MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 14/1991

Gewöhnliche Differentialgleichungen
24.03. bis 30.03.1991

The conference was organized by H. Knobloch (Würzburg), J. Mawhin (Louvain-la-Neuve), and K. Schmitt (Salt Lake City).

As on previous occasions the conference focused upon one area of current research interest in ODE. We had chosen

INVARIANT MANIFOLDS
and
Applications

as the conference title, hoping to bring together many of the important contributors to this exciting and extremely active area of research. The list of names to follow and the abstracts of lectures given show that we very much succeeded in our goal.

Forty scientists from ten different countries followed the Institute's invitation to attend the conference. There were a total of 30 lectures presented during the week. Most of the lectures were concerned with qualitative as well as quantitative aspects of Invariant Manifold Theory, and several presented applications of this theory to specific sets of evolutionary equations which are used as models of certain physical situations.

Several of the lectures addressed the important problems of the existence of finite dimensional dynamical systems whose flow already determines that of a given infinite dimensional system. Perturbation theory of invariant manifolds as well as "global" numerical schemes for their computation were topics of considerable interest. Applications to control theory were also present as were lectures on Hartman - Grobman type results and several talks on semilinear elliptic problems and their solution and stability structures.

In addition there was a lecture given by one of the organizers (Mawhin) on the history of the Tagung "Gewöhnliche Differentialgleichungen" and the important contributions which were first presented there.

Many of the lectures generated intense discussions which, after the lecture periods, were continued, in the Oberwolfach tradition, well into the evening hours.

We believe that the meeting was an excellent one, which by a large measure is also due to the outstanding service provided by the Institute's administration and staff.

The meeting was adjourned at 12:05 p.m., on Friday, March 29, 1991.

Vortragsauszüge

B. Aulbach. *Linearization and decoupling for noninvertible mappings.*

The talk deals with the question whether a mapping $f : \mathcal{B} \to \mathcal{B}$ on an arbitrary Banach space $\mathcal{B}$ is topologically conjugate near a fixed point to a linear mapping. For invertible f this problem is well understood, however, if f is noninvertible (e.g. as time-one map of an evolution equation) the linearization problem has not been touched yet in this abstarct setting. It is demonstrated by means of examples, that one can neither expect complete linearization nor decoupling. Results on partial linearization and decoupling are presented.

P. Bates. *Invariant manifolds for Cahn-Hilliard and Phase-Fields equations.*

The Cahn-Hilliard equation

$$(CH)\begin{cases} -\Delta(\epsilon^2\Delta u + f(u)) = u_t, & \text{in } \Omega \\ \frac{\partial u}{\partial n} = \frac{\partial \Delta u}{\partial n}, & \text{on } \partial\Omega. \end{cases} \qquad (1)$$

and the Phase-Field system

$$(PF)\begin{cases} \epsilon^2\Delta\phi + f(\phi) = \tau\phi_t, & in\ \Omega \\ \kappa\Delta\phi = (\theta + \frac{l}{2}\phi)_t, & in\ \Omega \\ \frac{\partial\phi}{\partial n} = \frac{\partial\theta}{\partial n}, & on\ \partial\Omega, \end{cases}$$

with f bistable, are models for phase transitions which can be usefully viewed as dynamical systems. In work with S. Zheng (PF) is shown to have an inertial manifold when $n = 1$ or $n = 2$. In joint work with N. Alikakos and G. Fusco (CH) is shown to have invariant manifolds of metastable states when $n = 1$. Both systems have conserved quantities and even when $n = 1$, the common set of steady states with a given value of this quantity is unknown. The structure of this set together with the stability of these points is conjectured and certain aspects verified in joint work with P. Fife.

F. Battelli. *Chaos and heteroclinic orbits in Nonlinear systems.*

This is joint work with K. Palmer (Miami). We consider the forced Duffing equation

$$z'' + a^2 \sin z = \sin t \quad (1).$$

It is known that, for any positive integer N, there exists a_N such that for $a > a_N$, equation (1) has at least N $2\pi-$ periodic solutions. This suggests that, for large a, the Poincaré map of (1) has a transverse homoclinic point (or heteroclinic cycle). Rewrite (1) as:

$$z'' + a^2 g_0(z) = f(t) \quad (2)$$

and assume that $z'' + g_0(z) = 0$ has a heteroclinic orbit. We show that, if $f(t)$ is T - periodic and has a simple zero, then, for $a >> 1$, equation (2) has a bounded solution whose corresponding variational system has an exponential dichotomy on R. As a consequence the Poincaré map of (2) has a transverse homoclinic orbit.

J. Bebernes. *Final time blow-up profiles for semilinear parabolic equations via center manifold theory.*

Consider the semilinear parabolic equation

$$u_t = \Delta u + f(u), \quad in \quad R^n \times (0, \infty),$$

where $f(u) = e^u$, or $f(u) = u^p$, $p > 1$. For any initial data that is a positive radially decreasing lower solution which causes the corresponding solution

$u(x,t)$ to blow up at $(0,T) \in R^n \times (0,\infty)$, we prove by using techniques from center manifold theory that the final time blow-up profiles satisfy:

$$u(x,T) = -2ln|x| + ln|ln|x|| + ln8 + o(1), \; f(u) = e^u,$$

and

$$u(x,T) = \frac{8\beta^2 p|ln|x||^\beta}{|x|^2}(1+o(1)), \;\; f(u) = u^p,$$

where $\beta = \frac{1}{p-1}$.

P. Brunowský. *The center manifold of a homoclinic.*

Assume that 0 is a hyperbolic equilibrium of the differential equation

$$x' = f(x,\mu), \;\; x \in R^n, \;\; \mu \in R^p,$$

admitting a homoclinic trajectory $\{h(t)\}_{t=-\infty}^{\infty}$ for $\mu = 0$. Further assume that $h(t)$ is tangent to the eigenvectors of the leading negative (positive) eigenvalue of $D_x f(0,0)$ for $t \to -\infty$ ($+\infty$, respectively). Then, for the augmented system

$$\begin{aligned} x' &= f(x,\mu) \\ \epsilon' &= 0 \end{aligned}$$

there exists a $p+2$ dimensional locally invariant manifold containing the homoclinic as well as all trajectories which stay in some neighborhood of the homoclinic for all times.

J. Carr. *Applications of invariant manifolds to metastable patterns.*

We study the extremely slow evolution of patterns in solutions of

$$u_t = \epsilon^2 u_{xx} - f(u), \; 0 < x < 1,$$

subject to Neumann boundary conditions and where for example $f(u) = u^3 - u$. The metastable states are characterised in terms of the global unstable manifolds of equilibria.

W. Everitt. *Nonlinear quasi-differential control systems.*

(Joint work with L. Markus, Minnesota.) The linear control system $x' = Ax + Bu$, *on* $I \subset R$ is known to be fully controllable on I when $A, B \in L^1_{loc}(I)$ if A generates a quasi-differential equation of order $n \geq 2$ on I and $B = (b_{rs})$, *with* $b_{rs} = 0$ $(r = 1,2,...,n-1;\; s = 1,2,...,m)$ and $\sum_{s=1}^m |b_{ns}(t)| > 0$, *pp* $t \in I$. This result is used to prove that the nonlinear control system $x' = f(t, x, u)$ is fully locally controllable at the point $(0, 0, 0,)$ if, when the matrices $A(t), B(t)$ are defined by

$$A(t) := \frac{\partial f}{\partial x}(t, 0, 0), \quad B(t) := \frac{\partial f}{\partial u}(t, 0, 0),$$

A, B satisfy the above conditions for control of the linear system. Additional conditions are required on f to ensure that solutions of the differential equation $x' = f(t, x, u)$ are evrywhere defined on the $t-$interval over which the control is taking place.

B. Fiedler. *An index for periodic orbits of time-reversible systems.*

Natural homotopy invariants for time-periodic solutions of autonomous systems can be based on the Brouwer fixed point index of the iterates of the associated Poincaré map Π. This is the underlying idea of the Fuller degree. Generically, however, the fixed point indices of the iterates are either identical or just alternate in sign. For reversible periodic orbits of reversible systems, we present a sequence of homotopy invariants which does not suffer from such a constraint. As a consequence, we indicate some results in analogy to Hamiltonian systems although variational structure is not known. A particular example are Neumann boundary value problems for second order systems

$$u'' + g(u, u'), \quad u \in R^n,$$

where g is even with respect to u'. The results are joint work with S. Heinze (Heidelberg and Georgia Tech.)

D. Flockerzi. *Parameter dependence of invariant tori.*

Given a smooth system of ordinary differential equations

$$x' = \epsilon[Ax + g(x, \phi, \epsilon) + \lambda\hat{g}(x, \phi, \epsilon)], \ x \in R^n,$$

$$\phi' = \omega + \lambda\hat{\omega} + \epsilon[h(x, \phi, \epsilon) + \lambda\hat{h}(x, \phi, \epsilon)], \ \phi \in \Pi^k,$$

with parameters $\lambda \in [0, 1]$, and $\epsilon > 0$, we give conditions for the existence of invariant tori of the form

$$M_{\lambda,\epsilon} = \{x = s(\phi, \lambda, \epsilon) : \phi \in \Pi^k\}$$

with s being smooth with respect to ϕ and with

$$\|s(., 0, \epsilon)\|_{C^j} \leq \chi_j \|g(0, ., \epsilon\|_{C^j}, \ j = 0, 1, 2,$$

for positive constants χ_j not depending upon $\epsilon \in (0, \epsilon_0]$. As the example

$$x' = \epsilon[-x + \sigma \cos \phi], \quad \phi' = \lambda \hat{\omega}$$

shows, the distance between $M_{1,\epsilon}$ and $M_{0,\epsilon}$ need not be small uniformly on $(0, \epsilon_0]$. We investigate in which precise sense the perturbations $\hat{g}, \hat{\omega}, \hat{h}$ have to be small for $M_{0,\epsilon}$ to be a uniformly close approximation to $M_{1,\epsilon}$ on $(0, \epsilon_0]$.

J. Gossez. *Characterization of nonresonance for some semilinear periodic problems.*

(Joint work with P. Omari, Italy.) We consider the problem

$$\begin{cases} -u'' &= g(u) + h(t), \quad in \;\; [0, \pi] \\ u(0) &= u(2\pi), \quad u'(0) = u'(2\pi), \end{cases}$$

where g is continuous on R and h is bounded on $[0, 2\pi]$. It is assumed that the nonlinearity g interferes at most with the first eigenvalue $\lambda_1 = 0$ of the associated linear problem. Precisely:

$$\limsup_{\pm\infty} \frac{g(s)}{s} \leq \lambda_2, \quad \limsup_{\pm\infty} \frac{2G(s)}{s^2} < \lambda_2,$$

where G is the primitive of g. We then show that a necessary and sufficient condition for the problem to be solvable for any forcing term h is that the function g be unbounded from above and from below. Jumping nonlinearities could also be considered.

M. Jolly. *Dissipativity of numerical schemes.*

We show that the way in which the finite differences are applied to the nonlinear term in certain PDEs can mean the difference between dissipation and blowup. For fixed parameter values and arbitrarily fine discretizations we construct solutions which blow up in finite time for semi-discrete schemes. We also show the existence of spurious steady states whose unstable manifolds, in some cases, contain solutions which explode. This connection between the blowup phenomenon and spurious steady states is also explored for Galerkin and nonlinear Galerkin semi-discrete approximations. Two fully discrete finite difference schemes derived from a third order semi-discrete scheme, shown in Foias and Titi (1990) to be dissipative, are analyzed. Both latter schemes are shown to have a stability condition which is independent of the initial data. A similar result is obtained for a fully discrete Galerkin scheme.

While the results are stated for the Kuramoto - Sivashinsky equation, most naturally carry over to other dissipative PDEs.

C. Jones. *Tracking invariant manifolds during the passage near a slow manifold.*

In singular perturbation problems, one is often interested in constructing solutions that pass near a slow manifold. For instance, an unstable manifold of some remote critical point may be "shot" at a slow manifold and the relevant issue is to determine the configuration of the manifold as it emerges from a neighborhood of the slow manifold. In joint work with N. Kopell, flows are induced using differential forms from the equation of variations that lead to the "Exchange Lemma." Fenichel's Invariant Manifold Theorems play a crucial rôle in producing a canonical form of the equation near a slow manifold.

H. Kielhöfer. *Nodal patterns of global bifurcation branches.*

For certain classes of quasilinear elliptic problems with homogeneous Dirichlet boundary conditions the nodal pattern of some eigenfunctions of the linearization at the bifurcation point is globally preserved along bifurcation branches. These patterns enjoy some specific symmetries. When embedded into the fixed point subspaces of their symmetry (isotropy subgroups) these branches are actually smooth curves such that secondary bifurcation is necessarily symmetry breaking.

U. Kirchgraber. *Invariant manifolds in the numerics of ODEs.*

This is a report on the ongoing research of D. Stoffer, K. Nipp and the author. We consider some aspects of the numerics of ODEs from a qualitative point of view, in particular in the light of invariant manifold theory (IMT). There are three types of results. The first is concerned with the relation between an ODE and its discretization. Consider a set of weakly coupled harmonic oscillators admitting an attracting invariant torus. We study the relation between the step size h and the strength ϵ of the coupling under the condition that the discretized system admit a companion torus. It turns out that most methods enforce $h \to 0$ as $\epsilon \to 0$; the only exceptions are so-called linearly canonical schemes. For these methods there is no relation between h and ϵ! As a second application of IMT to numerics we show that for a large class of step-size estimates the step size (asymptotically) becomes independent of the previous choice of the step size. The key to this result is the existence of a certain invariant manifold. The result means that many step size estimators are actually of a simpler nature than it appears. Finally we apply IMT to design efficient algorithms for the integration of certain FDEs and singularly perturbed ODEs. Connected with this is the design and study of efficient algorthims to compute (low dimensional) invariant manifolds.

H. Knobloch. *Invariant manifolds in control theory.*

Part 1. Extension of standard existence theorems for invariant manifolds (imf). The differential equation is written in the form

$$x' = g(t, x, y), \quad y' = h(t, x, y), \quad (1).$$

Main assumption: Dichotomy and exponential gap for the variational equation along solutions. The imf. is given in terms of an equation $y = S(t, x)$ and is uniquely determined by the natural boundary condition ('inflowing') and the initial condition $S(0, x) = s(x)$. *Part 2.* Application: Inverse problems in control theory. Given a solution $x(t), y(t)$ of (1) with known initial state and known approximation $\eta(t)$ for $y(t)$ (plus estimate) $\|y(t) - \eta(t)\| \leq \eta,\ t > 0$. The auxiliary system

$$\tilde{x}' = g(t, \tilde{x}, \tilde{y}), \quad \tilde{y}' = h(t, \tilde{x}; \tilde{y}) - \frac{1}{\epsilon}(\tilde{y} - \eta(t)),$$

satisfies all conditions imposed on (1) if the parameter $\epsilon > 0$ is small enough. Aprediction - correction scheme for estimating $y'(t)$ based on the data of the imf. is presented.

R. Lauterbach. *Forced symmetry breaking and heteroclinic cycles.*

We study the dynamics near a manifold of equilibria of equivariant equations if we perturb the symmetry. To be more precise, let $x' = g(x, \lambda)$ be equivariant with respect to a group G and suppose we perturb this equation to $x' = g(x, \lambda) + \epsilon h(x, \lambda)$, where h is equivariant with respect ot a subgroup K of G. Assume that M is a normally hyperbolic manifold of equilibria of the unperturbed equation. If M is a single group orbit all points on this manifold M have the same isotropy type σ, let H be a subgroup of G in σ. Due to normal hyperbolicity, for sufficiently small ϵ, there will be an invariant manifold M_ϵ near M. The flow on this manifold has a structure which is to some extend determined by the pair (K, H). We give examples of these flows for problems with spherical symmetries. A theorem of Schwarz on lifting properties of flows allows one to construct flows which have precisely the features which are predicted by the general theory. Especially we observe heteroclinic cycles.

N. Lloyd. *Cubic systems and conditions for a center.*

Hilbert's sixteenth problem is concerned with the number of limit cycles of classes of systems

$$x' = P(x,y), \quad y' = Q(x,y),$$

where P, Q are polynomials, and their possible configurations. In recent investigations, limit cycles which bifurcate from a critical point have been considered. To maximize the number of such limit cycles, the focal values at the critical point (say $x = 0 = y$) are computed. These are polynomials in the coefficients in P and Q such that there is a function V with $V' = \sum \eta_{2k} r^{2k}$, where $r^2 = x^2 + y^2$. It is necessary to know the conditions under which the origin is a center. Necessary and sufficient conditions for a center are known for various classes of systems. In this talk such conditions will be presented for some classes of cubic systems (i.e. systems for which P and Q are of degree three). Sufficiency is proved by sytematically searching for invariant algebraic curves and using them to transform the system to Hamiltonian form. The necessity of the conditions is proved by computing the variety of the ideal generated by the focal values. Particular attention will be devoted to the system

$$x' = y, \quad y' = -x + a_1 x^2 + a_2 xy + a_3 y^2 + a_4 x^3 + a_5 x^2 y + a_6 xy^2 + a_7 y^3.$$

Necessary and sufficient conditions for a "persistent" center will be given. Their derivation involves large scale use of computer algebra, and some of the computing difficulties will be described.

K. Lu. *Invariant manifold theory and invariant foliation theory and structural stability for parabolic equations.*

By using invariant manifold and foliation theory we prove that the flow nearby hyperbolic equilibria of parabolic equations is structurally stable.

A. Mielke. *Invariant manifolds for variational and Hamiltonian systems.*

We consider center, stable and center-stable manifolds for an equilibrium in a Hamiltonian system. It follows that the reduced system on the center manifold is again a Hamiltonian system. The flow on the (center-) stable manifold is (co-) isotropic. Moreover, we show that in most cases the center manifold flow of a variational (Lagrangian) problem can be described by a reduced variational problem. This leads to applications in the theory of elliptic variational problems on cylindrical domains, such as the beam problem of elastostatics. We show that all small deformations of a hyperelastic beam (3 dim. prismatic body) can be described by a hyperelastic rod model in a mathematically rigorous way.

V. Pliss. *Perturbations of attractors of differential equations.*

(Joint work with G. Sell, Minnesota.) Consider the system

$$\frac{dx}{dt} = X(x) + \mu Y(x), \quad (1)$$

where $x \in R^n$ and $X, Y \in C^1(R^n)$. *Definition:* An attractor $\mathcal{K}$ of (1) with $\mu = 0$ is normally hyperbolic, if: (i) There exist constants $a \geq 1, \lambda_1 > 0, \lambda_2 < \lambda_1$, and linear spaces $U^s(t, x_0), U^u(t, x_0), x_0 \in \mathcal{K}, dimU^u = k, dimU^s = n - k$, and if $\bar{x} \in U^s(\tau, x_0)$, then $|\Phi(t, x_0)\Phi^{-1}(\tau, x_0)\bar{x}| \leq a|\bar{x}|e^{-\lambda_1(t-\tau)}$, for $t \geq \tau$, and if $\bar{x} \in U^u(\tau, x_0)$, then $|\Phi(t, x_0)\Phi^{-1}(\tau, x_0)\bar{x}| \leq a|\bar{x}|e^{-\lambda_2(t-\tau)}$, for $t \leq \tau$, where Φ is a fundamental matrix solution of the system

$$\frac{dx}{dt} = \frac{\partial X(x(t, x_0))}{\partial x} x.$$

(ii) There exists an $r > 0$ such that for $x_0 \in \mathcal{K}$, ther exits a $k-$ dimensional disk $\mathcal{D}(x_0) \subset \mathcal{K}$ with center at x_0 and radius r, locally invariant and such that if $x \in \mathcal{D}(x_0)$, then the disk $\mathcal{D}(x_0)$ is tangent to $U^u(0, x_0)$ at x. *Theorem:* Let $\mathcal{K}$ be a normally hyperbolic attractor and assume that $U^u(0, x_0)$ is a Lipschitz continuous function on $\mathcal{K}$. For every $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < \mu < \delta$, then there exists a homeomorphism h of $\mathcal{K}$ into R^n, $|hx - x| < \epsilon$ and $\mathcal{K}_\mu = h\mathcal{K}$ is a normally hyperbolic attractor of (1).

R. Schaaf. *Semilinear elliptic problems with supercritical growth.* The equation

$$\Delta u + \lambda f(u) = 0, \quad in \quad \Omega \subset R^n, \quad u = 0, \quad on \quad \partial\Omega, \quad (1)$$

can be viewed as a pseudo steady state equation of a combustion process, where u is temperature, $\lambda \in R^+$ is the amount of unburned substance. The question about the number of solutions of (1) and their stability properties is then of interest for the time dependent process. In this context nonlinearities arise which grow supercritically for u large:

$$\limsup_{s \to \infty} \frac{(p_c + 1)F(s)}{s} < 1, \quad p_c = \frac{n+2}{n-2}, n \geq 3. \quad (A)$$

If the inequality in (A) holds for all $s \in R$ then no nontrivial solutions of (1) exists in a starshaped domain, a consequence of the Rellich - Pohozaev identity. Even if f is sufficiently well behaved near 0 and a positive solution branch exists, the shape of this branch can change rather drastically with the dimension n of the domain Ω. A famous example for this is the result of Joseph and Lundgren for $f(u) = e^u$ or $f(u) = (1 + \alpha u)^p$ and Ω a ball. A common feature of all of these examples is that in the superlinear but subcritical case there always exist at least two solutions for $0 < \lambda < \bar{\lambda}$, one stable and the other unstable, whereas in the supercritical case, there exists a $\lambda^* > 0$, such that (1) has a unique solution for $0 < \lambda < \lambda^*$, which is stable. We can show that this uniqueness for small λ always holds if: (a) $f(0) = 0, f'(0) > 0$, Ω *starshaped*, f *satisfies* (A), or (b) $f(0) > 0$, Ω *convex*, f *satisfies* (A), $f'' > 0$. However in the case $f'' > 0$ we can show that (1) admits at most one stable solution for any fixed λ, proving that among the multiple solutions which are present in the Joseph - Lundgren examples only the minimal solution is stable.

K. Schneider. *Perturbations of differential algebraic equations.*

The equation (*) $F(y, y', t) = 0$ is said to be a differential algebraic equation (DAE) if $dim\ ker F_{y'}(y, y', t) > 0$, $\forall (y, y', t) \in dom F$. Under certain assumptions a vector field v on some smooth manifold can be associated with (*). Such a DAE is called a regular DAE with degree d. The lecture addresses perturbations of autonomous DAE $F(y, y', \lambda) = 0$ depending on a parameter λ. We consider two types of perturbations. The first kind can be described by a perturbation of the corresponding vector field v. By this way, bifurcations of DAE can be described by bifurcations of vector fields. The second type of bifurcations can be characterized by singular perturbations, that is, a bifurcation point is connected with the change of the degree of the DAE. We give an approach to establish the bifurcation of relaxation oscillations which is based on the splitting of an invariant manifold at the "jumping point" which separates a stable and an unstable part of this manifold.

J. Scheurle. *Construction of invariant manifolds by the deformation method.*

The construction of invariant manifolds always leads to equations which have to be solved in one way or another. Usually one uses some version of the contraction mapping principle. However, there are other possibilities, e.g. the deformation method which has certain advantages. In this talk, I want to describe the basic ideas of this method by considering just the case of center unstable invariant manifolds. For details and remarks about other types of invariant manifolds I refer to the article by J. Marsden and myself in SIAM J. Math. Anal. 18(1987), 1261-1274. For simplicity, we consider maps rather than differential equations. But everything can be done for differential equations as well.

G. Sell. *Approximation dynamics and the Navier-Stokes equations.*

During the last few years several researchers have developed techniques for constructing *Approximate Inertial Manifolds* (AIMs) for studying the dynamics of nonlinear evolutionary equations

$$(1)\quad u' + Au = F(u).$$

We have shown that, in the case of the 2D Navier-Stokes equations, every such AIM is an actual inertial manifold for a perturbed equation

$$(2)\quad u' + Au = F(u) + E(u),$$

where the term E is known and has a suitable norm $\|E\|$ which tends to zero as the dimension of the AIM goes to ∞. In a different direction, we give a brief report on the University of Minnesota PhD thesis of M. Kwak, wherein he shows that the long time dynamics of the 2D Navier-Stokes equations (with periodic boundary conditions) can be described completely (and with no error) by a finite dimensional system of ordinary differential equations. Kwak introduces a nonlinear change of variables which regularizes the nonlinear term in the Navier-Stokes equations and reduces the problem to a suitable system of reaction diffusion equations.

R. Smith. *Poincaré - Bendixson theory for autonomous retarded functional differential equations.*

For autonomous functional differential equations, expressed in feedback control form, new sufficient conditions are discussed for the absence of chaotic motion and the existence of at least one stable periodic trajectory. The results are obtained by finding an invariant set in Banach space which is homeomorphic to a plane set. Application to the delayed Goodwin equation is mentioned.

D. Ulmet. *Nonlinear electrical circuits. Singularities and periodic solutions.*

We analyze some types of singular points that occur in nonlinear electrical circuits and their effect on the dynamics of such systems. First we show that global singularities (or forced degeneracy) can be reduced to isolated singularities, which are related to relaxation oscillations. Afterwards we address some problems that center around the regularization techniques that are available for the study of the isolated singular points. They are based formally on the addition of small parasitic inductances l and capacitors c to the circuit N so that the augmented system $N(l, c)$ is regular. We consider a regularization version which provides a gradient type function, in the sense that the averaged equations for the action variables form a gradient system. By an example of the coupled van der Pol oscillators, we show that we can obtain periodic solutions instead of expected invariant 2 tori for the full system. Finally the dynamics of the original system N is essentially obtained as the limit as $l, c \to 0$ of the dynamics of $N(l, c)$. Several questions concerning the structural stability of N are open problems.

A. Vanderbauwhede. *Center manifold theory in infinite dimensions.*

We describe some joint work with G. Iooss (Nice) on center manifolds in infinite dimensional systems. We first show how the existence of a pseudo inverse for a linear operator associated with the hyperbolic part of the equation and on a space of exponentially weighted functions leads to the existence and smoothness of center manifolds. In specializing to the local situation we pay special attention to the cut-off problem usually associated with the local center manifolds in Banach spaces. Next we describe a set of spectral properties which imply the existence of the pseudo inverse mentinoed above; these properties are weaker than those required for the existence of an analytic semigroup, and allow for example the treatment of certain elliptic problems. We conclude with some simple examples.

P. Volkmann. *Über die Invarianz linearer Mannigfaltigkeiten in Banachräumen - ein Gegenbeispiel.*

Es sei E_1 ein (abgeschlossener) Unterraum eines Banachraumes E, es sei $f: E \to E_1$ stetig und beschränkt, $f(E_1) \subset E_1$, und es werde vorausgesetzt, daß das Anfangswertproblem

$$u(0) = a, \quad u' = f(u)$$

für jedes $a \in E$ eine (lokal) eindeutige Lösung $u_a : [0, \infty) \to E$ besitze. An Hand eines Beispiels wird gezeigt, daß dann nicht notwendig $a \in E \Rightarrow u_a(t) \in E$ $(t \geq 0)$ gilt.

W. Walter. *Discretization of a parabolic free boundary problem.*

The following model occuring in models of groundwater contamination and statistical decision theory, among others, is discussed:

$$(P)\begin{cases} u_{xx} &= u_t + f(t,x,u,u_x), \quad for \;\; 0 < t \le T, \;\; 0 < x < s(t), \\ Bu &= au(t,0) - bu_x(t,0) = \alpha(t), \quad for \;\; 0 < t \le T, \\ u(t,s(t)) &= u_x(t,s(t)), \quad for \;\; 0 < t \le T. \end{cases}$$

Here $a, b \ge 0$ with $a + b = 1$; the free boundary is given by $x = s(t)$. Basic results: (P) is a well posed problem, i.e., existence, uniqueness plus continuous dependence of the solution (u, s) on f and α hold. A monotonicity theorem (comparison principle) states roughly that if: f is made smaller and α larger, then u and s become larger. Existence is proved by applying the Rothe method (discretization in t which leads to a problem)

$$(P_N)\begin{cases} u_n'' &= \delta u_n + f(t_n, x, u_n, u_n'), \quad in \;\; 0 < x < s_n, \\ Bu_n &= \alpha_n, \quad u_n(s_n) = u_n'(s_n) = 0, \quad n = 1, 2, ..., N. \end{cases}$$

Here $u_n(x) \approx u(t_n, x), t_n = hn, h = \frac{T}{N}, \delta u_n = \frac{u_n - u_{n-1}}{h}$.

H. Walther. *Unstable manifolds for* $x'(t) = -\mu x(t) + f(x(t-1))$.

Suppose $f(0) = 0, f' < 0$, and f is bounded from below or from above. If the zero solution is linearly unstable, then the global unstable set associated with the leading pair of eigenvalues is a smooth, Lipschitz graph of dimension 2 whose boundary is a periodic orbit. Also, unstable sets of hyperbolic periodic solutions, which are slowly oscillating, are smooth Lipschitz graphs of dimension 2; the boundaries consist of 2 periodic orbits (one of them may be 0).

F. Zanolin. *Time maps for the solvability of nonlinear equations.*

The solvability of various boundary value problems for a class of ordinary differential equations is obtained under suitable conditions on the time maps of associated autonomous equations. As a typical application, consider the second order nonlinear Duffing equation $x'' + g(x) = p(t)$, with the nonlinear term g satisfying the asymptotic condition $\lim_{|x|\to\infty} g(x) sgnx = +\infty$. For the associated autonomous equation $x'' + g(x) = 0$, we can define the following time maps: $T^+(c), [T_-(c')], T(c)$ which are, respectively, the distance of two consecutive zeros of a solution $x(t)$ of the autonomous equation with $\max\{x(.)\} = c >> 0, [with \;\; \min\{x(.)\} = c' << 0]$ and the minimal period of $x(.)$, with $\max\{x(.)\} = c >> 0$. Then the solvability of various BVPs associated to the nonautonomous Duffing equation (e.g., the periodic, two point, subharmonic, BVPs), is obtained via asymptotic conditions on the time maps. The classical superlinear - semilinear - sublinear conditions on g are thus improved.

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