MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 21/1991

Nonlinear Evolution Equations

12.5. bis 18.5.1991

Die Tagung fand unter der Leitung von Herrn S. Klainerman (Princeton) und Herrn M. Struwe (Zürich) statt. Die Teilnehmer kamen aus der Bundesrepublik Deutschland, den USA, der Sovjetunion, Australien, Japan und anderen Ländern und vertraten einen breiten Themenkreis aus dem Gebiet der nichtlinearen Evolutionsgleichungen. Schwerpunkte stellten unter anderem Regularitätssätze für semi-lineare Wellengleichungen, Evolution von Hyperflächen unter dem mittleren-Krümmungs-Fluss und Probleme der allgemeinen Relativitätstheorie dar.

Die Ergebnisse wurden in interessanter und verständlicher Weise vorgetragen. Sicherlich gaben auch die fruchtbaren Diskussionen vielerlei Anregungen.

Vortragsauszüge

Curvature Evolution of Hypersurfaces and Selfsimilarity

Gerhard Huisken, ANU, Canberra

Let $F(\cdot,t) : M^n \hookrightarrow R^{n+1}$ be a smooth family of hypersurface immersions such that

$$\frac{d}{dt}F(p,t) = f(\kappa_1,\kappa_2,\ldots,\kappa_n)\,\nu(p,t)\ ,\ t>0,\ p\in M^n\,. \qquad (*)$$

Here ν is the unit normal to the hypersurface $M_t = F(\cdot,t)(M^n)$ and f is a smooth homogeneous, symmetric function of the principal curvatures κ_i. The flow $(*)$ is parabolic and admits at least shorttime solutions under natural assumptions provided $\frac{\partial f}{\partial \kappa_i} > 0$. Typical examples are the mean curvature $H = \kappa_1 + \ldots + \kappa_n$, the Gauss curvature $K = \kappa_1\,\kappa_2\,\ldots\,\kappa_n$ and the harmonic mean curvature $H_{-1} = (\kappa_1^{-1} + \ldots + \kappa_n^{-1})^{-1}$.
It is demonstrated that in many cases solutions of $(*)$ approach selfsimilar solutions, either in finite time near a singularity, or in infinite time. Various examples are discussed and a classification of selfsimilar contracting solutions is given for the mean curvature flow, i.e. $f = H$.

Asymptotic Orbital Stability on Nonlinear Dispersive Systems

Michael I. Weinstein, Ann Arbor

I shall discuss work in progress on asymptotic orbital stability and scattering for a class of nonlinear Schrödinger systems which supports (nonlinear) bound states and dispersive solutions. These are nonintegrable Hamiltonian systems whose dynamics are characterized by interaction for all times among bound states and dispersive radiation. The System is studied using a decomposition of the solution, whose bound state part is modeled by "collective coordinates" or modulation of symmetries.

On the Nature of Singularities of General Relativity

Demetrios Christodoulou

This is a survey of my work on the Einstein equations in the spherically symmetric case with a scalar wave field as the material model. The main results are the following: The maximal class of wave functions for which the initial value problem is well posed are the functions whose derivatives are of bounded variation. The domain of outer communications U is defined to be the union of all future null geodesic cones with vertices at the center of symmetry which are complete and whose cross sectional area tends to ∞. For each such cone we can define the Bondi mass M. If the initial total variation is small then there is a global solution, U is complete and the final Bondi mass $M_1 = 0$. On the other hand, if there is, on the initial future cone, an annular region bounded by two spheres such that the mass contained in the region bears to the radius of the outer sphere a ratio which is large in comparison to the ratio of the radii minus 1, then a trapped sphere forms in the future whose mass is bounded from below in terms of the two initial radii. (For a trapped sphere S the outer component of the boundary of the future of S is contracting at S). $M_1 > 0$ and U has a boundary H, the event horizon, a complete future null geodesic cone whose cross sectional area tends to $16\pi M_1^2$. The trapped region T, defined to be union of all trapped spheres, is a future set whose past boundary A, the apparent horizon, is asymptotic to H and contains no incoming null segments. The future boundary of T is the singular boundary B minus its center B_0. This is the union of all spheres of zero radius and positive mass. $B \setminus B_0$ is strictly spacelike. The curvature blows up at $B \setminus B_0$ at least as fast as $(\text{radius})^{-3}$. Finally, all initial data except those in an exceptional set ϵ, contained in a hypersurface in the space of functions of bounded variation, lead either to a complete solution or to a strictly spacelike singular boundary preceded by a trapped region (Cosmic censorship). However, ϵ contains an infinite dimensional set of initial data which lead to the formation in the future of singular points contained in U and lying at the center of symmetry (naked singularities).

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Flow of Convex Hypersurfaces by Functions of Curvature

Ben Andrews, Canberra

This work generalizes the results on mean curvature flow of convex hypersurfaces to flows by other curvature functions in a large class. I will also discuss recent developments in Harnack inequalities for these flows, and some applications in geometry.

Blow Up for Semilinear Parabolic Equations

Pavol Quittner, Bratislava

We study stationary solutions and asymptotic behaviour of solutions of the following two problems:

Problem (P1):

$$\begin{aligned} u_t &= \Delta u - \lambda u^p && \text{in } \Omega \times (0,T) \\ \frac{\partial u}{\partial n} &= u^q && \text{on } \partial\Omega \times (0,T) \\ u(x,0) &= u_0(x) \geq 0 && x \in \Omega \end{aligned}$$

Problem (P2):

$$\begin{aligned} u_t &= \Delta u - |\nabla u|^q + \lambda u^p && \text{in } \Omega \times (0,T) \\ u &= 0 && \text{on } \partial\Omega \times (0,T) \\ u(x,0) &= u_0(x) \geq 0 && x \in \Omega \end{aligned}$$

Here Ω is a bounded domain in R^n, p and $q > 1$, $\lambda > 0$. If e.g. $N = 1$, then the blow up (in finite time in L^∞-norm) of solutions of (P1) or (P2) may occur iff $p \leq 2q - 1$ (and $\lambda < q$ if $p = 2q - 1$) or $p > q$ (and λ is sufficiently large if $q \geq \frac{2p}{p+1}$), respectively. The results for (P1) were obtained jointly with M. Chipot and M. Fila, the blow up results for (P2) are new only in the case $p > q > \frac{2p}{p+1}$.

Hölder Estimates for Quasilinear Doubly Degenerate Parabolic Equations

A. V. Ivanov, Leningrad

We consider quasilinear degenerate parabolic equations of the type

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x_i} a^i(x,t,u,\nabla u) + b(x,t,u,\nabla u) = 0$$

in a cylinder $Q_T = \Omega \times (0,T]$, $\Omega \subset R^n$, $n \geq 1$, where the functions $a^i(x,t,u,p)$ and $b(x,t,u,p)$ satisfy Caratheodory conditions and some growth conditions. The typical example is the equation of nonnewtonian polytropic filtration

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x_i}(a_0|u|^{\delta(m-1)}|\nabla u|^{m-2}\frac{\partial u}{\partial x_i}) = 0 \ , \ a_0 > 0 \, , \ b \geq 0 \, , \ m \geq 2 \, .$$

We proved

1. Inner and boundary Hölder estimates for positive weak solutions independent of their infimums.
2. Existence of nonnegative Hölder continuous in $\overline{Q}_T$ weak solutions of the Cauchy-Dirichlet problem.

Existence of Global Small Solutions in Nonlinear Thermoelasticity

Reinhard Racke, Bonn

First we characterize some typical existence respectively blow-up results for nonlinear wave equations in bounded and unbounded domains. Then the nonlinear hyperbolic-parabolic coupled system of thermoelasicity is considered.

1. The three-dimensional Cauchy problem is reviewed.
2. Results for one-dimensional models are presented, mainly on the existence of global smooth solutions for small data.

The Cauchy Problem for Semilinear Hyperbolic Equations

Lev Kapitanskii, Leningrad

The existence and uniqueness of global weak solutions to the Cauchy problem for a semilinear hyperbolic second-order (pseudo) differential equation with the critical Sobolev exponent (in particular, the equation $u_{tt} - \Delta u + |u|^{4/(n-2)}u = 0$, $t \in R^1$, $x \in R^n$, $n \geq 3$) is proved. The proof is based on certain new estimates for the solutions of the corresponding linear problems, which generalize in several directions the well known estimates of Strichartz, Brenner and Ginibre and Velo.

Some Remarks on the Regularity properties of the equivariant wave map in $2+1$ dimensions

Manoussos G. Grillakis, College Park

The wave map equations consist of a system of equations of the form

$$\Box u^i + \Gamma^i_{jk}(u)\partial^\alpha u^j \partial_\alpha u^k = 0 \ , \ i = 1, \ldots, n. \tag{1}$$

Here $u : R \times R^n \to (M^n, h_{ij})$ is a map from the flat Minkowski space-time to an n-dimensional Riemannian manifold M equipped with a metric h_{ij}, and $\Gamma^i_{jk}(u)$ are the Christoffel symbols on M. The space $R \times R^n$ is equipped with the standard metric $\eta^{\alpha\beta} = \text{diag}(-1, 1, \ldots, 1)$ and $\partial_\alpha = \frac{\partial}{\partial x^\alpha}$, $\partial^\alpha = \eta^{\alpha\beta}\partial_\beta$ while $\Box = \partial^\alpha\partial_\alpha$ is the D' Alembertian. Equations (1) describe the evolution of waves on the manifold M, and are "critical" if $n = 2$. Assuming that $n = 2$ and that the map is equivariant, i.e. it satisfies a certain symmetry, one can prove that the solutions of (1) are regular for all time provided that the metric satisfies a certain restriction. It is an interesting open problem to understand the behaviour of (1) when $n = 2$. If one assumes that M has negative curvature the solutions should be regular, it is not clear what happens if M has positive curvature, for example $M = S^2$ the two dimensional sphere.

Degenerate Semilinear Parabolic Equations

Andreas Stahel, Salt Lake City

On a smooth bounded domain Ω in R^N and on a given time interval $I = [0, T^+]$ we consider the equation

$$u_t - \text{div}(a\, \text{grad} u) + b\, \text{grad} u = f(u)$$

with Dirichlet, Neumann or mixed boundary conditions. For a subset S of Ω we require

$$a \in C^0(\overline{\Omega}, L(R^N)) \cap C^\infty(\overline{\Omega}\backslash S, L(R^N)), \ a\,|_{\overline{\Omega}\backslash S} > 0\,.$$

If $\underline{a}(x)$ is the smallest eigenvalue of the symmetric matrix a, then we have the essential assumption $\underline{a}^{-1} \in L_q(\Omega)$ for some $q > 1$.

If $s\,f(s) \leq c(1+s^2)$ and $u|_{t=0} \in W^{2,2}(\Omega) \cap L^\infty(\overline{\Omega})$ and $\mathrm{supp}(u|_{t=0}) \subset\subset \Omega \backslash S$ then there is a unique solution u of the problem with

$$u \in \mathrm{Lip}(I, L_2(\Omega)) \cap C^{\frac{1}{2}}(I, H^1_a(\Omega)) .$$

H^1_a is the weighted Sobolev space with the norm $\|u\|^2_{H^1_a} = \|u\|^2_{L_2} + \langle a\nabla u, \nabla u\rangle$.

On a Construction of Morse Flows for a Variational Functional of the Harmonic Map Type

Norio Kikuchi, Keio University, Yokohama

We try to construct Morse flows for the variational functional

$$F(u) = \int_\Omega A^{\alpha\beta}_{ij}(x, u(x)) \frac{\partial u^i}{\partial x^\alpha}(x) \frac{\partial u^j}{\partial x^\beta}(x)\, dx$$

in the Sobolev space $H^1(\Omega, R^M)$, $M \geq 1$, where Ω is a bounded smooth domain in Euclidian space R^m, $m \geq 2$, and the coefficients $A^{\alpha\beta}_{ij}$, $1 \leq \alpha, \beta \leq m$, $1 \leq i, j \leq M$, are bounded smooth functions satisfying Legendre-Hadamard condition. We adopt the following method. Let T be a given positive number and $u_0 \in H^1(\Omega, R^M)$ be a given initial mapping. For a positive integer N sufficiently large, we put $h = T/N$. Starting from u_0, we inductively construct two sequences of mappings u_n and functionals F_n, $n = 1, 2, \ldots, N$, as follows: For each n, $1 \leq n \leq N$, we define a functional f_n by

$$F_n(u) = F(u) + \int_\Omega \frac{1}{h}|u - u_{n-1}|^2\, dx$$

and fix u_n as a minimizer of F_n in the space $u_0 + H^1_0(\Omega, R^M)$. We notice that $\{u_n\}_{1\leq n\leq N}$ is an approximate Morse flow. We have an estimate, independent of N, of "Caccioppoli's type" for $\{u_n\}$.

For time-discrete linear parabolic equations related to the functionals F_n, $1 \leq n \leq N$, it is stated that the following estimates are valid:

1. Hölder estimate of Ladyzhenskaya-Ural'ceva's type.
2. Harnack inequality of Morse's type (by M. Misawa).
3. Campanato estimate for vector case.

Global Solutions of Nonlinear Wave Equations

Hans Lindblad, Princeton

We are concerned with conditions on G such that the Cauchy problem

$$\begin{aligned} \partial_t^2 u - \sum_{i=1}^{3} \partial_{x_i}^2 u &= G(u, u', u'') & \\ u &= \epsilon u_0 & \text{for } t = 0,\ u_0 \in C_0^\infty(R^3) \\ u_t &= \epsilon u_1 & \text{for } t = 0,\ u_1 \in C_0^\infty(R^3) \\ G(0) = G'(0) &= 0 & \end{aligned}$$

has global C^∞ solutions u for small $\epsilon > 0$. It is well known that the "null condition" of Klainerman implies global existence. However, we conjecture that this should be true also if G contains terms of the form

$$u \partial_{x_j} \partial_{x_k} u, \quad j, k = 0, 1, 2, 3.$$

We prove this conjecture in the radially symmetric case.

The Initial Value Problem for Self-Gravitating Fluid Bodies

Alan Rendall, Garching

I will discuss what is known concerning the initial value problem for a self-gravitating fluid body and in particular the question of local in time existence in the case of a perfect fluid described by the Euler equations. In the case of Newtonian gravitation there is an existence result due to Makino and I have generalized this to the corresponding situation within the framework of general relativity. An intersting intermediate case, that of post-Newtonian gravitation, will also be mentioned together with possible applications to the rigorous justification of approximation techniques in general relativity.

Global Existence and Asymptotics of Solutions of the Robinson-Trautman (2d-Calabi) Equation

Piotr Chrusciel, Canberra

To every equation of the Robinson-Trautman (RT) equation

$$\frac{\partial g_{ij}}{\partial u} = \frac{1}{12m} \Delta_g R \, g_{ij} \ , \quad m = \text{constant} \qquad (*)$$

where g_{ij} is a u-dependent family of Riemannian metrics on a smooth, compact, connected, orientable two dimensional manifold M^2, one can associate a vacuum solution of Einstein equations describing a space-time, with a Schwarzschildlike event horizon when $m > 0$ and $M^2 = S^2$. Global existence of solutions of $(*)$ can be established for H_4 initial data. A rather precise asymptotic expansion as $u \to \infty$ of solutions of $(*)$ is needed in order to understand smoothness of the space-time metric across the event horizon $\mathcal{H}^+$. In a joint work by this author and D. Singleton such an expansion has been shown to hold, which leads to the result that for generic RT space-times which start from "sufficiently small" initial data no extensions of C^{123} differentiability across the event horizon $\mathcal{H}$ exist, vacuum or otherwise. It is believed that the "small data" restriction is unnecessary.

Global Classical Solutions of Semilinear Wave Equations

Hartmut Pecher, Wuppertal

Consider Cauchy problems of the type

$$\begin{aligned} u_{tt} - \Delta u + f(u) &= 0 \\ u(x,0) &= \varphi(x) \\ u_t(x,0) &= \psi(x) \end{aligned}$$

where the data φ and ψ are not assumed to satisfy a smallness condition. If the energy functional is definite and the data have finite energy the existence of global weak solutions is well known. If one wants to have more regularity, further restrictions were necessary to establish such results. In the case of C^2-solutions f had to fulfill a growth condition at $\pm\infty$, namely $f(u) \sim |u|^{p-1}u$ as $u \to \pm\infty$, where $p < \frac{n+2}{n-2}$, and moreover $n \leq 9$. If $n = 3$, $p = \frac{n+2}{n-2}$ could be included . I am now able to show that in the case $n = 3$ also nonlinearities which only fulfill a "one-sided" condition are allowed. f has to be of the class $C^{2,\alpha}(R)$, and $|f(s)| \leq K$ as $s \rightarrow +\infty$, and $f(s) \geq -K \ \forall s \in R$, where K is an arbitrary real constant. For a more restricted class of nonlinearities such as $f(u) = 0$ for $u \geq 0$ and $f(u) = |u|^p$ for $u < 0$ and $p > 2$ arbitrary, a densely defined scattering operator in energy space exists that belongs to the pair of equations $u_{tt} - \Delta u + f(u) = 0$ and $u^0_{tt} - \Delta u^0 = 0$.

Regularity of the Flow defined by Quasi-linear Hyperbolic Equations and Applications to Qualitative Properties of the Flow

Herbert Koch, Heidelberg

The local behaviour of a flow near an equilibrium should be described by some kind of Taylor expansion. Bifurcation theorems establish such results in rather general situations–for differentiable flows.
An example is given, which shows that the flow generated by a hyperbolic equation is not differentiable. This flow has some differentiability properties if one considers a scale of spaces. This allows to prove a Hopf bifurcation theorem.

Evolution and Self-Organization Laws in Complex Systems

Sergey P. Kurdyumov, Moscow

New ideas and mathematical methods for the studying different nonlinear open systems and general classes of nonlinear differential equations are given. It is shown that properties of localization of different nonstationary dissipative processes in a nonlinear medium such as nonlinear heat conductivity and combustion are the most general ones and play an important role in the understanding "laws of evolution" and the structure of the attractors of nonlinear systems. The idea of so called "eigenfunctions" of nonlinear systems which are the only asymptotically stable trajectories is considered. It is shown that in many cases above "eigenfunctions" are some explicit solutions which are invariant under Lie or Lie-Bäcklund transformations or so called approximate self-similar solutions. The class of nonlinear heat equations with source of energy of the form $u_t = \Delta\phi(u) + Q(u)$ is discussed in detail.

Regularity of Scattering Operators for Nonlinear Klein Gordon Equations

Philip Brenner, Göteborg

In this talk we discuss mapping properties of the scattering operator

$$S: \quad (u_-, \partial_t u_-) \longrightarrow (u_+, \partial_t u_+)$$

where $u_\pm$ are solutions of the linear Klein-Gordon equation

$$\partial_t^2 u - \Delta u + m^2 u = 0 \;, \quad x \in R^n, \; t \in R, \qquad \text{(KG)}$$

such that they asymptotically are approached by the solution u of the equation

$$\partial_t^2 u - \Delta u + m^2 u + f(u) = 0 \qquad \text{(NLKG)}$$

as $t \rightarrow \pm\infty$. The main result is the following theorem.

Theorem: Let $n \geq 3$ and let $f(u) = |u|^{\rho-1}u$ where $1 + \frac{4}{n} < \rho < 1 + \frac{4}{n-2}$. Then S maps $H^s \times H^{s-1}$ into $H^s \times H^{s-1}$ for $1 \leq s \leq 2$ and

$$||u(t) - u_\pm(t)||_{H^s} \longrightarrow 0 \;, t \rightarrow \pm\infty .$$

Derivative Nonlinear Schrödinger Systems

Horst Lange, Köln

We consider some examples of nonlinear Schrödinger equations which contain derivatives of the wave function in the nonlinear part, and discuss the difficulties coming from that fact. The examples are e.g.

Problem (P1):

$$i\,u_t = -u_{xx} + i\,\alpha |u|_x^2\, u \,.$$

Problem (P2):

$$i\,u_t = -u_{xx} + \beta |u|_{xx}^2\, u \,.$$

Problem (P3):

$$\begin{aligned} i\,u_t &= -u_{xx} + \gamma\phi_u\,u \\ \phi_u &= \frac{\partial\phi}{\partial x}|_{y=0} \\ \phi_{xx}+\phi_{yy} &= 0 \\ \phi_y|_{y=0} &= \delta\,|u|_x^2 \\ \phi_y|_{y=-1} &= 0 \\ u(x+1,t) &= u(x,t)\,. \end{aligned}$$

Here α, β and γ are real numbers. The Problems (P1) and (P3) come from the theory of deep water waves (Dysthe's NLS), whereas (P2) describes the thickness and velocity of a thin layer of a superfluid film (also appears in Heisenberg ferromagnets). We discuss global existence and uniqueness results for weak finite energy solutions of various initial boundary value problems for (P1), (P2) and (P3) which are quite different in each case.

Sharp Estimates of Blow-Up for Degenerate Parabolic Equation with Gradient-like Diffusion Source

Sergey A. Posashkov, Moskow

The Cauchy problem for quasilinear parabolic equation with source

$$\begin{aligned} u_t &= A_0(u) := \nabla\cdot(|\nabla u|^\sigma\nabla u) + u^\beta && \text{for } t>0,\ x\in R^n \\ u(0,x) &= u_0(x) \geq 0 && x \text{ in } R^N \\ \nabla u_0 &\in C(R^N) \\ \sup u_0 &< \infty\,, \end{aligned}$$

where $\sigma > 0$ and $\beta > \sigma+1$ are fixed constants, is considered. Unbounded blow-up solutions $u = u(t,x) \geq 0$ to the above problem existing on a finite time interval $(0, T_0)$ such that

$$\limsup u(t,0) = +\infty\ ,\ t \to T_0^- < +\infty$$

is investigated. The main results are to prove estimates of $u(t,x)$ near finite blow-up time $t = T_0^-$. For solution $u = u(t,r)$, $r = |x| > 0$ under some hypotheses on $u_0(r) \geq 0$ the following upper estimate in $(0,T_0)\times\{r>0\}$

$$u(t,r) \leq \Big(\frac{\beta-(\sigma+1)}{\sigma+2}\Big(\frac{\sigma}{\sigma(N+1)+2}\Big)^{\frac{1}{\sigma+1}}\Big)^{-\frac{\sigma+1}{\beta-(\sigma+1)}}\cdot r^{-\frac{\sigma+2}{\beta-(\sigma+1)}}$$

and the following lower estimate for small $r > 0$

$$u(T_0^-, r) := \liminf_{t \to T_0^-} u(t, r) \geq C_1 r^{-\frac{\sigma+2}{\beta-(\sigma+1)}}, \ C_1 = C_1(\sigma, \beta, N)$$

are proved.

Global Solutions of the Dirichlet Problem in One-Dimensional Nonlinear Thermoelasticity

S. Jiang, Bonn

We consider the following initial boundary value problem for the equations of one-dimensional nonlinear thermoelasticity

$$\begin{aligned} u_t - v_x &= 0 \qquad\qquad x \in (0, \infty) \\ v_t - \sigma_x &= 0 \qquad\qquad t > 0 \\ (e + \frac{v^2}{2})_t - (\sigma v)_x + q_x &= 0 \end{aligned}$$

$$\begin{aligned} v|_{\partial\Omega} = \theta|_{\partial\Omega} &= 0 \\ u(0) = u^0, \ v(0) &= v^0, \ \theta(0) = \theta^0 \end{aligned}$$

u is the displacement gradient, v the velocity and θ the temperature difference. $\sigma := \hat{\sigma}(u, \theta) = \frac{\partial \psi(u,\theta)}{\partial u}$ is the stress and $\psi(u, \theta)$ the Helmholtz free energy, $q := q(\theta_x)$ is the heat flux, $e := \hat{e}(u, \theta) = \psi(u, \theta) - (\theta + T_0) \frac{\partial \psi(u,\theta)}{\partial u}$ the initial energy and T_0 the reference temperature.
Under the usual assumptions on ψ and q (such that the above system is hyperbolic-parabolic) we prove a global existence theorem for smooth small initial data by the L^2 energy method.

Almost Periodic Attractors for a Class of Reaction-Diffusion Equations on $\mathbf{R}^N$

Pierre-A. Vuillermot, Max-Planck-Institut für Mathematik, Bonn

Our talk was devoted to the presentation of some recent results concerning the long-time behaviour of classical solutions to certain parabolic partial differential equations which occur in population genetics. We showed how to construct almost-periodic attractors for those equations, and how to prove their uniform Liapounov stability by devising an appropriate center-manifold theory for nonlinear and non autonomous evolution equations on Sobolev spaces

Transitions in Convective Turbulence

Peter Constantin, Chicago

We offer an estimate of the average area of isotherms in convective turbulence as a possible signature of the observed transitions. The estimate predicts roughening above an inner scale. As this scale passes through the significant scales established in the medium (size of the domain, size of the mixing layer etc.) the signal changes qualitatively.

Weak and Strong Solutions of the Navier-Stokes Equations

Wolf von Wahl, Bayreuth

We consider the Navier-Stokes equations

$$\begin{aligned} \partial_t u - v\Delta u + u\nabla u + \nabla\pi &= f \\ \nabla u &= 0 \\ u|_{\partial\Omega} &= 0 \\ u(0,x) &= u_0(x) \end{aligned}$$

over a cylindrical domain $[0,+\infty) \times \Omega$, where Ω mostly is a smooth, bounded domain of R^3. First we eliminate the pressure π by applying the projection onto the divergence-free part of $L^p(\Omega)$. Then a local (in time) strong solution of the problem is constructed. Beside this solution there is a global weak solution of the problem. The connection between local strong and global weak solutions is discussed via Serrin's uniqueness theorem. Finally the regularity of weak solutions is studied.

Die Herren B. Chow und F. John waren leider verhindert an der Tagung teilzunehmen; sie haben jedoch ebenfalls Vortragszusammenfassungen geschickt, die wir nachstehend abdrucken.

The Ricci Flow on Surfaces and 2-Orbifolds

Bennett Chow, New York

The Ricci flow is the nonlinear parabolic equation obtained by deforming a Riemannian metric in the direction of its Ricci tensor. In light of Thurston's Geometrization Conjecture and the Calabi conjecture, it is of special interest to consider the Ricci flow on 3-dimensional Riemannian manifolds and n-dimensional Kähler manifolds, respectively. The study of the Ricci flow on surfaces and 2-orbifolds is necessary in analyzing the singularities which develop under the Ricci flow on 3-dimensional Riemannian manifolds. Moreover, surfaces are 1-dimensional Kähler manifolds. Thus, one hopes that the techniques on surfaces will generalize to Kähler manifolds. By the work of Richard Hamilton, Lang-Fang Wu and the speaker, there is a relatively complete understanding of the Ricci flow on surfaces. In particular, one has a new proof of the uniformization theorem and a generalization to orbifolds.

Lifespan of Finite Amplitude Waves in an Isotropic Homogeneous Hyperelastic Material

Fritz John, New York

Let $u = u(t, x) = u(t, x_1, x_2, x_3)$ denote the displacement vector, and $u' = (\frac{\partial u_i}{\partial x_k})$ the Jacobian matrix. The equations of motion are hyperbolic and of the form

$$\frac{\partial^2 u}{\partial t^2} = \sum_{r,s} c^{rs}(u') \frac{\partial^2 u}{\partial x_r \partial x_s} \,. \tag{1}$$

Here the matrices $c^{rs}(u')$ have as elements the second derivatives of the energy function $W(u')$. In our case $W(u')$ is a symmetric function of the eigenvalues of the strain matrix $\frac{1}{2}(u' + u'^t + u'^t u')$. The initial conditions are

$$u = f(x) \,, \; \frac{\partial u}{\partial t} = g(x) \quad \text{for } t = 0 \tag{2}$$

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with f, g in $C(R^3)$. We first show the "almost global" existence of waves arising from "small" initial disturbances. Let δ be the supremum of the L_2-norms of all derivatives of f and g of order ≤ 15. There exist then two positive constants δ_0 and A depending only on the choice of the function $W(u')$, such that the solution $u(t, x)$ of (1) and (2) exists for

$$x \in R^3, \ t \leq e^{A/\delta} \tag{3}$$

provided $\delta < \delta_0$. For a finer analysis of the solution we assume that the initial data (2) have the form

$$f(x) = \epsilon F(x), \ g(x) = \epsilon G(x) \tag{4}$$

with F and G fixed in $C_0^\infty(R^3)$ and a positive scalar parameter ϵ. We investigate the solution $u = u(t, x, \epsilon)$ for small ϵ. We write (1) as a first order system for the vector U with the 12 components $\frac{\partial u_i}{\partial x_k}$, $\frac{\partial u_i}{\partial t}$ for $i, k = 1, 2, 3$:

$$\frac{\partial U}{\partial t} = \sum_j B^j(u') \frac{\partial U}{\partial x_j} \tag{5}$$

We write $x = r\xi$ with $r = |x|$, $\xi \in S^2$. We introduce the characteristic matrix of the linearized equations

$$B = \sum_j B^j(0)\xi_j \tag{6}$$

We have 12 eigenvalues λ_a and corresponding eigenvectors η^a for the constant matrix B. if $0 < c_2 < c_1$ denote the speeds of transverse and longitudinal waves in the linear theory, then $\lambda = 0$ is an eigenvalue of multiplicity 6, $\lambda = \pm c_2$ have multiplicity 2, and $\lambda = \pm c_1$ have multiplicity 1.

The key is the expansion

$$\frac{d}{dr} U = \sum_a w_a \eta^a \tag{7}$$

It turns out that $w_a = O(\frac{\epsilon}{t^2})$ as long as $(\lambda_a + \frac{r}{t})^{-1}$ is bounded. The bulk of the transverse waves is concentrated near the cone $r = c_2 t$, and that of the longitudinal waves near the cone $r = c_1 t$. Surprisingly the transverse waves do not blow up before the longitudinal ones, due to the special nature of the function $W(u')$. Blow-up of the longitudinal waves does not occur before a time $e^{A/\epsilon}$. Here A depends on the choice of F, G and W. More precisely A depends on the values for large t of the solution of the linear equation

$$\frac{\partial^2 u}{\partial t^2} = \sum_{r,s} c^{rs}(0) \frac{\partial^2 u}{\partial x_r \partial x_s}$$

with initial values (4), and on the first derivatives of the $c^{rs}(u')$ for $u' = 0$.

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