat{P}_T^\lambda = \exp\left(\int_0^T ds \int_0^T dt\, \tfrac{1}{2}\, \lambda\, e^{-\lambda|t-s|}\, V(X_t, X_s)\right) dP \,/\, Z_T^\lambda \;,$$

where X_t, $t \geq 0$, is a Markov process and Z_T^λ the appropriate norming constants. The problem is to determine $\lim_{\lambda \to 0} \lim_{T \to \infty} \hat{P}_T^\lambda$. In the classical Fröhlich–polaron X is 3–dim Brownian motion and $V(x,y) = |x-y|^{-1}$. This is not yet solved. However, some examples can be treated by large deviation techniques. The analysis of these examples lead to some natural conjectures.

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Rolf Schaßberger (Braunschweig)

Some remarks on queuing networks and Markov chains

Recently, LeBoudec gave an extension of the class of queuing networks widely known as BCMP or Kelly networks. Krzesinski and myself gave a considerable generalization of this work, and I managed to mention this in the last 2 minutes of this talk, after having introduced the class of BCMP networks to an audience which could not be expected to be familiar with it. No time was left to make the intended remarks about Markov chains.

Enno Mammen (Heidelberg)

Empirical processes of residuals in high–dimensional linear model

In this talk we consider the problem of estimating the error distribution in a linear model. A possible estimator is the empirical distribution of residuals. To study this estimator we use an asymptotic approach where the dimension of the model may converge to infinity. This approach is appropriate for many applications in which the dimension is large compared with the number of observations.

It turns out that the asymptotic behaviour of the empirical distribution of residuals depends heavily on the used estimator of the parameter. In particular, if one uses an M–estimator whose ψ–function is motivated by a distribution function G , then the empirical distribution is biased towards G . For very high dimensions this effect is larger then the stochastic fluctuations. Therefore the statistical analysis may reproduce the assumptions imposed.

David Donoho (Berkeley)

Wavelets and function estimation

1. Renormalization for nonlinear cases.

Consider the white noise model $Y(dt) = f(t)dt + \epsilon W(dt)$, $t \in \mathbb{R}^d$. We are interested in the functional $T(f)$ and know a priori that $f \in \mathfrak{F}$. The difficulty of estimating T(f) is the minimax risk

$$R^*(\epsilon) = \inf_{\hat{T}} \sup_{\exists} \mathbb{E}(\hat{T}(Y) - T(f))^2 .$$

Under natural scaling hypotheses on T and $\mathfrak{F}$, it can easily be derived that

$$R^*(\epsilon) = R^*(1)\,(\epsilon^2)^r$$

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with an r deriving from the scaling arguments. If $T(a\, f(b\cdot)) = a^{d_3}\, b^{e_3}\, T(f)$ and $(U_{a,b}\, f)(t) = a\, f(b\, t)$, $U_{a,b}\, \mathfrak{F} = \mathfrak{F}$ if $a\, b^{e_1} = 1$. Then $r = (d_3 \cdot e_1 - e_3)/(e_1 + d_2)$. This result predicts successfully the optimal rate of convergence in nonlinear problems such as estimation of mode, of location of zeros, boundary of convex sets and horizons in images. Joint work with Charles Koopgrise and Marie Low.

2. Wavelets.

A computer program has been developed with the following properties: You supply a data file. It should consist of $y_i = f(t_i) + z_i$, $i=1,\dots,2^n$, $z_i \overset{i.i.d.}{\sim} N(0,1)$, f unknown. The programm transforms the data by an empirical wavelet coefficients with an adaptively–chosen nonlinear shrinker, the inverse wavelet transform supplies a function estimate $\hat{f}$ with several asymptotic optimality properties. In particular the estimate is simultaneously asymptotically within a few percent of minimax over all Besov spaces' balls $B_{s,p,g}(c)$ with $s > s_0$, $c \in (0,\infty)$. Joint work with Iain Johnstone; with appreciations to Diminique Picard, Catherine Laredo, Gerard Kerkyacharan.

Peter Green (Bristol)

Stochastic algorithms for some inverse problems in medical imaging

A class of large nonlinear inverse problems is considered,such as arise in certain medical imaging problems, including emission and transmission tomography, spatial epidemological modelling, and remote sensing, etc. The Bayesian paradigm provides an appealing framework for the regularisation that is necessary, using Gibbs distributions as priors. In the emission tomography context, for example, the data are generated by a Poisson linear model, and a non–Gaussian prior is needed to model the true isotope concentration as generally smooth but with some steep transitions.

Standard linear methods are thus inappropriate. As an alternative to deterministic methods built on EM algorithm for the maximum a posteriori reconstruction, we consider Markov chain simulation algorithms. Hastings' generalisation of the Metropolis method, with a carefully chosen Gaussian proposal distribution, produces sample reconstructions converging rapidly to the posterior distribution. This approach also allows estimation of the posterior distributions of arbitrary functionals of the isotope concentration, so that flexible inference is possible.

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Joe Whittaker (Lancaster)

Kalman filtering: a Rake's Progress

Conditional independence graphs in the sense of Darroch, Lauritzen and Speed and their generalisations to directed acyclic graphs are reviewed. The decomposability of the moralised graph is a key feature of message passing algorithms. The normal linear dynamic model of Harrison and Stephens has a graph with the topology of a (garden) rake and Kalman filtering can be regarded as a progression over the prongs: A generalisation is a non–Gaussian state space that leads to a non–linear filter which follows the same progression. Kalman filtering appears to be succesful because the moralized rake is decomposable.

Bernd Streitberg (Hamburg)

Cumulants and interactions

The joint cumulant k of a random vector $X = (X_1,\ldots,X_n)$ with $\mathbb{E}|X_i|^n < \infty$ can be characterised by five properties:

(1) $k(X_1,\ldots,X_n)$ is symmetric

(2) $k(X_1,\ldots,X_n)$ is multilinear

(3) $k(X_1,\ldots,X_n)$ is a moment functional

(4) The coefficient of $\mathbb{E}(X_1,\ldots,X_n)$ in k is unity

(5) $k(X_1,\ldots,X_n) = 0$ if $(X_1,\ldots,X_m)$ independent of $(X_{m+1},\ldots,X_n)$

An analogous construction can be done for measures P on a product measurable space, giving a new intergral representation of cumulants. The basic technical tool is a characterization of the Moebius function of a finite lattice via solving a simultaneous eigenproblem for all intersection functions $k_\tau = k_\tau(\beta,\gamma) = \{\beta = \gamma \Lambda \tau\}$.

Berichterstatter: E. Uhrmann–Klingen

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