Mathematisches Forschungsinstitut Oberwolfach

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Singuläre Störungsrechnung

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The meeting was organized by J. Hale (Atlanta), W. Jäger (Heidelberg) and L. Modica (Pisa). The topic "singular perturbations" had not been covered by a Oberwolfach meeting for years. Its techniques are used in a lot of mathematical fields, especially in those dealing with mathematical modelling and applications of mathematics. However, whereas a lot of analytic tools and techniques have been developed in different situations the theoretical treatment is up to now not in the state it should be. The recent developments in the theory of dynamical systems, the progress obtained in the theory of phase transition or in the growing field of homogenization stimulated the organizers to bring together scientists from this areas using or working on singular perturbations methods. The aims were:

(1) Information about the progress in these fields concerning singular perturbations.

(2) Discussion of a more unified approach to singular perturbation and the perspectives for theory opened up e.g. by techniques used in dynamical systems theory.

The list of 34 lectures reflects the variety of applications of singular perturbation methods and the necessity to have an exchange of knowledge. There is no doubt about the success in reaching goal (1). More important is progress in theory. At least the discussion during or after the lectures contributed essentially to a better theoretical understanding. The different groups could profit from each other. Here the rule to leave enough time for discussions again proved to be important. The stimulating atmosphere of the Forschungsinstitut is essential for meetings of this type, where the field and also the group of participants is not so standard. All participants aggreed that this workshop was very usefull and that the different groups have to stay in contact. Only few lectures addressed the important relation of the field to numerical analysis. There is a special need for research in this area.

The following alphabetical list of contributions can be ordered according to topics: Dynamical systems and singular perturbations (geometric methods, bifurcation, attractors, ordinary and partial differential equations, functional equations); nonlinear waves; singular perturbations of variational problems; homogenization; integral equations; computational methods.

Vortragsauszüge

Nicholas Alikakos

On the Spectrum of the Cahn-Hilliard Operator

The nonlinear Cahn-Hilliard equation $u_t = \left(-\varepsilon^2 \Delta u + W'(u)\right)$, $\frac{\partial u}{\partial n} = \frac{\partial \Delta u}{\partial n} = 0$, on a bounded domain $\Omega \leq R^N$, W double well potential, $\varepsilon << 1$, is used as a model for phase separation and coarsening. Formal arguments due to Pego and others show that the formed interfaces Γ for small ε evolve according to the non-local law $V = \left[\frac{\partial \mu}{\partial n}\right] =$ jump of the normal derivative of μ across Γ, where μ is determined by the Dirichlet Problem

$$\Delta\mu = 0 \text{ for } x \notin \Gamma,\ x \in \Omega,\ \mu = -\varepsilon a K \text{ on } \Gamma \text{ and } \frac{\partial \mu}{\partial n} = 0 \text{ on } \partial\Omega.$$

Here V stands for the normal velocity, K is the mean curvature and a,b are constants determined by W. The lecture presents joint work with Giorgio Fusco and aims towards a rigorous connection between the Cahn-Hilliard equation and the reduced geometrical law. The following two theorems are useful in this direction. To state these linear results we introduce the linearized Cahn-Hilliard elliptic operator L about a layered function u_ε (not necessarily an equilibrium) with interface Γ:

$$Lh = \Delta\left(-\varepsilon^2 \Delta h + W''(u_\varepsilon) h\right),\ \forall \frac{\partial h}{\partial n} = \frac{\partial \Delta h}{\partial n} = 0$$

__Theorem 1__

$\sigma(L) \leq C\varepsilon$, C constant independent of ε .

__Theorem 2__

If Γ is spherical then

(i) $\sigma(L) \leq C e^{-\frac{c}{\varepsilon}}$

(ii) L has exactly N exponentially small eigenvalues at least one of which is positive and the remaining of the spectrum is bounded away from zero by $-C^2\varepsilon$.

Guy Bouchitté

Limit Analysis Problems in Electromagnetic Waves Theory

The limit analysis of the Maxwell system is performed in the case of a scattering body whose thickness h tends to 0 while the permittivity and the permeability, ε_n and μ_n, vary with h and possibly tend to ∞ in modulus.

A new interface condition is obtained which takes account of the original shape of the scatter. The application of this to the numerical study of the diffraction by a two-dimensional array of thin conductory plates is given.

Lia Bonsard

Front Propagation for Bistable Reaction-Diffusion Equations

We study the asymptotic behaviour as $\varepsilon \to 0$ of solutions to

$u_t - \varepsilon\Delta u + \frac{1}{\varepsilon} w'(u) = 0$ in $\mathbf{R}^n \times \mathbf{R}^+$, where w is a bistable potential. A typical bistable potential is given by $w'(u) = (u^2 - 1)(u - \mu)$ where -1 or $\mu < 1$. Formal analysis suggests that $\mathbf{R}^n$ is divided in regions where $u^\varepsilon \approx +1 < u^\varepsilon \approx -1$ and that u^ε is asymptotically given by the travelling wave q associated to the equation. Here its speed α is proportional to the difference in height between the two wells of the potential. Using the technique of viscosity solutions for Hamilton-Jacobi equations, we prove that $u^\varepsilon(x,t) = q\left(\frac{v(x,t) + o(1)}{\varepsilon}\right)$ as $\varepsilon \to 0$, where v is the viscosity solution to a first order Hamilton-Jacobi equation with discontinuous hamiltonian. In particular this says that the interface moves with constant speed α.

Pavol Brunovsky

Analysis of the Flow of a Viscoelastic Fluid by Geometric Singular Perturbation Theory

We consider a simplified model of the flow of a viscoelastic fluid discussed by J. Nohel (see his lecture). By adding a small diffusion term representing capillarity we obtain a system of partial differential equations admitting a global attractor.
Employing techniques developped by N. Fenichel, X. B. Lin and Ch. Jones we establish the existence of a unique equilibrium for small and large values of the driving pressure gradient, as well as of three equilibria for pressures in an intermediate interval. We also determine stability properties of the equilibria for the reduced system.
We prove that both the full and the reduced problems admit finite dimensional internal manifolds of equal dimension. The internal manifold of the full system approaches that of the reduced system in the C^1 sense as the parameter tends to zero. Since the inertial manifold restriction of the reduced system is structurally stable, it is topologically conjugate to the inertial manifold restriction of the full system.
The results have been obtained jointly with D. Sevcovic.

Giuseppe Buttazzo

A Singular Perturbation Problem with a Compact Support Semilinear Term

For every $\varepsilon > 0$ we consider the minimum problem

$$(P_\varepsilon) \qquad \min \left\{ F_\varepsilon(u) + \int_\Omega g(x,u)\,dx : u \in H^1(\Omega) \right\}$$

where Ω is a bounded open subset of $\mathbf{R}^n$, $g : \Omega \times \mathbf{R} \to \mathbf{R}$ is a suitable Carathéodory integrand, and F_ε is given by

$$F_\varepsilon(u) = \int_\Omega \left[\varepsilon |Du|^2 + \varepsilon^{-3} \beta\left(\frac{u}{\varepsilon}\right) \right] dx.$$

Here β is a nonnegative function with a compact support. It is proven that the limit functional (in the variational sense given by Γ-convergence) of F_ε as $\varepsilon \to 0^+$ is

$$F(u) = \min\{cP(B,\Omega) : \{u > 0\} \subset B \subset \{u \geq 0\}\}$$

being P (B,Ω) the perimeter of B in Ω and

$$c = 2\int_{R} \beta^{1/2}(s)ds.$$

Therefore, the limit problem of (P_ε) is

$$\text{(P)} \qquad \min\left\{cP(B,\Omega) + \int_B g_+(x)dx + \int_{\Omega\setminus B} g_-(x)dx : B \text{ Borel set}\right\}$$

where

$$g_+(x) = \min_{t \geq 0} g(x,t) \qquad g_-(x) = \min_{t \leq 0} g(x,t)\,.$$

Shui-Nee Chow and Ying-fei Yi

Dynamical Systems and Singularly Perturbed Equations

We study singularly perturbed ODES and its related dynamics in three aspects:

1. Geometric theory of singular perturbations: We derive center, center stable-unstable manifolds and their foliations by looking at so called reduced spectra and reduced flow. The smoothness of those invariant manifolds is also obtained.
2. Dynamics of singularly perturbed systems: We study the existence of compact motions (such as quasi-periodical motions) and their stabilities by making use of regularity of the center manifold.
3. Using dynamical systems to study singular perturbation problems: We have shown by applying our invariant manifolds theory that dynamical system can be successfully built into problems such as inner-outer expansions, matching principles, turning points, existence of layers, initial and boundary values, etc.

Bernold Fiedler, Jürgen Scheurle

Discretization of Homoclinic Orbits, Rapid Forcing, and "Invisible" Chaos

One-step discretizations of order p and step size ε of ordinary differential equations can be viewed as time-ε maps of

$$\dot{x}(t) = f(\lambda, x(t)) + \varepsilon^p g(\varepsilon, \lambda, t/\varepsilon, x(t)), \qquad x \in \mathbf{R}^n, \lambda \in \mathbf{R},$$

where g has period ε in t. This is a rapidly forced nonautonomous system.
We study the behaviour of a homoclinic orbit Γ, ε = 0, λ = 0 under discretization. Under generic assumptions we show that Γ becomes transverse for positive ε. The transversality effects are estimated from above to be exponentially small in ε. For example, the length $\ell(\varepsilon)$ of the parameter interval of λ for which Γ persists can be estimated by

$$\ell(\varepsilon) \le C\exp(-2\pi\eta/\varepsilon)$$

where C, η are positive constants. The coefficient η is related to the minimal distance from the real axis of the poles of Γ(t) in the complex time domain.
Likewise, the region where complicated, "chaotic" dynamics prevail is estimated to be exponentially small, provided $x \in \mathbf{R}^2$ and the saddle quantity of the associated equilibrium is nonzero.
Our results are visualized by high precision numerical experiments. The experiments show that, due to exponential smallness, homoclinic transversality becomes practically invisible under normal circumstances, already for only moderately small step sizes.

Dietrich Flockerzi

Applications of Invariant Manifolds in Nonlinear Control Theory

By studying some problems of nonlocal stabilization for nonlinear control systems

$$\dot{x} = f(x) + g(x)u \ , \ y = h(x) \ [f(0) = 0] \ .$$

We show how the theory of invariant manifolds for singularily perturbed O.D.E.s can be successfully employed in attacking control theoretical tasks. The main problems can be phrased the following way. Given a neighbourhood U of 0 and a compact set K in state space. Can one find a feedback or even an output-feedback control u such that the solutions of the closed loop starting in K eventually are confirmed to U? Does there exist such a control that these solutions asymptotically tend to 0?

Irene Fonseca

Phrase Transitions and Surface Energies

A phase transition problem where the energy density $W : \mathbf{R}^n \to [0, +\infty]$ has two potential wells of equal depth is considered. As in the Van-der-Waals-Cahn-Hilliard gradient theory of phase transitions, a family of singular perturbations (possibly anisotropic)

$$E_\varepsilon(u) := \int_\Omega W(u(x)) + \varepsilon^2 h^2 (\nabla u(x))^2 \, dx$$

is introduced and the Γ-limit of the rescaled functionals

$$J_\varepsilon(u) := \frac{1}{\varepsilon} E_\varepsilon(u)$$

is obtained. Directly related to this problem is the question concerning the integral representation of the relaxation F(.) in $BV(\Omega; \mathbf{R}^n)$ of a functional

$$u \to \int_\Omega f(x, u(x), \nabla u(x)) \, dx.$$

Assuming that $f(x,u,\cdot)$ is quasiconvex and it has linear growth, in joint work with S. Müller the integral representation of F(.) was identified. This result has been obtained recently in the more restrictive case where $f(x,u,\cdot)$ is convex, by Ambrosio & Pallara, P. Rybka & I. F.

Leonid S. Frank

Coercive and Dispersive Singular Perturbations and Applications.

The singular perturbations appearing in linear and non-linear elasticity theory are coercive (the coerciveness conditon is an algebraic condition on the principal symbol of the linearized operator and of the boundary operators, which was introduced in 1976 by L. S. Frank, CRAC, t. 282, série A, p. 1109-1111. Typical applied problems addressed are plates and rods with clamped, hinged or free boundaries, plates and rods supported by elastic media (treated by using a version for Garding`s inequality for singular perturbations, L.S. Frank, Singular Perturbations I, North-Holland, 1990, pp. 437-485), deformations of a long circular vessel (trough) under normal load, bending of a circular hinged plate subjected to a tensile stress at its edge and a uniform transverse load (L. S. Frank, Singular Perturbations II, Elsevier Science Publishers, North-Holland, to appear soon) and the existence of solitary travelling waves for the refined Korteweg-De Vries equation in the non-linear water wave theory under the gravity and capillarity effects combined together (to appear in "Asymptotic Analysis").
The stability for $t \to +\infty$ of such solitary waves is a research in progress.

Giorgio Fusco

Equilibria with Spherical Interior Layers for the Nonlinear Cahn-Hilliard Equation

The nonlinear Cahn-Hilliard equation

$$1)\quad u_t = \Delta\left(-\varepsilon^2\Delta u + W(u)\right),\quad x \in \Omega;\quad \frac{\partial u}{\partial n} = \frac{\partial \Delta u}{\partial n} = 0,\quad x \in \partial\Omega.$$

(where: u= concentration; W a double well potential, $0 < \varepsilon << 1$) models separation and coarsening phenomena that take place when a binary alloy, initially homogeneous, is quenched at a temperature where equilibrium corresponds to two separated phases. Formally it can be shown that, in the late stages of the separation, fronts develop and their evolution is determined by

2) $\Delta\mu = 0, x \notin \Gamma;\ \mu = -\varepsilon a K, x \in \Gamma,\ \frac{\partial \mu}{\partial n} = 0,\ x \in \partial\Omega\ ,\quad v = b\left[\frac{\partial \mu}{\partial n}\right]_\Gamma$

where: Γ is the front; K the mean curvature of Γ; v the velocity of Γ; $[\phi]_\Gamma$ the jump of the function ϕ across Γ and a, b constants. The talk presents joint work with N. Alikakos in the direction of a rigorous understanding of the relationship between (1) and (2). Since spheres are equilibria of (2) one expects that solutions of (1) with spherical interface should either be equilibria or persist for a long time $T_\varepsilon(T_\varepsilon \to \infty$, as $\varepsilon \to 0)$. The questions discussed are: a) the construction of a criteria for determining the particular spherical solutions of (2) which correspond to equilibria of (1) for $1 < \varepsilon << 1$. b) the estimate $T_\varepsilon = 0(e^{-c/\varepsilon})$, $c > 0$. The first question is solved via a reduction similar to the classical Liapounov-Schmidt reduction which allows for the construction of an approximate "slow motion manifold" (an invariant manifold for (1) with order $0(e^{-c/\varepsilon})$ dynamic). The estimate $T_\varepsilon = 0(e^{-c/\varepsilon})$ is obtained by using the approximate slow motion manifold and spectral estimates for the linearized Cahn-Hilliard operator $\Delta(-\varepsilon^2\Delta + W''(\bar{u}^\varepsilon))$, $\bar{u}^\varepsilon$ being a function with a spherical interface, of the type $\sigma(\lambda) < Ce^{-2c/\varepsilon}$. These estimates are presented in N. Alikakos talk.

Jack Hale

Period Doubling in a Singularly Perturbed Delay Equation

We consider the equation

$$(1)\quad \varepsilon x(t) + x(t) = f_\lambda(x(t-1))$$

where $\varepsilon > 0$ is a small parameter, $f_\lambda(0) = 0$ and the fixed points 0 of f_λ undergoes a generic period doubling bifurcation at $\lambda = 0$. If this bifurcation is supercritical, we prove that, for fixed $\lambda \sim 0$ small, there is a periodic orbit of (1) of period ~2 which goes from a sine wave to a square wave as $\varepsilon \to 0$. In the subcritical case, we prove that the periodic orbit goes from a sine wave to a pulse type periodic solution. The method of proof is the reduction to a two-dimensional center manifold of an auxiliary system of delay equations.

Frank C. Hoppensteadt

Stability of Quasi-Static State Approximations in the Presence of Noise

Quasi-static manifolds for systems of ordinary differential equations can be derived under various stability conditions on the system's vector field. Stability of these approximations over short time and long time intervals is described in three cases. First, we review results for the non-random case that give approximations to the solutions of

$$\frac{dx}{dt} = f_0(t,x,y) + \varepsilon f_1(t,x,y,\varepsilon), \quad \varepsilon\frac{dx}{dt} = g_0(t,x,y) + \varepsilon g_1(t,x,y,\varepsilon)$$

The result is that the solution can be approximated for small values of ε by a quasi-static state over time intervals up to $t_0 \le t \le \infty$, depending on suitable stability conditions of f_0 and g_0. Next, we study the persistence of these states under small amplitude random perturbations. Finally, we consider the persistence of the manifold of quasi-static states when the system is subjected to large deviation perturbations.

Ulrich Hornung

Homogenization of Flow and Transport through Porous Media

A periodic porous medium is considered with periodicity all of length $\varepsilon > 0$. In the interior of the pores steady state flow of an incompressible fluid is assumed. Within the fluid a chemical substance is dissolved that undergoes diffusion and convection. In addition, the substance may be absorbed onto the surfaces of the grains of the solid material. On the surfaces certain chemical reactions take place which may be influenced by catalysts. Finally, migration of the substances on the surfaces is taken into consideration which is of diffusion type. The corresponding mathematical model is studied, and the limit problem for $\varepsilon \to 0$ is derived. Convergence of the solution of the micro-model to that of the macro-model is shown. This is joint work with Willi Jäger.

Bernhard Kawohl

Eigenvalues of Clamped Plates and Related Questions

A clamped plate embedded in an elastic medium with elasticity constant a is laterally compressed, and buckling occurs for compressions of magnitude $\gamma_i(a)$. The buckled deformation is described by

$$\Delta\Delta u + \gamma(a)\,\Delta u + au = 0 \quad \text{in } \Omega \quad , \quad u = 0,\ \nabla u = 0 \text{ on } \partial\Omega$$

The dependence of the first eigenvalues $\gamma_i(a)$ on a is investigated. In particular the behaviour as $a \to \infty$ (stiffening of the ambient medium) is considered.
The mathematical tools are then applied to other eigenvalue problems such as vibrating plates under tension, linear elasticity systems and to nonlinear variational problems of type

$$\text{minimize} \ \ \varepsilon J_2 + J_1$$

over a set of admissible functions. There are examples for which the limiting problem has many, one or no solutions. The latter ones are most interesting, because for small ε the solutions exhibit rapid oscillations.
The results were obtained jointly with Howard Levine (Ames) and Waldemar Velte (Würzburg).

Jim P. Keener

The Core of the Spiral - A Free Boundary Problem

An important feature of spiral patterns in Belousov-Zhabotinsky reagent (as well as other excitable media) is that their period, wavelength and shape are uniquely determined by chemical properties of the medium. In contrast, target patterns arise with a continuous array of periods and wavelength. In this presentation, we use asymptotic and numerical methods to show that the shape and rotation frequency of the spiral in excitable media (as described by a system of diffusion reaction equations) can be found by solving a sequence of nonlinear ordinary differential equation eigenvalue problems, solvable numerically with shooting methods. To obtain this solution, the governing diffusion reaction system is reduced using singular perturbation methods to a free boundary problem (introduced originally by P. Fife) in which the spatial domain is divided into two regions where different "outer" dynamics apply, seperated by an interface whose motion is governed by an "eikonal-curvature" equation (An eikonal-curvature equation described the normal velocity of an interface as the sum of its plane wave velocity and a diffusion coefficient times its mean curvature). In a steadily rotating coordinate system, the outer dynamics and interface motion are described by a system of ordinary differential equations which can be solved iteratively using shooting techniques. Numerical evidence is that iterates converge rapidly to a solution that agrees quite well quantitatively with a solution of the original partial differential equation system in the limit that ε (the natural small parameter of the problem) goes to zero.

Klaus Kirchgässner

Nichtlineare Dynamik, die von Fronten erzeugt wird

Für die Kolmogorov Gleichung $u_t - u_{xx} = u - u^2$ wird die Dynamik der Lösungen untersucht, die einer Frontlösung benachbart sind. Der Front zugeordnet ist eine Eichform, deren selbstadjungierte Linearisierung die räumliche Asymptotik der zugelassenen Lösungen definiert. Die Eichform kann eigebettet werden in eine einparametrige Schar von Eichgleichungen, deren Linearform lautet $U_t - U_{xx} - H_{\alpha,\beta}U_x = O(\varepsilon^2\delta_0 U^2)$. Hierbei ist $H = \alpha$ für $x < 0$ und β für $x > 0$, und α, β werden durch die räumliche Asymptotik der Front bestimmt. Es zeigt sich, daß die Lösungen durch zwei dissipative Wellen beschrieben werden, die auf kompakten Mengen exponentiell, jedoch auf Streifengebieten in der (x,t)-Ebene wie $t^{-1/2}$ gegen 0 gehen.

Hans Wilhelm Knobloch

Global Center Manifolds and Geometric Singular Perturbation Theory

The usual understanding of a ´singular perturbed´ ODE. as a pair of coupled DE´s

$$\dot{y} = g(y,z,\varepsilon),\ \varepsilon\dot{z} = h(y,z,\varepsilon),\ \ \varepsilon > 0 \quad \text{and 'small'} \qquad (1)$$

implies that the role of ´slow´and ´fast´variable is fixed once and for all. Rescaling of time allows to write (1) in the symmetric form

$$\dot{x}=f(x,\varepsilon) \qquad (2)$$

We discuss (2) under the hypothesis that the vectorfield $f(x,0)$ has a smooth k-dimensional manifold M_0 of zeros, $k \leq n = \dim x$. In the special case (1) this manifold is defined in terms of the equation $h(y,z,0) = 0$ and it is then assumed that M_0 admits a global representation z = z(y). The standard geometric approach to singular perturbation analysis amounts to an extension of z(y) to a mapping $S(y,\varepsilon)$ such that $z = S(y,\varepsilon)$ represents an invariant manifold for (1). Fenichel was the first to establish a analogous result for (2). A different approach to the same problem is proposed. Its advantage compared with Fenichel's work is twofold: Relaxed hypotheses concerning the geometry of M_0 and a greater variety for the choice of ε.

John Mallet-Paret

Conjugacy of Singularly Perturbed Vector Fields

With P. Brunovsky and S.-N. Chow we study the question of conjugacy between singularly perturbed vector fields, and obtain results in the case of a fold.

Two vector fields

$$\dot{z} = h_i(z,\varepsilon)\,, \quad i = 1,2$$

for $z \in \mathbf{R}^n$, are said to be uniformly conjugate if there exists neighbourhoods $U,V \subseteq \mathbf{R}^n$ of z = 0, and a parameterized family of homeomorphisms varying continuously in ε for $0 \le \varepsilon \le \varepsilon_0$, such that (1) $\Phi_\varepsilon(U) \supseteq V$ for all ε, and (2) Φ_ε takes solution curves of the first vector field (i = 1) to those of the second (i = 2).

Typically

$$z = (x,y) \in \mathbf{R}^m \times \mathbf{R}^k$$
$$h_i = (\varepsilon f_i, g_i)$$

for functions $f_i(x,y,\varepsilon)$ and $g_i(x,y,\varepsilon)$, therefore representing a singularly perturbed vector field. Note the homeomorphism Φ_0 is defined even at ε = 0.

We say the conjugacy is uniformly Lipschitz if each Φ_ε and its inverse has a Lipschitz constant independent of ε, and uniformly C^1 if each Φ_ε is a diffeomorphism varying continuously in ε in the C^1 topology (including at ε = 0).

We consider the simplest planar (m = k = 1) degenerate case in which the critical manifold $g^{-1}(0)$ for ε = 0 is a smooth curve for which normal hyperbolicity fails at the origin, but in generic way. We assume specifically that $g = g_y = 0$ but $f, g_{yy}, g_x \neq 0$ at x = y = ε = 0. After a smooth change of variables we obtain

$$\dot{u} = 2uv + \varepsilon q(u,v,\varepsilon)\ , \quad \dot{v} = u$$

where $q(0,0,0) = 1$. Let $q^0(v) = q(0,v,0)$, let

$p(v,c) = (v^2 - c^2)^{-1}\left[q(v^2 - c^2,v,0) - q(0,v,0)\right]$, and let

$r(v) = \left[vq^0(v)\, q_u(0,v,0) - v^2 q_\varepsilon(0,v,0)\right] / \left[q^0(v)\right]^2$. Finally let

$$I(c) = \int_{-c}^{c} p(v,c)\,dv - \frac{2q^0(c)}{c}\int_0^c r(v)\,dv - \frac{2q^0(-c)}{c}\int_0^{-c} r(v)\,dv .$$

Theorem. Consider two systems, with q_1 and q_2, as above. Then they are uniformly conjugate. If the functions q_1^0 and q_2^0 are identical on some neighbourhood of the origin, then the conjugacy is uniformly Lipschitz. If in addition the functions I_1 and I_2 are identical then the conjugacy is uniformly C^1.

Moshe Marcus

A Variational Problem Arising from a Model in Thermodynamics

We consider a model (proposed by Coleman) for determining the equilibrium state of a body at constant temperature. This is a second order model in which the average energy is given by

$$J_\Omega[u] = \frac{1}{|\Omega|}\int_\Omega \left(|\Delta u|^2 - \alpha|\nabla u|^2 + \psi_0(u)\right) dx$$

where $\alpha > 0$ and ψ_0 is a double-well potential. We wish to minimize $J_\Omega[u]$ subject to the constraint

$$\frac{1}{|\Omega|}\int_\Omega u = a .$$

In recent joint work with Coleman and Mizel we studied this problem in one dimension when $\Omega = \mathbf{R}$. It is expected that the study of this problem will shed light on the behaviour of minimizers in large intervals. Denote,

$$J_\infty[u] = \lim_{T\to\infty} J_{(-T,T)}[u], \quad \langle u\rangle_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u$$

and

$X = \{u \in H_2(\mathbf{R}) \cap C_b^1(\mathbf{R}) : J_\infty[u]$ and $\langle u \rangle_\infty$ are well-defined$\}$.

Consider the problem,

$$(P_a) \qquad \inf\{J_\infty[u] : u \in X, \langle u \rangle_\infty = a\}$$

Theorem (Coleman, Marcus, Mizel). For every real a, problem (P_a) possesses a solution which is either periodic or "pseudo-periodic". Denote,

$$(P_{a,T}) \quad \inf\left\{J_{(-T,T)}[u] : u \in H_2(-T,T), \frac{1}{2T}\int_{-T}^{T} u = a\right\}$$

Theorem For every real a,

$$\inf(P_{a,T}) \to \inf(P_a) \text{ as } T \to \infty.$$

Alexander Mielke

Reduction of PDEs in Domains with Several Unbounded Directions

Often systems are studied which have, in addition to the time variable, one or more unbounded space directions. If a spacially homogeneous solution becomes unstable the system can develop patterns which are modulations of the basic critical mode. To describe this modulation equations, like the Ginzburg-Landau equation, are derived by multiple scaling methods. We try to give this method a rigorous bases by using integro-differential equations. Thus, we obtain a reduced PDE on the"unbounded cross-section" involving nonlocal terms in the nonlinearity. As example we consider the Bénard problem in an infinite strip.

Konstantin Mischaikow

Conley Index Theory on Thin Domains

One of the most important properties of the Conley index for isolated invariant sets is that it is invariant under continuous perturbations. This invariance is due primarily to two facts:

1. The index depends on the unstable set of the invariant set being compact.

2. As a function of continuous parameters the invariant sets are upper semi-continuous.

Recent work of J. Hale and G. Raugel shows that on thin domains the attractors for parabolic and hyperbolic equations are upper semi-continuous. In particular the attractor for a thin enough domain in $\mathbf{R}^{n+k}$ is a neighborhood of the attractor for the corresponding equation on the lower dimensional domain in $\mathbf{R}^n$. They have proven similar results comparing the attractor for

$$u_t = \Delta u + f(u)$$

with the attractors for

$$\varepsilon u_{tt} + u_t = \Delta u + f(u).$$

Joint work with G. Raugel showing that the Conley index is invariant unter perturbation for parabolic equations on thin domains is presented, along with a discussion of the difficulty in obtaining similar results for hyperbolic equations on thin000 domains and the singular perturbation from parabolic to hyperbolic.

Luciano Modica

Approximation of the Capillarity Problem by Plateau Problems

Let Ω be an open, bounded, strictly convex subset of $\mathbf{R}^n$ with smooth boundary, $\lambda \in]0,1]$, $\psi \in L^\infty(\partial\Omega)$. Consider the problem of minimizing the functional

$$F(u; \psi, \lambda) = \int_\Omega \sqrt{1 + |\nabla u|^2}\, dx + \lambda \int_{\partial\Omega} |u - \psi|\, dH^{n-1}$$

for $u \in BV(\Omega)$. The case $\lambda = 1$ corresponds to the Plateau problem for graphs on Ω; ψ gives the prescribed boundery values. The case $\lambda < 1$ is often called "capillarity problem" because the minimizer does not generally attain the boundary values ψ, and a bound on the contact angle between the graph of the minimizer and the cylinder $\partial\Omega \times \mathbf{R}$ exists in dependence on $\lambda < 1$.
Nevertheless, under some conditions on ψ, we have proved in a joint work with S. Baldo that the minimizers of $F(u;\psi\varphi_h,1)$ form a relatively compact sequence in $L^1(\Omega)$ and $C^\infty(\Omega)$, and each limit point is a minimizer of $F(u;\psi,\lambda)$ with $\lambda < 1$. The se-

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quence (φ_h) must be chosen in such a way that it performs a homogenization process on ψ. Appropriate assumptions are:

$$|\varphi_h| = 1 \text{ a.e. on } \partial\Omega\,;\ \varphi_h \to \lambda \text{ weakly in } L^1(\partial\Omega)\,.$$

We used this result by ourselves to construct a non-uniqueness example for the non-parametric Plateau problem on the unit disk in the plane.

Yasumasa Nishiura

Stability of Interfaces in Higher Dimensional Space

It is well-known that reaction-diffusion systems such as activator-inhitor systems and phase field model exhibit a variety of special patterns.
We consider the relation between very complicated connected pattern (snaky pattern) and simple constant mean curvature solutions such as planar and spherical ones. Through the stability analysis of an arbitrary interfàcial pattern, we found that there are no interfacial patterns which remain stable when ε (=width of interface) goes to zero. This suggests that the complicated patterns can be obtained via successive bifurcation of tip-splitting type starting from simple patterns. A precise stability analysis of planar and spherical solutions shows that their stability properties heavily depend on the parameters such as diffusion rates. Numerically it is confirmed that simple spherical patterns deform into complicated patterns via instability.

John A. Nohel

Nonlinear, Singularly Perturbed Systems of PDE's for Unsteady Flows of Non-Newtonian Fluids

We study the initial-boundary value problem for a singularly perturbed system of quasilinear PDE's in one space dimension modelling shear flow of a highly elastic and viscous non-Newtonian fluid driven by a pressure gradient under incompressible, isothermal conditions. The non-Newtonian contribution to the shear stress is assumed to satisfy a Johnson-Segalman-Oldroyd differential constitutive law. The key feature is a non-monotone relation between the total steady shear stress and shear strain-rate that results in steady states having, in general, discontinuities in the strain rate. The reduced problem is a quadratic system of ODE's which is analyzed completely by the phase-plane techniques phenomena. When the singular parameter is sufficiently small, the dynamics of the PDE's is similar to that of the quadratic system. For the PDE's, we identify steady states that are nonlinearly stable under perturbations of initial data, and we show that every solution tends to a steady state as $t \to \infty$ if the singular parameter is sufficiently small. It appears extremely difficult to determine the region of stability of such stable solution (i. e. the governing system does not appear to possess a compact global attractor).

Olga Oleinik

An Asymptotic Behaviour of Solutions of Some Nonlinear Elliptic Equations in Unbounded Domains

For the model equation $\Delta u - |u|^{p-1}u = 0$, $p = \text{const} > 1$, in the halfcylindrical domain $\Omega = \{x : x' \subset \omega, 0 < x_n < \infty\}$, $x' = (x_1, \ldots, x_{n-1})$ it is proved that

1) If $u > 0$ in Ω, $\frac{\partial u}{\partial \nu} = 0$ on $S = \{x : x' \in \partial\omega,\ 0 < x_n < \infty\}$

then

$$u(x', x_n) = C_p\, x_n^{\frac{2}{1-p}} + v(x', x'_n),$$

where

$$C_p = \left(\frac{2(1+p)}{(p-1)^2}\right)^{\frac{1}{p-1}},\quad |v(x', x_n)| \le C \exp\{-\alpha x_n\},\quad C, \alpha = \text{const} > 0,$$

$\frac{\partial u}{\partial \nu}$ is a normal derivative on S.

2) If $u(x', x_n)$ changes the sign in Ω (this means that for any $x_n > 0$ the solution takes on positive as well as negative values), then
$|u(x', x_n)| \le C' \exp\{-\beta x_n\}$, $C', \beta = \text{const} > 0$.

These results are obtained jointly with V. A. Kondratiev.
For the model equation $\Delta u + |u|^{p-1}u = 0$, $p = \text{const} > 1$, in Ω it is proved that

3) A solution $u(x',x_n)$ with condition $\frac{\partial u}{\partial \nu} = 0$ on S and condition $u \geq 0$ in Ω does not exist.

4) If $\max_{x_n = 0} |u|^{p-1} < \lambda_1$, where λ_1 is a first eigenvalue of the problem

$\Delta_{x'} V + \lambda V = 0$ in ω, $V = 0$ on $\partial\omega$, and $u = 0$ on S, then

$$|u(x',x^n)| \leq C \exp\{-\alpha x_n\}, \quad C,\alpha = \text{const} > 0.$$

These results are obtained jointly with L. Peletier.
Results, similar to 1) -4) are proved also for some classes of nonlinear elliptic equations.

Robert E. O´Malley

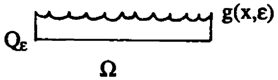
Regularization of Differential-Algebraic Equations

Differential algebraic equations (DAEs) have been of great recent interest to numerical analysts who seek efficient computational methods based on initial value codes for ODEs (cf. the books of Brenan, Campbell, and Petzold (1989) and Hairer, Lubich, and Roche (1989)). One of the primary difficulties involves the selection of appropriate consistent initial conditions. A successful solution technique for DAEs involves introduction of an artificial singular perturbation via a small parameter (or "viscosity") $\varepsilon > 0$. One determines the DAE´s solution as the limiting solution as $\varepsilon \to 0$ away from an initial layer region. Gear integration codes can be easily used. Details require paying attention to the stability hypotheses of the Tikhonov-Levinson theory for singular perturbations, with the nature of the initial impulses depending on the index of the DAE. Explicit examples are provided and applications are included. We note the close relationship to the cheap control approach to singular controls as developed by Jameson and O´Malley.

Geneviève Raugel

Dynamics on Thin Bounded Domains

We consider, for instance, a bounded regular domain Ω in $\mathbf{R}^n$, n = 1,2 and a bounded domain $Q_\varepsilon \subset \mathbf{R}^{n+1}$, $\varepsilon > 0$, which converges in some sense to Ω as $\varepsilon \to 0$.



More precisely, let $g : \overline{\Omega} \times [0,\varepsilon_0] \to \mathbf{R}$ be a function of class C^3 such that

$$\begin{cases} g(x,0) = 0, \quad g_0(x) \equiv \dfrac{\partial g}{\partial \varepsilon}(x,0) > 0\,, \;\; x \in \overline{\Omega} \\[2ex] g(x,\varepsilon) > 0 \quad \text{for } x \in \overline{\Omega}\,, \varepsilon \in (0,\varepsilon_0] \end{cases}$$

We set: $\qquad Q_\varepsilon = \{(X,Y) \in \mathbf{R}^{n+1};\ 0 < y < g(x,\varepsilon),\ x \in \Omega\}$

Let ν_ε be the outer normal to ∂Q_ε, $\tilde{Q} = \Omega \times (0,\delta)$ which contains Q_ε, for $0 < \varepsilon \le \varepsilon_0$; and let G be a function in $W^{1,\infty}(\tilde{Q})$.

For $\alpha > 0$, we consider the equation

$$(1)_\varepsilon \begin{cases} u_t - \Delta u + \alpha u = -f(u - G) \;\; \text{in } Q_\varepsilon\,, \dfrac{\partial u}{\partial \nu_\varepsilon} = 0 \\[2ex] u(0) = u_0 \;\; \text{given in } H^1(Q_\varepsilon) \end{cases}$$

where $f : \mathbf{R} \to \mathbf{R}$ is a given C^2-function such that

$$(2)\ \overline{\lim_{|s| \to +\infty}} \frac{-f(s)}{s} \le 0 \;\;, \quad (3)\ |f'(s)| \le C\left(1 + |s|^{\tilde{\gamma}}\right)\,, \; s \in \mathbf{R}\,,$$

where $\quad 0 \le \tilde{\gamma} < +\infty \;\; \text{if } n = 1, \quad 0 \le \tilde{\gamma} \le 2 \;\; \text{if } n = 2\,.$

Making the change of variables $X = x$, $Y = g(x,\varepsilon)y$, we change the domain Q_ε into the fixed domain $Q = \Omega \times (0,1)$. In these new variables, the Laplacian becomes the operator $L_\varepsilon u = -\frac{1}{g}\,\mathrm{div}\, B_\varepsilon u$ where

$$B_\varepsilon u = \begin{bmatrix} g u_{x_1} - g_{x_1} y\, u_y \\ g u_{x_2} - g_{x_2} y\, u_y \\ -g_{x_1} y u_{x_1} - g_{x_2} y u_{x_2} + \frac{1}{g}(1 + (g_{x_1} y)^2 + (g_{x_2} y)^2)\, u_y \end{bmatrix}.$$

If we let $G_\varepsilon(x,y) = G(x, g(x,\varepsilon)y)$, the problem $(1)_\varepsilon$ becomes on Q,

$$(4)_\varepsilon \begin{cases} u_t + L_\varepsilon u + \alpha u = -f(u) - G_\varepsilon \text{ in } Q\,, \quad \dfrac{\partial n}{\partial \gamma_{B_\varepsilon}} \equiv B_\varepsilon u.\nu = 0 \quad \text{in } \partial Q \\ u(0) = u_0 \quad \text{given in } H^1(Q) \end{cases}$$

The formal limit problem to $(4)_\varepsilon$ is the following :

$$(4)_0 \begin{cases} v_t - \dfrac{1}{g_0} \displaystyle\sum_{i=1}^{n} (g_0 v_{x_i})_{x_i} + \alpha v = -f(v) - G_0 \quad \text{in } \Omega\,, \quad \dfrac{\partial v}{\partial n} = 0 \qquad \text{in } \partial\Omega \\ u(0) = u_0 \quad \text{given in } H^1(\Omega) \end{cases}$$

If $T_\varepsilon(t)$ (resp. $T_0(t)$) is the semigroup on $H^1(Q)$ (resp. $H^1(\Omega)$) defined by $(4)_\varepsilon$ (resp. $(4)_0$), we show that, if $\|u_0\|_{H^1(Q)} \leq r$, then, for $t > 0$,

$$(5)\quad \|T_\varepsilon(t)u_0 - T_0(t)\, Mu_0\|_{H^1(Q)} + t\|DT_\varepsilon(t)u_0 - DT_0(t)\, Mu_0\|_{L(H^1(Q);\, H^1(Q))} \leq C\varepsilon^{1/2} K(r,t)$$

where $K(r,t)$ is a positive, increasing function of r and t, and

$$Mu_0(x) = \int_0^1 u_0(x,y)\, dy.$$

Let A_ε (resp. A_0) be the global attractor of $(4)_\varepsilon$ (resp. $(4)_0$). Using the etimates (5) and results of [Hale, Raugel] , we show that, if the equilibrium points of $(4)_0$ are hyperbolic, then the attractors $(A)_\varepsilon$ are continuous at $\varepsilon = 0$.

Moreover, if $(4)_0$ is a Morse-Smale system, $(4)_\varepsilon$ is also Morse-Smale, for $0 < \varepsilon \leq \varepsilon_1$.

Hans-Georg Roos

The Numerical Solution of Singularly Perturbed Elliptic Boundary Value Problems

Let us consider the boundary value problem

$$-\varepsilon\Delta u + b\nabla u + cu = f \quad \text{in } \Omega \subset \mathbf{R}^2 \text{ , } u = 0 \text{ on } \partial\Omega$$

under the assumption $c - \frac{1}{2}\operatorname{div} b \geq \beta > 0$ in a polygonal domain.
In the singularly perturbed case $0 < \varepsilon << 1$ classical numerical methods cannot work due to stability problems and the nonboundedness of the consistency or interpolation error.
From the theoretical point of view, uniformly with respect to the parameter ε, convergent methods seem to be very nice. But until now only in very simple situations uniformly convergent methods are available. For instance, under the assumption $b = (b_1,b_2) > 0,\ b_1 = b_1(x),\ \ b_2 = b_2(y)$ a standard Galerkin-technique based on L-splines over a rectangular grid yields

$$\|u - u_n\|_\varepsilon^2 := \varepsilon|u - u_n|_1^2 + |u - u_n|_0^2 \leq C\,h \qquad (C \text{ independent of } \varepsilon \text{ !}).$$

In practice, upwind schemes are used. It is possible to construct an upwind finite element method on weakly acute triangulations which preserves the inverse-monotonicity of the problem and satisfies

$$\|u - u_n\|_\varepsilon \leq C\varepsilon^{-1/2}h\ \|u\|_2 \text{ .}$$

The combination with an adaptive procedure based on local error estimations seems to be very promising.

Kunimochi Sakamoto

Geometric Approach to Singular Perturbation Problems for Ordinary Differential Equations

Many of dynamical features of multi-dimensional relaxation systems can be profitably studied by dynamical system approaches, especially by invariant manifold theory. In many cases, an important part of the dynamics of singularly perturbed systems for ODEs behaves in a rather regular manner when approached from a perspective of invariant manifold theory. In this context, singular perturbation problems are treated as a gene-ralized bifurcation problem.

In this talk, after developing a general theory for singularly perturbed systems by using invariant manifold theory, cases are studied in which slow variable passes through elementary codimension one bifurcation points, i.e., transcritical, pitch-fork, and Hopf-bifurcation points. Once the general theory is applied, everything else turns out to be regular perturbation problems.

Jan A. Sanders

On the Computation of Versal Normal Forms

The problem of computing the (versal) normal form of an ordinary differential equation at equilibrium is a very practical one with applications in dynamics and bifurcation theory. Since the computations necessary to do this are both simple and extensive, this looks like the ideal kind of problem to do with computer algebra. This talk describes the inner workings of a program, written by the author in Maple, that computers versal normal forms for vectorfields and Hamiltonians at equilibrium. It emphasizes the use of direct methods, exploiting the multilinear character of formal expansions. To handle the normalization with respect to the semisimple part of the linear field, one uses the avera-ging method, while the nilpotent part is treated using embedding in an $SL_2(\mathbf{R})$ and a splitting algorithm developed by R. H. Cushman and the author.

To express the final result in symmetric coordinates, use has been made of the Gröb-ner-basis package available in Maple. Partially open problems include the treatment of constant terms in the nilpotent case and the computation of the normal form of the linear deformation.

Klaus Schneider

Singularly Perturbed ODE - Geometrical and Numerical Approaches

We prove the existence of invariant manifolds for autonomous singularly perturbed ODEs by means of which we can reduce the original system either to a slow or fast manifold. We derive conditions guaranteeing that the dynamics on the slow manifold determines the dynamic of the full system near that manifold.
For nonautonomous ODE with several time scales we propose a special waveform iteration procedure to solve the initial value problem in the boundary layer numerically.
We describe a method for establishing unique relaxation oscillation to autonomous systems with several time scales. As an example the Belousov-Zhabotinsky reaction is considered.
We propose a new approach to prove the existence of unique stable relaxation oscillations for two-dimensional systems.

Donald Smith

Singularly Perturbed Integral Equations

The linear Volterra or Fredholm vector equation

$$\int_0^1 K(t,s)\, w(s)\, ds = h(t,\varepsilon) + \varepsilon w(t),\ 0 \leq t \leq 1 \quad ,$$

with kernel possessing a jump discontinuity along $t = s$ is discussed for small $\varepsilon > 0$. An asymptotic splitting permits the construction of approximate solutions of boundary-layer type, and then a suitable error estimate leads to the existence of a (unique) solution $w = w\,(t,\varepsilon)$ that is well-approximated by the given approximate solutions. (Jointly with C. Lange)

Luc Tartar

A Boundary Layer Effect in Optimal Design

In a ball Ω of radius R in R^N, we consider a variational elliptic equation $-\text{div}\{a(x)\,\text{grad}\,u_a(x)\} = 1$, with $u_a = 0$ on the boundary $\partial\Omega$. The function a is only allowed to take the values α or β (with $0 < \alpha < \beta$) and the average of a on Ω should be $(1-\varepsilon)\alpha + \varepsilon\beta$ where ε is given a small number. The optimal design problem consists in seeking a function a which minimizes $J(a) = \int_\Omega u(x)dx$. In the language of heat conduction, one wants to minimize the average temperature in a domain Ω where there is a uniform source of heat, by placing in an adequate way two conducting materials given in a precise amount.

For $\varepsilon > 0$, is there an optimal position for an interface between the two different conducting materials ? Is there a simple analysis of what to do when ε is small ?

It appears that there is no optimal interface that seperates the two conducting materials and the best way to use them is to create a mixture in an annular region, the mixing using layers in the radial direction, the proportion of the good conductor varying in an affine way with respect to the distance to the center. When ε is small, the mixing occurs in an annulus of size of order $\varepsilon^{1/2}$ near the boundary of Ω, the proportion of the good conductor being 0 on the interior boundary of the annulus and being of order $\varepsilon^{1/2}$ on the exterior boundary of the annulus, i.e. the boundary of Ω. The optimal value $\text{Inf}_a J(a)$, which is only attained by an optimal mixture, is $J(\alpha) - O(\varepsilon^{1/2})$.

One can visualize a way to approximate the "optimal" design by considering a large number m of triangular spikes of the good conductor, each triangle having a basis of the order $aR\varepsilon^{1/2}/m$ on the boundary of Ω and having height $bR\varepsilon^{1/2}$ in the direction of the center.

The method of attack for this type of problem has been developed many years ago in joint work with François Murat: it is based on homogenization results.

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