MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20/1991

Darstellungstheorie endlich-dimensionaler Algebren

5.5. bis 11.5.1991

Die Tagung wurde von V. Dlab (Ottawa) und C.M. Ringel (Bielefeld) organisiert. Es würde allgemein begrüßt, daß zum ersten Mal seit 1986 wieder eine Oberwolfach-Tagung zu diesem Thema stattfand. Eine Vielzahl Mathematiker auch benachbarter Gebiete hatte ihr Interesse an einer solchen Tagung bekundet, so daß die Einladungsliste drastisch beschränkt werden mußte. Dennoch nahmen mehr Mathematiker als eigentlich absehbar an der Tagung teil, da es nur ganz wenige Absagen gab.

Die sehr arbeitsintensive, aber trotzdem gelöste Atmosphäre wurde sehr gelobt (nicht immer war es bei früheren Tagungen spannungsfrei gewesen). Das Programm konzentrierte sich auf die wichtigsten Themen und es gab jeweils längere Pausen, um Gesprächs- und Kontaktmöglichkeiten zu schaffen. Davon wurde reger Gebrauch gemacht. Neben den Vorträgen, die unten dokumentiert sind, gab es vielfältige informelle Berichte in den Mittagspausen und auch am Abend.

Im Mittelpunkt der Tagung standen neben den neuerlichen Entwicklungen der eigentlichen Darstellungstheorie von Algebren ihre Anwendungen in Lie-Theorie und Singularitätentheorie; berichtet wurde vor allem über den Einsatz homologischer, algebraisch-geometrischer und kombinatorischer Methoden.

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Vortragsauszüge

M. Auslander: Cotilting modules: Real and Imaginary
A method of using relative homological algebra for constructing cotilting modules will be discussed.

V.J. Bekkert: Representations of quivers with relations
The tame and finite growth criterions for trees with relations are given. Furthermore the tame criterion for vector space categories and the tame criterion for quivers with zero-relations.

K. Bongartz: On degenerations and extensions of finite dimensional modules
(1) We derive a cancellation theorem for degenerations of modules and apply it to characterize orbit closures of modules living on preprojective components.
(2) We show that any minimal degeneration of modules over a representation-directed algebra comes from a short exact sequence. The same holds true for Kronecker-modules.
(3) We construct lots of exact sequences using degenerations. In particular, we prove that any indecomposable non-simple over a tame concealed algebra is an extension of an indecomposable and a simple.

J.F. Carlson: Periods for periodic modules
We briefly describe some recent work with David Benson on the periods of periodic modules. Among other things we prove that for any number n there exists a finite 2-group such that for any field K of characteristic 2 there exists a periodic KG-module whose period is larger than n. A theorem of R.C. Andrews permits us to restrict our attention to extra special groups. Here the theorem is proved by the computation of the varieties of certain modules. The varieties are obtained from a calculation of the partial inflations of the spectral sequence associated to the central subgroup. These

computations may be of independent interest. The results in the odd characteristic case are incomplete.

C. Cibils: The syzygy quiver of a monomial algebra

Let $\Lambda = kQ/ < Z >$ be a monomial algebra. The set of vertices Ω_0 of the syzygy quiver Ω is the set of non-zero paths of Q. There is an arrow in Ω from α to β if

1) β and α are composable

2) $\beta\alpha$ contains only one subpath ς of Z and $\beta\alpha = \varsigma\varepsilon$.

Theorem a) The homological dimension $hd(\Lambda)$ is finite iff Ω has no cycles. In that case $hd(\Lambda)$ is one plus the maximal length of a path of Ω starting at an arrow.

b) Let N be the maximal length of a directed path of Ω starting at arrow. The injective dimension of a f.g. Λ-module is either infinite or bounded above by $N + 1$.

Part a) lies on an explicit description of the minimal projective resolutions of the simple modules which avoids coverings.

Part b) is proved following Igusa-Zacharia's proof of the finitistic conjecture for monomial algebras. The bound $N + 1$ improves the bound $\dim_k rad\Lambda$ previously obtained. 2-nilpotent algebras are good examples of this improvement.

W. Crawley-Boevey: Characters for modules of finite endolength

The endolength of a module M over an associative ring R with 1 is the length of M over its endomorphism ring. In this lecture we presented a sort of character theory for the modules of finite endolength.

As motivation we recalled the fact that a finite dimensional algebra over an algebraically closed field has tame representation type if and only if the endomorphism ring of every indecomposable module of finite endolength is a P.I. ring.

We say that a function χ from the set of finitely presented right R-modules is a character if it is additive on direct sums and if $\chi(Z) \leq \chi(Y) \leq \chi(X) + \chi(Z)$ whenever $X \to Y \to Z \to 0$ is a right exact sequence. This is adapted from Schofield's notion of a Sylvester rank function. We say that a non-zero character is irreducible if it cannot be written as a non-trivial sum of characters.

We sketched part of the proof (which uses the functor category) of the following theorem

Theorem (1) The assignment $M \mapsto \chi_M$ where $\chi_M(X) = Length_{\mathrm{End}_R(M)}(X \otimes_R M)$ induces a bijection between the isomorphism classes of finite endolength indecomposable left R-modules and the irreducible characters.

(2) Every character is a sum of irreducibles.

(3) The irreducible characters are independent over $\mathbb{Z}$.

E. Dieterich: Tame curve singularities with large conductor

Let $(C,0)$ be an affine-algebraic curve singularity over an algebraically closed field. We study the category cmΛ of Cohen-Macaulay Λ-modules over its complete local ring $\Lambda = \hat{O}_{C,0}$. Our main interest is to "solve its classification problem", i.e. to produce a complete and irredundant system of representatives for the set Is(indΛ) of isomorphism classes of indecomposable Cohen-Macaulay Λ-modules.

The "representation-finite" curve singularities (these are curve singularities $(C,0)$ such that Is(indΛ) is finite) are known and their classification problems are solved. On the other hand, very little was known so far about "tame" curve singularities (these are representation-infinite curve singularities $(C,0)$ whose classification problem admits a solution by means of finitely many one-parameter families of indecomposable Cohen-Macaulay Λ-modules, in each rank). However, for curve singularities whose conductor is "large" (in the sense that it contains the radical squared of its normalization), there is now a complete description of those which are tame, together with solutions of their respective classification problems.

P. Dräxler: Decomposing points of tame algebras

Let $A = k[\Delta]/I$ be a finite dimensional algebra over an algebraically closed field k. Fix a point s of Δ and denote by $e(s)$ the induced idempotent of A. Denote by P_s the indecomposable projective A-module belonging to s and by K_s the subcategory of A-mod induced by all indecomposable modules V satisfying $\mathrm{Hom_A(P_s, V)} \neq 0 = \mathrm{Hom_A(P_s, \tau_A^- V)}$. Assume $\dim_k \mathrm{End}(P_s) = 1$.

We use the sign $\check{\mathcal{U}}$ for the subspace category of the vectorspace category $(K_s, \mathrm{Hom_A(P_s, -)})$ and prove that the fiber sum functor $F_s : \check{\mathcal{U}} \to A$-mod is dense if Fac P_s has only finitely many indecomposable objects. Moreover F_s preserves wildness and also tameness under some additional assumptions.

Our results above show that one can use $\check{\mathcal{U}}$ for the study of A-mod provided $\check{\mathcal{U}}$ can be computed. This is actually possible if s is a so-called decomposing point. Such points are defined by the property that the support of each indecomposable module over the factor algebra $A/Ae(s)A$ does not contain simultanously predecessors and successors of s in Δ. We denote by A^s respectively A_s the subalgebra of A given by all points which are not predecessors respectively successors of s in Δ. With these notations we prove that for decomposing points the vectorspace category $(K_s, \mathrm{Hom_A(P_s, -)})$ can be explicitely constructed from the module categories A^s-mod and A_s-mod which are easily accessible in many cases.

We give several examples of algebras for which our methode can be used to prove their tameness.

Yu.A. Drozd: Open subcategories and Cohen-Macaulay modules

Let $\mathfrak{A}$ be a bocs, $\mathfrak{X} \subset \mathrm{Rep}(\mathfrak{A})$ a full subcategory closed under isomorphic copies, direct sums and summands. Call $\mathfrak{X}$ open if for any vector dimension $\underline{d}$ the subset $\mathfrak{X} \cap \mathrm{Rep}_{\underline{d}}(\mathfrak{A})$ is Zariski open in the set of representations $\mathrm{Rep}_{\underline{d}}(\mathfrak{A})$ of dimension $\underline{d}$. For open subcategories of representations of free triangular bocses the tame/wild dichotomy is proved.

This notion is applied to Cohen-Macaulay modules over semiprime Cohen-Macaulay algebras over $K[|X|]$, K an algebraically closed field. Namely, for each such algebra Λ an open subcategory $\mathfrak{X}$ is constructed classifying Cohen Macaulay Λ-modules. As a consequence, the tame/wild dichotomy for Cohen-Macaulay modules is obtained.

J. Feldvoss: The module type of some classes of algebras

In my talk I consider the problem of determining the restricted Lie algebras of finite (resp. tame) module type. I give a complete solution of the first problem over arbitrary fields and of the second problem over algebraically closed fields of characteristic $p > 2$. Moreover, I indicate some problems for more general classes of algebras which perhaps could be used to solve the case $p = 2$ and to get rid of the assumption of the base field.

G.-M. Greuel: On tame and wild curve singularities (Report on results with Y.A. Drozd, A. Kotlov, A. Schappert)

The following results show that there is a really surprising coincidence between the classification of singularities by internal properties on one side and by max. (CM-)Cohen-Macaulay modules on the other side.

1st result: Let R be the complete local ring of a reduced plane curve singularity, then:

(i) R is of finite CM-type $\Leftrightarrow$ R is elliptic

(ii) R is of tame CM-type $\Leftrightarrow$ R is parabolic

(iii) $\mathrm{par}(R, 1) = 1 \Leftrightarrow R$ is strictly unimodal

The elliptic resp. parabolic resp. strictly unimodal singularities are the most basic classes in Arnold's classification. Elliptic resp. parabolic refers to the associated quadratic form on the Milnor fibre. The elliptic ones are just the "simple" or $A - D - E$-singularities, the parabolic curve singularities are

$$\tilde{E}_7 = T_{244} = x^4 + y^5 + ax^2y^2 \quad a^2 \neq 4$$

$$\tilde{E}_8 = T_{236} = x^3 + y^6 + ax^2y^2 \quad 4a^3 + 27 \neq 0.$$

(i) resp. (iii) resp. (ii) "$\Leftarrow$" was previously known and due to Greuel/Knõrrer resp. Schappert resp. Dieterich and Kahn. (par(R, n) denotes the max. dimension of a family of pairwise non isomorphic CM-R-modules of rank n).

2nd result: Let R be as before but no necessarily plane. Then R is CM-tame $\Rightarrow$ par$(R, \Lambda) = 1 \Leftrightarrow R$ dominates a strictly unimodal plane curve singularity.

J.Y. Guo: Isomorphisms of Hall algebras

In the talk we sketch the proofs of the following theorems:

Theorem 1 Let R, R' be representation directed, connected finite algebras (over fields k and k', respectively). The following conditions are equivalent

(1) $H(R) \simeq H(R')$.

(2) $k \simeq k'$ and $T_R \simeq T_{R'}$ and they have the same symmetrizations

(3) There is a bijection $\sigma : S_i \to \sigma(S_i)$ of simple R modules to the simple R' modules such that $\mathrm{End}_R S_i \simeq \mathrm{End}_{R'}\sigma(S_i) =: F_i$ (as finite fields), and $\mathrm{Ext}^t_R(S_i, S_j) \simeq \mathrm{Ext}^t_{R'}(\sigma(S_i), \sigma(S_j))$, as F_i modules and as F_j modules, for $t = 1, 2$ and all i, j where $H(R)$ is the Hall algebra of the algebra A and T_R the Auslander-Reiten quiver of R.

Theorem 2 Let R, R' be finitary rings satisfying the following conditions

(a) $\mathrm{Ext}^1_R(S_i, S_i) = 0$ for all finite simple R modules S_i.

(b) If $\mathrm{Ext}^1_R(S_i, S_j) \neq 0$, then $\mathrm{Ext}^1_R(S_j, S_i) = 0$ for simple all finite R modules S_i and S_j.

(c) R is connected and there are simple R-modules S_i, S_j such that

$\dim_{\mathrm{End}_R S_i} \mathrm{Ext}^1_R(S_i, S_j) = 1$ or $\dim \mathrm{Ext}^1_R(S_i, S_j)_{\mathrm{End}_R S_j} = 1$.

Then $H(R) \simeq H(R')$ implies that there is a bijection $\sigma : S_i \to \sigma(S_i)$ such that $\mathrm{End}_R S_i \simeq \mathrm{End}_{R'}\sigma(S_i) =: F_i$ and $\mathrm{Ext}^1_R(S_i, S_j) \simeq \mathrm{Ext}^1_{R'}(\sigma(S_i), \sigma(S_j))$ as F_i modules and as F_j modules, for all simple R modules S_i and S_j.

D. Happel: Generalized Nakayama Conjecture for algebras with $J^{2l+1} = 0$ and A/J^l representation-finite

This is a report on some joint work with Peter Dräxler.

Let A be a finite-dimensional algebra over some field k. Let us denote the Jacobson radical of A by J. We consider finitely generated left A-modules.

The generalized Nakayama conjecture says that in a minimal projective resolution of the injective cogenerator ${}_AD(A_A)$ each indecomposable projective A-module occurs as a direct summand, or equivalently that for each simple A-module ${}_AS$ there is an integer $i \geq 0$ such that $\mathrm{Ext}^i_A({}_AD(A_A), {}_AS) \neq 0$.

Theorem The generalized Nakayama conjectures holds for algebras A with $J^{2l+1} = 0$ and A/J^l representation-finite.

Observe that an algebra A with $J^3 = 0$ satisfies the assumptions of the theorem. In case $l = 2$ it is very easy to construct examples of algebras A with A/J^2 representation-finite.

A.V. Jakovlev: Torsion-free abelian groups of finite rank and theory of representations

For every prime p and for $p = 0$ we construct a category M_p containing the category M of all torsion-free abelian groups of finite rank.The category M_p for p prime is typical for representation theory: the Krull-Schmidt theorem holds in this category, and the problem of classification of indecomposable objects is equivalent to the classical 2-matrices problem (and so this problem is wild). The category M_0 is more complicated.

We say, that two groups of M belong to the same genus, if they become isomorphic in all categories M_p. A one-by-one correspondence between genae of groups of M and vectors of positive cones of certain finite-dimensional lattices is established. This correspondence preserves indecomposability and direct sums.

B. Keller: A remark on tilting and DG algebras

Differential graded algebras provide an alternative approach to the problems which J. Rickard has solved in his papers

[1] Morita theory for derived categories, J. London Math. Soc. **39**, 1989,

[2] Derived equivalences as derived functors, preprint.

Let k be a com. ring, A and B k-projective k-algebras , and T a tilting complex [1] over A such that $B \cong \mathrm{End}_{\mathcal{D}^b A}(T)$. Then T is a $\tilde{B} - A$-bimodule where $\tilde{B} =$ END (T) is the diff. graded alg. with $\tilde{B}^n = \prod_{p\in\mathbf{Z}} \mathrm{Hom}_A(T^p, T^{p+n})$. We show how one can modify T to obtain a $B - A$-bimodule complex X (note that $B = H^0\tilde{B} = \mathrm{End}_{K(A)}(T)$) such that ${}_BX$ and X_A have projective components and X_A is homotopy-equivalent to T.

O. Kerner: Regular stones and height one modules over wild hereditary algebras

Let A be a finite dimensional wild hereditary algebra over some algebraically closed field k. An indecomposable module X is called *stone*, if $\mathrm{Ext}(X, X) = 0$.

Theorem 1 If A is wild hereditary there exists only finitely many non sincere regular components containing stones of quasi-length bigger than one.

A regular module X is called ***height one*** module, if X is not the middle term of a short exact sequence $0 \to U \to X \to V \to 0$ with $U, V \neq 0$ and regular.

Theorem 2 If A has a height one module with selfextensions, then there do not exists sincere regular components containing stones of quasilength bigger than one.

"Normally" wild hereditary algebras have height one modules with selfextensions. But there exist (infinitely many) wild hereditary algebras such that: (a) All height one modules are stones, (b) there exist only finitely many τ-orbits of height one modules.

H. Krause: On a polynomial bound for endomorphism rings over some tame algebras

Let p be a polynomial. A k-algebra E is called polynomial bounded by p, if for any factor algebra E' with $\dim_k(rad/rad^2 E') \leq 2$ and $rad^n E' = 0$ $\dim_k E' \leq p(n)$ holds.

Let $Q = (Q_0, Q_1)$ be a quiver. For each word w in Q there is canonically defined a module $M(w)$ over the path algebra kQ.

Theorem Let $M(w)$ be the module corresponding to a word w in Q. Then $\mathrm{End}_{kQ}(M(w))$ is polynomial bounded by $p(n) = 2n^2 - 2n + 1$.

Corollary Let A be a string algebra and M be an indecomposable A-module of the first kind. Then $\mathrm{End}_A(M)$ is polynomial bounded by $p(n) = 2n^2 - 2n + 1$.

This result leads to the following conjecture:
There exists a polynomial p such that a fin. dim. k-algebra is of tame representation type iff $\mathrm{End}_A(M)$ is polynomial bounded by p for every indecomposable A-module of finite dimension.

H. Lenzing: Frobenius numbers, distinguished components and growth reciprocity

The talk reports on joint work with J.A. de la Peña and deals with the representation theory of a wild canonical algebra Λ of weight type $p = (p_1, \ldots, p_t)$ and the corresponding wild hereditary star algebra Λ_0 obtained by deletion of a suitable vertex of Λ.

All but one of the indecomposable projective Λ-modules lie in the preprojective component of Λ, the component of the remaining one is the *distinguished component* $\mathcal{D}$ of Λ. $\mathcal{D}$ agrees on a τ^--cone with a regular component $\mathcal{D}_0$ of Λ_0, the distinguished component of Λ_0.

The Hilbert-Poincaré series P_Λ of Λ (resp. P_{Λ_0} of Λ_0) measures the growth of the Auslander-Reiten translation τ_Λ, which is linear, and τ_{Λ_0} which is exponential. They are related by the reciprocity law $P_{\Lambda_0} = T/(1 - P_\Lambda)$ and further to the Coxeter polynomials ψ_Λ and ψ_{Λ_0} by the rule $P_{\Lambda_0} = \psi_\Lambda/\psi_{\Lambda_0}$. In particular this allows to derive the formula

$$\psi_{\Lambda_0} = \left[(T+1) - T\sum_{i=1}^{t} \frac{1-T^{p_i-1}}{1-T^{p_i}}\right] \cdot \prod_{i=1}^{t} \frac{1-T^{p_i}}{1-T}.$$

Among other items it is further shown that the set of integers n such that $\mathrm{Hom}(M, \tau_\Lambda^n M) \neq 0$ holds for each indecomposable Λ-module M of positive rank form a numerical semigroup, whose *Frobenius number* $n = \alpha(p)$ is the largest integer such that $\mathrm{Hom}(M, \tau_\Lambda^n M) = 0$ shows up. It is shown that always $\alpha(p) \leq 43$ (also for the hereditary star algebra Λ_0) and by associating Dynkin labels D_n, E_6, E_7, E_8 the wild canonical (resp. hereditary star) algebras are partitioned into four groups, with the maximal quasi-length of exceptional modules bounded by the numbers 1,2,3,5 resp. and equality attained in the case of the canonical algebras.

S.-X. Liu: Isomorphism problems for tensor algebras over valued graphs

Let Σ be a valued graph with a modulation (D, M). We denote by $T(\Sigma) = T(\Sigma, D, M)$ the tensor algebra over Σ. First we prove the following isomorphism theorem:

Theorem 1 Let Σ and Σ' be two valued graphs. If $T(\Sigma) \simeq T(\Sigma')$, then $\Sigma \simeq \Sigma'$.

In particular, we have

Theorem 2 Let Δ and Δ' be two quivers and F a field. If their path algebras $F(\Delta), F(\Delta')$ are isomorphic, then $\Delta \simeq \Delta'$.

Next we consider tensor products of path algebras and we have

Theorem 3 Let Δ, Δ' and F as in Th.2. Then

1. $F(\Delta)$ and $F(\Delta')$ are prime algebras $\Leftrightarrow$ $F(\Delta) \otimes_F F(\Delta')$ is prime,
2. $F(\Delta)$ and $F(\Delta')$ are semi-primitive algebras $\Leftrightarrow$ $F(\Delta) \otimes_F F(\Delta')$ is semi-primitive.
3. $F(\Delta)$ and $F(\Delta')$ are right noetherian algebras $\Leftrightarrow$ $F(\Delta) \otimes_F F(\Delta')$ is right noetherian.

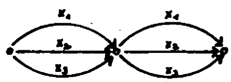
M.P. Malliavin: Lie group representations

My talk was concerned with a paper in preparation of A. Guichardet who connects the socalled admissible algebras to admissible categories. Admissible algebras (resp. categories) are special cases of Gabriel's pseudocompact rings (resp. length categories) [P. Gabriel: Indecomposable representations II, Symposia Mathematicia XI, Indam 1973]. As a result of this, we gave the quiver founded by Y. Gaiffard associated to the admissible category of Harish-Chandra modules with trivial infinitesimal character and trivial central character for Spin$(n, 1)$.

H. Meltzer: On tilting modules over the truncated symmetric algebra

This is a report on a joint work with L. Unger

In this talk we study tilting modules over the wild algebra $A = kQ/I$ where Q is the quiver



and I is the ideal generated by all $x_i x_j - x_j x_i$. We classify the tilting modules $M = M_0 \oplus M_1 \oplus M_2$ having the property that the endomorphism rings of all M_i are the ground field k. These tilting modules will be called exceptional.

Our result uses the classsification of the exceptional triples of coherent sheaves over $\mathbb{P}^2$ by Rudakov. Recall that a triple (E_0, E_1, E_2) of sheaves (respectively modules) is called exceptional if $\mathrm{End}(E_j) = k$, $\mathrm{Ext}^r(E_i, E_j) = 0$ for $0 \le i, j \le 2$, $r > 0$ and $\mathrm{Hom}(E_i, E_j) = 0$ for $j > i$. The classification of these exceptional triples is strongly related to the solutions of the diophantic equation $X^2 + Y^2 + Z^2 = 3XYZ$, called the Markov equation. It is known that its solutions can be obtained from the trivial solution $(1, 1, 1)$ by two standard transformations which allow us to associate with the set of all solutions an infinite tree of valency 3, the so-called Markov tree. Rudakov showed that all exceptional triples can be constructed from the tripel $(\mathcal{O}, \mathcal{O}(1), \mathcal{O}(2))$ by two operations called left and right mutations corresponding to the standard transformations mentioned above.

We consider A as the endomorphism algebra of the tilting sheaf $\mathcal{T} = \mathcal{O} \oplus \mathcal{O}(1) \oplus \mathcal{O}(2)$. Then the functor $\mathrm{Hom}(\mathcal{T}, \cdot) : \mathrm{coh}\mathbb{P}^2 \longrightarrow \mathrm{mod}A$ induces an equivalence of the derived categories $\mathrm{D}^b(\mathrm{coh}\mathbb{P}^2) \longrightarrow \mathrm{D}^b(\mathrm{mod}A)$. In the talk we show in which way all exceptional tilting modules can be constructed from the projective A-modules using "mutations" and the Auslander-Reiten translation in $\mathrm{D}^b(\mathrm{mod}A)$. We obtain a countable number of infinite trees similar to the Markov tree.

An important role in the proof is played by the fact that for two modules M and N with $\mathrm{End}(M) = k = \mathrm{End}(N)$ and $\mathrm{Ext}^i(M, M) = 0 = \mathrm{Ext}^i(N, N)$ for $i \neq 0$ we have complete information about $\mathrm{Hom}(M, N)$ and $\mathrm{Ext}^i(M, N)$ by purely numerical data corresponding to the slopes associated to vector bundles.

G.O. Michler: On deformations of modular group algebras

Let (F, R, S) be a splitting p-modular system for the finite group G. Let X be an indeterminate, $A = R[[X]]$, $P = \pi A$, and A_P the localization of A at P. Then $A_P/PA_P = F((X))$, A_P and $S((X))$ is also a splitting p-modular system for G. Most of the known examples of semisimple deformations $(F((X)), *)$ of the modular group algebra FG are liftable. In particular, each p-block B of FG with cyclic defect group $\delta(B)$ has a semisimple liftable deformation $(B \otimes_F F((X)), *)$. The main result of this lecture asserts that FG has a semisimple liftable deformation if and only if the group ring RG has a deformation $(A_P G, \hat{*})$ which is a maximal A_P-order in the semisimple group algebra $S((X))G$. It follows that the decomposition matrix of the semisimple liftable deformation $(F((X)), *)$ equals the ordinary decomposition matrix of G with respect to (FG, RG, SG).

J.A. de la Peña: Tame algebras with a sincere directing module

Let Λ be a finite dimensional, basic and connected algebra over an algebraically closed field k.

An indecomposable Λ-module X in said to be directing if X does not belong to a cycle $X = X_0 \xrightarrow{f_1} X_1 \rightarrow \cdots \xrightarrow{f_n} X_n = X$ of non zero non isomorphisms with X_i indecomposable, $1 \leq i \leq n$. The module X is sincere if $\mathrm{Hom}_\Lambda(P, X) \neq 0$ for every $0 \neq P$ projective.

Algebras with a sincere directing module were classified in the representation-finite situation, they are relevant in the polynomial growth tame situation.

Theorem Let Λ be a tame algebra with a sincere directing module. Then

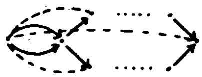
a) Λ is domestic in at most 2 parameters ($\mu_\Lambda \leq 2$)

b) If $\mu_\Lambda = 1$, then Λ is a (co-)enlargement of a domestic (co-) tubular algebra.

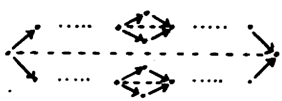
c) If $\mu_\Lambda = 2$, then Λ is the glueing of two domestic one parametric algebras.

Now, let Λ be a tame algebra with a sincere directing module and assume that $\mu_\Lambda = 2$. Then Λ is a tilted algebra, say of type Δ. We get the following

Proposition a) If Δ in not a tree, then Λ (or Λ^{op}) is of the form



b) If Δ is a tree, then Δ has at most 5 terminal points. If Δ has 5 terminal vertices, then Λ (or Λ^{op}) is of the form



J. Richard: Derived categories of blocks of symmetrie groups

According to conjectures of Michel Broué, there are many situations in modular representations theory where two related blocks of related group algebras should have equivalent derived categories. In all cases where these conjectures were previously known to be true, the proof is unsatisfactory in that it follows from a complete knowledge of the algebra structure of the blocks. I shall discuss the case of blocks of symmetrie groups, where many equivalences of derived categories occur, and where for the first time there is a proof that does not use a complete description of the algebra structure.

Ch. Riedtmann: Lie algebras generated by indecomposables

Given a finite dimensional associative $\mathbf{C}$-algebra Λ having only finitely many indecomposable modules, there is a Lie algebra structure on the free $\mathbf{Z}$-module generated by the indecomposables. For $\Lambda = \mathbf{C}\vec{Q}$, where Q is a Dynkin diagram A_n, D_n, E_6, E_7 or E_8, this Lie algebra is the positive part of a $\mathbf{Z}$ form of the simple Lie algebra associated with Q.

A. Schofield: Representations of algebras and a path to Lie algebras

Given an arbitrary f. d. associative $\mathbf{C}$-algebra, A, we associate a Lie algebra, $L^+(A)$, to its representation theory. In the case of $A = \mathbf{C}\vec{Q}$, the Lie algebra is the positive part of the Kac-Moody Lie algebra associated to Q.

D. Simson: Matrix problems, coverings and Cohen-Macaulay modules

Theorem 1 (Kasjan, Simson) Let I be a finite poset having exactly two maximal elements $*, +$ and let K be a field. The quadratic form

$$q_I(z) = \sum_{j\in I} z_j^2 + \sum_{\substack{p<q \\ q\neq *,+}} z_p z_q - \sum_{p<*} z_p z_* - \sum_{q<+} z_q z_+$$

is weakly nonnegative iff the posets $*^{\nabla} := \{j \in I; j < *\}$ and $+^{\nabla}$ do not contain (1,1,1,1,1), (1,1,1,2), (2,2,3), (1,3,4), (N,5), (1,2,6) and I does not contain certain 41 critical posets with two maximal elements. If the category $mod_{sp}KI$ of socle projective representations of I is of tame type then q_I is weakly nonnegative. The converse is proved for $\tilde{\mathbb{A}}_3$-free posets of finite growth.

Theorem 2 (Lenzing, Simson) Let A be a tiled $K[[t]]$-order of index $s \geq 1$. Then there is a $\mathbb{Z}$-graded $K[t]$-algebra S_A and an infinite poset I_A such that $\hat{S}_A \cong A$ and the Auslander-Reiten completion functor $\hat{\bullet} : CM^{Z}(S_A) \to CM(\hat{S}_A)$ between the categories of Cohen-Macaulay modules is equivalent to a covering functor $\mathsf{F} : I_A - \tilde{sp} \to latt(A)$ of Roggenkamp-Wiedemann type.

An application to curve singularities of finite lattice type will be discussed.

A generalization of Theorem 2 for nontiled orders will be indicated in a connection with Theorem 1, representations of bipartite posets and socle projective representations of associated bound quivers.

A. Skowroński: Algebras of polynomial growth

Let k be an algebraically closed field and A be a finite dimensional k-algebra. Then A is of polynomial growth if there exists a natural number m such that the indecomposable finite dimensional A-modules occur, in each dimension d, in a finite number of discrete and at most d^m one-parameter families. Moreover, A is called standard if there exists a Galois covering $R \to R/G = A$ with R simply connected. Here, by a simply connected k-category we mean a locally bounded k-category R such that: (a) R is Schurian and $\tilde{A}$-free;

(b) the quiver of R is connected, interval-finite and without oriented cycles;

(c) $H_1 R = 0$.

Applying the Galois covering techniques developed several years ago by Dowbor and the speaker we shall give a criterion for polynomial growth of standard algebras and describe the indecomposable finite dimensional modules over such algebras.

S.O. Smalø: Short chains and short cycles

Let Λ be an artin R-algebra. An indecomposable f.g Λ-module C is

i) said to be the middle of a short chain if there exists an indecomposable module X with $\mathrm{Hom}_\Lambda(X, C) \neq 0 \neq \mathrm{Hom}(C, DTrX)$, and

ii) said to belong to a short cycle if there exists an indecomposable Λ module X and $f : X \to C$ and $g : C \to X$ with $f \neq 0 \neq g$ and both f and g nonisomorphisms. With this notion we prove the following

Theorem Let M and N be indecomposable Λ-modules having the same composition factors.

a) If M does not belong to any short cycle then $M \simeq N$

b) If M is not the middle of any short chain and N is not on an A_∞^∞-sectional chain, then $M \simeq N$

c) If Λ is of finite representation type and M is not the middle of a short chain, then $M \simeq N$.

L. Unger: A family of non-selfextending generic bricks

This is a report on some joint work with Dieter Happel.

Let A be a finite-dimensional, hereditary k-algebra over an algebraically closed field k. An A-module Q is called a brick if $\mathrm{End}Q$ is a division ring. An infinite-dimensional A-module is called generic if it is of finite length over its endomorphism ring.

It is known that A has indecomposable generic modules if and only if A is not representation-finite.

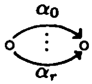
Our considerations were motivated by the following result of Ringel:

Theorem A tame algebra has up to isomorphism a unique non-selfextending generic brick.

In the talk the following questions will be considered:

If A is wild, do there exist modules with the same properties?

If yes, is there a 'canonical' way to construct them?

To attack these questions we consider the algebra $\mathcal{K}_r$: 

which for $r > 1$ is wild, and construct one non-selfextending generic brick explicitely.

For $r > 1$ we give an algorithm how to construct from this module an infinite family of modules having the same properties. This is then used to show the following theorem:

Theorem For a wild finite-dimensional, hereditary k-algebra A there exists an infinite family of pairwise non-isomorphic non-selfextending generic bricks.

P. Webb: The Structure of Mackey Functors

I will survey some key points in recent work of J. Thévenaz and myself in which we develop a theory of Mackey functors as representations of a certain quiver with relations, or equivalently as representations of an algebra we call the "Mackey algebra". We proceed by considering simples, projectives etc., and it turns out that a development which parallels the usual development of group representation theory is possible. I will emphasize the theorem which characterizes when the Mackey algebra has finite representation type.

D.J. Woodcock: The relationship between cohomology and combinatorics in the Schur algebras for GL_n and its parabolic subgroups.

G a reductive group over an algebraically closed field k, B a Borel subgroup. Kempf's vanishing theorem can be stated in the form:

If λ is a dominant weight the simple rational B-module of weight λ is Ind_B^G-acyclic.

It is a corollary that if V, W are finite dimensional rational G-modules then $\mathrm{Ext}^i_G(V, W) = \mathrm{Ext}^i_B(V, W)\ \forall i \geq 0$.

I am studying analogous questions with the categories of rational $G \ltimes B$-modules replaced by the categories of modules for

Schur algebras for $G = GL_n$ and $B \leq G$ a Borel subgroup. The methods used are combinatoric: The modules I consider have bases parametrized by various types of tableaux.

Ch. Xi: Symmetric algebras as endomorphism rings of large projective modules over quasi-hereditary algebras

In this talk the following theorem is proved:

Theorem 1 Let A be a basic, connected algebra. Then the following are equivalent:

(1) A is symmetric, and there is an indecomposable module M such that $\mathrm{End}_A({}_AA \oplus M)$ is quasi-hereditary.

(2) A is isomorphic to the trivial extension of a serial algebra with radical square zero and finite global dimension.

As a consequence of the above theorem we have

Theorem 2 Let $S_K(n, r)$ be a Schur algebra with $n \geq r$. Assume that the field K has a prime characteristic p. Then $S_K(n, r)$ is of finite type if and only if $r < 2p$.

K. Yamagata: Construction of algebras with large global dimensions

E. Green gave an example of a family of finite dimensional algebras A_n $(n \geq 0)$, over an algebraically closed field, with exactly two isomorphism classes of simple modules and having global dimension n. The algebra A_0 in the family is a semi-simple algebra with two simple modules and A_n is obtained by adding one arrow and zero-relations to the quiver of A_{n-1}. The aim of my talk is to generalize the construction to arbitrary algebras. More precisely, for an algebra A we shall construct a family of algebras $A_n(n \geq 0, A_0 = A)$ having the same number of simple modules as the number of simple A-modules, so that the global dimension of A_n is greater than that of A_{n-1} provided that the global dimension of A is finite, and the Cartan determinants of all A_n are the same as that of A. Although our construction depends on the decomposition of A and the order of decomposition factors, the global dimension of every A_n is determined by the number n and the number of decomposition factors provided that the ring A is semi-simple or hereditary.

B. Zimmermann Huisgen: Homological domino effects and the first finitistic dimension conjecture

We refute the first finitistic dimension conjecture. This conjecture asserted that $\operatorname{fin}\dim \Lambda = \operatorname{Fin}\dim \Lambda$ for every finite dimensional algebra Λ, where $\operatorname{fin}\dim \Lambda$ is the supremum of the projective dimensions of those finitely generated left Λ-modules which have finite projective dimension, and $\operatorname{Fin}\dim \Lambda$ is the analogous supremum obtained on waiving the restriction 'finitely generated'. The class of examples we present rests on a theory that provides thorough understanding of both finitistic dimensions for a certain class of finite dimensional algebras. This theory allows us not only to construct algebras Λ with $(\operatorname{fin}\dim \Lambda, \operatorname{Fin}\dim \Lambda) = (n, n+1)$ for each $n \geq 2$, but also to exhibit a variety of other homological frills.

The examples are counterbalanced by positive insights, e.g. concerning the question of when the two finitistic dimensions do coincide. Moreover, the positively graded modules over certain graded algebras are shown to be rather manageable from a homological point of view in that they are no more difficult to handle than finitely generated modules; this is in sharp contrast to the behavior of the $\mathbb{Z}$-graded modules in general.

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