MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematische Optimierung
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Wie in den Jahren zuvor stieß auch diese Tagung über mathematische Optimierung auf reges Interesse im In- und Ausland. Insgesamt 35 Teilnehmer aus 11 Ländern nahmen an der Tagung teil. 32 Voträge wurden gehalten. Obwohl die Tagung dieses Mal bewußt einen kleineren Teilnehmerkreis hatte und viele junge Wissenschaftler teilnahmen, wurde wegen der internationalen Zusammensetzung das gesamte Spektrum der diskreten und nicht-linearen Optimierung angesprochen. Viele Vorträge gingen auch auf besonders interessante methodische Entwicklungen und Anwendungen ein, die in den letzten beiden Jahren stattgefunden haben.

Im Bereich der stetigen Optimierung wurden neue Verfahren zur Basissuche in interior-point Algorithmen, zur active-set Methode und bei Quasi-Newton Algorithmen vorgestellt. Vorträge struktureller Art diskutierten neue Verbindungen zwischen nicht-linearer Optimierung, der Variationsrechnung und der Theorie gewöhnlicher Differentialgleichungen. Die Vorträge über Anwendungen der stetigen Optimierung erstreckten sich auf Molekular-Modelling Probleme, ingenieurwissenschaftliche Probleme der Tragwerk-Konstruktion (z.B. Elektrizitätsmasten) und Transportprobleme.

Die Vorträge über diskrete Optimierung stellten neue Resultate und neuartige Methoden der polyedrischen Kombinatorik, der Netzwerk-Fluß und Scheduling Theorie, der Graphentheorie und randomisierter Algorithmen vor und gaben neue algorithmische Anwendungen im Design von VLSI-Chips. In verschiedenen Vorträgen wurde die erfolgreiche Anwendung von Methoden aus benachbarten mathematischen Disziplinen (wie z.B. Wahrscheinlichkeitstheorie, Gruppen- und Kohomologietheorie, Knotentheorie) bei der Lösung kombinatorischer Probleme demonstriert.

Die gute Arbeitsatmosphäre der Tagung zeigte sich in den intensiven Diskussionen während der zeitlich großzügig gestalteten Mittagspause und an den Abenden. Die Tagung reflektierte nicht nur ein breites Themenspektrum, sondern gab neue methodische Impulse, die schon jetzt zu Lösungsansätzen für eine Reihe von Problemkreisen in der diskreten und stetigen Optimierung geführt haben.

Der besondere Dank der Veranstalter und Teilnehmer der Tagung gilt dem Direktor des Mathematischen Forschungsinstituts, Herrn Professor Dr. M. Barner, und seinen Mitarbeitern für deren herzliche Gastfreundschaft und die ausgezeichnete Betreuung.

Vortragsauszüge

A. Bachem: *Simulated trading - a parallel approach for solving vehicle routing problems*

Starting with the optimal tours $T_1, \ldots, T_n$ of a sequential heuristic the simulated trading concept assigns the data of each tour T_i to a single processor i (the tour-manager) of a parallel (MIMD) computer. In addition an extra processor $n+1$ (the stock manager) is linked to all tourmanagers receiving at specific levels buy and sell offers of the tourmanagers at insert resp. saving costs. The stockmanager builds up an admissible trading graph where the nodes correspond to sell or buy offers of customers and the edges link sell to buy nodes when fulfilling a level constraint. A matching (with some additional constraints) of cardinality k in this trading graph corresponds to a feasible k-interchange of customers between tours.

Although in general finding a maximal weighted trading matching is NP-complete even for trading trees with only 3 levels, we show that for fixed number of customers a dynamic programming algorithm easily solves the stockmanager's problem.

Finally we report on computational experience using this heuristic for real applications. .

M. J. Best: *Active set algorithms for isotonic regression: a clarifying framework*

We consider a variety of algorithms for the solution of the isotonic regression problem subject to chain constraints. This problem arises in applications in statistics, production planning and inventory control. We show that many known solution algorithms can be formulated as either primal or dual active set quadratic programming algorithms. Complexity results are established and a new algorithm is presented. Extensions form chain constraints to the arborescence case are given. (Joint work with N. Chakravarty)

B. Bixby: *Recovering an optimal LP basis from an interior-point solution*

A practical method is presented for constructing an optimal basic solution from an optimal LP solution produced by an interior-point method. The algorithm is based on carefully constructing a well-conditioned initial superbasic solution and then applying a version of the simplex method that accomodates superbases. Computational comparisons are given to Meggido's strongly-polynomial basis recovery algorithm.

R. E. Burkard: *Quickest flows in networks*

The *quickest flow problem* can be described as follows: Let $G = (N, A)$ be a directed network with node set N, arc set A and arc capacities $c(x)$, $x \in A$. For any arc $x \in A$ let $l(x)$ be the *lead time* for a flow through this arc, i.e. a flow originating at time T in the tail-node of arc x arrives at time $T + l(x)$ in the head-node of this arc. Given a source s and a sink t we ask for the shortest possible time that a fixed amount of flow can be routed through this network from s to t. Chen and Chin investigated recently the *quickest path problem* and solved it by considering all possible capacities of paths from the source to the sink. We allow here that the flow splits to different paths and get a faster solution, since the objective function turns out to be unimodal and therefore suited for binary search. A polynomial optimization algorithm can be found for the quickest flow problem using dynamic flows. Finally we consider the disjoint quickest flow problem where the flow has to use node or arc disjoint paths from s to t. We develop a solution routine also for this problem and show that it is polynomial, if the underlying network is acyclic and the number of disjoint paths is fixed. (Joint work with K. Dlaska and B. Klinz)

W. H. Cunningham: *k-restricted 2-factors*

A k-restricted 2-factor of the complete graph $K_n = (V, E)$ is a 2-factor for which no component circuit has $\leq k$ nodes. We show that the bipartition inequalities for the TSP in which each tooth has size at most k are valid inequalities for the polytope Q_k^n of k-restricted 2-factors. We also show a necessary condition for such an inequality to be facet-inducing, and a necessary and sufficient condition when $k = 3$.

J. E. Dennis: *Parameter identification for ordinary differential equations*

Many important situations must be modeled by a system of differential equations which involve some parameters, perhaps themselves functions of the independent variable. This talk presents a domain decomposition approach to the problem of identifying the parameters form data off the solution curve for the process. The approach we take is midway between the "initial value" approach that computes data residuals by integrating the system with the current value of the parameter and the "altogether" approach that uses collocation to replace the differential equations by a nonlinear algebraic system and then treats the solution components and parameters together as independent variables.

J. Fonlupt: *Characterization of minimal non-integer extreme points of some polytope defined by cut-constraints associated to the travelling salesman problem*

Let $G = (V, E)$ be an undirected connected graph. We denote by $\delta(S)$ $(S \subset V)$ the set of edges with one end in S and the other end in $V \setminus S$. Consider the following relaxation of the travelling salesman problem, given by the polytope

$$P(G) = \{x \mid x \geq 0, x(\delta(S)) \geq 2 \ \forall S \subset V; x \leq 1\}$$

We say that a non integer extreme point $\bar{x}$ of $P(G)$ is dominated by another non integer extreme point $\bar{y}$ if:

$$\bar{x}(e) = 0 \Rightarrow \bar{y}(e) = 0$$

$$\bar{x}(e) = 1 \Rightarrow \bar{y}(e) = 1.$$

There exists at least one edge with $0 < \bar{x}(e) < 1$ and $\bar{y}(e) = 0$ or 1. An extreme point is critical if it is a non integer extreme point and if it is not dominated by another extreme point. We give a complete characterization of critical extreme points of $P(G)$.

A. Frank: *Strongly conservative weightings*

A ± 1 weighting of a graph is called *conservative* (*strongly conservative*) if the total weight of every circuit is non-negative (positive). What is the maximum number of negative edges in a (strongly) conservative weighting? Denoting these parameters by μ and $\mu^<$, we have:

Theorem.

(1) $\mu(G) = \frac{|V|-1}{2} + \frac{\varphi(G)}{2}$

(2) $\mu^<(G) \leq \frac{|V|-1}{2} - \frac{\varphi(G)}{2}$

where $\varphi(G)$ denotes the minimum cardinality of a subset of edges whose contraction leaves a factor-critical graph. While computing $\mu^<(G)$ was proved recently (Fraenkel-Loebl) to be NP-complete, we develop a polynomial time algorithm to decide whether $\mu^<(G) = \frac{|V|-1}{2}$ when G is bipartite. A by-product of these investigations is the following.

Theorem. A strongly connected digraph $G = (V, E)$ contains a spanning tree so that every fundamental circuit belonging to T is a directed circuit iff the number of directed circuits of G is precisely $|E| - |V| + 1$.

This is a joint work with T. Jordán and Z. Szigeti.

R. Kannan: *Chance constrained optimization*

We consider the problem of minimizing the cost of raw materials/components subject to the stochastic constraint that the probability of meeting certain demands (which are random variables) is at least certain amount. By the Brunn-Minkowski theorem, it follows that the feasible set is convex. This set admits a membership oracle if we can sample according to the density of the demands. This is possible to do in time polynomially bounded when the density is log-concave. We describe the sampling method that uses a rapidly mixing random walk and an optimization algorithm. This is joint work with J. Mount, S. Tayar, D. Applegate.

V. Kovačevic-Vujčic: *Interior point methods for transportation problems*

We will discuss the application of the primal interior method of Freund to transportation problems. It is shown that due to the special structure of the transportation problem the stopping criterion can be improved, which reduces the complexity bound of Freund's method. The main computational effort per step in Freund's method comes from computing the projection of certain vectors to a subspace induced by a scaled transportation matrix. We propose the solution to this problem via a successive projection method, which takes the advantage of the special structure of the transportation matrix. It is shown that the projection can be obtained with $O(\min\{p,k\}^2 \max\{p,k\})$ operations, where p and k are dimensions of the given instance of the transportation problem. This approach also may have advantages over standard approaches in the degenerate case, when numerical instability is likely to occur. (Joint work with M. Ašic.)

B. Korte: *Min-max matchings for balanced clock trees with zero skew*

This paper deals with a new application of bottleneck (and min-sum)- matching to the construction of balanced clock trees on VLSI-Chips. Since the cycle time of chips gets smaller clock trees with zero skew are most essential. A tree $T = (V, E)$ with root $v \in V(T)$ is called balanced if every path from the root to a leaf has the same length, it is called totally balanced if all subtrees (of the same height) have the same length. It is well known that the best possible tree (namely a Steiner tree) on n uniformly distributed points in the unit square has length $O(\sqrt{n})$. We give different algorithms for top-down and bottom-up construction of balanced and totally balanced trees for uniformly and arbitrarily distributed points in the plane which have length $O(\sqrt{n \log n})$ and $O(\sqrt{n})$. The most advantageous algorithm constructs bottom-up a totally balanced tree pairing two leaf nodes to a parent node by min-max matchings. The parent nodes are treated recursively the same way. This is joint work with Karsten Muuss.

M. Laurent: *Hypercube embedding of metrics*

A metric d is h-embeddable if it can be embedded in some hypercube or, equivalently, if it can be written as a nonnegative integer sum of cuts. The problem of testing h-embeddability is NP-complete (Chvatal 1980). A good characterization of h-embeddability permitting a polynomial time algorithm was given for several classes of metrics, including path metrics of graphs (Djokovic 1973), metrics with values in $\{1,2\}$ (Assuoad and Deza 1980), metrics on $n \geq 9$ points with values in $\{1,2,3\}$ (Avis 1990). In fact, for these classes, the even and hypermetric conditions, that are always necessary for h-embeddability, are also sufficient. We consider here generalized bipartite metrics, i.e. metrics d such that $d(i,j) = 2$ for all $i,j \in S$ or $i,j \in T$ for some bipartition S,T of the points. We characterize h-embedabble generalized bipartite metrics by their distance matrix and derive a polynomial recognition algorithm. We have examples of such metrics that satisfy the even condition and the hypermetric conditions (in fact, belong to the cut cone) but are not h-embedabble.

This is joint work with M. Deza.

C. Lemaréchal: *Large-scale problems in molecular modelling*

The general problem is to figure out the shape of a given molecule. There are two cases:

* Theoretical, non-destructive approach: the formula of the molecule is known and one can compute the energy associated to a given distribution of the atoms. The problem is then simply to minimize this energy $E(x)$, where $x \in (\mathbb{R}^3)^N$, and N is the number of atoms. Usually, a good starting guess is available.

* X-ray crystallography. The molecule is synthetized, crystallized, and X-ray diffracted through it. Let $p(x)$ be the electron density at $x \in \Omega$ (Ω is the crystal unit); and let $F_k(p)$ be the k^{th}-th Fourier coefficient of p. The above X-ray experiment furnishes the moduli $|F_k(p)| = m_k$ of a number of Fourier coefficients (viewed as complex numbers). The problem is then to compute a "likely" distribution p matching the data $\{m_k\}$. To eliminate under-determination, one chooses to maximize the entropy, say $\int_\Omega p(x) \log p(x) dx$ subject to $|F_k(p)| = m_k$. The resulting methodology uses duality, and amounts to solving a minimax problem - internal maximization is done over the Lagrange multipliers, and external minimization over the unknown phases. (Joint work with A. Decarreau, D. Hilhorts, J. Navaza)

J. K. Lenstra: *Complexity of approximation*

We discuss performance bounds for combinatorial optimization problems that cannot be achieved in polynomial time unless $P = NP$. More precisely, consider a constrained minimization problem with a positive integer value for each feasible solution. If there exists a constant $c > 0$ such that the question "is there a feasible solution with value $\leq c$?" is NP-complete, then there is no polynomial approximation algorithm with performance ration smaller than $\frac{c+1}{c}$, unless $P = NP$. We illustrate the application of this theorem to the bin packing problem, precedence constrained scheduling, scheduling unrelated machines, and open shop scheduling.

Th. M. Liebling: *Optimization problems concerning strongly connected graphs*

Given a connected digraph $G = (V, E)$ with arc weights, the following three problems each consisting in finding a subset A of E such that the corresponding graph G' is strongly connected have been widely studied:

SCS (spanning subgraph): $G' = (V, A)$

JOIN (adjunction of set A': arcs antiparallel to A): $G' = (V, E \cup A')$

OSCAR (arc reversal): $G' = (V, E - A \cup A')$.

Problem SCS is NP-complete. A. Frank gave an $O(|V|^5)$ primal-dual algorithm for JOIN, yielding a linear description of the corresponding polyhedron and proving the weighted version of the Lucchesi/Younger Theorem. This also yields a linear description of the OSCAR polyhedron and the corresponding theorem by Nash-Williams.

For series-parallel graphs there are $O(|V|)$ algorithms from which follows linear descriptions of the three polyhedra using recent results by Margot/Schaffers. Numerical experiences using Simplex algorithm and separation for OSCAR (or JOIN) on arbitrary graphs indicate that such an approach is promising.

(Based on joint work with P. Freymond, F. Margot and A. Prodon.)

L. Lovász: *Interactive proofs and quadratic optimization*

We study a very simple 2-prover interactive proof system for problems in NP. The error probability is the optimum of a quadratic program. Replacing products of variables by new ones, we obtain a relaxation in the form of optimizing a linear objective function over positive semidefinite matrices, subject to linear side constraints. This relaxation can be solved in polynomial time. Solving this relaxation gives a general scheme for proving that the given NP-property does not hold.

As an application, we get another general example of pairs of disjoint NP-classes that can be separated by a P-class. (Joint work with U. Feige)

L. McLinden: *Finding all solutions to nondegenerate montone complementarity problems*

We consider the problem

$$(C) \qquad \text{Find } \Omega := \{(x, y) \in \mathbf{R}^n \times \mathbf{R}^n | x \geq 0, y \geq 0, y \in Tx, \langle x, y \rangle = 0\}$$

where $T : \mathbb{R}^n \to \mathbb{R}^n$ is a maximal monotone multifunction satisfying

$$\exists\ (\tilde{x}, \tilde{y}), \tilde{x} \succ 0, \tilde{y} \succ 0, \tilde{y} \in T\tilde{x}.$$

If $(x^0, y^0) \in \Omega$ is strictly complementary, then

$$\Omega := \{(x,y) \in \mathbb{R}^n \times \mathbb{R}^n | x \geq 0, y \geq 0, y \in Tx, \langle (x,y), (y^0, x^0) \rangle = 0\}.$$

We develop a general method of approximating such an (x^0, y^0), if any exists, by means of the problems

(C_z) Find $\{(x,y) \in \mathbb{R}^n \times \mathbb{R}^n | x > 0, y > 0, y \in Tx, x_k y_k = z_k \forall\ k = 1, \ldots, n\}$

where $z > 0$ is a parameter which is sent to the origin in the limit. Certain well-defined continuous trajectories of approximate solutions (x^z, y^z) are studied. They are shown to converge to a unique point of Ω which is Pareto optimal with respect to a direction which describes the asymptotic approach of z to the origin.

R. H. Möhring: *Approximation algorithms for gate matrix layout and PLA folding*

Gate matrix layout and PLA folding are special VLSI layout technologies. The respective area minimization problems can be formulated as augmenting a given graph by adding edges to an interval graph of small chromatic number (gate matrix layout) and as matching problems with side constraints (PLA folding). We report on approximation algorithms for these problems developed in Berlin by Parkey, Möhring, and Müller, respectively.

For gate matrix layout, the algorithm is based on on-line interval graph augmentation, and it uses modified PQ-trees as data structure. For the PLA folding problem, we consider a Lagrangian relaxation for the so-called constrained block folding problem. Solving the Lagrangian dual leads to a min cost flow problem with special cost coefficients (only on edges incident to source and sink) that is shown to be solvable in $O(\max(n \log n, nm))$ time, where $n = |V|$ and $m = |E|$.

S. Poljak: *Nonlinear relaxation of the max-cut problem*

This lecture deals with the following topics:

> an upper bound on the max-cut problem, its combinatorial properties, dual characterization, complexity, and estimates of the quality (joint work with Ch. Delorme).

report on developing a code and experiments on various data sets (joint work with F. Rendl).

theoretical and experimental comparisons with the polyhedral approach.

previous and recent work of other authors (Fiedler, Donath-Hoffman, Boppana, Rendl-Wolkowicz) for related partition problems.

B. Reed: *When is the TSP easy ?*

We consider random instances of the TSP on complete directed graphs with n nodes where the cost of each arc is drawn uniformly from $\{0,1,...,p_n\}$ for some integer p_n and these choices are made independently. We show that if $p_n = o(n)$ then the optimum tour cost equals the optimum cycle cover cost, almost surely. Conversely, we show that if $n = o(p_n)$ then these two optima are almost surely different. We also discuss some concentration results for the TSP and cycle cover optima in this model. This is joint work with A. Frieze and R. Karp.

S. M. Robinson: *The normal-map approach to variational problems*

Normal maps are single-valued, generally nonsmooth, functions useful for modeling variational problems, including optimality conditions for nonlinear optimization. Zeros of the normal maps correspond precisely to solutions of the variational problems. Moreover, many techniques from smooth calculus can be carried over with little change to the case of normal maps. Therefore these functions offer a powerful tool for analysis and computation in optimization. We will describe some recent results about these maps, and their applications.

A. Schrijver: *Paths in graphs*

We present two results on paths in graphs. The first result is jointly obtained with P.D. Seymour (BELCORE). It shows that the odd path polytope (= convex hull of the characteristic vector of the odd $s-t$ paths in an undirected graph (fixing s and t)) is determined by the so-called 'slice inequalities' (they all have $0, \frac{1}{2}, 1$ coefficients). The second result is the polynomial-time solvability of the k disjoint paths problem in directed planar graphs, fixing k. The method is based on homology over free groups.

W. Schwärzler: *A generalized Tutte polynomial for signed matroids*

A new polynomial is defined on signed matroids. It contains as special cases the Kauffman bracket polynomial of knot theory, the Tutte polynomial of a graph, the partition function of the anisotropic Ising model and the Kauffman-Murasugi polynomial of signed graphs. It also leads to new proofs and generalizations of theorems of Lickorish and Thistlethwaite about adequate and semi-adequate link diagrams. This lecture is based on joint work with D. J. A. Welsh.

A. Sebő: *On the structure of shortest paths in graphs*

In this talk I am presenting a joint work with Michael Lomonosov (Beer Sheba, Israel) about the following questions:

What are the non-negative edge-weightings of a graph according to which all shortest paths are minimum cardinality paths of the graph. When does this set of edge-weights consist only of the constant functions.

These questions are initiated in terms of cones of metrics by Lomonosov (1985), motivated by multiflows. Here we give the complete answers. The most attractive results concern bipartite graphs, which are the most important from the point of view of multiflow applications.

From a polyhedral viewpoint our results characterize the minimal face of the cone of metrics which contains the metric induced by the distances (minimum cardinalities of paths) in a given graph; in particular they characterize the case when such a metric is an extreme ray of this cone.

As a byproduct we get a characterization of graphs in which there exists a partition of the edge-set into disjoint cuts with the property that every pair of vertices is separated by as many cuts as their distance. We also show other applications and connections in particular to multiflow problems.

B. Shepherd: *Edge colouring extensions of the four colour problem*

A polytope P has the integer decomposition property if for each integer $k \geq 1$ and each $x \in P$ such that kx is integral, x is expressable as the sum of integer vectors in P. In particular, P is integral. We consider a conjecture of Seymour which states that the matching polytope of a planar graph has the integer decomposition property. This contains a special case, the conjecture due to Grötzsch which asserts that any fractionally 3-edge-colourable graph is 3-edge-colourable. Seymour's conjecture also implies that any cubic graph without isthmus is 3-edge-colourable. This latter statement is equivalent to the four colour problem. We consider extensions of these problems to minor closed classes. Specifically we consider Tutte's conjecture that any cubic, 2-edge-connected graph without a minor isomorphic to the Petersen graph P_{10}, is 3-edge-colourable. We prove a relaxed form of the conjecture where P_{10} is replaced by $P_{10} - e$ for some edge of P_{10}.

A. Srivastav: *Derandomization of randomized algorithms*

We consider the problem of derandomization of a randomized algorithm: Given an $f : \Omega \to \mathbb{R}$, $\Omega=$ discrete $2n$-dimensional cube, and suppose that there is an algorithm A finding $w \in \Omega$ such that $|f(w) - E(f)| \leq \lambda$ with high probability ($E(f)$ is the expectation of f under the probability distribution considered). The task is to find w in a deterministic way. Such a deterministic conversion was previously known only for linear functions f (J. Spencer, P. Raghavan). The derandomized method now can be extended to cover nonlinear functions with bounded martingale differences and yields an application to the relationship between the integer and rational optimum of the graph bisection problem, formulated as an zero-one quadratic program.

T. F. Sturm: *A Quasi-Newton method by Hermite interpolation*

The new attempt to solve the problem of the local minimization of a continuously differentiable function $F : \mathbb{R}^n \to \mathbb{R}$ without constraints is the modelling of the objective function by Hermite interpolation.

The function values and gradients of old iteration points should serve as information for the points of support. One step in the algorithm constructs an interpolant, optimizes that interpolant, and uses its minimizer as new iteration point. The considered interpolant has $n+1$ points of support, denoted by $z_1, ..., z_{n+1}$.

$D := \{z_1, ..., z_{n+1}\}$ is called a set of vertices, $H := conv(D)$ is called a search hull to D.

$y_D(F) : \mathbb{R}^n \to \mathbb{R}$ is a Hermite interpolant of F at D, if

$$\forall\, z_i \in D \begin{cases} F(z_i) = y_D(F)(z_i) \\ \nabla F(z_i) = \nabla y_D(F)(z_i) \end{cases}$$

There exists a polynomial interpolant $y_D(F) \in \mathbb{P}_3(x)$ for all "regular" search hulls resp. sets of vertices with such properties, such that the described optimization algorithm converges with convergence order 2. These theoretical results are supported by numerical test examples.

G. Tinhofer: *Bin packing and threshold graphs*

A bin packing problem is as follows: Given n items having weights $x_1, x_2, ..., x_n$ we want to pack them into bins such that the total weight of the content of every bin does not exceed 1 and such that the number of used bins is as small as possible. To every instance of such a problem we may associate a threshold graph $G(x)$ with vertex set $V(x) = \{1, 2, ..., n\}$ and with edge set

$$E(x) = \{\langle i, j \rangle \mid x_i + x_j \leq 1\}.$$

A feasible solution to the problem instance defines a subgraph $G'(x)$ consisting of disjoint weighted cliques where the total weight of every clique is less or equal to 1. One can prove:

(1) For every bin packing problem $x = (x_1, ..., x_n)$ there is an optimal solution such that the corresponding subgraph $G_{opt}(x)$ of $G(x)$ contains the edges of some maximum matching in $G(x)$. Hence, any bin packing problem can be solved by carefully selecting maximum matchings in a sequence of threshold graphs where the first graph is $G(x)$ and each further graph of the sequence is obtained by shrinking the edges of the maximum matching in its predecessor.

(2) In the probabilistic model where the weights $x_1, ..., x_n$ are realizations of n independent random variables $X_1, ..., X_n$ each of which is uniformly distributed on $(0, 1]$ the event "$G(x) = G$" has probability 2^{-n} for every threshold figure G (= threshold graph together with a fixed split partition). Hence, these figures are uniformly distributed. One can use this observation in order to find the upper bound $c \cdot \sqrt{n}$ for the expected waste of optimal solutions.

D. de Werra: *Chromatic scheduling with simulaneity constraints*

Edge colouring provides usually a useful model for several types of scheduling problems ranging from automated production systems to school timetabling. We consider here the situation where, in addition to the classical requirements of colorings, some equality constraints are imposed (in order to express simultaneity requirements).

Complexity results are given (showing that some versions are NP-complete) and polynomially solvable cases are presented.

L. Wolsey: *Packing cardinality constrained subtrees of trees*

Given a tree $G = (V, E)$ and node $i \in V$, we associate a family $\mathcal{T}^i$ of i-rooted subtrees $\{T^i\}$, and obtain the constrained subtree problem: $\max\{\sum_{j \in T^i} c_j^i : T^i \in \mathcal{T}^i\}$. When $\mathcal{T}^i$ is the set of subtrees with at most k nodes, we derive valid and facet-defining inequalities for the convex hull of the set of feasible incidence vectors. We completely characterize these polyhedra when $k \leq 4$.

We then consider the constrained subtree packing problem: $\max\{\sum_{i \in V} \sum_{j \in T^i} c_j^i : T^i \in \mathcal{T}^i, \{T^i\}_{i \in V}$ are node disjoint $\}$. We show that if X^i is the set of incidence vectors of $\mathcal{T}^i$, the polyhedron $\{x : \sum_i x_j^i \leq 1, x^i \in conv(X^i)\}$ is integral.

U. Zimmermann: *Balanced flows*

Balanced flows are flows in a network $G = (V, E)$ with lower and upper capacities satisfying an additional reliability constraint on every arc $e \in E$. Various applications can be modelled in this way, including sharing problems, and several telecommunication problems. For given rational numbers $\alpha(e), \beta(e), \alpha'(e), \beta'(e) \in [0, 1]$, $e \in E$, the reliability constraints have the form

$$\alpha'(e) \cdot v - \beta'(e) \leq x(e) \leq \alpha(e) \cdot v + \beta(e), \qquad e \in E,$$

where v denotes the total flow from source to sink. While there are strongly polynomial but impracticable algorithms for maximizing the total flow in the case of rational valued flows, in the integral case the problem is NP-hard. We discuss a dual method which finitely solves the problem in both cases. That method is polynomial for rational flows, and pseudopolynomial for integral flows. Compared to other known methods, complexity bounds are better in general. Numerical investigations show that an optimal solution is usually constructed after 3-10 iterations, which consist in a mincut and in a new upper bound calculation. For integral problems the effort spent for bound calculation exceeds the effort spent for the min-cut-construction. The converse holds for rational problems. The same is true for integral problems when no lower reliability constraints are present. Nevertheless, that problem is conjectured to be NP-hard, too.

J. Zowe: *Truss topology optimization*

Truss topology optimization deals with the optimization of pinjointed frameworks like electricity masts, cantilever arms, arched bridges etc. The design variables are the bar volumes which should be chosen such that the structure becomes as stiff as possible. By working with a large number of bars and joints the model also includes elements of geometry and shape optimization.

This approach leads to a constrained nonconvex optimization problem with up to 10^5 variables for a 2D-model. By applying duality results the dimension of the problem can be drastically reduced for the price of having to deal with a nondifferentiable problem. We show that standard nonsmooth software has no difficulty to deal with such problems. Our numerical results compare very favourably with those obtained by engineering heuristics.

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