MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Brauer Groups and Representation Theory of Finite Groups

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Die Tagung fand unter der Leitung von H. Opolka (Braunschweig), F. Van Oystaeyen (Wilrijk) und W. Scharlau (Münster) statt.

In zahlreichen Diskussionen und in 31 Vorträgen beschäftigte sich die Tagung vornehmlich mit linearen und projektiven Darstellungen endlicher Gruppen, einfachen Algebren und Azumaya - Algebren, Brauergruppen von Körpern, Ringen und Varietäten, Galoiskohomologie sowie mit Zusammenhängen zwischen diesen Themen und mit Anwendungen auf Probleme der algebraischen Zahlentheorie und der algebraischen Geometrie.

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E. ALJADEFF:

On the semisimplicity of crossed products and cohomology of groups

If K is a field of char$K = p > 0$ and Γ is a finite group with $p \mid |\Gamma|$, then the group ring is not semisimple. If Γ acts on K via a group G namely $t : \Gamma \longrightarrow G \subset \mathrm{Aut}(K)$ and H is the kernel of this action then the skew group ring $K_t\Gamma$ is not semisimple iff $p \mid |H|$. Assuming this is the case, we consider the question whether there exists $\alpha \in \mathrm{H}^2(\Gamma, K^*)$ such that the crossed product is semisimple.

A necessary condition is: a p-Sylow subgroup of H intersects trivially H' (the commutator subgroup). We construct such 2-cocycles for the nontrivial action case $H = \mathbb{Z}_p,\ G = \mathbb{Z}_{p-1}$.

R. BOLTJE:

On canonical and explicit Brauer induction

Let G be a finite group and $\mathrm{R}(G)$ its character ring. Then, for an arbitrary finite dimensional $\mathbb{C}G$-module V,

$$V = \sum_{\substack{\sigma = (K,\psi) < \ldots < (H,\varphi) \\ \mathrm{mod} G}} (-1)^{|\sigma|} \mathrm{ind}_K^G V^{(H,\varphi)} \in \mathrm{R}(G)$$

is an explicit version of Brauer's induction theorem. Here the sum runs over G-orbits of chains of pairs (U, μ) where $U \leq G$ and $\mu \in \mathrm{Hom}(U, \mathbb{C}^*)$ with the obvious partial order. $V^{(H,\varphi)} := \{v \in V \mid hv = \varphi(h)v \text{ for all } h \in H\}$ is the φ-homogeneous component of $\mathrm{res}_H^G V$, hence also a $\mathbb{C}G$-module. The above formula is natural in the following sense:

Let $\mathrm{R}_+(G)$ be the free abelian group on the G-orbits $\overline{(H,\varphi)}^G$ of pairs (H, φ) and let $b_G : \mathrm{R}_+(G) \to \mathrm{R}(G)$ be given by $\overline{(H,\varphi)}^G \mapsto \mathrm{ind}_H^G \varphi$. As for $\mathrm{R}(G)$ we have induction and retriction maps between $\mathrm{R}_+(G)$ and $\mathrm{R}_+(H)$ whenever $H \leq G$. Then the above formula comes from a splitting map $a_G : \mathrm{R}(G) \to \mathrm{R}_+(G)$ for b_G ($b_G a_G = $id) which commutes with restriction. Another splitting map $\tilde{a}_G : \mathbb{Q} \otimes_{\mathbb{Z}} \mathrm{R}(G) \to \mathbb{Q} \otimes_{\mathbb{Z}} \mathrm{R}_+(G)$ with rational coefficients is given, which commutes with restriction and induction.

D. BURNS:

On the module structure of units in number fields

We discuss certain integral representations of a finite group G naturally defined when G is realized as the Galois group of a finite extension of number fields F/E. In particular we considered certain S-unit groups of F.

The techniques of investigation are due to Fröhlich and proceed via his construction of an integral regulator (Crelle (1989); see also Proceedings of 1989 Durham Symposium on "L-functions in Arithmetic"). In case G abelian, by using the known validity of the Strong Stark Conjecture (à la Tate) in case $E = \mathbb{Q}$ or quadratic imaginary one obtains new structure results. In case $E = \mathbb{Q}$ Fröhlich showed how explicit results then followed from the known validity of the Gras Conjecture in this case. In case E quadratic imaginary similar results can be obtained by means of the recent techniques of Kolyvagin (and Rubin) using "Euler systems".

S. CAENEPEEL :

The Brauer-Long group of a Hopf algebra

Let R be a commutative ring, and H a finitely generated, projectiv, commutative and cocommutative Hopf algebra. Long has introduced a Brauer group $\mathrm{BD}(R,H)$ classifying equivalence classes of H-Azumaya algebras (these are R-algebras A with a H-action and H-coaction satisfying the following isomorphism: $A\sharp\bar{A} \cong \mathrm{End}_R(A)$ and $\bar{A}\sharp A \cong \mathrm{End}_R(A)^{opp}$). $\mathrm{BD}(R,H)$ may be described as follows:
1) Let $\mathrm{BD}(R,H) = \bigcup_S \mathrm{BD}(S/R,H)$. Then

$$\mathrm{BD}^S(R,H) = \mathrm{H}^1(R, G(H^* \otimes .)) \times \mathrm{H}^1(R, G(H \otimes .)) \times \mathrm{H}^2(R, G_m)_{tors}$$

2) We have an exact sequence

$$1 \to \mathrm{BD}^S(R,H) \to \mathrm{BD}(R,H) \xrightarrow{\beta} \mathrm{Aut}_{Hopf}(H \otimes H^*).$$

The image of β is an orthogonal subgroup of $\mathrm{Aut}_{Hopf}(H \otimes H^*)$. Our theory allows us to compute some explicit examples. In particular, we have that
$\mathrm{BD}(\mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{2}]/(x^2 - \sqrt{2}x)) \cong D_8$.

F. DEMEYER:

The Brauer group of toric varieties II

A fan is a combinatorial object in $\mathbb{R}^n$ which consists of a union of cones such that all the faces of a cone and the intersection of any two cones in the fan are in the fan. To each fan Δ can be associated a "toric" variety $X = T_N\mathrm{emb}(\Delta)$. A support function on Δ is a function defined on the points of $\mathbb{R}^n$ which are in cones in Δ and is linear on each cone in Δ. Let $\mathrm{SF}(\Delta)$ be the group of support functions on Δ. Topologize Δ by letting the open sets be the subfans. Define a sheaf $\mathcal{L}$ on Δ by letting $\mathcal{L}(\Delta') = \mathrm{SF}(\Delta')$ for each Δ' open in Δ. We conjecture: If K is the function field of X then $\mathrm{H}^2_{et}(K/X, G_m) \cong \check{\mathrm{H}}^1(\Delta, \mathcal{L})$.

B. FEIN:

Finite groups and relative Brauer groups

In this talk we discuss some recent joint work with David Saltman and Murray Schacher concerning the nontriviality of certain Brauer groups. Let K be an arbitrary field, let L be a finite separable extension of K, and let x be transcendental over K. We prove that the relative Brauer group, $\mathrm{Br}(L(x)/K(x))$, is infinite. Although this statement may seem rather innocuous, we also show that this is equivalent to the following purely group-theoretic statement: if $\mathcal{H}$ is a subgroup of a finite group $\mathcal{G}, \mathcal{H} \neq \mathcal{G}$, then there is an element in $\mathcal{G}$ of prime power order having no conjugates in $\mathcal{H}$. The only known proofs of this result make essential use of the classification of the finite simple groups.

B. T. FORD:

The Brauer group of toric varieties I

(Joint with F. DeMeyer.) If X is a normal toric variety defined over an algebraically closed field of characteristic zero and the singular locus of X has codimension at most 2, then we give a reasonably complete description of the cohomological Brauer group of X.

E. FORMANEK:

The group determinant

Let $G = \{g_1, ..., g_n\}$ be a finite group, and let $\{X_{g_1}, ..., X_{g_n}\}$ be commuting variables indexed by G. The group determinant is

$$D_G = \det(X_{g_i g_j^{-1}}).$$

Kenneth W. Johnson asked whether the group determinant determines the group. Following a suggestion of H.-J Hoehnke, David Sibley and I showed that this was the case. Later, Johnson and Hoehnke showed that the coefficients of X_1^{n-2} and X_1^{n-3} in D_G are enough to determine the group, and still later Richard Mansfield produced a completly elementary proof of their result.

J. GRÄTER :

Extensions of valuation rings in central simple algebras

Let Q be a central simple algebra with center F. An order B (i.e. a prime PI-ring B with quotient ring Q) is called Dubrovin-valuation ring if there exists a maximal ideal M of B such that for all $q \in Q \backslash B$ there exist $b_1, b_2 \in B$ such that $b_1 q, q b_2 \in B \backslash M$. It turns out that $M = J(B)$. A main result concerning D-valuation rings is that each valuation ring V of F has such an extension to Q (Dubrovin). Now, D-valuation rings $B_1, \ldots, B_n$ have the intersection property (IP) if $B_1 \cap \ldots \cap B_n$ is a Bézout order in Q. It is shown that each valuation ring V of F can be extended to a Bézout order R of Q that is integral over V. This order is unique up to inner automorphisms of Q and is the intersection of n_V extensions of V. This number n_V depends only on V and is called the extension number of V. Furthermore, $[Q : F] = f_B \cdot e_B \cdot (n_V)^2 \cdot p^d$ where f_B is the residue degree, e_B the ramification index, $p = \mathrm{char} V/J(V)$, and $d \in \mathbb{N} \cup \{0\}$, where B denotes an extension of V to Q.

D. HAILE :

Clifford algebras and relative Brauer groups

If $f(u, v)$ is a binary form of degree n one can form the Clifford algebra C_f of f given by $C_f = F\{x, y\}/I$ where $F\{x, y\}$ is the free algebra and I is the ideal generated by the elements $(\alpha x + \beta y)^n - f(\alpha, \beta)$ for all $\alpha, \beta \in F$. In particular if f is a nondegenerate binary cubic form this algebra is Azumaya of rank nine over its center and its center is the affine ring of the elliptic curve $Y^3 = X^3 - 27D$ where D is the discriminant of f. The simple homomorphic images of C_f with center F are then in one-to-one correspondence with the F-rational points on this elliptic curve and in fact one obtains a homomorphism from $E(F)$, the group of F-rational points, to $\mathrm{Br}(F)$, the Brauer group of F. We examine this map and its generalizations to forms of higher degree.

I. HAMBELTON:

Cancellation of lattices and hyperbolic forms over orders

In joint work with M. Kreck, we obtain cancellation theorems for certain modules over R-orders in separable F-algebras, where R is a Dedekind ring with quotient field F. If L is an A-lattice, and $\varepsilon : A \to B$ is a surjective ring homomorphism of R-orders, we say that L has $(A - B)$ free rank ≥ 1 at a prime $p \in R$ if $L_{(p)} \oplus B^r_{(p)}$ has a direct summand $A_{(p)}$ for some $r \geq 0$.

Theorem 1. *Let L be an A-lattice and $M = L \oplus A$. If there exists $\varepsilon : A \to B$ such that L has $(A - B)$ free rank ≥ 1 at at all but finitely many primes, and $\mathrm{GL}_2(A)$ acts transitively on unimodular elements in $B \oplus B$, then for any A-lattice N which is locally a direct summand of M^n (some n), $M \oplus N \cong M' \oplus N \Rightarrow M \cong M'$.*

If $B = 0$, this implies the Bass cancellation theorem. For quadratic forms there is the analogous notion of "$(A - B)$ hyperbolic rank ≥ 1" at a prime $p \in R$ if $M_{(p)} \perp H(B^r_{(p)})$ contains $H(A_{(p)})$.

Theorem 2. *Let V be a (λ, Λ)- quadratic module over a ring with form parameter (A, Λ), and set $(M, [h]) = V \perp H(A)$. Suppose that there exists $\varepsilon : A \to B$ such that V has $(A - B)$ hyperbolic rank ≥ 1 at all but finitely many primes. If $U_2(A)$ acts transitively on the set of unimodular elements in $H(B \oplus B)$ of fixed length, then $M \perp H(A^r) \hat{=} M' \perp H(A^r) \Rightarrow M \cong M'$.*

F. Jonas

The conductor of the character field

Let G be a finite group and $D : G \to GL(n, \mathbb{C})$ an irreducible linear representation with character χ. Define $M(D) = \{m \in \mathbb{N} \mid D \text{ is realizable in} \mathbb{Q}(\xi_m)\}$. We are looking for properties of $M(D)$, for example is it true that

(*) $$m, m' \in M(D) \Rightarrow \gcd(m, m') \in M(D)?$$

One can show that (*) holds if and only if $f \in M(D)$, where f is the conductor of the character field $\mathbb{Q}(\chi)$.

If n is an odd prime, it follows by theorems of Schur and Solomon that $f \in M(D)$, i.e. in this case $M(D) = f\mathbb{N}$. (For $n = 2$ the Schur index $m_k(\chi) = 1$ if $\sqrt{-1} \in k$, i.e. $\mathrm{lcm}(4, f) \in M(D)$.)

B. Kahn

The degree of a division algebra over a C_i-field

A classical theorem asserts that any division algebra over a global field has degree equal to its exponent. This is conjectured to hold for C_2-fields, where C_i is Lang's diophantine condition, but so far it is known only when the exponent has the form $2^a 3^b$ (Artin-Tate, Yanchersky). We relax this property as follows:

Definition. *Let F be a field and $n \geq 1$. We say that F has property Br_n if central simple algebras over F of exponent dividing n have bounded index (the bound depending on n and F).*

Conjecture. *If F is a C_i-field, then F has property Br_n for any $n \geq 1$.*

The field $\mathbb{C}(x, y)$ is C_2 but this conjecture does not seem to be known even for $n = 5$! The aim of the talk was to prove the following theorem:

Theorem. *Let F be a field with finite u-invariant. Then F has property Br_{2^e} for any $e \geq 1$.*

The u-invariant of a field F is the least integer $u(F)$ such that any quadratic form over F in $u(F) + 1$ variables represents 0: if F is C_i, then $u(F) \leq 2^i$. The proof uses the Merkurjev-Suslin theorem, a lemma of Merkurjev on quadratic forms and the permanence of finiteness of $u(F)$ under finite extensions (Elman-Lam).

I. KERSTEN

Generic splitting of reductive algebraic groups

(Joint work with U. Rehman.) Let G be a reductive linear algebraic group over a field k. Then a field $F \supset k$ is called a splitting field of G, if G_F is a Chevalley F-group. A splitting field F of G is called generic, if every splitting field of G is a specialization of F. We construct a generic splitting field of a semisimple connected group G which generalizes known results of Witt, Amitsur, Roquette, Knebusch, Petterson, and others. In many cases this generic splitting field is the function field of a projective variety G/P with a suitable parabolic subgroup P of G. If G is a k-torus then G has a generic splitting field being finite and separable over k.

F. LORENZ

On the Schur multiplier of fields

For a field K with absolute Galois group $G_K = Gal(C/K)$ the group $\mathrm{Sr}(K) = \mathrm{H}^2(G_K, \mathbb{C}^*) = \mathrm{H}^2(G_K, \mathbb{Q}/\mathbb{Z})$ is called the Schur multiplier of K. In an obvious way $\mathrm{Sr}(K)$ is connected with projective representations and corresponding field theoretical and arithmetical questions. There is also a connection with the Brauer group $\mathrm{Br}(K) = \mathrm{H}^2(G_K, C^*)$ of K: Assuming $\mu_n \in K$, we conclude from the exact seqence $\chi(G_K) \to \mathrm{Br}(K)_n \to \mathrm{Sr}(K)_n \to 1$ that the following are equivalent:
(i) $\mathrm{Sr}(K)_n = 1$, (ii) Each element of $\mathrm{Br}(K)_n$ can be represented by a cyclotomic algebra of the special type (E, σ, ζ), where $\zeta \in \mu_n$.

For a global field K the Schur multiplier of K is trivial (by a theorem of Tate, which actually can be conceived as a basic fact of the "theory of norm residues", which started with A. Scholz in 1940), so we get an interesting result concerning the Brauer group of a global field; it is related to - but different from - the "Brauer-Hasse-Noether Hauptsatz der Algebrentheorie".

Therefore there seems to be some good reasons to study the Schur multiplier also of other types of fields. For instance it follows from the (elementary algebraic) theory of Tsen that the rational function fields $C(X)$ and $R(X)$ in one variable over an algebraic or real closed field have trivial Schur multiplier. The same is true for the power series fields $C((X))$ and $R((X))$. Next we study the more difficult case of a rational function field $K = k_0(X)$ over a global (or local) field of characteristic 0. Here we find $\mathrm{Sr}(K) \cong \bigoplus_\alpha \mathrm{H}^1(G_\alpha, \mu^*)$, where α runs through the conjugacy classes of the $a \in C_0$, G_α is the absolute Galois group of $k_0(a)$ and μ^* denotes the Tate twist $\mathbb{Q}/\mathbb{Z}(-1)$ of the group μ of roots of unity. So, for an arbitrary global (or local) field one is lead to study the groups $\mathrm{H}^1(G_K, \mu^*)$. It contains the groups $\mathrm{H}^1(\Gamma, \mu)$ - with $\Gamma = Gal(k(\mu)/k)$ - as a subgroup (and from this we also see that $\mathrm{Sr}(k_0, X)$ is not trivial). There is also some connection between $\mathrm{H}^1(G_K, \mu^*)$ and the Schur group of K since the latter can be discribed as the image of $\mathrm{H}^2(\Gamma, \mu)$ in $\mathrm{H}^2(\Gamma, k(\mu^*))$.

We conclude with the observation, that the above function field $K = k_0(X)$ satisfies a simple Global-Local-Principle, namely $\mathrm{Sr}(K) \cong \bigoplus_{\mathcal{P}} \mathrm{Sr}(K_{\mathcal{P}})$. But the proof for this is done only by way of a kind of direct inspection as above, and it remains open whether the statement remains valid also in the non rational case.

L. LEBRUYN

Rational identities of generic division algebras

A short proof is presented of the fundamental results of Bergman on rational identities. As a corollary the following extension of the classical p.i. result is obtained: if $(a+1)m < n$ and if z is a rational identity for $UD(n)$ of "inversion depth" $\leq a$ then z (or one of its subexpressions) holds in $UD(m)$ too.

P. MAMMONE

On the tensor product of division algebras

In a recent work, Tignol and Wadsworth constructed finite dimensional division algebras D_1, D_2 over a field F such that $D_1 \otimes D_2$ is not a division algebra but D_1 and D_2 have no common subfield properly containing F. However Saltman observed that the division algebras constructed by Tignol and Wadsworth satisfy the property that $D_1 \otimes D_2^{op}$ is a division algebra. So, Saltman's observation raised the following new tensor product question: If $\mathrm{index}(D_1 \otimes D_2^{op}) < \mathrm{index} D_1 \cdot \mathrm{index} D_2$ for all $1 \leq n \leq \exp(D_2)$ then do D_1 and D_2 have necessarily a common subfield?

We make some observations on the symmetry of the new tensor product hypothesis and give two families of negative answers to the question.

R. MASSY:

Numerical decompositions of cohomology classes

Let K be a field, $\widehat{K}$ its separable closure and $\Omega_K = Gal(\widehat{K}/K)$. Assume that K contains a primitive νth root of unity ζ_ν, with ν invertible in K. For $a \in K^\times$, let $(a) \in \mathrm{H}^1(\Omega_K, \mathbb{Z}/\nu\mathbb{Z})$ be the map defined by $\omega(a^{1/\nu})/a^{1/\nu} = \zeta_\nu^{(a)(\omega)}$ for $\omega \in \Omega_K$. It follows from work of Merkur'ev and Suslin that any $\epsilon \in \mathrm{H}^2(\Omega_K, \mathbb{Z}/\nu\mathbb{Z}) \cong \mathrm{Br}_\nu(K)$ can be expressed as a sum:

$$(*) \qquad \epsilon = \sum_{i=1}^{n} (a_i, b_i)$$

where $(a,b) = (a) \cup (b)$ and $\cup : \mathrm{H}^1(\Omega_K, \mathbb{Z}/\nu\mathbb{Z}) \times \mathrm{H}^1(\Omega_K, \mathbb{Z}/\nu\mathbb{Z}) \longrightarrow \mathrm{H}^2(\Omega_K, \mathbb{Z}/\nu\mathbb{Z})$ is the cup-product.

We consider the question of obtaining a decomposition (*) when $\widehat{K}$ is replaced by a finite Galois extension E of K. Take for E/K an abelian p-extension (p any prime). Then it may be necessary to add a supplementary term to the sum (*). This term is induced by only one element a_0 of K and defined via another cup-product. When the elements a_i, b_i ($i = 1, \ldots, n$) are linearly independent in the Kummer group of E/K, we prove that there are only three kinds of new decompositions (*), and we get the following property:
Every class $\epsilon \in \mathrm{H}^2(Gal(E/K), \mathbb{Z}/\nu\mathbb{Z})$ is of one, and only of one, of these three kinds.

P. Morandi:

Maximal orders over valuation rings

In the talk we discuss maximal orders over valuation rings in central simple algebras. The case of maximal orders over discrete valuation rings is classical and well known. We are particularly interested in maximal orders that are either Bézout or semihereditary. By using results of Gräter we see that a maximal order R is Bézout iff R is an intersection of Dubrovin valuation rings satisfying the intersection property. Thus there is an unique up to isomorphism Bézout maximal order over a valuation ring V in any central simple algebra.

For semihereditary orders, taking into account the classification of hereditary orders over a DVR we are able to construct a class of block matrix orders which are semihereditary maximal orders. For V a valuation ring with value group $\mathbb{Z}^n$, any semihereditary maximal order is of this block matrix form.

Using defective field extensions we give a third construction of maximal orders. For $S = \mathrm{M}_2(F)$ we see any maximal order is one of these three types.

P. Nelis:

Schur and projective Schur groups over number rings

Notation: K a number field, R its ring of integers, R_S localization of R.
The talk considered a conjecture by Riehm. Let

$$S(R_S) = \{[A] \in \mathrm{Br}(R_S) : \exists G,\ \text{finite group}\ , \exists\ \text{epimorphism}\ \pi : \mathbb{Q}G \to A\}.$$

$S(R_S)$ is a subgroup of $\mathrm{Br}(R_S)$, called the Schur group. The diagram:

$$\begin{array}{ccc} S(R_S) & \hookrightarrow & S(K) \\ \cap\downarrow & & \cap\downarrow \\ \mathrm{Br}(R_S) & \hookrightarrow & \mathrm{Br}(K) \end{array}$$

shows that $S(R_S) \subseteq \mathrm{Br}(R_S) \cap S(K)$.
Conjecture (Riehm, 1989): $S(R_S) = \mathrm{Br}(R_S) \cap S(K)$.

We explained the proof of this equality in the case $R = R_S$, i.e. for number rings. The proof makes use of the representation theory of $\mathrm{GL}_2(\mathbb{F}_p)$ and of a method of combining elements in $S(R)$ and $S(R')$ to obtain an element in $S(R'')$, where R, R', R'' are number rings of three different fields K, K', K''.

G. PAZDERSKI:

On the number of irreducible representations of an algebra

Following Brauer's approach in counting the irreducible representations of a finite group over a splitting field we give a formula for the number of irreducible representations of an algebra over an arbitrary field. As an application to the group rings the known Berman-Witt result is obtained.

E. PEYRE:

Unramified cohomology in degree 2 or 3 and rationality problems

Let k be an algebraically closed field of characteristic 0 and K a function field over k. I note $P(K)$ the set of discrete valuation rings A such that $k \subset A \subset K$ and $\mathrm{Fr}(A) = K$.

$$\mathrm{H}^i_{\mathrm{nr}}(K, \mu_n^{\otimes j}) = \bigcap_{A \in P(K)} \ker(\delta_A : \mathrm{H}^i(K, \mu_n^{\otimes j}) \to \mathrm{H}^{i-1}(\mathbb{K}_A, \mu_n^{\otimes j}))$$

where δ_A is the residue map associated to A.

By a proposition of Colliot-Thélène and Ojanguren, if K is stably rational over k (this means that there exist indeterminates $V_1, \ldots, V_m$ and $T_1, \ldots, T_l$ such that $K(V_1, \ldots, V_m) \cong k(T_1, \ldots, T_l)$ over k), then $\mathrm{H}^i_{\mathrm{nr}}(K, \mu_n^{\otimes j}) = \{0\}$.

Let V be a finite $\mathbb{F}_p$ - vector space for an odd prime number p and $\phi : V^* \to \mathrm{H}^1(K, \mu_p)$ a morphism. This induces a map $\phi^j : (\bigwedge^j V)^* \to \mathrm{H}^j(K, \mu_p^{\otimes j})$. Let $S^j \subset \bigwedge^j V$ be the orthogonal of its kernel. Let $S^j_{\mathrm{dec}} = \langle u \wedge v,\ u \in V,\ v \in \bigwedge^{j-1} V\rangle$. Then $(S^j / S^j_{\mathrm{dec}})^* \hookrightarrow \mathrm{H}^j_{\mathrm{nr}}(K, \mu_p^{\otimes j})$.

If V and $S^3 \subset \bigwedge^3 V$ are given, with $S \neq S_{\mathrm{dec}}$, we may, using a recent result of Suslin, construct in some cases a unirational field K and a map $V^* \to \mathrm{H}^1(K, \mu_p)$ such that $(\ker \phi^3)^\perp$ is the chosen S. The field K is therefore not stably rational.

M. REGAUER:

Weak solvability of embedding problems defined by the second Stiefel-Whitney class

Let $G_k = Gal(\bar{k}/k)$ be the Galois group of an algebraic closure of a number field k and G the Galois group of a finite normal extension K/k. If r_G denotes the regular representation of G, we can define the second Stiefel-Whitney class $e_G := w_2(r_G)$ of r_G as an element in $\mathrm{H}^2(G, \mathbb{Z}/2\mathbb{Z})$.

The corresponding central embedding problem $\mathcal{E} = (G, \mathbb{Z}/2\mathbb{Z}, e_G)$ is called weakly solvable if $\inf_G^{G_k}(e_G)$ becomes trivial for some $i \in \mathbb{N}$ under the map $\mathrm{H}^2(G_k, \mathbb{Z}/2\mathbb{Z}) \to \mathrm{H}^2(G_k, \mathbb{Z}/2^i\mathbb{Z})$ which is induced by the natural map $\mathbb{Z}/2\mathbb{Z} \to \mathbb{Z}/2^i\mathbb{Z}$. The minimal i such that $\mathcal{E}$ is weakly solvable is called the index of $\mathcal{E}$. This is a finite number which will be estimated.

Therefor the problem is firstly reduced to a 2-Sylow subgroup of G and secondly to local fields. An important role plays the formula of Serre which relates $\inf_G^{G_k}(e_G)$ to the Hasse invariant of the trace form $tr_{K/k}(x^2)$ of K/k.

An upper bound of the index of $\mathcal{E}$ is $r+2$, where 2^r is the largest order of the 2-power roots of unity in the completitions of k with respect to the ramified places of K/k. But there are many cases in which this index is less than $r+2$.

C. RIEHM:

Complements to some theorems of A. Fröhlich

In his paper in Crelle, vol. 360, Fröhlich gave several formulas involving invariants of orthogonal representations of finite and profinite groups. In addition he gave analogues of some of them in the case of projective representations.

In this lecture I outlined proofs for the "missing" formulas in the projective case. In fact one of them can be done more generally for a representation in a category with direct sums relative to "admissible systems" of morphisms.

The result is that the K-objects $\mathrm{Cor}_{L/K}X^s$ and $(\mathrm{ind}X)^{\mathrm{ind}\, s}$ are isomorphic, where L/K is a finite separable extension, $s : \Delta \to \mathrm{Aut}X$ is a representation of the subgroup $\Delta = Gal\,(K^{sep}/L)$ of finite index in $\Gamma = Gal\,(K^{sep}/K)$, and the subscripts s and $\mathrm{ind}\, s$ represent twisting by s and $\mathrm{ind}\, s$ interpreted as 1-cocycles.

The other formula, in the case of projective representations, gives the Brauer class $PN_2[\mathrm{ind}s]$ in terms of $PN_2[s]$ and the discriminant of L/K (in Fröhlich's notation).

J. RITTER:

On two integrality properties of group representations

1. It is well known that each absolutely irreducible character χ of a finite Group G is the trace of a representation $T : G \to \mathrm{GL}_n(\mathbb{Q}(\zeta))$, where $n = \chi(1)$ and ζ is a

root of unity of order exp$G$. It is known that T can in fact be chosen to take values already in $\mathrm{GL}_n(\mathbb{Z}[\zeta])$, provided that G is solvable. By means of an example one sees that in general $\mathbb{Q}(\zeta)$ is the minimal splitting field for χ enjoying the stated integrality property. The result comes from joint work with G. Cliff and A. Weiss.

2. A. Fröhlich, in connection with his definitions of regulators and resolvents in Galois module structure theory, has asked whether there exist Galois-stable lattices M on irreducible FG-modules V, where F is a splitting field for V, which is Galois over the character field belonging to V; Galois-stability meaning that M and M^σ are G-isomorphic lattices.

From joint work with A. Weiss the following theorem is presented, which not only gives the answer to Fröhlich's question, but also provides uniquely defined resolvents and regulators in the non-commutative situation.

Theorem. *If A is a central simple K-algebra split by F, a finite Galois extension of K, then to each maximal $\mathcal{O}_K$-order Λ in A there exists a unique Galois-stable maximal $\mathcal{O}_F$-order in $A \otimes F$ containing Λ, provided that the local Schur indices of A divide the local ramification indices of F/K.*

The main tool of the proof is the concept of generalized crossed product orders.

D. Saltman:

A little result about generic division algebras

Let $UD(F,n,r)$ be the generic division algebra over F of degree n in r variables. If A/F is central simple, let $BS(A)$ be the Brauer-Severi variety.

The main theorem is that $K = Z(UD \otimes_Z (A \otimes_F K)^o)$ is rational over F where Z is the center of UD and $Z(C)$ is the function field of $BS(C)$. We use to give three applications.

First, that there exists $Z \subsetneq L$ with L/F rational and $\mathrm{Br}(Z) \to \mathrm{Br}(L)$ is injective.

Second, if $n = ab$ for $(a,b) = 1$ then $Z(F,n,r)$ is stably isomorphic to $Z(F,a,r')Z(F,b,r'')$. This fact was first proved by Schofield.

Third, that if F is Hilbertian, $f \in Z[x]$ irreducible, and A/F central simple of degree n, then there exists $\phi : C \to F$ realizing A ($C \subsetneq Z$, B/C Azumaya, C smooth, $BZ = UD$, $B \otimes_\phi F \cong A$) such that $\phi(f(x))$ is irreducible.

M. Schacher:

Subfields of division rings

We discuss joint work with Fein and Saltman concerning the following problem: Suppose L/F is a finite dimensional separable extension of fields, and

$$L_n = L(t_1, \ldots, t_n), \quad F_n = F(t_1, \ldots, t_n).$$

We ask:

1) Is L_n a maximal subfield of a division ring central over F_n, some n ?

2) Same as in 1) without maximality.

3) What is the minimum n ?

The answer in 1) is yes when L/F is Galois. This makes the answer to 2) yes also. 1) is false for general non-Galois L/F. Let $G = Gal(L/F)$ when L/F is Galois. Methods of permutation modules, using the induction theorem of Moody, accomplish 1) in a large number of variables. Methods using corestriction require fewer variables.

Let r = maximum cyclic length among Sylow subgroups of G. Then r variables suffice for 3). If F is a number field, $r-1$ variables suffice. As a consequence, if Sylow subgroups of G are metacyclic and F is a number field, then the construction can be accomplished over F(t).

V. SNAITH:

Conductors in the non-seperable case

This is a report on joint work with R. Boltje and G. M. Cram. Let L/K be a finite Galois extension of complete, discrete valuation fields with group $G(L/K)$.

When $\bar{L}/\bar{K}$ - residue extension - is seperable we have the Artin conductor $f_K : R(G_0(L/K)) \to \mathbb{Z}$. f_K is inflative, $\Omega_{\mathbb{Q}}$-fixed and inductive in dimension zero (up to a factor $f_{L/K}$).

It is a problem of J. P. Serre to extend f_K to the general case. Using Explicit Brauer Induction we do this with the exception of inductivity. We reduce inductivity to cases when $G(L/K) = G_1 = \mathbb{Z}/p^n$ or $\mathbb{Z}/p \times \mathbb{Z}/p^n$ and give infinitely many examples where our conductor is inductive in the same sense as is f_K.

J. SONN:

Brauer groups and embedding problems over function fields

Let K be a field, K_λ, $\lambda \in \Lambda$, a family of extension fields of K. If

$$\mathrm{Br}(K) \to \prod_\lambda \mathrm{Br}(K_\lambda)$$

is injective, then for certain important types of embedding problems over K, a solution exists if and only if a solution exists for the induced embedding problems over K_λ for all $\lambda \in \Lambda$.

The classical example of this injectivity is K a number field and Λ the set of all places of K.

Similary, the Faddeev-Auslander-Brumer Theorem gives a second example with $K = K_0(t)$ a rational function field over a field K_0 (but "keep away" from char(K_0)), Λ the set of all places of K trivial on K_0.

A third example is given by the following theorem:

Let k be a global field, p a prime $\neq \mathrm{char}(k)$, Λ the set of primes of k. Then

$$\mathrm{Br}(k(t)) \to \prod_{v\in\Lambda} \mathrm{Br}(k_v(t))$$

is injective.

J.P. TIGNOL:

Linkage of division algebras over Laurent series fields

Joint work with Bill Jacob. A field F is called p-linked for some prime number p if for every Brauer class $\alpha \in \mathrm{Br}(F)_p$ the following equation holds: $\exp(\alpha) = \mathrm{ind}(\alpha)$.

By an easy argument of Albert, this condition is equivalent to the following: if D_1, D_2 are division algebras of degree p, $D_1 \otimes D_2$ is either 1 or similar to a division algebra of degree p. Using the Witt exact sequence for the Brauer group of a field of Laurent series, it is easy to see that, when $\mathrm{char}(k) \neq p$, the field $k((t))$ is p-linked iff every division algebra of exponent p over k is split by every cyclic extension of degree p of k.

This condition holds in particular when k is a local field (of char $\neq p$). Using induction on the height of division algebras of degree p over $\mathbb{F}_p((x))((y))$, it is shown that this field is p-linked.

For $p = 2$, this yields a counterexample to a theorem of Arf, which claims that the u-invariant of a 2-linked field of characteristic 2 is at most 4.

T. YAMADA:

The formula of Schur index over the 2-adic field

Theorem 1. *Let $k \supset \mathbb{Q}_2$. Let $B = (\beta, k(\varepsilon)/k)$ be a cyclotomic algebra over k. Let $\beta(\sigma,\tau) \in \langle\zeta\rangle$, where $\langle\zeta\rangle$ is the 2-part of the group of the roots of unity in k.*

If $n < 2$, then $B \sim k$.

Assume that $n \geq 2$. If the inertia group $\mathcal{I}$ of $k(\varepsilon)/k$ does not contain ι such that $\zeta^\iota = \zeta^{-1}$, then $B \sim k$. Suppose that $\iota \in \mathcal{I}$. Then $\beta(\iota,\iota) = \pm 1$, and $\mathcal{I} = \langle\iota\rangle \times \langle\tau\rangle$, $\zeta^\tau = \zeta^{5^{2^\lambda}}$.

Let η be a Frobenius automorphism and $\zeta^\eta = \zeta^{5^\mu}$. Let $\dfrac{\beta(\tau,\eta)}{\beta(\eta,\tau)} = \zeta^a$, $\dfrac{\beta(\iota,\eta)}{\beta(\eta,\iota)} = \zeta^b$, $\dfrac{\beta(\tau,\iota)}{\beta(\iota,\tau)} = \zeta^c$, $h = \{2a + (5^{2^\lambda} - 1)b + (5^\mu - 1)c\}/2^n$.

Then $h \in \mathbb{Z}$, and $\mathrm{inv}B \equiv h/2 + [k : \mathbb{Q}_2]\cdot\delta/2 \pmod 1$, $\delta = 0$ if $\beta(\iota,\iota) = 1$, $\delta = 1$ if $\beta(\iota,\iota) = -1$.

Theorem 2. *Suppose that the residue class degree f of $k(\varepsilon)/k$ is odd. Then there exists a Frobenius automorphism η of $k(\varepsilon)/k$, such that $\zeta^\eta = \zeta$. Furthermore, $2|h$ and so $\mathrm{inv}B \equiv [k : \mathbb{Q}_2]\cdot\delta/2 \pmod 1$.*

Proposition 3. *Suppose that* $2|f$ *and there exists* η *of* $k(\varepsilon)/k$ *such that* $\zeta^\eta = \zeta$. *Then there exists* $B = (\beta, k(\varepsilon))/k)$ *such that* $h/2 \not\equiv 0 \pmod 1$, *and so* $\mathrm{inv}B \equiv \frac{1}{2} + [k : \mathbb{Q}_2] \cdot \delta/2 \pmod 1$.

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