MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 15/1991

Arbeitsgemeinschaft zu einem
aktuellen Thema:

Connections Between Mathematical Physics (Quantum Groups, von Neumann Algebras) and the Theory of Knots.

31. März - 6. April 1991

Leitung: Prof. V.G. Turaev, Université Louis-Pasteur, Strasbourg

In recent years a number of fundamental ideas and methods of mathematical physics has penetrated through psychological barriers between physics and topology. In knot theory this development was initiated by V. R. F. Jones who used von Neumann algebras to construct a new polynomial invariant of knots and links in the 3-dimensional sphere S^3. Jones' discovery gave impetus to an enormous development in knot theory, 3-dimensional topology and related domains. This development was the main subject of the meeting.

Since 1984 when Jones introduced his polynomial, the main line of attack was the explanation of the nature of this polynomial and its generalization, say, to links in other 3-manifolds. To the moment of writing, several different points of view on the Jones polynomial have been developed basing on various techniques coming from algebra and mathematical physics. Here is a short but impressive list of theories more or less directly involved in the subject: theory of quantum groups, conformal field theory in dimension 2, representation theory of symmetric groups and Hecke algebras, theory of exactly solvable models of statistical mechanics etc.

The Arbeitsgemeinschaft considered four different though related lines of study forming the main body of the theory:

I. The first and most algebraic approach stems directly from the original paper by Jones. It involves Temperley-Lieb algebras, Hecke algebras and their natural modifications due to Birman-Wenzl. The core of this approach is the theory of braid groups and their linear representations.

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II. The second approach is concerned with Witten's ideas, relating the Jones polynomial to conformal field theories in dimension 2 and Chern-Simons invariants. From the topological viewpoint, the important achievement of Witten is the inclusion of the Jones polynomial in a more general picture of topological quantum field theories.

III. This approach is based on models in statistical mechanics and the theory of quantum groups. A state sum model for the Jones polynomial was introduced by Kauffman. More general vertex models associated with R-matrices lead to other related poynomials. This line is crowned with a construction of the topological quantum field theory in dimension 3 extending the Jones polynomial (as predicted by Witten).

IV. The last approach uses another kind of state sum models producing topological invariants of knots and 3-manifolds. These are the so-called face models and simplicial models based on quantum $6j$-symbols associated with quantum groups.

Furthermore, the Arbeitsgemeinschaft considered various aspects of the subject which either have drawn considerable attention in the recent time or seem to be good starting points for further research. Of course, quite a number of interesting problems were left outside the schedule.

Abstracts of the talks

Serge Ochanine
The Jones Polynomial

This introductory talk presented the original construction of the Jones polynomial via von Neumann algebras. Main topics:
(1) Links and Braids, the Alexander-Markov reduction.
(2) Von Neumann algebras, II_1 factors, Jones' index theorem.
(3) Temperley-Lieb algebras and the trace invariant.

Uwe Kaiser
Hecke Algebras and the HOMFLY Polynomial

One can consider a certain class of finite dimensional (quadratic) group representations of braid groups factors through algebra representations of classical Hecke algebras $H_n(q) = KB_n/(\sigma_i^2 - (q-1)\sigma_i - q)$, where KB_n is the group algebra of the braid group B_n, K is a field, $q \in K$ and B_n is generated by $\sigma_1, \dots \sigma_{n-1}$; $(\sigma_i^2 - (q-1)\sigma_i - q)$ is the two-sided ideal generated by the indicated elements for all $1 \leq i \leq n-1$. Ocneanu proved for each $z \in K$ the existence of normed trace functions $tr : H_n(q) \to K$ which are compatible with the inclusions $H_n(q) \to H_{n+1}(q)$ and satisfy $tr(x\sigma_n) = ztr(x)$ for $x \in H_n(q)$. Thus, by Markov's theorem, there are maps $B_n \to K = \mathbb{C}(q,z)\sqrt{\frac{qz}{w}}, w = 1-q+z$ which induce a function on the set of the isotopy classes of links in S^3. A change of variables leads to the HOMFLY polynomial taking values in the Laurent ring $\mathbb{Z}[l^{\pm 1}, m^{\pm 1}]$ which turns out to be the universal linear skein invariant.

Wolfgang Müller
Modular Functors and Conformal Field Theories (CFT)

One important motivation for studying CFTs in physics comes from string theory where one considers strings (something $\cong S^1$) moving in some background manifold M. Whereas a point particle moving in M sweeps out a curve, a string evoluting in time sweeps out some Riemann surface Σ

From the physical point of view, it is highly plausible that physics should not depend on the parametrisation of the string but only on the conformal structure of Σ. So, thinking of the corresponding quantum mechanical theory, we define a CFT as a functor from the category $\mathcal{C}$ (defined below) to the category of Hilbert spaces. The objects of $\mathcal{C}$ are the 1-dimensional compact manifolds S_i and a morphism from S_0 to S_1 is a Riemann surface Σ with $\partial\Sigma = S_0 \sqcup S_1$. If $\mathcal{H}_S$ denotes the Hilbert space that is attached to a 1-manifold S, we postulate that $\mathcal{H}_{S_0 \sqcup S_1} = \mathcal{H}_{S_0} \otimes \mathcal{H}_{S_1}$. Furthermore, let Σ be a surface with exactly two boundary circles so that we have an operator $T_\Sigma : \mathcal{H}_{S_0} \to \mathcal{H}_{S_1}$ and let $\hat{\Sigma}$ be $\Sigma/(S_0 = S_1)$ then $T_{\hat{\Sigma}} = trace\, T_\Sigma$. From these axioms we get that the partition function Z_T of the theory is modular invariant. By a similar construction but attaching to each boundary circle a representation of some fixed goup G, one obtains the modular functor.

Maxim Kontsevich
Topological Quantum Field Theories (TQFT)

One defines TQFT as follows: Let $\mathcal{C}_d$ be the category whose objects are the d-dimensional oriented closed manifolds and whose morphisms are the $(d+1)$-dimensional bordisms. Then, a TQFT in $(d+1)$ dimensions is a $\otimes$-functor from $\mathcal{C}_d$ to the category of finite-dimensional vector spaces.

Some examples (and counterexamples) coming from physics were considered: the non-linear σ-model, Witten-Jones theory, Floer-Donaldson theory.

Finally, the equivalence of notions of Witten's (2+1)-dimensional TQFT and the modular functor coming from CFT was presented.

A. Szűcs
Kauffman's State Model for the Jones Polynomial

First part: The Kauffman bracket in knot theory.

(1) Reidemeister moves.

(2) Kauffman bracket $\langle K \rangle$ and its state model.

(3) $f[K](A) = \langle K \rangle \cdot (-A)^{-3w(K)}$ is the Jones polynomial if $A = t^{-\frac{1}{4}}$.

Application: Tait's conjecture is true, namely, for any two simple alternating links the numbers of crossings are equal.

Second part: The Kauffman bracket in graph theory and statistical mechanics.
Here, we have shown that the Kauffman bracket, the dichromatic polynomial of a graph, and the partition function of the Pott's model are essentially the same.

Jaap Kalkman
Jones-Witten Theory ((2+1)-dimensional Topological Quantum Field Theory (TQFT))

Also called Chern-Simons (CS) theory, since it is based on the CS functional given by $CS_k(A) = \frac{k}{4\pi}\int_M Tr(AdA + \frac{1}{3}A[A,A])$, where $k \in \mathbb{Z}$ and $A \in \Omega^1(M)\otimes\mathfrak{g}$, the space of connections on the bundle $M \times G$ (M a 3-manifold, G a compact Lie group).
The talk was divided in two parts: In the first part, some evidence was presented that the partition function of the CS-theory, $Z = \int_{\mathcal{A}} \mathcal{D}A \exp(iCS_k(A))$, gives topological invariants of 3-manifolds. This was done using a stationary phase approximation ($k \to \infty$). In the second part, it was shown that one can obtain the Jones polynomial by calculating $Z_L = \int_{\mathcal{A}} \mathcal{D}A\, W_L(A) \exp(iCS_k(A))$, where the Wilson-loop $W_L(A)$ is the trace of the holonomy along a link L. The computation was done using surgery properties of TQFTs and additional data from conformal field theories (WZW-models).

Alan Durfee
The Kauffman Polynomial and Birman-Wenzl Algebras

The Jones polynomial $V(t)$ of an oriented link can be defined in terms of the Kauffman bracket, an invariant of regular isotopy classes of unoriented link diagrams. The Kauffman polynomial $K(l,m)$ is a two-variable generalization of $V(t)$ which is similarly defined in terms of a two-variable generalization Λ of the bracket polynomial. The Kauffman polynomial is different from the HOMFLY polynomial $P(l,m)$, another two-variable generalization of $V(t)$. For instance, K is almost independent of the orientation of the link.
Just as $P(l,m)$ can be defined in terms of a trace on the Hecke algebra, so can $K(l,m)$ be defined as a trace on an algebra constructed by Birman and Wenzl.

Dirk Siersma
Peter Schauenburg
Categories of Tangles and Their Linear Representations

Tangles are local versions of links. They form a category which can be modified attaching certain additional structures (orientation, framing) to the tangles. R-matrices produce linear representations of these categories, i.e. covariant functors in the category of vector spaces. For links, this construction yields invariants generalizing the Jones polynomial without having to use the Alexander-Markov reduction. There are also relations with the HOMFLY and Kauffman polynomial.

The linear representations of the categories of tangles can be understood and constructed via ribbon quasitriangular Hopf algebras. The latter are algebras whose categories of representations have the same properties as the categories of tangles. Thus, functors from the categories of tangles (oriented and 'coloured') into the categories of representations of a ribbon quasitiangular Hopf algebra A can be constructed in a very natural way: The tensor product of tangles is mapped to the tensor product of representations, the braiding of tangles is mapped to the braiding of A-modules (induced by the comultiplication and by the quasitriangular structure on A, respectively), turning of the projection of some tangle corresponds to taking the dual of a representation (defined via the antipode of A). The notion of a ribbon Hopf algebra can be defined to amend some deficiencies of the natural dual representation.

Johannes Huebschmann
Poisson Brackets on Representation Spaces and Quantization

Let π be the fundamental group of a closed surface S and let G be a Lie group. In a series of papers Goldman introduced and examined certain symplectic structures on the representation space $\mathrm{Rep}(\pi, G) = \mathrm{Hom}(\pi, G)/G$, the G-action on $\mathrm{Hom}(\pi, G)$ coming from conjugation in G. These symplectic structures give rise to Poisson structures on a suitable smooth submanifold of $\mathrm{Rep}(\pi, G)$. Goldman also introduced certain Lie algebras of closed curves together with homomorphisms of Lie algebras into a Poisson algebra of the kind just mentioned.
Turaev introduced a structure of a non-commutative algebra on certain skein modules defined over $S \times I$, and he showed that these skein algebras furnish a quantization of the Lie-Poisson algebras over the Goldman Lie algebras mentioned before, in a suitable sense. In this way a rigorous notation of quantization of Wilson loop observables in Chern-Simons gauge theory over $S \times I$ is obtained.

Maxim Kontsevich
Higher Associativiy, Higher Invariants and Cohomology of the Moduli Space of Curves

Invariants of manifolds are functions from the set of manifolds to some field K. Higher invariants are proposed to be elements of $\bigoplus_{n=0}^{\infty} H^{2n}(B\,\mathit{Diff}(X), K)$ where X is a manifold. Usual invariants are just 0-components of higher invariants. It is possible to describe a machinery which gives higher invariants for the case of oriented surfaces with boundaries.
Let A be any finite-dimensional associative algebra with a non-degenerate scalar product $(\cdot, \cdot)$ such that $(xy, z) = (x, yz)$. Then, it is possible to define some invariant of surfaces with boundaries in the following way: Any such surface can be cut into pieces looking like a ribboned three-star. This defines a way to convolute the tensor of structure constants of A and the scalar product. There is some homotopy analogue, to associative algebras, the so-called A_∞-algebras. The main statement is that homotopy analogs of the notion of associative algebras with scalar product produce elements of the cohomology of the moduli space of curves with marked points.

Christian Kassel
The Yang-Baxter Equation and Quantum Groups

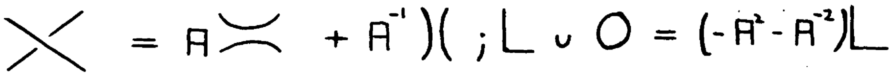
In this talk examples of R-matrices, i.e. solutions of the Yang-Baxter equation, were given. All these examples arise from representations of certain Hopf algebras A together with a specific $R \in ISO(A \otimes A)$. These Hopf algebras introduced by Drinfeld are called quasi-triangular Hopf algebras.
The case of the quantized universal enveloping algebra $U_q(sl(2))$ of $sl(2)$ was presented in some detail, along with its representation theory, and its universal R-matrix which can be obtained by Drinfeld's double construction (where a quasi-triangular Hopf algebra is associated to any Hopf algebra).

Tammo tom Dieck
Quantum Invariants of Three-manifolds

The talk was a report on the work of Reshetikhin and Turaev. The invariants are constructed via the following scheme:
(1) An oriented, connected, closed 3-manifold M can be constructed by Dehn-surgery on framed links in S^3. The Kirby moves tell under which conditions two framed links give the same manifold.
(2) The representation theory of a suitable quasi-triangular Hopf algebra yields a functor $\mathcal{F}$ from the category of ribbon tangles to the category of vector spaces.
(3) Under suitable additional conditions and by applying the functor $\mathcal{F}$ to the framed links in S^3, the invariant of the manifold is constructed. The problem is whether this invariant is invariant under Kirby moves.
(4) The additional conditions under (3) lead to the notion of a modular Hopf algebra. Specific modular Hopf algebras are constructed from the quantum group $U_q(sl(2))$ by specializing the generic parameter q to a root of unity.
The main intention of this work is to relate representation theory of quantum groups to geometry of 3-manifolds, links in 3-manifolds and topological quantum field theories.

G. Masbaum
A Construction of 3-Manifold Invariants from the Kauffman Bracket

Reshetikhin and Turaev have constructed new non-trivial invariants from any modular Hopf algebra (such algebras can be constructed from any classical Lie algebra). Recently, Lickorish has shown how to express these invariants, in the special case of $SU(2)$, in terms of Kauffman's one variable bracket



More precisely, if M^3 is obtained by surgery on a bounded link L in S^3 then the invariants can be expressed as linear combinations of Kauffman brackets of cablings of L where A is a primitive root of unity of order divisible by 4.

The talk presented recent work of C. Blanchet, N. Habegger, P. Vogel and the speaker who further developped Lickorish's approach. It was discussed how to find all those linear combinations of brackets of cablings which yield 3-manifold invariants. It was shown that A can also be a primitive root of unity of any even order, yielding new non-trivial invariants. However, no other evaluations are possible within this approach. (The invariants at roots of order $\equiv 2$ modulo 4 should correspond, in some sense, to quantum $SO(3)$.) The proof uses only Kirby calculus and elementary linear algebra but no representation theory of quantum groups or Temperley-Lieb algebras. Moreover, this approach gives an example of the explicit geometric meaning of *colors* and *Verlinde algebras*.

Johan van de Leur
Quantum 6j-Symbols

The representation theory of $U_q(sl(2))$ (quantum $sl(2)$) is discussed. For the finite dimensional irreducible representations V^j, $0 \leq j \in \frac{1}{2}\mathbb{Z}$, a complete orthonormal basis is given. The decomposition of tensor products of two such irreducible representations yields a direct sum of irreducible ones. The q-analogs of the Clebsch-Gordan coefficients (CGC) are described. These CGCs, the universal R-matrix as well as all sorts of relations between them are presented in some graphical notation.
Decomposing tensor products of three irreducible representations in two different ways into irreducible components, $(V^{j_1} \otimes V^{j_2}) \otimes V^{j_3}$ and $V^{j_1} \otimes (V^{j_2} \otimes V^{j_3})$, gives two complete orthonormal bases in $V^{j_1} \otimes V^{j_2} \otimes V^{j_3}$. The matrix elements connecting these bases are the q-analogs of the $6j$-symbols which were presented graphically introducing the so-called shadow world. At the end of the talk, it was shown that any graphical configuration which describes relations between CGCs and R-matrices can be transformed into the shadow world. So, any identity between CGCs and R-matrices gives an identity between $6j$-symbols.

Günther Harder
The Turaev-Viro-Invariant of a Three Dimensional Manifold

Using the q-$6j$-symbol Turaev and Viro attach an invariant to any three dimensional manifold M without (or with triangulated) boundary. The invariant depends on the choice of an integer $r \geq 3$ and a choice of a $2r$-th root of unity q_0 such that $q = q_0^2$ is a primitive r-th root of unity.
To any triangulation of M one defines *admissible* colourings

$$\phi : \{\text{edges}\} \longrightarrow I = \left\{0, \frac{1}{2}, \ldots, \frac{r-2}{2}\right\}.$$

To each such colouring one defines a number

$$|M_\phi| = w^{-a} \prod_{E \in \{\text{edges}\}} w_{\phi(E)} \cdot \prod_{T \in \{\text{tetrahedra}\}} |T^\phi|$$

where the w_i are certain weights and

$$|T^\phi| = \text{weight factor} \begin{Bmatrix} i & j & k \\ l & m & n \end{Bmatrix}$$

and where $\left\{ \begin{matrix} i & j & k \\ l & m & n \end{matrix} \right\}$ is the q-$6j$-symbol built from the colours of the tetrahedra. The theorem of Turaev-Viro says that this number is an invariant of M. The proof depends on the well known identities among the q-$6j$-symbols.

Jens Hoppe
The Tetrahedron Equation and Zamolodchikov's Solution

Starting with a quick passage from Newtonian mechanics to n-particle states of a relativistic quantum field theory, the Yang-Baxter equation was reviewed as a consistency condition for the S-matrix factorization in a (1+1)-dimensional relativistic quantum field theory. In analogy, the tetrahedron equations arise as consistency conditions in the (2+1)-dimensional scattering theory of 'straight strings'. The (very large) number of independent functions appearing in these functional equations can be substantially reduced by an appropriate ansatz, due to Zamolodchikov. Following the proof of Baxter - involving a variety of non-trivial observations, and spherical trigonometry - this particular ansatz can be shown to lead to an explicit solution. The general d-simplex equations were shortly mentioned.

(P.S. Will membrane theories be related to invariants of 3-manifolds ?)

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