MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1991

Differential Geometry in the Large

19.5. bis 25.5.1991

Die Leitung der Tagung hatten die Herren W. Ballmann (Bonn), J.P. Bourguignon (Palaiseau), W. Klingenberg (Bonn) und W. Ziller (Philadelphia) inne. In den 23 Vorträgen wurden aktuelle Ergebnisse vorgestellt, deren Themen mit den Stichworten negative Krümmung, homogene Räume, dynamische Systeme, metrische Differentialgeometrie und Analysis auf Mannigfaltigkeiten nur grob umrissen sind.

Beeindruckend war das enge Zusammenspiel geometrischer und analytischer Methoden, das zu zahlreichen vertiefenden und anregenden Diskussionen Anlaß gab.

Die familiäre Atmosphäre in Oberwolfach bot Gelegenheit zum Knüpfen neuer Kontakte über Generationen und Nationen hinweg, besonders erfreulich war die Anwesenheit einiger russischer Mathematiker.

VORTRAGSAUSZÜGE

M. ANDERSON (Stony Brook)

MINIMIZING THE L^2 NORM OF CURVATURE ON 3-MANIFOLDS

Consider the functional $\int_M |R|^2(g)dv_g$ on the space of metrics $\mathcal{M}_1$ of volume 1 (mod diffeomorphisms) on a compact 3-manifold M. We study the convergence/degeneration of sequences approaching critical values of $\int |R|^2 dv$ — e.g. the infinum.

If (g_i) is a sequence in $\mathcal{M}$, with $\int |R|^2_{g_i} dv_{g_i} \leq k$, we define a thick/thin decomposition of (M, g_0). The thick part is precompact in the $C^\alpha \cap L^{2,\alpha}$ topology, for $\alpha < \frac{1}{2}$; in particular there are only finitely many diffeomorphism types. The thin part admits an f-structur — is hence a union of Seifert fibred manifolds.

If (g_i) is a sequence in $\mathcal{M}_1$ approaching a critical value of $\int |R|^2$, we prove convergence on a (maximal) thick part $\Omega \subset M$ to a smooth metric g_∞ on Ω which is critical for $\int |R|^2$ on Ω.

If one restricts $\int |R|^2 dv$ to the submanifold $\mathcal{S} \subset M_1$ of constant scalar curvature metrics it is remarked that critical metrics for the restricted functional are critical also on $\mathcal{M}_1$. This leads to existence of critical metrics which are complete, finite volume, of constant scalar curvature and with $|R|^2 =$ const. The conjecture was made that such critical metrics are complete hyperbolic. This is true if the Ricci-curvatrue is negative at some point.

I. BABENKO (Moscow)

ASYMPTOTIC INVARIANTS OF RIEMANNIAN MANIFOLDS AND CLOSED GEODESICS

The function of the volume of a geodesic ball of a large radius for some covering of the Riemannian manifold is considered. New homotopical invariants of the given manifold are determined by means of changing the initial metric. These invariants are not the invariants of the fundamental group only, they characterize the given manifold as a whole. Call these invariants "absolute asymptotic volumes" They are closely connected with some other useful geometrical characteristics of the manifold. In particular they allow to get asymptotic estimates of the number of closed geodesics depending on the "topological constants".

Ch. BÄR (Bonn)

REAL KILLING SPINORS AND HOLONOMY

We classify all compact simply connected Riemannian spin manifolds carrying real Killing spinors. A real Killing spinor is a spinor field ψ for which there is a constant $\alpha \in \mathbb{R}, \alpha \neq 0$, such that for all tangent vectors X the equation $\nabla_X \psi = \alpha \cdot X \cdot \psi$ holds. In particular, we show that in even demension $n \neq 6$ only the standard sphere occurs. The proof yields new examples of manifolds with exceptional holonomy G_2 and Spin (7).

V. BANGERT (Freiburg)

CLOSED GEODESICS ON S^2: RECENT PROGRESS

In a preprint from February 1991 John Franks presents a proof for the existence of infinitely many (geometrically distinct) closed geodesics for a large class of Riemannian metrics on S^2 including all those with positive curvature. This is the class where — following G.D. Birkhoff's idea — one can reduce the study of the geodesic flow to the investigation of the dynamics of an area preserving map of an annulus. Relying on results by W. Thurston, M. Handel and by himself John Franks shows that every area-preserving diffeomorphism of the closed annulus which has an interior (fixed or) periodic point has infinitely many interior periodic points. Prior to this work the existence of infinitely many closed geodesics was only known for C^2 metrics on S^2.

If Birkhoff's annulus map is not defined one can use results by Bangert or Bangert/Klingenberg to prove the existence of infinitely many closed geodesics by variational methods.

Put together this shows that there exist infinitely many closed geodesics on every Riemannian 2-sphere.

G. BESSON (Grenoble)

ENTROPY OF RIEMANNIAN MANIFOLDS

We (G. Courtois & S. Gallot) study the volume growth of a negative curved metric in the universal cover of a compact manifold. The main question (asked by M. Gromov) is whether a locally symmetric metric is a minimum, for this invariant, among metrics of fixed volume.

Our result extends a result due to Katok which says that the answer is true when we restrict ourselves to metrics in the conformal class of the locally symmetric one.

To do so we developped a technique consisting of embedding the universal cover in the unit sphere of a Hilbert space, and then introduce a conformal invariant and estimate it in terms of known quantities such as the bottom of the spectrum in the universal cover and the entropy of the universal cover. Then we finish by a question involving the volume of ideal simplices in the hyperbolic geometry.

O. BIQUARD (Palaiseau)

STABLE PARABOLIC BUNDLES AND SINGULAR CONNECTIONS

We consider parabolic bundles over a Riemann surface in the sense of Seshadri. We construct functional spaces of connections, singular over the marked points, and gauge groups, which are canonically associated to the parabolic structure. As a consequence, one can prove the Mehta-Seshadri theorem by Donaldson's method and give constructions of 1) the moduli space of stable parabolic bundles 2) the moduli space of parabolically stable Higgs bundles with the Higgs field meromorphic at the punctures and at most simple poles, with residue preserving strictly the parabolic structure.

S. BUYALO (Leningrad)

SIMPLICIAL SPACES ASSOCIATED TO A HADAMARD MANIFOLD AND ITS ISOMETRY GROUP

We introduce in a natural way a series of simplicial spaces associated to a Hadamard manifold and its isometry group. This construction is very useful in many questions concerning the manifolds of nonpositive sectional curvature. I mean such questions as what we can say about such manifolds with small inscribed ball radius, how do look like the ends of manifold with negative sectional curvature. For example, this construction plays a crucial role in the proof of the following theorem: There exists a constant $\varepsilon > 0$ such that if an n-dimensional, $n = 2, 3, 4$, closed smooth Riemannian manifold M with sectional curvature $-1 \leq K \leq 0$ has the inscribed ball radius $\sigma(M) \leq \varepsilon$, then there exists a Cr-structure on M and the metric of M contains locally an Euclidean factor.

E. CALABI (Philadelphia)

ISOSYSTOLIC PROBLEMS

Given a closed manifold $M(\text{class}\, C^\infty)$ various classes of ***systoles*** can be defined as functionals on the space of all Riemannian metrics on M : for any such metric g, for instance:

i) the ***general systole*** $L_0(g)$ = infimum length of all closed geodesics

ii) the ***homotopy systole*** (if $\pi_1(M) \neq \{e\}) L_\pi(g)$ = infimum length of all non-contractible curves on M;

iii) the ***homological systole*** (if $H_1(M, \mathbb{Z}) \neq 0)$, $L_{H_\mathbb{Z}}(g)$ = infimum length of all nontrivial cycles represented by closed curves in M.

iv) the ***rational homological or orientable homology systole***, $L_{H_\mathbb{Q}}(g)$ resp. $L_{H_t}(g)$, if $H_1(M)$ satisfies corresponding conditions, represented by the infimum length of all closed curves representing a nontrivial rational homology class (resp. a nontrivial orientation preserving integral cycle) on M.

Related to these systoles (especially to $L_{H_\mathbb{Q}}(g)$ is the "dual" systole $L^*(g)$ defined as the infimum L_∞-norm (with resp. to g) of any closed 1-form β on M representing a de Rahm class on M with nontrivial periods, all integral valued.

All the above may be compared with the total volume $V(g)$ of M via the scale-invariant ***systolic ratio*** $\rho = \rho(g)$

$$\rho(g) = \frac{V(g)}{L_*(g))^n}, n = dim_{\mathbb{R}} M$$

For certain manifolds (notably $K\Pi(1)$-manifolds) the systolic ratio has an ***absolute lower bound*** (Blatter, Accola, Berger, Gromov et al.) for all g on M called the ***extremal isosystolic ratio***, $\sigma(M)$: Gromov proved that it can be achieved by some (possibly degenerate or singular) metric.

General estimates of the exremal isosystolic ratio are rather crude, because of lack of sufficiently useful criteria to construct or test metrics for closeness to an extremal one: a few notable exceptions form the following short list:

i) the real projective spaces $\mathbb{P}^n_{\mathbb{R}}$

$$\rho(\mathbb{P}^n_{\mathbb{R}}) \text{ (in terms of } L_\pi) :$$

achieved by the elliptic geometry metric (Pu, Chavel).

ii) For the torus T^2: the flat metric on $\mathbb{C}/\mathbb{Z}[\frac{-1+\sqrt{-3}}{2}]$ (C. Löwner) and in higher dimension by $\mathbb{R}^n/\Lambda_0$ where Λ_0 is an n-lattice representing an optimal sphere-packing by a lattice pattern.

iii) For the Klein bottle: a special, singular metric (Bavard) defined by glueing together two Möbius band on the boundary where each has the metric defined by taking the euclidean spherical zone $\{(x,y,z) e \mathbb{R}^3 : x^2+y^2+z^2=1, |z| \leq \frac{1}{\sqrt{\pi}}\}$ and identifying antipodal points: here

$$V(g) = 2\pi \cdot \sqrt{2},\ L(g) = \pi \quad \rho(M) = \frac{2\cdot\sqrt{2}}{\pi}$$

For other surfaces the problem is open. Conjecture (by Gromov): For a surface Σ of genus g (orientable), $g \gg 2$,

$$\rho(\sum) \sim c \cdot \sqrt{g}$$

The problem is being investigated for $g = 2,3$ with varying results and upper and lower bound estimates for $\rho(\sum)$. The Euler-Lagrange equation for systems of closed geodesics achieving the systole on $\sum$ and intersecting in a common local domain are derived and studied; this is an application of a global theory of Riemannian (geodesic) webs of geodesic foliations.

Chr. CROKE (Pennsylvania)

VOLUMES OF BALLS IN MANIFOLDS WITHOUT CONJUGATE POINTS

We consider volumes $\tilde{V}(x,r)$ of balls of radius r in the universal cover of a compact Riemannian manifold M which has no conjugate points. We compare the average $\underset{x\in M}{AVE}\ \tilde{V}(x,r)$ (with respect to the Riemannian measure) to $V_\delta(r)$, the volume of a ball of radius r in the simply connected space of constant curvature δ.

a) $\underset{x\in M}{AVE}\ \tilde{V}(x,r) \geq V_0(r)$ with equality iff M is flat.

b) If $\liminf_{r\to\infty} \frac{\tilde{V}(x,r)}{V_0(r)} = 1$ then M is flat.

Further if h_μ is the measure entropy of the geodesic flow of M and $\delta = -(\frac{h_\mu}{n-1})^2$.

c) $\underset{x\in M}{AVE}\ \tilde{V}(x,r) \geq V_\delta(r)$ with equality iff M has constant curvature δ.

We also use b) to prove a rigidity result for geodesic flows of flat manifolds.

P. FOULON (Palaiseau)

ENTROPY RIGIDITY AND GODBILLON-VEY INVARIANTS

There are several rigidity problems in the context of contact Anasov flows φ_t on a compact manifold. For the differentiable rigidity we have the following result (Benoist-Labourie- –). Theorem. Let M^{2n+1} be a compact manifold, φ_t an C^∞ contact Anasov flow. If

($\star$) stable and unstable foliations are C^∞

then there exist $[\alpha] \in H^1(M, \mathbb{R})$ such that the new spray $Y = \frac{X}{1+\alpha(x)}$. is C^∞ conjugated (up to finite covers) to the geodesic flow of a locally symmetric space of rank 1. For the rigidity entropy problem I prove: Theorem. Let M^3 be a compact 3-manifold, φ_t a contact C^∞ Anasov flow. If the metric entropy and the topological entropy coincide ($h_{top} = h_\mu$), then the flow φ_t is C^∞ conjugated to the geodesic flow of a surface with constant negative curvature (up to finite covers). In higher dimension the question is not solved, even for the geodesic flows. I introduce new invariants, which I hope will help for some understanding of the problem. The stable (unstable) foliation are Lagrange foliations (F, A), to such a structure are associated $(n+1)$ classes $GV(P)$ that extend the Godbillon-Vey classes. For surfaces these classes can be used as well as Gauss-Bonnet formula to gain rigidity, the advantage is that $GV(2)$ exists in any dimension and I provide for it a Mitsumatsu formula.

C. GORDON (Hannover)

ISOSPECTRAL, NON-ISOMETRIC MANIFOLDS

Two closed Riemannian manifolds are said to be isospectral if the associated Laplace operators have the same eigenvalue spectrum. All known examples of isospectral manifolds have a common Riemannian cover. We consider a number of constructions and examples of isospectral manifolds and examine their geometry.

D. GROMOLL (Stony Brook)

ISOMETRIC DEFORMATIONS OF COMPACT EUCLIDEAN SUBMANIFOLDS IN CODIMENSION 2

Jointly with M. DAJCZER (IMPA) we have developed a structure theory for isometric deformations of compact submanifolds $f : M^n \to \mathbb{R}^{n+2}$, $n \geq 5$. Roughly speaking, any isometric deformation is almost everywhere induced by isometric deformations of hypersurfaces. This is a global result which is not true already in the complete case. As a particular application, if p_1 is the first geometric Pontrjagin class of M, then $p_1^2 = p_1 \wedge p_1$ must vanish wherever f is deformable. So for example, when f is analytic and topologically $[p_1]^2 \neq 0$, then f must be rigid. This answers a question of M. Gromov.

U. HAMENSTÄDT (Bonn)

EQUIVALENCE OF HARMONIC AND LEBESGUE MEASURES IMPLIES ASYMPTOTICAL HARMONICITY

We consider compact Riemannian manifolds M of negative sectional curvature. The universal covering $\tilde{M}$ of M is diffeomorphic to $\mathbb{R}^n$, and for every $x \in \tilde{M}$, Brownian motion on $\tilde{M}$ starting at x gives rise to a Borel-probability measure ω^x on the ideal boundary $\partial\tilde{M} \sim T_x^1\tilde{M}$ The measures ω^x project to measures on the fibres of $T^1M \to M$. We then obtain a Borel-probability ω on T^1M by defining $\omega(A) = \int \omega^x(A \cap T_x^1M)dx$. This measure is the unique harmonic measure for the Laplacian along the leaves of the stable foliation.

Using a suitable differential complex we show: Thm: If ω is contained in the Lebesgue measure class then $\tilde{M}$ is asymptotically harmonic, i.e. asymptotically the Green's function σ on $\tilde{M}$ only depends on the distance of points.

J. HEBER (Augsburg)

RANK RIGIDITY OF HOMOGENEOUS SPACES OF NONPOSITIVE CURVATURE

We consider homogeneous spaces of nonpositive curvature which do not split as a Riemannian product. Such spaces can always be identified with a solvable Liegroup $S = \mathcal{A} \ltimes \mathcal{N}, \mathcal{N} = [S, S]$, which is equipped with a left invariant metric $\langle \cdot, \cdot \rangle$. The abelian factor $\mathcal{A}$ is a flat, i.e. a totally geodesically and isometrically embedded Euclidean space in $(S, \langle \cdot, \cdot \rangle)$ (results of Wolf, Aleksewskii, Azencott/Wilson and Heintze). In this situation

(i) the foliation $S = \bigcup_{n \in \mathcal{N}} (n \cdot \mathcal{A})$ by flats naturally induces a polyhedral decomposition of dimension $(dim\mathcal{A} - 1)$ for the ideal boundary $S(\infty)$. This structure completely describes Gromov's Tits-distance at infinity and coincides with the Tits-building at infinity if $(S, \langle \cdot, \cdot \rangle)$ is symmetric;

(ii) as an application, (i) helps to prove higher rank rigidity, i.e. if $(S, \langle \cdot, \cdot \rangle)$ is higher geometric rank (every geodesic is contained in a flat of $dim \geq 2$), the space is necessarily symmetric of noncompact type and higher rank (unless it is a product).

D. JOYCE (Oxford)

QUOTIENT CONSTRUCTIONS IN QUATERNIONIC GEOMETRY

The subject of this talk is hypercomplex, hyperkähler, quaternionic and quaternionic Kähler manifolds. (See e.g. Salomon, 'Riemannian geometry and holonomy groups', Pitma Research Notes 201 (1989) for definitions). We describe a quotient construction for hypercomplex and quaterionic manifolds that generalise constructions of Hitchin et al. for the hyperkähler case, and Galidcki and Lawson for the quaternionic Kähler case. The important point is the definition of 'moment map', which, in the hypercomplex case is 3 functions $\mu_i : \mathcal{M} \to g^*$, $i = 1, 2, 3$, satisfying the differential equation $I_1 d\mu_1 = I_2 d\mu_2 = I_3 d\mu_3$, and also a 'tranversality condition', and which should be equivariant with the adjoint action of the quotient group G on g^*.

We then state some results, that self-dual metrics on $n \cdot \mathbb{C}P^2$ can be constructed as quaternionic quotients of $\mathbb{HP}^{n+1}$ by $U(1)^n$, that include all of the explicit solutions given by Poon and LeBrun and also some others; further, that in dimensions higher than four, using quotient techniques and other techniques we may construct compact, simply-connected hypercomplex and quaternionic manifolds, homogeneous and non-homogeneous, in every diversion > 4.

This is a strong contrast to the hyperkähler and quaternionic Kähler case, where there are very few compact examples known other than Riemannian symmetric spaces; indeed it is open whether there exist compact quaternionic Kähler manifolds of positive scalar curvature that are not locally symmetric.

A. KATOK (University Park)

RIGIDITY OF SMOOTH ACTION OF 'LARGE' GROUPS
(Super-rigidity and hyperbolic dynamics)

We give a survey of recent (and on-going) work by S. Hurder, J. Lewis, R. Zimmer and the author dealing with differentiable rigidity and global classification of actions of lattices on compact manifolds. There are two approaches: one based on hyperbolic dynamics, Stowe's theorem and recovering the rational structure (started by Hurder and continued by Lewis and the author) and the other based on the one of Zimmer's super-rigidity, property T and normal hyperbolicity.

B. KLEINER (Pennsylvania)

A THREE DEMINSIONAL ISOPERIMETRIC INEQUALITY

If $D \subseteq \mathbb{E}^n$ is a compact domain with smooth boundary then its boundary area and its volume satisfy the isoperimetric inequality area $(\partial D) \geq C_n[\text{volume}(D)]^{\frac{n-1}{n}}$ where $C_n = \frac{\text{vol}_{n-1}(S^{n-1}(1))}{[\text{vol}_n(B(1))]^{\frac{n-1}{n}}}$.

It was conjectured some time ago that the same inequality would be valid for compact domains sitting in a complete, 1-connected Riemannian manifold of nonpositive curvature. Our main result is the following comparison therorem, which settles the three dimensional case of the conjecture.

Theorem 1: Let (M^3, g) be a complete, 1-connected, threedimensional Riemannian manifold with sectional curvature $K^3_M \leq k \leq 0$, and let N^3_k be the model space of constant sectional curvature k. If $D \subseteq M^3$ is a compact domain with smooth boundary, and $B \subseteq N^3_k$ is a geodesic ball having the same volume as D, then area $(\partial D) \geq \text{area}(\partial B)$. Moreover, if area $(\partial D) = \text{area}(\partial B)$ then D is isometric to B.

The analysis of the equality case in Theorem 1 leads to a rigidity result for compact subdomains of Hadamard manifolds of dimension $n \geq 3$:

Theorem 2: Let (M^n, g) be a complete, 1-connected Riemannian manifold with sectional curvature $K_{M^n} \leq k \leq 0$, and let $D \subseteq M^n$ be a compact domain with smooth boundary. If the sectional curvature of M^n is equal to k for every two-plane tangent to ∂D, then D has constant sectional curvature k.

F. LABOURIE (Palaiseau)

AFFINE ANASOV-DIFFEOMORPHISMS

We (Y. Benoist and I) study Anasov diffeomorphisms preserving a volume form and a connection. We prove

Theorem 1: Let φ be an Anasov diffeomorphism preserving a connection and a volume form. If the stable and unstable foliations are C^∞, then φ is C^∞ conjugated to a smooth automorphism of an infranilmanifold. *Corollary:* Every symplectic Anosov diffeomorphism with smooth stable and unstable foliations is C^∞ conjugated to a smooth automorphism of an infranilmanifold.

The starting point of this is a theorem of Gromov which establishes the existence of a locally homogeneous structure on an open dense set, for a geometric structure having a topologically transitive pseudo-group of transformations.

The problem then is to study Anasov actions on homogeneous manifolds and to identify the group in question. Using deep results of Margulis-Goldsheid and Guivarc'h-Rangi we also prove

Theorem 2: let G be a Zariski-dense subgroup of $\mathbf{SL}(n,\mathbb{R})$ (to simplify) then there exists a subgroup G_1 on G such that (i) G_1 is Zariski-dense (ii) free and finitely generated (iii) consists only of real diagonalisable elements.

C. OLMOS (Córdoba)

ISOPARAMETRIC SUBMANIFOLDS AND THEIR HOMOGENEOUS STRUCTURES

We give a simple and geometric proof of the result of Thorbergsson 'Every compact irreducible isoparametric submanifold of rank (i.e. codeminsion) ≥ 3 of the Euclidean space is an orbit of an s-representation (i.e. isotropy representation of semisimple symmetric spaces)'. The proof relies on the construction of a canonical connection on the submanifold such that the second fundamental form is parallel with respect to the normal connection and this new connection. This connection satisfies also that the difference tensor between itself and the Levi-Civita connection is parallel (with the new connection). From a joint work of C. Sánchez with the author follows the homogenity.

A. RESNIKOV (Trieste)

SYMPLECTIC TWISTOR SPACES

We construct closed 2-forms in twistor spaces over a compact manifold M. If M is symplectic itself, or if $\dim M = 4$ and M is Einstein and $K \neq 0$, or if M is selfdual and Ricci curvature is pinched, then these forms are symplectic. This gives a vast collection of new examples of symplectic manifolds of different homotopical type.

We prove that any selfdual manifold M with positive pinched curvature is conformally equivalent to S^4 or $\mathbb{C}P^2$ with standard metric. We also study intersections of totally geodesic manifolds of positively curved M. We prove that two such submanifolds of a pinched M^{2n} of dimension n intersect at least in 2 points, using Lagrangian intersection theory.

M. RUMIN (Strasbourg)

A HOMOGENEOUS COMPLEX ON CONTACT MANIFOLDS

We construct on contact manifolds a complex of differential forms 'modulo contact forms'. This complex is a resolution and leads, in the C.R. strictly pseudoconvex case, to strongly hyperelliptic Laplacians which are homogeneous with respect to some natural rescaling of the metric. This allows us to represent the cohomology by harmonic forms in this sense. By the use of the Webster connection, and associated Weitzenböck formulas, we can prove that if the Ricci pseudohermitian curvature tensor R is big enough compared to the Webster torsion A (and it's horizontal covariant derivative in the three deminsional case), then the first Betti number vanishes. In fact, at least in dimension three, one can prove, by studying the Carnot geodesics, that the manifold is compact when R is big enough against A and two horizontal covariant derivatives of it.

G. TIAN (SUNY, Stony Brook)

KÄHLER-EINSTEIN METRICS AND STABILITY

We consider the relation between the existence of Kähler-Einstein metrics and various stability conditions on a compact Kähler manifold with positive first chern class. A concept of strong stability of the tangent bundle is introduced. Such a strong stability is the same as the stability of the extension of the tangent bundle by a trivial one with extension class equal to a multiple of the first chern class. We proved that the existence of Kähler-Einstein metrics implies the strong semi-stability of the tangent bundle. We also proved that in case of complex surfaces with positive first chern class the existence of Kähler-Einstein metrics is equivalent to the strong stability of the tangent bundle. Other related topics were also discussed in the lecture.

McKenzie WANG (Hamilton, Canada)

NON-SIMPLY CONNECTED RIEMANNIAN MANIFOLDS WITH NON-NEGATIVE ISOTROPIC CURVATURE

This is a report on joint work in progress with M. Micallef. A Riemannian manifold (M,g) has non-negative (resp. positive) isotropic curvature if for all isotropic 2-planes $P \subset TM \otimes \mathbb{C}P = \{Z \wedge w\}, \langle \check{R}(Z \wedge w), Z \wedge w \rangle \geq 0$ (resp. > 0), where $\check{R}$ is the curvature operator extended $\mathbb{C}$-linearly to $\wedge^2 TM \otimes \mathbb{C}$ and $\langle\ ,\ \rangle$ is the hermitian inner product induced by g.

Suppose that (M,g) is compact (without boundary) with non-negative isotropic curvature. First we give a structure theorem for the universal cover $\tilde{M}$. Next, we show that all harmonic 2-forms on an even-dimensional M (with the above properties) are parallel. If g is locally irreducible, we show that either $b_2(M) = 0$ or else (M,g) is Kähler with $C_1(M) > 0$ and $H^2(M;\mathbb{Z}) \approx \mathbb{Z}$.

Specializing to 4 dimensions, we prove that connected sums of manifolds with positive isotropic curvature admit metrics of positive isotropic curvature. We apply Hamilton's Ricci flow to refine the statement for the structure of manifolds with non-negative isotropic curvature. A Tachibana-type theorem is also deduced, with the corallary that a quarter variably pinched Einstein metric on S^4 is isometric to the standard metric. Some information about manifolds with $\frac{s}{\sigma} \cdot 1 - W_1 \geq 0$ or $\frac{s}{\sigma} \cdot 1 - W_1 \geq 0$ & Einstein are also obtained using the heat flow.

BERICHTERSTATTER: MATTHIAS WEBER

Tagungsteilnehmer

Prof.Dr. Uwe Abresch
Mathematisches Institut
Universität Münster
Einsteinstr. 62

4400 Münster

Christian Bär
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

5300 Bonn 1

Prof.Dr. Michael T. Anderson
Department of Mathematics
State University of New York

Stony Brook NY 11794-3651
USA

Prof.Dr. Marcel Berger
IHES
Institut des Hautes Etudes
Scientifiques
35, Route de Chartres

F-91440 Bures-sur-Yvette

Prof.Dr. Ivan K. Babenko
Chair of Higher Geometry & Topology
Dept. of Mathematics and Mechanics
Moscow State University

Moscow 11 98 99
USSR

Prof.Dr. Gerard Besson
Mathematiques
Universite de Grenoble I
Institut Fourier
Boite Postale 74

F-38402 Saint Martin d'Heres Cedex

Prof.Dr. Werner Ballmann
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

5300 Bonn 1

Dr. Olivier Biquard
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Victor Bangert
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

7800 Freiburg

Prof.Dr. Jean Pierre Bourguignon
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Jochen Brüning
Institut für Mathematik
Universität Augsburg
Universitätsstr. 8

8900 Augsburg

Prof.Dr. Keith Burns
Max-Planck-Institut für Mathematik
Gottfried Claren Str. 26

5300 Bonn 3

Prof.Dr. Sergei V. Buyalo
Leningrad Branch of Steklov
Mathematical Institute - LOMI
USSR Academy of Science
Fontanka 27

Leningrad 191011
USSR

Prof.Dr. Eugenio Calabi
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Bruno Colbois
Ch. Sout 6

CH-1028 Préverenges

Prof.Dr. Chris B. Croke
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Patrick B. Eberlein
Dept. of Mathematics
University of North Carolina
Phillips Hall CB # 3250

Chapel Hill , NC 27514-3175
USA

Prof.Dr. Jost-Hinrich Eschenburg
Institut für Mathematik
Universität Augsburg
Universitätsstr. 8

8900 Augsburg

Prof.Dr. Patrick Foulon
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Carolyn Gordon
Dept. of Mathematics
Dartmouth College
Bradley Hall

Hanover , NH 03755
USA

Prof.Dr. Detlef Gromoll
Department of Mathematics
State University of New York

Stony Brook , NY 11794-3651
USA

Dominic Joyce
Mathematical Institute
Oxford University
24 - 29, St. Giles

GB- Oxford , OX1 3LB

Prof.Dr. Ursula Hamenstädt
Mathematisches Institut der
Universität Bonn
Beringstr. 1

5300 Bonn 1

Thomas Kakolewski
Mathematisches Institut
Universität Bonn
Beringstr. 1

5300 Bonn 1

Dr. Jens Heber
Institut für Mathematik
Universität Augsburg
Universitätsstr. 8

8900 Augsburg

Prof.Dr. Anatole B. Katok
Department of Mathematics
Pennsylvania State University
218 McAllister Building

University Park , PA 16802
USA

Prof.Dr. Ernst Heintze
Institut für Mathematik
Universität Augsburg
Universitätsstr. 8

8900 Augsburg

Prof.Dr. Bruce Kleiner
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Jürgen Jost
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof.Dr. Wilhelm Klingenberg
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

5300 Bonn 1

Prof.Dr. Francois Labourie
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Carlos Enrique Olmos
Facultad de Matematica
Astronomia y Fisica
Univ.Nac. de Cordoba
Av. Valparaiso y R. Martinez

5000 Cordoba
ARGENTINE

Dr. Bernhard Leeb
Department of Mathematics
University of Maryland

College Park , MD 20742
USA

Dr. Jean-Pierre Otal
Max-Planck-Institut für Mathematik
Gottfried-Claren-Straße 26

5300 Bonn 3

Prof.Dr. Wolfgang T. Meyer
Mathematisches Institut
Universität Münster
Einsteinstr. 62

4400 Münster

Prof.Dr. Ulrich Pinkall
Fachbereich Mathematik - MA 7-2
Technische Universität Berlin
Straße des 17. Juni 136

1000 Berlin 12

Prof.Dr. Maung Min-Oo
Department of Mathematics and
Statistics
Mc Master University
1280 Main Street West

Hamilton Ontario L8S 4K1
CANADA

Dr. Hans-Bert Rademacher
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

5300 Bonn 1

Prof.Dr. Igor G. Nikolaev
Dept. of Geometry and Analysis
Institute of Mathematics of the
Siberian Branch Acad. Sci. USSR
Universitetskii pr. 4

630090 Novosibirsk
USSR

Prof.Dr. Alexander Reznikov
Department of Mathematics
I.C.T.P.
P.O. Box 586

I-34100 Trieste

Prof.Dr. Ernst Ruh
Department of Mathematics
Ohio State University
231 West 18th Avenue

Columbus Ohio 43210-1174
USA

Prof.Dr. Gang Tian
Department of Mathematics
State University of New York

Stony Brook NY 11794-3651
USA

Michel Rumin
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. McKenzie Y. Wang
Department of Mathematics and
Statistics
Mc Master University
1280 Main Street West

Hamilton Ontario L8S 4K1
CANADA

Dr. Paul Schmutz
Departement de Mathematiques
Ecole Polytechnique Federale
de Lausanne

CH-1015 Lausanne

Matthias Weber
Mathematisches Institut
Universität Bonn
Beringstr. 1

5300 Bonn 1

Prof.Dr. Viktor Schroeder
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

7800 Freiburg

Prof.Dr. Wolfgang Ziller
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Gudlaugur Thorbergsson
Dept. of Mathematics
University of Notre Dame
P. O. Box 398

Notre Dame , IN 46556
USA