MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Variationsrechnung und Optimalsteuerung
Optimal Control

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Die Tagung wurde von
R. Bulirsch, TU München,
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52 Wissenschaftler aus dem In- und Ausland haben an der Tagung teilnehmen können. Davon 11 Teilnehmer aus den Vereinigten Staaten, und zum ersten Mal hat eine erhebliche Anzahl (10) aus den neuen Bundesländern und aus der Sowjetunion (5) teilnehmen dürfen. Darüber hinaus nahmen auch Gäste aus Österreich, Frankreich und Polen teil.

In den Vorträgen wurden Grundlagenfragen ebenso behandelt wie Probleme der Praxis. Vorträge, die eher Grundlagenfragen zuzurechnen sind, befaßten sich mit abstrakten optimalen Steuerungsproblemen, mit Anwendungen auf verteilte Parametersysteme (Beñker), der Zeltmethode in der Steuerungstheorie (Boltyanski), Zerlegung und suboptimaler Rückkopplungssteuerung bei nichtlinearen dynamischen Systemen (Chernousko), numerischen Fragen bei der Lösung von Steuerungproblemen mit Zustandsraumbeschränkungen (Dikusar), numerischen und theoretischen Ergebnissen über Stekloff-Eigenwerte (Dittmar), Konvexifizierung von Steuerungsproblemen parabolischer Differentialgleichungen (Kirillova), Pontryagin's Maximumprinzip bei Mehrfachintegralen (Klötzler), optimaler Steuerung vererbbarer Systeme (Kolmanovskii), notwendigen Bedingungen bei singulären Oberflächen (Melikyan), Maximumprinzip für mehrdimensionale relaxierte Steuerungsprobleme (Pickenhain), der Anwendung des analytischen Zentrums (Sonnevend), hinreichenden Bedingungen für nichtlineare parabolische Steuerungsprobleme (Tröltzsch), Konvexifizierung von Steuerungsproblemen bei parabolischen Differentialgleichungen (Kampowsky) und Differenzierbarkeit von optimalen Lösungen bei Störungen (Maurer).

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Eine große Anzahl von Beiträgen waren den Anwendungen gewidmet, zum Beispiel: Zeitoptimale Steuerung singulär gestörter Systeme (Ardema), Gleichungslösen als Problem optimaler Steuerung (Bittner), Berechnung optimaler Roboterbahnen mit Hilfe der direkten Mehrzielmethode (Bock), Sensitivitätsanalysen in der Elastomechanik (Brokate), numerische Konditionierung von Steuerungsproblemen (Burns), Grenzzyklen bei streng konkaven Steuerungsproblemen (Feichtinger), Steuerungsprobleme bei der Kristallisation (Hoffmann), Zeitoptimale Steuerung von Industrierobotern (Kraft), Steuerungsprobleme unter Unsicherheiten (Leitmann), die optimale Gestaltung von elastischen Stäben (Mikulski), Spline-Interpolation als Steuerungsproblem (Oberle), Stabilitätsprobleme bei einem optimalen Steuerungsproblem (Bourdache-Siguerdidjane).

Über effiziente numerische Verfahren wurde berichtet, so über iterative Methoden bei Integralprozessen (Schmidt), über die SQP-Methode bei parabolischen Steuerungsproblemen (Sachs), Schranken für die globale Lösung bei optimalen Steuerungsproblemen durch semiinfinite lineare Programmierung (Rudolph), singuläre Störungsmethoden mit Anwendung auf optimale Steuerungsprobleme (Mease), Trajektorienoptimierung durch sequentielle quadratische Programmierung (Betts) und über die numerische Lösung von optimalen Steuerungsproblemen mit Hilfe der direkten Kollokation (von Stryk).

Eine große Anzahl von Vorträgen befaßte sich mit Anwendungen auf die Luft- und Raumfahrt, so über singuläre Störungen bei optimalen Steuerungen (Calise), optimaler Entwurf einer Mission zum Planten Neptun (Callies), optimaler Aufstieg eines hypersonischen Raumfahrzeugs (Chudej), Flugzeugoptimierung (Cliff), optimale Flugbahnen unter Staudruckbeschränkungen (Grimm), optimale Unterstufenrückkehr eines transatmosphärischen Fahrzeugs (Jänsch), Steuerbarkeitsuntersuchung für Wiedereintritt und Aufstieg eines Raumtransportersystems (Kugelmann), Optimierung eines Reichweitenflugs eines Hängegleiters (Pesch) und optimale periodische Flugbahnen mit singulärer Steuerung (Sachs, Lesch), notwenidge Bedingungen für Minimax- und Maximaxprobleme bei Aerogleitern (Vinh).

Vortragsauszüge.

M. D. Ardema

Time-Optimal Control of Singularly Perturbed Linear Systems with Application to Disc Drives

A computer disk drive actuation is modeled as a singularly perturbed linear system. A time optimal control law for such a system is developed from the analysis of two reduced order systems. Attention is focused on the dependence of the control switching times on the small parameter of the problem. Simulation results show that the control law gives improved results relative to existing methods, except when movements on the disk are very short. Adding more boundary layer switches and including more terms in the asymptotic expansions are expected to solve the remaining problems.

H. Benker

An Algorithm for Abstract Optimal Control Problems Using Maximum Principle and Applicaton to a Class of Distributed Parameter Systems

Two algorithms using maximum principle for computing lumped parameter optimal control problems are extended to the following abstract optimal control problem in Hilbert spaces:

$$J(u) = j(x(u), u) \overset{!}{=} \min_{u \in U \subset H}, \quad Ax = F(x, u).$$

Furthermore the application to concrete distributed parameter systems with non-convex functional is considered.

J. T. Betts

Path Constrained Trajectory Optimization Using Sparse Sequential Quadratic Programming

One of the most effective numerical techniques for the solution of trajectory optimization and optimal control problems is the direct transcription method. This approach combines a nonlinear programming algorithm with a discretization of the trajectory dynamics. The resulting mathematical programming problem is characterized by matrices, which are large and sparse. Constraints on the path of the trajectory are then treated as algebraic inequalities to be satisfied by the nonlinear program. This lecture describes a nonlinear programming algorithm, which exploits the matrix sparsity produced by the transcription formulation. The optimization algorithm employs a quadratic programming subproblem solved using a Schur-Complement method in conjunction with a symmetric indefinite multifrontal sparse linear algebra package. Numerical experience is reported for trajectories with both state and control variable equality and inequality path constraints.

L. Bittner

Das Lösen von Gleichungen als Problem optimaler Steuerung

Im Vortrag wird die geeignete Wahl von freien Funktionen in Homotopieansätzen als Steuerproblem aufgefaßt. Für die Gleichung $F(x) = 0$ mit einer Näherung x_0 ist der folgende Trajektorienansatz

$$x(0) = x_0,\ F(x(t)) = F(x_0)\,v(t) \text{ bzw. } \dot{x} = F'^{-1}(x)\,F(x_0)\,\dot{v},\ 0 \le t \le 1$$

ein Beispiel, wobei v eine hinreichend glatte Funktion der Eigenschaften $v(0) = 1$, $v(1) = 0$ ist. Wenn $x(1)$ existiert, löst $x(1)$ die Gleichung. Es werden Sätze dafür angegeben, daß die Trajektorie bis zu $t = 1$ hinauf definiert ist. Die Fehlerabschätzung für ein Prediktor-Korrektor-Verfahren, angewendet auf das Anfangswertproblem führt auf die folgende Aufgabe zur Wahl von $u = \dot{v}$: Wenn $\dot{x} = g(x, u(t)), x(0) = x_0$ ein dynamisches System ist und q eine durch das P-K-Verfahren bedingte natürliche Zahl ist, dann wähle man u als $C^{(q-1)}$-Funktion aus einer gewissen Klasse so, daß das Funktional $\varphi(u) = \sup_{0 \le t \le 1} \|x^{(q)}(t)\|$ minimal wird. Diese Aufgabe wird behandelt und dafür werden Lösungsaussagen für konkrete Spezialfälle hergeleitet.

H. G. Bock

Robot Trajectory Optimization by a Direct Multiple Shooting Method

Some desired properties of a direct multiple shooting method were briefly discussed. Time-optimal trajectories for an elbow robot with 2 respectively 3 degrees of freedom were presented.

V. G. Boltyanski

The Tent Method in the Optimal Control Theory

The importance of the maximum principle consists not only in the result itself, but also in its proof and the possibilities of generalization. There are many similar optimization problems and each one differs from others by the specific conditions involved. A general method for obtaining optimization criteria is much more important than a concrete, individual result. The tent method is just a general tool for finding necessary conditions in different extremal problems. I found it by generalizing my proof of the maximum principle and applying different arguments in the field of mathematical programming. In the tent method, the generality is coupled with clear geometric ideas and intuitive comprehension. The main ideas and results of the tent method and some examples of its application will be given.

H. Bourdache-Siguerdidjane

Stability Conditions in Terms of Eigenvalues of a Nonlinear Optimal Controlled System

The purpose of this lecture is to show that, for some nonlinear systems, it is possible to give algebraic conditions for the stability in terms of characteristic values. We have recently shown that for a certain class of nonlinear systems, one may define characteristic values and characteristic vectors, which satisfy an algebraic nonlinear equation. That equation is equivalent to the known characteristic linear equation of eigenvalues. This theory is illustrated by the determination of the analytic solutions of the trajectories of an optimal feedback controlled spacecraft angular momentum. From this result, the stability conditions are deduced.

M. Brokate

Sensitivity Analysis in the Rigid Punch Problem

We consider an elastic seat deformed by a rigid body from above. Its indentation depth is not given, but has to be determined from its weight. This leads to a noncoexive variational inequality. We prove that the solution of the variational inequality has a directional derivative with respect to parameters. Also some results of numerical simulations are presented.

J. A. Burns

Control System Radii and Numerical Conditioning of Control Problems

We discuss questions of convergence and ill-conditioning of computational algorithms for control design of distributed parameter systems. A simple thermo-elastic control system is used to motivate the basic issues. We present an example to illustrate that even convergent finite element methods can produce non-convergent control designs. Control systems radii are introduced and applied to finite difference and finite element models of control systems governed by parabolic and hyperbolic partial differntial equations. These radii are shown to provide a measure of numerical ill-conditioning and are computable for systems that have the special structure common to finite element approximations. The lecture ends with a conjecture.

A. J. Calise

Singular Perturbations in Optimal Control Problems with State Variable Inequality Constraints

The established necessary conditions for optimality in optimal control problems that involve state-variable inequality constraints are applied to a class of singularly perturbed systems. The distinguishing feature of this class of two-time-scale systems is a transformation of the state-variable inequality constraint, present in the full order problem, to a constraint involving states and controls in the reduced problem. It is shown that, when a state constraint is active in the reduced problem, the boundary layer problem can be of finite time in the streched time scale. Thus the usual requirement for asymptotic stability of the boundary layer system is not applicable and can not be used to construct approximate solutions. Several alternative solution methods are explored and illustrated with simple examples.

R. Callies

Optimal Design of a Mission to Neptune

For an interplanetary spacecraft from Earth to the planet Neptune trajectory optimization and the optimization of spacecraft design are tightly coupled to increase overall system performance.
For each set of boundary conditions and constraints the problem of trajectory optimization is transformed into a multi-point boundary value problem. The spacecraft is described by a very accurate model. Model parameters and model functions are integrated into the framework of differential equations. The solution of the resulting problem is by the multiple shooting method.
For a journey to Neptune with a launch in the year 2001 the optimal spacecraft needs 18 years without swing-bys and has an initial mass of about 1.7 tons. Launch window covers the full year of 2001.

F. L. Chernousko

Decomposition and Suboptimal Feedback Control for Nonlinear Dynamic Systems

A nonlinear dynamic system governed by Lagrange equations and subject to bounded control forces is considered. Under a certain condition the feedback control is chosen in such a way that the system reaches the prescribed terminal state in a finite time. This control is obtained through decomposition of the system into subsystems with one degree of freedom each and applying the approach of differential games. The obtained feedback control is robust with respect to small disturbances and parameter variations and time-suboptimal. Applications to robot control are discussed.

K. Chudej

Optimal Ascent of a Hypersonic Space Vehicle

The optimal ascent of a hypersonic Sänger typ space transportation system is investigated. The focus of this presentation lies on the optimal ascent trajectory of the lower stage, which is turbo and ramjet powered. Realistic models, which inhibit all of the numerical problems are derived by a nonlinear least squares approach from given table data. Due to its importance on the flight path the dynamic pressure is constrained. Very accurate solutions are obtained by multiple shooting. Due to the complicated solution structure a good first estimate of state and especially adjoint variables is needed to start the multiple shooting algorithm. This estimate is calculated by a direct method, which solves a nonlinear programming version of the original problem. Therefore in fact a reliable hybrid solution technique is used to solve this state and control constrained optimal control problem.

E. M. Cliff

Aircraft Cruise-Dash Optimization

We consider vertical plane motions of an aircraft and minimize a performance index, which is a weighted sum of time and fuel $J = W_f(t_f) + \theta t_f, \theta \geq 0$. A steady flight solution is possible; in this case the altitude, velocity, path angle, load factor and throttle-setting are each constant. It is known that with $\theta = 0$ the steady extremal is not minimizing. For a given aircraft model we examine the family of solutions for $\theta \geq 0$. Periodic extremals are found with bang-bang throttle and for some θ values with bang-singular throttle. For $\theta > \theta_c$ the steady solutions are the minimizers.

V. V. Dikusar

Numerical Methods for Solving Control Problems with Control-State Constraints

Availability of control-state constraints leads to a complication of the formulation of the maximum principle. New objects emerge: measure and functional Lagrange multipliers. For the computation it becomes necessary to examine the properties of these objects. The list of examined questions include: canonical problems (Dubovitsky and Milutin), integration of ordinary differential equations, the nonlinear programming, preliminary estimation of the type of contact of the optimal trajectory with inequality constraints and the determination of Lagrange multipliers. Optimization of reentry body distance with full loading constraints and of a problem of flight-dynamic serve as examples.

B. Dittmar

Numerical and Theoretical Results about Stekloff Eigenvalues

The aim of this lecture is a classical eigenvalue problem for harmonic functions (In C. Bandle: Isoperimetric Inequalities and Applications, 1980). The Stekloff eigenvalue problem is defined by the conditions

$$\Delta u = 0 \text{ in } G \quad , \quad \frac{\partial u}{\partial n} = \lambda u \text{ on } C = \partial G$$

G is a simply-connected domain and n the outward directed normal. Let $\lambda_1 = 0 < \lambda_2 \leq \lambda_3 \ldots$ be the eigenvalues of this problem. It is easy to show, that we must distinguish between the Stekloff eigenvalue of the exterior domain $\lambda_{2,e}$ and the one of the interior domain $\lambda_{2,i}$. We can derive the following sharp estimates with the first non-trivial

Fredholm eigenvalue $\lambda > 1$

$$\frac{\lambda - 1}{\lambda + 1} \leq \frac{\lambda_{2,e}}{\lambda_{2,i}} \leq \frac{\lambda + 1}{\lambda - 1}$$

This equation is often usefull in order to obtain numerical estimates for $\lambda_{2,e}$, for example if we have good estimates for $\lambda_{2,i}$ and λ. We describe briefly a new procedure for the numerical evaluation of λ_2 (ZAMP 39 (1988), S. 351 – 366), which works only with upper bounds. Results of numerical experiments are given and all examples show a monotonicity of the eigenvalues, which was derived in ZAMP 40 (1989).

G. Feichtinger

Limit Cycles in Strictly Concave Optimal Control Models

The purpose of the talk is to identify economic mechanisms implying stable limit cycles as optimal solutions of deterministic and autonomous control systems with infinite time horizons. Three rather simple models are presented: Rational addiction and cyclical consumption patherns, the corrupt politician under popularity constraints and a non-zero-sum differential game for the dynastic cycle of the ancient chinese history. The Hopf bifurcation theory is applied on the first order necessary conditions of optimality. Three open questions are formulated: 1. In which way can Hopf bifurcation in two-state optimal control models with path constraints occur? 2. Which are the sufficient conditions, when the Hamiltonian is not concave? 3. Do there exist more complex attractors, i.e. chaotic ones, as limit sets of continuous-time optimal control models, which are economically meaningful?

W. Grimm

Optimal Aircraft Trajectories with Constrained Dynamic Pressure

The influence of the dynamic pressure constraint on the optimal control of a high performance aircraft is examined, which is modelled as a point mass in threedimensional space. Two different optimal control problems are considered: 1) maximum range in fixed time and 2) minimum time approach maneuvers. First, the different types of optimal control for all possible combinations of active control and state constraints are determined. The optimal switching structure for 1) and 2) is determined with multiple shooting for various boundary conditions. Along the dynamic pressure constraint a) full, b) partial and c) minimum throttle control occur in the sequence as mentioned. b) is a singular, c) is a chattering control. The junction a)-b) is a particular theoretical problem. Finally the accuracy of approximate solutions (parameterized optimal control) is discussed.

K.-H. Hoffmann

Optimal Control of a Phase Field Model Describing Crystallization

Starting from the well known multidimensional Stefan type problem for solidification of a liquid the phase field model is introduced and its physical meaning is discussed in detail. Existence and uniqueness is proved as well as stability under mild assumptions. But the main goal of our presentation is to establish results for the distributed optimal control problem for the phase field equations. Besides existence of an optimal control necessary optimality conditions of first order were proved. The control problem is nonlinear and nonconvex.. Nevertheless we are able to prove uniqueness of the optimal control function local in time. From the necessary conditions we derive a numerical algorithm to compute a controlled crystallization process. Some computed pictures of a star shaped growing crystal were presented.

C. Jänsch

Optimal Lower-Stage Return Trajectories of a TSTO Transatmospheric Vehicle

This lecture presents optimal trajectories of a two-stage to orbit (TSTO) transatmospheric launch vehicle. In particular, the fuel optimal return flight of the lower-stage from the stage separation point subject to several state and control inequality constraints is investigated. The suboptimal trajectories are obtained by a direct collocation method that discretizes the state and control histories and introduces collocation constraints to satisfy the state differential equations. It is shown that the algorithm is able to converge from rather "bad" starting estimates for the controls. The optimal solution is also compared to steady-state flight mechanics considerations and to vehicle performance data. Then it is seen that the quasi-cruise flight segment takes place at $(L/D)_{\max}$ and the initial turn takes place near the corner velocity.

W. Kampowsky

Convexification of Control Problems in Parabolic Differential Equations

The lecture presents a common work of U. Raitums (Latvian State University, Riga, Latvia) and the author. We consider an optimal control problem governed by a nonlinear initial-boundary value problem for a parabolic differential equation of second order of divergence type and by a cost functional of integral type. The control functions appear in the coefficient functions of the differential equation and of a Neumann boundary condition. Using the so called decomposition condition and a basic lemma of convexification it's possible to consider extensions of the control problem, which are defined by the convex hull and an extended convex hull of the elliptic differential operators and cost functionals given by the admissible control functions. Necessary optimality conditions in form of a minimum principle are derived by applying a special extension of this kind.

F. M. Kirillova

Feedback Optimal Controls

The lecture deals with optimal control problems and gives a brief account of a course of lectures by R. Gabasov and F. Kirillova on feedback optimal controls. The course is based on a new point of view on feedback optimal control problems and results of the authors and their collaborators dealing with constructive theory of extremal problems obtained during 1975-1990 in Minsk. All the typical optimal control problems under conditions of uncertainty and restrictions on controls and state of control systems are considered. Algorithms of constructing optimal identificators, estimators and controllers are described. The form of realization is oriented on using microprocessor's elements.

R. Klötzler

Pontryagin's Maximum Principle for Multiple Integrals

The lecture deals with the development and application of a modified form of Pontryagin's maximum principle. This version prove correct too for problems of optimal control by multiple integrals and functions of several independent variables, more precisely for problems of Dieudonné-Rashevsky type.

V. B. Kolmanovskii

Optimal Control of Hereditary Systems

An optimal control problem for a predator-prey system is considered. As controls may be used different substances, which may act only on preys or on predators or on both species simultaneously. The optimal control problem consists of the systems attainment its rest state from arbitrary initial state at minimum time. Existence of an admissible control is established and existence of the optimal one is proved by using the maximum principle. Switching curves of the optimal control are constructed for all cases. Dependence of the optimal time on system parameters is investigated numerically.

D. Kraft

Computation and Animation of Time-Optimal Constrained Controls of an Industrial Robot

Industrial productivity can further be raised if the corresponding robots perform faster motions. Therefore, a study has been performed to compare robot controls as applied by an industrial control system with calculated optimal controls on the basis of an identified model of the real robot. An average time-saving of 50% has been gained. The time-optimal controls have been calculated with a direct shooting method including control and state constraints in the joint coordinate system. To visualize the cartesian motion an animation program has been written to support the numerical simulations.

B. Kugelmann

Controllability Investigations for Re-Entry and Ascent of Space Transportation Systems

Dynamic systems, which are governed by ordinary differential equations, often describe the trajectories of space vehicles. For the guidance of those vehicles it is not sufficient to only calculate the nominal path before launch-time but it is also important to have a correction method in order to handle perturbations from the nominal path, which may be due to the mathematical approximation of the physical process. In this lecture a feedback algorithm is presented, which allows to compute control corrections in minimal time during the process, while satisfying optimality conditions to the first order. This method can be applied to various kinds of optimal control problems including state-control-inequality constraints, interior point constraints and discontinuity conditions. The corresponding algorithm is applied to the re-entry problem of a Space-Shuttle-Orbiter type vehicle and the two-stage ascent of a Sänger configuration.

G. Leitmann

A Discrete Stabilizing Study Strategy for a Student Related Problem under Uncertainty

We consider a lazy, forgetful but ambitious student, who wishes to reach a desired knowledge level while learning and forgetting:

$$N(k+1) = [b_1 + b_2 N(k)]W(k) - c\,N(k), \quad k = 0, 1, \ldots, \eta - 1,$$

where $N(k)$ is the knowledge level at time k (%), $W(k)$ is the study effort (days/week) $0 \leq W(k) \leq \bar{W}$, η the number of weeks with $b_i = b_i^* + \Delta b_i(k)$ $(i = 1, 2)$, $c = c^* + \Delta c(k)$ with given $|\Delta b_i(k)| \leq \sigma_i$ $(i = 1, 2)$ and $|\Delta c(k)| \leq \sigma_3$. The task is now to determine $W(k)$, $k = 0, 1, \ldots, \eta - 1$ that $N(\eta) = N^*$ (desired) for all realizations (all b_i, c).

H. Maurer

Solution Differentiability for Perturbed Nonlinear Control Problems

We consider perturbed nonlinear control problems with data depending on a vector parameter. Using second-order sufficient optimality conditions it is shown that the optimal solution and the adjoint multipliers are differentiable functions of the parameter. The proof of such a second-order sensitivity result exploits the close connections between solutions of a Riccati differential equation and shooting methods for solving the associated boundary value problem.

K. D. Mease

A Computational Singular Perturbation Method Applied to Optimal Control Problems

The Hamiltonian systems associated with optimal control problems often exhibit the behavior of a boundary layer. Computational singular perturbation (CSP) is a methodology that leads to an algorithm for the solution of systems of ordinary differential equations that exhibit boundary layer type behavior. We discuss the application of CSP to the two-point boundary value problems of optimal control. Some illustrative examples are presented.

A. Melikyan

Necessary Conditions for a Singular Surface in the Form of Synthesis (Generalization of Kelley's and Kopp-Moyer's Conditions)

For the optimal control problem (e. g. time-optimal):

$$\dot{x} = f(x, v), \; u \in U, \; x(0) = x^0, \; x(T) \in M \subset \mathbb{R}^n, \; T \to \min_u \tag{1}$$

the necessary conditions in terms of Poisson's brackets are obtained

$$[F_0 F_1] = O, \; [[F_0 F_1] \, F_0] \leq 0, \; [[F_1 F_0] \, F_1] \leq 0 \tag{2}$$

for optimality of a singular surface; $F_i(x, p)$ are Hamiltonians, corresponding to each side of surface, p is the gradient of Bellman function, which is proved to be twice differentiable. The Kelley condition is shown to have the form $[[F_0 F_1] \, F_0] + [[F_1 F_0] \, F_1] \leq 0$, which is the corollary of (2). Conditions (2) include also restrictions on controls, i. e. using (2) for computation one needs not to check the inclusion $u \in U$. The same is true for Kopp-Moyer's conditions. The method of singular characteristics and smooth continuation approach for Bellman function are used.

L. Mikulski

Optimale Gestaltung von elastischen Stäben

In der Arbeit wird die Gestaltoptimierung des elastischen Stabes mit dünnwandigem Profil betrachtet. Das Ziel dieser Arbeit ist die Bestimmung der optimalen Form von elastischen Bögen unter verschiedenen Beschränkungen. Die dünnwandigen Träger sollen so gestaltet werden, daß das Volumen minimiert oder der 1. bzw. 2. Eigenwert maximiert wird. Als Steuervariable wählen wir die Breite eines Trägerprofils. Probleme der Gestaltoptimierung von elastischen Bögen lassen sich mathematisch als optimale Steuerungsprobleme formulieren. Zur numerischen Lösung dieser Aufgabe wurde die Mehrzielmethode verwendet. Erzielte Ergebnisse werden mit dem konstanten Profil verglichen.

H. J. Oberle

Spline-Interpolation and Histopolation as State-Restricted Optimal Control Problems

An interpolation scheme applied to positive function values in general will give approximations, which are not necessarily entirely positive. Therefore it is useful to search for an interpolation function, which minimizes the (linearized) strain energy under the constraint of non-negativity. A similar situation arises, if one looks for area-matching approximations of positive histograms. It is shown that both problems can be formulated as optimal control problems with a second-order inequality constraint. The corresponding necessary conditions are derived showing that the minimizer of the first problem is a natural cubic spline function with respect to an augmented grid whereas the minimizer of the second problem is a quartic spline function. The necessary conditions also reveal the corresponding junction conditions for contact points and boundary subarcs. These imply certain smoothness properties of the minimizing spline-functions. The criteria are applied to derive numerical algorithms to compute the minimizers. Numerical examples are presented, which illustrate these methods.

H. J. Pesch

Optimierung des Reichweitenfluges eines Hängegleiters mit Hilfe eines Kollokationsverfahrens zur Startdatenschätzung für die nachfolgende Anwendung des Mehrzielverfahrens

Bei der numerischen Lösung von optimalen Steuerungsproblemen mit Hilfe indirekter Verfahren, wie zum Beispiel der Mehrzielmethode, liegt eine der Hauptschwierigkeiten in der Bereitstellung geeigneter Startdaten; das schließt auch die Schätzung der adjungierten Variablen und der erwarteten Schaltstruktur der optimalen Lösung ein. Der Zugang zum Mehrzielverfahren wird dadurch erschwert und seine Akzeptanz beeinträchtigt, obwohl gerade diese Methode sehr genaue Resultate liefert und auf Grund der aus den notwendigen Bedingungen der Steuerungstheorie fließenden reichlichen Informationen auch eine weitgehende Überprüfung der Lösung auf Optimalität gestattet. Direkte Verfahren bieten dagegen einen einfacheren Zugang, da sie leicht zu handhaben sind, freilich verbunden mit geringerer Genauigkeit und weniger Einsicht in die Schaltstruktur der optimalen Lösung. Nicht selten werden auch Scheinlösungen erzeugt, z.B. dann, wenn das endlich dimensionale Optimierungsproblem das unendlich dimensionale Steuerungsproblem nur unzureichend beschreibt. In dieser Arbeit werden die Vorteile beider Verfahren dadurch vereinigt, daß es gelingt aus der Näherungslösung des verwendeten direkten Kollokationsverfahrens eine Schätzung für die adjungierten Variablen und die optimale Schaltstruktur zu erhalten, deren Güte ausreichend für die Konvergenz des Mehrzielverfahrens ist. Numerische Resultate werden für den reichweitenoptimalen Flug eines Hängegleiters in einem Aufwind präsentiert.

S. Pickenhain

The Maximum Principle for Multidimensional Relaxed Control Problems

In this lecture multidimensional control problems and corresponding dual problems are investigated. Results of strong duality between dual and generalized problems are obtained by using compactness properties of the set of relaxed controls and linear programming methods in Banach spaces. A weak Maximum Prinziple is shown for a class of generalized problems.

H. Rudolph

Bounds for Global Solutions in Optimal Control via SILP

The lecture deals with the relaxation approach to classical control problems, basing on ideas of L. C. Young in variational calculus and developed by J. Rubin for control problems. This method consists in substituting a control problem by an LP over a measure space. Recently the author pointed out how to use methods of semiinfinite linear programming (SILP) to get estimates for global optimal solutions of the control problem, especially lower bounds for the global minimal value by use of the semiinfinite simplex method. The globality seems to be one of the advantages of the relaxation method; most of the numerical methods, basing on first or higher order necessary conditions like Pontryagin's maximum principle, give only local solutions for the control problem. The approach is demonstrated by the numerical treatment of the minimal fuel Earth-Mars transfer.

E. Sachs

Parabolic Control Problems Solved by SQP-Methods

We review some of the recent results on the convergence of reduced SQP-methods. In the framework of parabolic control problems we illustrate the advantage of these methods in comparison to full SQP-methods. We give numerical results, which compare this method to recent publications.

G. Sachs , K. Lesch

Optimal Periodic Trajectories with Singular Control for an Aircraft with High Aerodynamic Efficiency

Aircraft cruise for maximizing range per fuel is considered as an optimal periodic control problem. Optimality conditions for flight trajectories with singular arcs and state variable constraints are derived. Based on the equivalence between singular and chattering control, it is shown that chattering arcs exist for a more realistic fuel consumption model. With this model significant reductions in fuel consumption are achievable, if the maximal altitude of the fligth path is constrained. The computation of these trajectories presents strong requirements to the numerical algorithm. They increase with the aerodynamic efficiency of the aircraft and the decreasing of the wing loading. This behaviour of the numerical system will be presented as well as a view on the measures to get a better convergence (lecturer : K. Lesch).

W. Schmidt

Iterative Methods for Integral Processes

There will be derived necessary optimality conditions for several control problems governed by integral equations (Volterra and Fredholm integral equations, integral equations with delay, integro-differential equations). Lagrange-functionals or generalized Mayer-functionals are considered. On principle there is the possibility of iterative computing of optimal controls by means of these optimality conditions (maximum principle). A version of the method of Chernousko and Luybushin will be given. In special cases the sequence of controls computed in this way converges in the sense of the functional.

G. Sonnevend

Application of Analytic Centers for the Solution of Control and Observation Problems with L-Infinity Bounds on the Variables

We consider the problem of constructing measurement feedback control for tracking a given output function $y_1(t)$, $0 \le t \le T$ in the system — under (concave) state constraints $h_j(x) \ge 0$, $j = 1, \dots, k$ —

$$\dot{x} = Ax + B_1u_1 + B_2u_2, \quad y_i = C_ix + v_i, \; i = 1,2,$$

where pointwise in t bounds $||u_i|| \le \rho_i$, resp. $||v_i|| \le \sigma_i$, $i = 1,2$ are prescribed, resp. known for the control u_1 and tracking error v_1 resp. for the disturbance u_2 and measurement error v_2 variables. The feedback control system has the form

$$\begin{aligned} \hat{x}' &= A\hat{x} + \sum_{i=1}^{2} B_i \frac{B_i^*\Psi}{||B_i\Psi||}\sqrt{z_i - \sqrt{z_i^2 - \rho_i^4}}, \quad \text{where } z_i = \rho_i^2 + \frac{2\gamma_i}{||B_i\Psi||^2} \\ \Psi_i' &= -A^*\Psi_i + \frac{C^*(C_i\hat{x} - y_i)\gamma_{i+2}}{\sigma_i^2 - ||C_i\hat{x} - y_i||^2} - \sum_{j=1}^{k} \frac{\gamma_{j+4}}{h_j(\hat{x})}\frac{\partial h_j(\hat{x})}{\partial x}, \quad i = 1,2 \end{aligned}$$

where $\hat{x}(0)$, $\Psi_1(0)$, $\Psi_2(0)$ are determined by $y_1(\cdot)$; these – as well as the explicitly computed functions F, G – depend on $k + 4$ penalty parameters γ_i, which are the weights for the logarithmic penalty terms in the two auxiliary Lagrange functions used to define the "central" feasible solutions of our feasibility problem. We motivate this solution concept based on the concept of a "central" solution of the convex feasibility problem identifying special cases for its successful application.

O. von Stryk

Numerical Solution of Optimal Control Problems by Direct Collocation

By an appropriate discretization of control and state variables a constrained optimal control problem is transformed into a finite dimensional nonlinear program. Convergence properties of this discretization can be established. The nonlinear program is solved by a SQP-method due to Gill, Murray et al.. From a solution of this good convergent direct method reliable estimates of adjoint variables can be obtained even in the presence of active path constraints. These estimates of state and adjoint variables are well suited as initial values for the numerical solution of the boundary value problem of the necessary first order conditions by multiple shooting. New numerical results are presented for very complicated problems: Minimum fuel ascent of the lower stage of a two-stage space transporter system and minimum time and minimum energy trajectories of an industrial robot with three degrees of freedom subject to state and control variable inequality constraints.

F. Tröltzsch

Sufficient Second Order Conditions for Nonlinear Parabolic Control Problems and Their Application

In the lecture a class of nonlinear optimal control problems for a parabolic partial differential equation is discussed. The equation of state is semilinear, controls are acting within the n-dimensional domain under consideration and on its boundary. Moreover, constraints on the control and the state variables are given. This problem is non-convex as the equation of state is nonlinear. Therefore, the known first order optimality conditions are not sufficient for optimality. In the lecture, second order sufficient optimality conditions are derived. It is shown, how they can be applied to show strong convergence of optimal controls for certain numerical approximations of the optimal control problem.

N. X. Vinh

Necessary Conditions for Minimax and Maximax Problems with Aeroglide

The necessary conditions for solving Chebyshev minimax (or maximax) problems with bounded control are presented. The jump conditions obtained are applicable to problems with single or multiple maxima. By using contenson domain of maneuverability, it is shown that when the maxima are isolated single points, the control is generally continuous at the jump point in the minimax problems and discontinuous in the maximax problems, in which the first time derivative of the maximax function contains the control variable. The theory is applied to the problem of maximizing the flight radius in a closed circuit glide of a hypervelocity vehicle and to a maximax optimal control problem, in which the control appears explicitly with the first time derivative of the maximax function.

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