MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 16/1991

Algebraische Gruppen

7.4. bis 13.4.91

An der unter der Leitung von P. Slodowy (Hamburg), T.A. Springer (Utrecht) und J. Tits (Paris) stattfindenden Tagung nahmen 29 Mathematiker aus 12 Ländern teil. Inhaltlich lassen sich die 20 gehaltenen Vorträge den folgenden Themenbereichen zuordnen:

1. Theorie und Anwendungen der sogenannten Quantengruppen.
2. Algebraische Transformationsgruppen (Kompaktifizierungen, Linearisierungen).
3. Diskrete Untergruppen algebraischer Gruppen.
4. Schubertvarietäten und symmetrische Varietäten.
5. Spezielle Untergruppen und Zerlegungen der Liealgebra.
6. Darstellungstheorie.
7. Kombinatorische Aspekte.

Dabei sind einige der Vorträge mehreren Bereichen zuzuordnen. Von besonders aktuellem Interesse, und daher recht umfangreich vertreten, waren die Vorträge zum Thema "Quantengruppen", deren Untersuchung und Anwendungen in den letzten beiden Jahren eine enorme Ausbreitung erfahren haben.

Vortragsauszüge

E. AKYILDIZ:

Schubert calculus.

The purpose of the talk was:

i. to explain the recent results on the cohomology ring of Schubert varieties in G/P

ii. to report on D. Peterson's work on the Schubert calculus for the Kac-Moody setting.

M. BRION:

Cohomology of quotient spaces.

Let G be a connected reductive algebraic group over $\mathbf{C}$. Let X be the projective space of a G-module, X^{ss} its open subset of semi-stable points, and Y the (Mumford) quotient of X^{ss} by G. Then a presentation was given of the equivariant cohomology algebra $H^*_G(X^{ss}, \mathbf{Q})$, (when every semi-stable point is stable it is the cohomology of Y). The proof relies on results of F. Kirwan and G. Ellingsrud - S.A. Strømme. Also some applications were given.

A. BROER:

Generating functions in invariant theory.

Let G be a connected, linearly reductive group and V a G-module, then G acts linearly on $A := K[V]$, respecting the natural grading. For a dominant weight χ let A^G_χ be the isotopical component of A corresponding to the simple module of highest weight χ; it is a finitely generated module over the ring of invariants A^G. The generating function $\mathcal{H}(A^G_\chi; t) = \sum \dim(A^G_\chi)_i \cdot t^i$, where $(A^G_\chi)_i$ is the homogeneous part of degree i, is useful in the study of these so-called modules of covariants. This generating function is actually the Taylor expansion of a rational function.
A new method was sketched for computing $\mathcal{H}(A^G_\chi; t)$ for small χ; in partic-

ular, $\mathcal{H}(A^G;t)$. This method was used to check on a computer all explicit results obtained earlier by Weyl's transcendental method of integration over compact groups. For example: the case of seven copies of the adjoint representation of SL_3, eight copies of the spin representation of Spin (7) and, also the SL_4-modules $V_{[1,1,0]}$ and $V_{[3,0,0]}$.
The method also yields many modules of covariants which are not Cohen-Macauley.

J.B. CARRELL:

Singularities of Schubert varieties.

(The talk was about joint work with D.H. Peterson.) Let G be a simple algebraic group over an algebraïcally closed field k, $B \supset T$ a Borel subgroup of G containing a maximal torus, and W the Weyl group of (G,T). For a positive root α and an x in W, let $C(x,\alpha) := \overline{xU_{-\alpha}B}/B$, a T-invariant curve in G/B. Let X_w be the Schubert variety $\overline{BwB}/B$ in G/B. If was shown that $C(x,\alpha) \subseteq X_w$ iff ($x \leq w$ and $xr_\alpha \leq w$). Moreover, for each $x \in W$ with $x \leq w$, there are at least $\ell(w)$ curves of type $C(x,\alpha)$ in X_w. (This verifies the Weyl group case of a conjecture of Deodhar made for all Coxeter groups.) The main step in the proof is:
Lemma. Let Y be a closed T-invariant subset of U_-, and put $\Phi(Y) = \{\alpha > 0 \mid U_{-\alpha} \subseteq Y\}$. Then $\#\Phi(Y) \geq \dim Y$, and equality holds if Y is smooth at 1.
By counting lines in X_w the following inequality was derived:
Proposition. For all $w \in W$, set a $a(w) = \frac{1}{K}\sum_{x\leq w}\ell(x)$, where ℓ is the length function and $K = \#\{x \leq w\}$. Then $a(w) \geq \frac{1}{2}\ell(w)$. Moreover, $a(w) = \frac{1}{2}\ell(w)$ whenever X_w is smooth. By a result of Lakshmibai and Seshadri one has in the case $G = S\ell_n(k) : X_w$ is smooth $\Leftrightarrow a_w = \frac{1}{2}\ell(w)$. There is good evidence that the latter holds in the D and E cases as well.

C. DECONCINI and C. PROCESI:

The coadjoint action for quantum groups at roots of unity.

After introducing the quantum groups associated to a Cartan matrix, the main features of the theory were discussed. The case in which q is not a root of unity was given first by means of results of Drinfeld-Jimbo-Rosso etc. Then the case in which q is an ℓ-th root of unity (ℓ odd) was discussed.

In this case the model which was used has a large centre over which it is a finite module (and, in fact, a maximal order in a division algebra of dimension $\ell^{2|\phi+|}$ over its centre). The centre Z is composed of two pieces Z_0 and Z_1. The Z_0 turns out to be the coordinate ring of a Poisson Lie group which comes from a Manin Triple associated to the classical Lie algebra with the same Cartan matrix. Also, an action was given of an infinite dimensional group on the quantized algebra preserving Z_0 whose orbits on Spec (Z_0) coincide with the Poisson leaves. It results in a certain qualitative description of the representations of the quantized enveloping algebra.
All these results are joint work with V. Kăc.

V. DEODHAR:

A combinatorial formula for Kazhdan-Lusztig polynomials.

Let (W, S) be a Coxeter group, and let $y = s_1 \cdot s_2, .., s_k$ be a reduced expression for an element y in W. A combinatorial setting involving subexpressions of this reduced expression is developed, including a notion of a good element. It is proved that all elements in a group for which the Kazhdan-Lusztig polynomials are non-negative are good. If y is good then an algorithm was developed to compute these polynomials in an efficient way. It was further proved that in these cases the coefficients are actually the sizes of certain subsets of subexpressions. Thereby providing an explicit setting for various questions regarding these polynomials and related topics. Also similar results were obtained for the so-called parabolic case.

F. GRUNEWALD:

Affine crystallographic groups.

G. HARDER:

The $q - 6j$ symbols and invariants of 3-manifolds (after Turaev and Viro).

Starting from a $2r$-th root of unity q_0 for which $q := q_0^2$ is a primitive r-th root of unity, Taraev and Viro have attached an invariant to any 3-dimensional manifold. It is a number in the cyclotomic field of $2r$-th roots of unity, and it is obtained from a triangulation. More precisely, the invariant is obtained by forming a state sum over all colourings of a number defined for each colouring using the $q - 6j$ symbols which come from coupling of three tensor factors. The purpose of this talk was to explain the above.

A.G. HELMINCK:

Symmetric varieties.

Let G be a connected reductive algebraic group defined over a field k of characteristic not two. Let $\mathcal{O} \in Aut(G)$ be a k-involution and let H be the fixed point group of $\mathcal{O}$. Write G_k and H_k for the set of k-rational points of G respectively H. Then G_k/H_k is called a symmetric k-variety. One can associate with such a symmetric variety a root system in a natural way, it is the root system of a maximal $(\mathcal{O}, k)$-split torus, (this is a torus which is both $\mathcal{O}$-split and k-split).
In the talk a characterisation theorem for the k-isomorphism classes of the k-involutions was discussed. Combined with Tits' classification of semi-simple k-groups this leads to a partial classification of k-involutions for certain types of fields including finite fields, number fields and p-adic fields. This classification also includes a classification of the restricted root systems with their Weyl groups and multiplicities.

J.-C. JANTZEN:

Representations of algebraic groups in characteristic p and quantum groups.

Lusztig has conjectured a character formula for the irreducible representations of a semi-simple algebraic group G in characteristic $p \neq 0$, and for those of a quantized enveloping algebra ("quantum group") at a root of unity. It was observed that when both conjectures are true then the reduction mod p of an irreducible representation of a quantum group at a p-th root of unity will be an irreducible representation of G, as long as the highest weight is small with respect to p^2. The work of Lusztig, Anderson-Polo-Wen and Scott and others provides evidence for this conjecture. Also an example was described (joint work with Anderson) of an irreducible representation with restricted heighest weight of the quantum group such that the reduction mod p is not irreducible for G.
Furthermore, some work in progress was discussed (joint work with Anderson and Soergel). It concerns a comparison of projective indecomposable modules (PIM's) for the restricted enveloping algebra with those for an analoguous finite dimensional subalgebra of the quantum group. One of the

results is that these PIM's can be lifted to modules over suitable local rings which are PIM's in an analoguous category.

F. KNOP:

B-orbits on spherical varieties.

Let G be a connected reductive group, $B \subset G$ a Borel subgroup and X a spherical variety, i.e. B has a dense orbit in X. By a theorem of Vinberg and Brion, B has only finitely many orbits in X.
In this talk an action of the Weyl group W on the set of B-orbits $B \backslash X$ was constructed using the action of a minimal parabolic subgroup on X. Also the W-orbit of the open B-orbit was determined. Its isotropy group is determined by the "little Weyl group" of X, which is defined using the compactification theory of X.

H. KRAFT:

G-vector bundles and non-linearizable actions.

In 1989, G. Schwarz gave the first examples of non-linearizable actions of complex reductive algebraic groups an affine n-space $\mathbf{C}^n$. These examples came from non-trivial G-vector bundles on representation spaces of G. In this talk a classification (i.e. a moduli space) for G-vector bundles on G-representations V with one-dimensional quotient was given, (this is joint work with Schwarz). One thus obtains non-linearizable actions for all classical groups, the spin groups, G_2, E_6 and E_7. Using these results, F. Knop has constructed for every reductive group G such that G^0 is not a torus, non-trivial G-vector bundles on Lie G and also non-linearizable actions. Masuda and Petrie have given a different construction and found an invariant for G-vector bundles on arbitrary representations, which enabled them to discuss new non-trivial G-vector bundles and non-linearizable actions, in particular for some finite groups. This work was discussed as well.

V. LAKSHMIBAI:

Quantum deformations of Schubert varieties.

Let G be a semi-simple algebraic group over k, B a Borel subgroup and X a Schubert variety in G/B. Let L be an ample line bundle on G/B, and $R[X] = \bigoplus_{L} H^0(X, L)$. Then $R[X]$ is $\mathbf{Z}^\ell$-graded, where $\ell =$ rank (G). Further $R[X]$ is a left $k[B]$-comodule, where $k[B]$ is the coordinate ring of B.
In this talk a quantization $R_q[X]$ of $R[X]$ was given, with q a parameter taking values in k^* and G being of type A_n, B_n, C_n, D_n, E_6 or G_2. This $R_q[X]$ is again $\mathbf{Z}^\ell$-graded; further $R_q[X]$ has a canonical left $k_q[B]$-comodule structure. And if $R_q[X] = \bigoplus_{\underline{a}} (R_q[X])_{\underline{a}}, \underline{a} \in (\mathbf{Z}^+)^\ell$, then the standard monomials in $(R_q[X])_{\underline{a}}$ form a $k(q)$-basis for this space. When $X = G/B$ then $R_q[X]$ is also a $k_q[G]$-comodule. Here $k_q[G]$ and $k_q[B]$ are the quantizations of $k[G]$ respectively $k[B]$ as constructed by Fadeev, Reshetikhin and Takhtajan.

P. LITTELMAN:

Some remarks on tensor product decompositions.

Let G be a simply connected simple algebraic group defined over an algebraically closed field of characteristic zero. Consider maximal parabolic subgroups $P_1, P_2 \subset G$ and the associated fundamental weights w_1 respectively w_2. Let C_i be the affine cone over $G/P_i \subset \mathbf{P}(V_{w_i}^*)$. In case w_1 and w_2 are miniscule it was shown that the G-module structure of the coordinate ring $k[C_1 \times C_2]$ of $C_1 \times C_2$ is very simple. In fact, if $U \subset G$ is a maximal unipotent subgroup then $k[C_1 \times C_2]^U$ is a polynomial ring. As a consequence one obtains very simple (multiplicity-free) decomposition formulas for tensor products of type $V_{nw_1} \otimes V_{mw_2}$ resp. $S^2(V_{w_1})$. The result holds also for a few other cases, it seems that the assumption $G/P_1 \times G/P_2$ is spherical may be sufficient.

A. LUBOTZKY:

On Groups of Polynomial Subgroup Growth.

Let Γ be a finitely generated group. Then Γ is called

a. residually finite if $\cap\{H|H$ a finite index subgroup of $\Gamma\} = \{1\}$,

b. a group of polynomial growth (PSG) if $a_n(\Gamma) = \{H \leq \Gamma|$ $[\Gamma : H] = n\}$ grows polynomially.

Theorem (A. Lubotzky, A. Mann, D. Segal). A finitely generated residually finite group is PSG if and only if Γ is almost solvable of finite rank. Here "almost" means that it contains such a finite index subgroup, and "finite rank" means that every finitely generated subgroup of Γ is generated by $\underline{c}$ (constant) elements.
The proof uses methods from algebraic groups, arithmetic groups, the prime number theorem, p-adic Lie-groups and the classification of finite simple groups.
The above theorem and its proof were discussed in this talk.

D. LUNA:

Solvable spherical subgroups.

Let G be a connected semi-simple algebraic group over $\mathbf{C}$. An algebraic subgroup H of G is called spherical if a Borel subgroup of G has an open orbit in G/H; it is called special if the group $N_G(H)/H$ is finite. In this talk a classification of all connected solvable special spherical subgroups of G was given.

PHAM HUU TIEP:

Irreducible orthogonal decompositions of Lie algebras, Jordan subgroups and associated lattices.

Many of the local maximal subgroups of finite group of Lie type are closely related to the so-called Jordan subgroups of complex Lie group discovered by A.V. Alekseevskii. The latter subgroups give in turn examples of irreducible orthogonal decompositions ($IOD's$) of complex Lie algebras by some standard construction. In the case for which the Lie algebras are of type A_n, B_n or exceptional, one can show that all $IOD's$ can be constructed this way. In this talk it was shown that Lie algebras of type B_n have $IOD's$ iff $n = [\frac{p^m-1}{2}]$ for some prime power p^m. And Lie algebras of type A_n have $IOD's$ iff $n = p^m - 1$ for some prime power p^m. Also a description was given of the automorphism groups of the associated integral lattices. Some of these groups are finite groups of Lie type or sporadic simple groups.

O. SCHWARZMANN:

A non-arithmetic cocompact reflection group in complex hyperbolic geometry.

Using some elementary results from the topological theory of branched coverings and the deep theorem of Yau-Miayoka, an example of a cocompact discrete group Γ was constructed, acting on the complex ball $\mathbf{B}^2$. It was proved that Γ is a non arithmetic subgroup of $Aut(B^2)$. This gives a geometric point of view on one of the Terada-Deligne-Mostow examples of discrete reflection groups in $\mathbf{B}^2$.

D.M. TESTERMAN:

Overgroups of unipotent elements in semi-simple algebraic groups.

In this talk the following theorem was presented, together with its proof and some applications:
Theorem. Let G be a semi-simple algebraic group defined over a field of characteristic $p > 0$. Assume p is good prime for G, and let $u \in G$ with $u^p = 1$. Then there exists a closed connected subgroup $X < G$ with X of type A, and with $u \in X$.
It was observed that a similar result holds without the restriction $u^p = 1$ when $p > 3(h-1)$, with h the Coxeter number. The latter follows from Pommerening's generalisation of the Jacobson-Morosov theorem to characteristic p with p a good prime.

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