MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Applied Dynamics and Bifurcation

12.1.92 - 18.1.92

The conference was organized by G. Iooss (Nice), H. Kielhöfer (Augsburg) J.E. Marsden (Berkeley) and J. Scheurle (Hamburg). It was one of the bigger Oberwolfach conferences. An unexpected high ratio of people followed the institute' s invitation to this conference. In fact, many more people asked to be invited, but could not be considered.

The aim of the conference was to discuss recent and new developments and techniques of the theory of dynamical systems, in particular bifurcation theory, that are useful to analyze the dynamical behaviour of concrete model systems from various applied areas. Another issue was the modelling aspect itself. Special emphasis was given to infinite dimensional systems governed by infinitely many ordinary differential equations, partial differential equations or differential delay equations. Both dissipative as well as conservative (Hamiltonian) systems have been considered. The methods which have been discussed include asymptotic expansions, Ginzburg-Landau technique, reduction by invariant manifolds, normal forms, singularity theory, topological index theories, numerical path following algorithms, conservative simulation algorithms. Many of the proposed techniques employ symmetries of a problem. Also, the usefulness of computer algebra to handle such methods has been demonstrated.

Dynamical issues which have been addressed are, for instance, existence of global attractors, existence of Ljapunov exponents, existence and stability of solitary waves (pulses) and wave fronts, complete integrability, homoclinic and heteroclinic bifurcation, bifurcation and stability of relative equilibria, preservation of nodal patterns, symmetry breaking, pattern selection, chaos, quasiperiodic transport, and reconstruction from data.

Finally, we mention concrete systems and physical phenomena, models for which have been discussed: double spherical pendulum, vehicle dynamics, KdV-like equations, water waves, vortex dynamics, Couette-Taylor problem, Bernard problem, interfacial tension between miscible liquids, elastic rods, plates and shells, viscous and dissipative effects in fluids and elasticity, coagulationfragmention dynamics, and the Cahn-Hilliard equation.

Abstracts

J. ALEXANDER:

Existence and stability of travelling wave solutions of fully diffusive parabolic systems

We consider existence and stability of travelling wave solutions of the parabolic system

$$U_t = U_{xx} + F(U) \quad , \qquad F(0) = 0. \qquad (*)$$

In particular, under standard technical assumptions, if (*) admits a linearly stable wave solution $u(\xi) = u(x - ct)$ which is asymptotically oscillatory at $\pm\infty (u(\xi) \sim K_\pm Re\ e^{i\alpha_\pm \xi}$ as $\xi \to \pm\infty)$, then there is an infinite family of double travelling waves (i.e. approximately two concatenated copies of $u(\xi)$) $u_i(x - c_i t)$ with $c_i \to c$. These are alternately stable and unstable. The argument consists of three parts:

I. General algebraic topological machinery for locating spectral elements.

II. Dynamical systems argument for existence.

III. More precise argument concerning eigenvalues near 0.

S. ANTMAN:

Quasilinear Hyperbolic-Parabolic Equations of Mechanics

The quasilinear equation

$$w_{tt}(s,t) = n(s, w_s(s,t), w_{st}(s,t))_s,$$

where $[0,1] \times (0,\infty) \times \mathbb{R} \ni (x,y,z) \mapsto n(x,y,z) \in \mathbb{R}$ is prescribed, governs the longitudinal motion of a nonlinearly viscoelastic rod provided that

$$n_z \geq const > 0 \text{ and } n(x,y,z) \searrow -\infty \text{ as } y \searrow 0.$$

A new set of physically reasonable restrictions on the function n ensure that $w_s(s,t) \geq c(t) > 0$ for all $t > 0$, so that the rod cannot suffer total compressions. This estimate supports both a global existence theory for initial-boundary-value problems for this equations and a rigorous asymptotic analysis of the problem describing the longitudinal motion of heavy mass on a light spring:

$$\varepsilon w_{tt} = n(s, w_s, w_{st})_s, w(0,t) = 0, -n(1, w_s(1,t), w_{st}(1,t)) = m w_{tt}(1,t).$$

The reduced problem with $\varepsilon = 0$ exhibits surprising effects. The much more complicated problem of the motion in space of a rigid body attached to a light nonlinearly viscoelastic rod is formulated, and the analysis of the reduced problem is carried out. The results suggest that it is possible to describe a variety of large snapping motions.

D. ARMBRUSTER:

Reconstructing phase space from PDE simulations

We propose the Karhumen-Loéve decomposition as a tool to analyze complex spatio-temporal structures in PDE simulations in terms of concepts from dynamical systems theory. The Kuramoto-Sivashinsky equation and simulations of Kolmogorov flow are used as examples. It is shown that the Karhumen-Loéve decomposition can extract stable and unstable manifolds of hyperbolic limit sets and determines linear spanning spaces for periodic, quasiperiodic, and chaotic dynamics. It is shown for Kolmogorov flow that the stream function data demonstrate a low-dimensional phase space dynamics of large scales whereas the vorticity data show an enstrophy cascade. A software package K-L tool which allows to do these analyses in an easy and controlled way is introduced.

J. BALL:

Attractors for nonlinear wave equations

A simple proof is given of the existence of a global attractor in the energy space $X = H_0^1(\Omega) \times L^2(\Omega)$ for the nonlinear wave equation

$$u_{tt} + u_t - \Delta u + F'(u) = 0$$
$$u\big|_{\partial\Omega} = 0.$$

Here $\Omega \subset \mathbb{R}^n$ is bounded and open with strongly Lipschitz boundary, and $F \in C^1(\mathbb{R})$ satisfies the hypotheses

$$\frac{F'(u)}{u} \geq -\lambda \text{ for } |u| \geq K, \text{ where } \lambda < \lambda_1 = \text{first eigenvalue of } -\Delta$$

with Dirichlet boundary conditions, and

$$|F'(u)| \leq c_0(|u|^{\frac{n}{n-2}} + 1) \quad \text{if } n \geq 3,$$
$$|F'(u)| \leq c_0 exp\psi(u), \qquad \frac{\psi(u)}{u^2} \to 0 \text{ as } |u| \to \infty, \text{ if } n = 2.$$

The result is a slight technical improvement of previous result of Babin & Visih, Arrieta, Cavalho & Hale, Hale & Raugel. It would work for growth conditions of the type

$$|F'(u)| \leq c_0(|u|^{\gamma-1} + 1)$$
$$F(u) \geq c_1(|u|^\gamma - c_2)$$

with $\gamma \leq \frac{2n}{n-2}$ (and even for larger γ under further hypotheses), were it drawn that the solution of these equations (proved to exist, e.g. by the Galerhin method) satisfy the energy identity.

The strategy of the proof is different from previous approaches, employing a device for improving weak to strong convergence. A global attractor is a compact invariant set that attracts bounded sets.

H.W. BROER:

Examples of quasi-periodicity and chaos, with dissipation

Persistant occurrence of quasi-periodic dynamics in non-conservative systems is knwon to happen, provided that external parameters are present. Although in phase-space the quasi-periodic dynamics often has measure zero, in the parameter-space it projects to a set of positive measure. Apart from the normally hyperbolic situation, which includes families of quasi-periodic attractors, also cases are of interest where the normal behaviour bifurcates. Of this we give two examples. First we consider a family of vector fields $\dot{x} = w + 0(|z|), \dot{z} = \lambda z + 0(|z|^2)$, as $z \to 0$, for $(x, z) \in T^2 \times \mathbb{C}$ and parameters $(w, \lambda) \in \mathbb{R}^2 \times \mathbb{C}$. The 0-terms depend on x. Generically, near $\lambda_1 = 0$ invariant 3-tori branch off from $z = 0$, with (quasi)-periodic or chaotic dynamics. Second (with F. Takem) we consider $S0(2, \mathbb{R})$-symmetric diffeomorphisms on $T^1 \times \mathbb{C}$, w.r.t. rigid rotation in the second factor. The general form is $(x, z) \mapsto (x + 2\pi\alpha + 0(|z|), z(c\, e^{ikx} + 0(|z|)))$, where (α, c) are parameters — α diophantine. Now, for c near 1 invariant 2-tori branch of, the dynamics in which has mixed spectrum.

F.H. BUSSE:

Tertiary and Quarternary States of Fluid Flow

Some fluid systems characterized by a high degree of external symmetries can be readily investigated with respect to sequences of subsequent bifurcations through the use of broken symmetries of the fluid flow. Of particular interest are those systems like the Rayleigh-Bénard layer and the Taylor-Couette system in which bifurcation occur supercritically and the onset of time-dependence is delayed. Various patterns of steady tertiary states and steady quarternary states will be discussed. Of particular interest are three-dimensional steady flow states in the case of plane Couette flow which does not exhibit a bifurcation from the primary state.

J. CARR:

Coagulation Fragmentation Dynamics

The coagulation-fragmentation equation describes the kineties of cluster growth in which clusters can coagulate via binary interactions to form larger clusters or fragment to form smaller ones. There are numerous applications of these equations, varying from cluster formation in galaxies to the dynamics of phase transitions in binary alloys. This talk will focus on the role of fragmentation. These are two classes of fragmentation. (1) Weak fragmentation, which corresponds to situations in which surface effects are not important: (2) Strong fragmentation, in which surface effects are not important (e.g. polymers). For case (1), all solutions conserve density but phase transitions may occur. For case (2), there are bad solutions which do not conserve density. Because of these spurious solutions we have to restrict the class of solutions. For this restricted class of solutions, the strong fragmentation acts as a dumping mechanism and there are no phase transitions.

P. CHOSSAT:

Forced reflectional symmetry breaking of an 0(2)-symmetric homoclinic cycle

Structurally stable heteroclinic cycles have been observed in systems with 0(2) symmetry (e.g. Kuramoto-Shivasinsky equation by Nicolaenko 88) and have been studied with bifurcation techniques in the 1-2-mode interaction case by Armbruster-Guckenheimer-Holmes (88). Suppose a perturbation in the model breakes 0(2) to S0(2). What does happen? "Standard" techniques are not directly applicable, because there is a continuous group orbit of heteroclinic cycles (a so-called homoclinic cycle). We show that the problem can be handled by going to the orbit space, and that generically the homoclinic cycle breaks to a quasi-periodic flow on a 2-torus.

P.G. CIARLET:

Existence theory for two-dimensional shell equations

Using the same ideas as in a recent joint work with B. Miara, we show how the ellipticity of the bilinear form associated with either the model of W.T. Koiter or with the membrane model can be derived in a systematic way. The method of proof relies on two main tools: one is a lemma due to J.L. Lions (which he used for proving three-dimensional Korn' s inequality); the other is an "infinitesimal rigid displacement lemma", which was proved in a joint work with M. Bernardou for the model of W.T. Koiter, and in a recent work of G. Geymonat and E. Sanchez-Palencia for the membrane model.

G. DANGELMAYR:

Codimension two bifurcations in ($\ell = 4$)-representation of 0(3)

Stationary bifurcations with spherical symmetry typically lead to reduced bifurcation equations that are posed in a space of spherical harmonics of some order ℓ. For ℓ even the generic bifurcation gives solutions with maximal isotropy subgroup which are all unstable. Stable branches can only be detected in degenerate bifurcation of codimension two or higher. In this talk results of an analysis of the codimension-two-bifurcation for $\ell = 4$ are presented. Two branches with maximal isotropy subgroups (octohedral group 0 and rotational group 0(2)) are determined which undergo saddle node bifurcations and further secondary bifurcation to branches with submaximal symmetries D_3 and D_4. In contrast to $\ell = 2$ these submaximal branches are unbounded. Moreover, there are tertiary bifurcations to solutions with lower symmetry D_2. Their stability could not be determined yet completely and requires further analysis. Also the question of possible heteroclinic cycles still has to be answered.

W. ECKHAUS:

On water waves for Froude number slightly higher than one and Bonds number less than 1/3

We consider the problem for $y(x), x \in (-\infty, \infty)$, solutions of the ODE

$$(*) \qquad \varepsilon^2 \frac{d^4 y}{dx^4} + \frac{d^2 y}{dy^2} - y + y^2 = \varepsilon^2 P(y, (\frac{dy}{dx})^2, \frac{d^2 y}{dx^2}; \varepsilon).$$

For $P \equiv 0$ it is a model problem proposed by Kirchgässner and Amick. For $\varepsilon = 0$ there exists a homoclinic orbit in the (y', y) plane which corresponds to a solitary wave that tends to zero for $|x| \to \infty$. The question put forward by Klaus Kirchgässner was, whether the solitary wave still exists for $0 < \varepsilon << 1$.
We first show that the perturbed model problem (*) indeed is a relevant discription under conditions stated in the title. Next, using results of a companion paper "Singular perturbations of homoclinic orbits in $\mathbb{R}^4$", we show that for the model problem the homoclinic orbit splits by an exponentially small amount. However, we show, that there exist perturbed problems with $\varepsilon^2 P = 0(\varepsilon^m)$, m arbitrarily large, for which the orbit does not split (and hence a solitary wave exists). Finally, in the case of splitting, we construct solutions which over very large distances are approximated by the non-regular solitary wave with an exponentially small error.

P. FIFE:

Cahn Hilliard dynamics

The Cahn-Hilliard Equation, which arises as a model for phase changes in binary alloys, generates a dynamics operating on various successive time scales. This dynamics mirrors the complex process of solidification of an alloy. It also generates many unsolved mathematical problems. Recent progress will be reviewed.

P.M. FITZPATRICK:

A Bifurcation Invariant for Families of Fredholm Operators and Topological Degree

(joint work with J. Pejsachowicz, and with him and P. Rabier)

If X and Y are infinite dimensional real Banach spaces, then $\Phi_0(X, Y)$, the space of linear Fredholm operators of index 0 between X and Y, will be connected, but not simply connected, if $GL(X, Y)$ is contractible. In particular, this is so if X is a Hilbert space, a Hölder space or a Sobolev space. An intersection index, the parity, is assigned to each path in $\Phi_0(X, Y)$ with invertible end-points. Generically, the parity counts, mod2, the intersection of the curve with the set of singular operators.

The parity is used to describe a degree-theory for C^2 nonlinear Fredholm maps from X to Y, using a construction by which the parity transports orientation between regular points of the map $f : X \to Y$.

The most important property of the degree is that there is an effective homotopy formula: no degree can be homotopy invariant in this general setting.

The parity of a closed path is defined and is shown to be a free homotopy invariant which coincides with the Atiyah-Jänich Index Bundle of the path. This is useful for computating the parity. A bifurcation result for the zeros of nonlinear families parametrized by S^1 is also discussed.

L.E. FRAENKEL:

The capacity of slender toroidal sets in $\mathbb{R}^N$

The main result is a formula which, to a specified approximation, expresses the electrostatic capacity of a slender toroidal set in $\mathbb{R}^N, N \geq 3$, in terms of the logarithmic capacity of its cross-section. The cross-section is to be compact and small, but need not be connected or smooth. When the outer boundary of the cross-section is pleasant (in a certain precise sense) the capacity can be calculated, in principle, to arbitrarily order in the small parameter measuring the slenderness of the toroidal set.

J.-M. GHIDAGLIA:

Estimates on the Lyapunov exponents for KdV like p.d.e.' s

We show that the sum of the m first Lyapunov exponents on the acttractor is negative for sufficiently large m by using a time dependent family of norms. We use the hamiltonian structure of the unperturbed problem.

M. GOLUBITSKY:

Planforms in two and Three Dimensions
(joint work with Benoit Dionne)

When solving systems of PDE in two dimensions it is often assumed that the solution is spatially doubly-periodic. The assumption is usually made in systems such as the Boussinesq equation or reaction-diffusion equation where the equation have Euclidean invariance. We use group theoretic techniques to determine a large class of spatially doubly periodic solutions that are forced to exist near a steady-state bifurcation from a translation-invariant equilibrium.

Typically, studies of this question begin by choosing a planar lattice type (square, hexagonal etc.). Our focus is different in that we attempt to find all equilibria that bifurcate on all possible lattices. We do this by looking only for translation free solutions - solutions that have no nontrivial translations as symmetry.

We find the known solutions (rolls, hexagons (higher dimensional ones are due to Kirchgässner) squares). We find higher-dimensional versions of squares.

The corresponding problem has been solved by Dionne on the three-dimensional Bravais lattices.

J.K. HALE:

Transversality in periodic 1-d parabolic systems

For the equation

$$u_t = u_{xx} + f(t, x, u, u_x), \quad 0 < x < 1,$$
$$f(t+1, x, u, v) = f(t, x, u, v),$$

and Dirichlet boundary conditions and certain regularity properties on f, we show the following theorem:

If π_f is the Poincaré map and all fixed points are hyperbolic, then the stable and unstable manifolds of fixed points are **always** transversal.

After showing that the recurrent set of π_f is the fixed point set, this result implies that hyperbolicity of fixed points is equivalent to Morse-Smale. In this latter statement, we assume that f is dissipative so that there exists a global attractor.

T.J. HEALEY:

Preservation of Nodal Structure in Global Bifurcation Problems for Elliptic Equations and Systems with Symmetry
(based upon joint with Hansjörg Kielhöfer)

We consider general classes of quasi-linear, second-order elliptic equations and diagonal systems of such equations in the presence of symmetry. In particular, we present generalizations of the classical results of Crandall and Rabinowitz (for nonlinear Sturm-Liouville eigenvalue problems) to these problems as follows: **the precise nodal set of a the eigenfunction** (of the linearization at the trivial solution) **is preserved along the associated global bifurcating solutions branch.** Our results follow from group-theoretic arguments (relying crucially upon the presence of reflection-inversion symmetries) and a subtle minimum principle. For systems we need stronger hypotheses (regarding the type of coupling) than we require for scalar-valued equations.

S. HEINZE:

Homotopy invariants for reversible periodic orbits
(with B. Fiedler)

For time reversible systems of ODE' s we associate to a symmetric periodic orbit an infinite sequence of invariants given by the degrees of the iterated half period map. We give relations between these degrees involving the Floquet multipliers. Vice versa we can recover the Floquet-multipliers on the unit circle from the sequence of degrees in nonresonant cases. For second order systems we prove the equality of the degree above and the Leray-Schauder degree.

Furthermore similarities to hamiltonian systems are discussed and a reversible Krein signature is defined.

For reversible systems we applied these invariants in order to get local and global bifurcation results.

P.J. HOLMES:

Homoclinic Orbits for Eventually Autonomous Planar Flows
(with C.A. Stuart)

We prove theorems giving conditions sufficient for the existence of homoclinic orbits for two dimensional, time-dependent vector fields which are autonomous for all sufficiently large values of the independent variable. We give applications to second order equations such as those arising in waveguide studies as well as explicit examples which illustrate our assumptions.

G. IOOSS:

Solitary waves for small surface tension
(Extension of a work done with K. Kirchgässner)

We consider the case when the Bond number is $< 1/3$ and the Froude number is close to $1 (> 1)$. All bounded solutions belong to a 4-dimensional center manifold, the linearized part of the vector field having a double zero eigenvalue (Jordan black of size 2) and a simple pair of pure imaginary eigenvalues. We discuss all solutions given by the normal form, and we give a geometric way to understand what solutions persist, under the full vector field (not truncated to the normal form).

We prove that there exist 2 families fo reversible solutions, homoclinic to periodic solutions. The family of periodic solutions (cnoidal waves) forms a 2-dimensional manifold containing the origin (new result). The family of homoclinic solutions tend to a limit when the limiting radius of the periodic solution diminishes, but the open problem is whether or not this limit is reached for a 0 radius (in their case there would exist a solitary wave of elevation).

J. IZE:

Equivariant Degree

If a compact Lie group acts linearly on a Banach space, one may look at equivariant problems and more particularly at bifurcation problems of the form:

$$B(\lambda)X + g(\lambda, X) = 0, \lambda \text{ in } R^k,$$

where both terms commute with the action of the group.

A complete classification of the possible linear terms enables one to give, with the aid of the equivariant degree, necessary and sufficient conditions for the equivariant bifurcation.

C.K.R.T. JONES:

Stability of Viscous Profiles

Viscous profiles are travelling wave solutions of conservation laws with added viscosity. The viscosity "smooths" out the shocks which occur in physical problems as discontinuous transitions between distinct asymptotic states. Their stability was first established by I' lin and Oleinik in 1956 by maximum principle methods. The rate of decay to the wave is significant as it describes the phenomenon of transfer from spatial information into temporal decay. It is shown, in joint work with Kapitoln and Gardner, that for a convex nonlinear term, algebraic spatial decay leads to algebraic temporal decay to the viscous profile with a precise corresponding rate.

D.D. JOSEPH:

Uncompressible miscible liquids and gradient stresses

Simple mixtures of uncompressible miscible liquids are described by an equation of state which in the isothermal case determines the density as a linear expression in the volume fraction of one of the constituents. The density of such a mixture changes with volume fraction so that the density of a simple mixture is not constant when there is diffusion. This means that the usual assumption about the uncompressibility of such mixtures is false. A theory for such mixtures is developed which allows also for stresses due to gradients of the volume fraction. When the gradients are large, at the border of freshly mixed liquids, the stresses give rise to a singular force propertional to $\sqrt{D/t}$, where t is time and D the diffusion coefficient, which can be interpreted as transient interfacial tension.

H.B. KELLER:

Stabilization of Unstable Procedures

Fixed point iteration of the form $u^{n+1} = F(u^n, \lambda)$ with $F : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}^N$ can be used to solve a variety of problems, including for example steady states of initial value problems: $\partial u/_{\partial t} = G(u, \lambda)$. When the procedure converges, $\{u^n(\lambda)\} \to u(\lambda)$, we construct a good approximation to $u(\lambda + \delta\lambda)$ and attempt to continue in this manner. However, as λ varies the solution and procedure may become unstable. It does so as a result of a few eigenvalues of $G_u(u(\lambda), \lambda)$ having their real parts change from negative to positive, or equivalently some eigenvalues of $F_u(u(\lambda)\lambda)$ depart from the unit disk, $K_1(0)$. We show how to use the same iterative procedure to construct a Krylov basis for the unstable subspace and then project out the unstable modes. We use a Lyapunov-Schmidt procedure and Newton' s method on the unstable subspace while retaining the contraction iteration on the stable subspace. Examples are given.

K. KIRCHGÄSSNER:

Dynamics of Nonlinear Fronts

Nonlinear waves in dissipative and conservative systems generate their own dynamics. A gauge transform is presented which brings the original equations into a form which contains the given wave are an isolated steady solution. The spaces to set the dynamics in follow then in a natural way. For the case of the Kolmogorov equation the long time asymptotics in described in some detail.

R.J. KNOPS:

Behaviour of solutions to equations on the half-strip involving the minimal surface operator

An analogue of the Phragmen-Lindelöf principle is established to the equation for the surface of constant mean currature on the plane half-strip. The method depends upon the derivation and integration of a first order linear differential inequality and shows, for example, that the cross-sectional L_2 norm of the solution grows at least to the order κ_1^2 as κ_1 tends to infinity (where κ_1 is the length along the strip), or the solution in weighted energy measure tends to that of the corresponding equation over the cross-section. The decay is at most exponential.

Generalisations of the method are possible for the equations of the minimal surface, the meniscus or capillary, and the thin extensible film, as well as for related equations. Extensions to the half-strip in $\mathbb{R} \times \mathbb{R}^n, n \geq 3$ may also be included in the analysis, provided the cross-section is convex.

The work describes was done jointly with L.E. Payne.

R. LAUTERBACH:

Forced Symmetry Breaking and the Spherical Brusselator
(with Mark Roberts)

In the first part we explain forced symmetry breaking in finite dimensions in the context of spherical symmetry. We assume that we have a S0(3) equivariant vectorfield in $\mathbb{R}^n$. Assuming the existence of a normally hyperbolic manifold M of equilibria we study the flow on a perturbed manifold near M if the equivariance of the equation is destroyed by adding a small term which is equivariant only with respect to a subgroup K of S0(3). Our results consist of a description and a construction part. The description consists in determining an invariant complex on the manifold together with possible flows on this complex. In the construction part we show the existence of flows with various features.

The application to infinite dimensional problems is more difficult. Both parts have no simple extension to infinite dimensions. Therefore we use a bifurcation to get a normally hyperbolic manifold of equilibria. Then we perturb the equation. It is possible to compute approximations of the invariant complex explicitely and therefore to discuss aspects of the flow on the perturbed manifold. In the last part we apply these methods to the equations of the spherical brusselator.

S. MAIER:

Symmetry-breaking at Non-Positive Solutions of Semilinear Elliptic Equations

(joint work with Reiner Lauterbach)

We consider symmetry-breaking bifurcations at non-positive, radially symmetric solutions of semilinear elliptic equations on a ball with Dirichlet boundary conditions. For nonlinearities which are asymptotically affine linear, we find solutions at which the symmetry breaks. The kernel of the linearized equation at these solutions will be an irreducible representation of the group $O(n)$. For this kind of equation a transversality condition is satisfied if the perturbation of the affine linear problem is small enough. Thus we obtain, by the equivariant branching lemma for instance, a great variety of isotropy subgroups of $O(n)$ which occur as symmetries of the bifurcating solutions branches.

J. MALLET-PARET:

An Estimate for the Morse Sets of a Singularly Perturbed Deley Equation

With Roger Nussbaum we study the singularly perturbed differential-delay equation

$$\varepsilon\dot{x}(t) = -x(t) + f(x(t-1)), \qquad (*)$$

where $x = 0$ is an unstable fixed point of the map $f : \mathbb{R} \to \mathbb{R}$, and where $xf(x) < 0$ for $x \neq 0$, and $f : [-B, A] \to [-B, A]$ for some $A, B > 0$.

We establish the estimate

$$\lim\ \inf_{t\to\infty} \|x_t\| \geq C$$

for slowly oscillating solutions of $(*)$, where $\|x_t\| = \sup_{-1\leq\theta\leq 0} |x(t+\theta)|$, and $C > 0$ is **independent** of small ε. In dynamical terms this means

$$dist(S_1, \{0\}) \geq C$$

where S_1, is the Morse set of slowly oscillating solutions.

J.E. MARSDEN:

The dynamics of the double spherical pendulum
(joint with J. Scheurle)

The double spherical pendulum is a mechanical system which has the circle as its symmetry group. All of the relative equilibria of this system and their stabilities, both in terms of the energy-momentum method and spectral stability, are described. There is a Hamiltonian transcritical bifurcation of relative equilibria as dimensionless system parameters, depending on the mass and the length ratios, are varied. In addition, we found a Hamiltonian Krein-Hopf bifurcation as these parameters and the angular momentum are varied. Also, a regularization method is proposed to study the dynamics of the double spherical pendulum close to the straight-down state which has the full symmetry of the circle and is therefore singular.

A. MIELKE:

Reduction of PDE' s on domains with several unbounded directions

In many applications one encounters systems which have, in addition to the time varible, one or more unbounded space directions (cylinders, plates). If a spacially homogeneous solution becomes unstable pattern formation can be observed. Often it can be described by modulations of the amplitude of the critical cross-sectional mode. Usually the associated modulation equation is derived by multiple scaling methods and leads to singular perturbations. We try to give an alternative approach using integrodifferential equations.

As an example we consider the Bénard problem in an infinite strip.

M. PULVIRENTI:

Some Problems on the Vortex Sheet Dynamics

The Vortex Sheet Dynamics is a subject of large interest in the applications. The purpose of my talk is to give a review (short) of the vortex sheet dynamics, its physical and mathematical relevance, the connections with the weak solutions of the Euler equation for incompressible flows.

Finally I present a recent result concerning the convergence of a thin vortex layer to the vortex sheet dynamics, illustrating the relevance of this result in numerical applications.

G. RAUGEL:

Wave equation with local damping
(joint work with J.Hale)

For $\alpha_0 > 0$, we consider the equation

$$(1) \qquad \begin{cases} u_{tt} + \beta(x)u_t - u_{xx} + \alpha_0 u = f(u) + G & \text{in } \Omega = (0,1) \\ \frac{\partial u}{\partial x} = 0 & \text{in } \partial\Omega \end{cases}$$

where $f \in C^2(\mathbb{R};\mathbb{R})$ satisfies

$$(2) \qquad \overline{\lim}_{|s|\to+\infty} \frac{f(s)}{s} < \alpha_0$$

and $G \in L^2(\Omega)$. One further assumes that β is a continuous function satisfying

$$(3) \qquad \beta(x) \geq 0, \qquad \omega = \text{ support } \beta \text{ contains a subinteval } (a,b), a \neq b.$$

We set: $Y^1 = H^1(\Omega) \times L^2(\Omega)$. Let $T_0(t)$ the solution operator on Y^1 associated with (1).

Theorem. Assume that (2) and (3) hold, then $T_0(t)$ is a gradient system, has a global attractor $\mathcal{A}_0$ in Y^1 and the ω-limit set of any point (φ_1, ψ_1) in Y^1 is a single equilibrium point. If only (3) holds, then the ω-limit set of any bounded positive orbit in Y^1 is a single equilibrium point.

Extensions of this theorem to thin two-dimensional domains are also given.

M. RENARDY:

Pattern Selection in the Bénard Problem for Viscoelastic Fluids

We discuss pattern selection in the Bénard problem for a viscoelastic fluid. Double periodicity of the solutions with respect to a hexagonal lattice is assumed. Both steady and oscillatory onset of instability are considered.

J.A. SANDERS:

Computational aspects of versal normal form theory

A description is given of a Maple program to compute the versal normal form of a vectorfield or Hamiltonian system at equilibrium.

This involves Jordan decomposition $(S+N)$ of the linear problem, Jacobson-Marozov exclusion (which M, N such that N, M and H form a $SL(2,\mathbb{R})$) and computation of τ^S and t^H.

An (explicit) algorithm is given to determine the normal form transformations if the given system is in linear versal normal form, and if the versal parameter can be taken small. Using Gröbner normalform the final result can be written in terms of invariants and equivariants of the induced group actions (of $\mathbb{C}^*$ and $SL(2,\mathbb{R})$).

D.H. SATTINGER:

Action-angle variables for completely integrable systems
(joint work with Richard Beals)

Action-angle variables are constructed to completely integrable systems generated by an $n \times n$ first order isospectral operator. When $n = 2$ the flows are linear on the scattering side; but for $n \geq 3$, flows which are nonlinear in the scattering data must be added for complete integrability. For the 3 wave interaction, two flows are linear, and the third is generated by the nonlinear equation $\ddot{\omega} + c\, sin\, \omega = 0$.

K. SCHMITT:

Asymptotics for Solution Continua of Semilinear Elliptic Problems

We present results about the behavior of solution continua $\{(\lambda, u)\}$ of solutions of semilinear equations

$$\Delta u + \lambda f(u) = 0 \quad \text{in } \Omega \quad (\Omega \text{ bounded})$$
$$u = 0 \quad \text{on } \partial\Omega$$

whose u-component is positive and bifurcates to infinity. Results are discussed for superlinear and sublinear f. The most detailed information is available for f of the form $f(u) = u + g(u)$, with g bounded. Here it is shown that for certain nonlinearities the asymptotics depends upon the dimension of Ω as well as its geometry. Explicit formulae for the asymptotics are derived.

J.C. SIMO:

Conserving Algorithms for Nonlinear Elastodynamics

In the absence of external loads or in the presence of symmetries (i.e., translational and rotational invariance) the nonlinear dynamics of continuum systems preserves the total linear and the total angular momentum. Furthermore, under assumption met by all classical models, the internal dissipation in the system is non-negative. The goal of this work is the systematic design of conserving algorithms that preserve exactly the conservation laws of momentum and inherit the property of positive dissipation for *any* step-size. In particular, within the specific context of elastodynamics, a second order accurate algorithm is presented that exhibits excact conservation of both total (linear and angular) momentum and total energy. This scheme is shown to be amenable to a completely straightforward (Galerkin) finite element implementation and ideally suited for long-term/large-scale simulations. The excellent performance of the method relative to conventional time-integrators is conclusively demonstrated in numerical simulations exhibiting large strains coupled with a large overall rigid motion.

I. STEWART:

Hidden symmetries in boundary value problems

Fujii, Mimura, and Nishiura, and Armbruster and Dangelmayr, have observed that reaction-diffusion equations on the interval with Neumann boundary conditions (NBC) can be viewed as restrictions of similar problems with periodic boundary conditions (PBC). This property changes, in a subtle way, the *generic* behaviour of codimension two steady-state mode interactions. There is a straightforward group-theoretic explanation: NBC can be interpreted under Euclidean translations and reflections, posed on multi-dimensional rectangular domains with either Neumann or Dirichlet boundary conditions. This structure imposes group-theoretic restrictions, which in turn imply invariant-theoretic restrictions on Liapunov-Schmidt reduced bifurcation equations. We describe a fundamental result, due to Gomes: a description of the generic bifuration equations for mode interactions in this setting. We indicate extensions to hemispherical domains (joint work with Field and Golubitsky), and discuss applications of the ideas to specific problems such as Lapwood convection in a porous block (joint work with Impey and Roberts).

H. TRUE:

Bifurcations and Chaos in a model of a rolling railway wheelset

We present the results of numerical investigation of the dynamics of a model of a suspended wheelset in the speed range between 0 and 55 km/h. A nonlinear relation between the creepage and the creep forces in the ideal rail-wheel contact point is used. The effect of flange contact is modelled by a very stiff spring with a dead band. The suspension elements have linear characteristics, and the wheel profile is assumed to be conical. All other parameters than the speed are kept constant.

Both symmery and asymmetric oscillations and chaotic motion are found. The results are presented as bifurcation diagrams and time series. We apply bifurcation and continuation routines to support the calculations.

Finally we examine one of the chaotic regions with the help of symbolic dynamics. The numbers of periodic solutions with integer period up to and including 20 are given.

A. VANDERBAUWHEDE:

Global Shilnikov variables and homoclinic bifurcation

One of the tools frequently used when studying homoclinic and heteroclinic bifurcations are the Shilnikov variables, introduced locally near an equilibrium and describing the orbits near such equilibrium. We obtain a general result on global solutions passing nearby a hpyerbolic equilibrium; this result can be interpreted in terms of what we call global Shilnikov variables. We show on two examples how such global Shilnikov variables can be used to study bifurcations near a homoclinic orbit; these examples are (1) period blow-ups in conservative systems, and (2) bifurcation of homoclinics near a given degenerate homoclinic orbit. This second example has codimension 3 and finally leads to a Whitney umbrella describing the parameter values corresponding to homoclinics. (Part of this work was done in collaboration with Bernold Fiedler.)

H.-O. WALTHER:

Two invariant 2-manifolds for a delay equation

Consider

$$x'(t) = -\mu x(t) + f(x(t-1)) \qquad (\mu \geq 0),$$

i.e., the simplest differential equation for a system governed by delayed feedback, and suffering losses.

Assume f is C^1, bounded (from above or from below), and

$$xf(x) < 0 \qquad \text{for } x \neq 0,$$

i.e. we have negative feedback with respect to a single steady state. We present two results on planar dynamics within the global attractor A, in case f is monotone:

1. If the steady state is linearly unstable then there is smooth invariant graph $W \ni 0$; $\dim W = 2$; the boundary $\overline{W} \backslash W$ is a periodic orbit.
   An example is given where $\overline{W} = A$.
2. Unstable sets of hyperbolic periodic orbits in the most stable set of the Morse decomposition of A are annulus-like graphs, bordered by 2 periodic orbits.
   Examples with such hyperbolic periodic orbits are given. The latter is joint work with A.F. Ivanov and B. Lani-Wayda.

B. WERNER:

Secondary Symmetry Breaking Bifurcation

Given a Γ-equivariant steady state bifurcation problem $\underline{g}(\underline{x}, \lambda) = \underline{0}$ we are interested in secondary bifurcation as the result of an interaction of two bifurcation points of different symmetry types by variation of a second control parameter. The symmetry types are given by irreducible respresentations δ and η of the group Γ acting on kernels.

Given a bifurcation subgroup $\Sigma \subset \Gamma$ for δ we formulate interaction conditions which involve decompositions of δ and η into irreducible components considered as representations of Σ. These conditions lead to secondary bifurcations on the Σ-branch.

The proof is based on a numerical concept of test functions for detection and computation of bifurcation points. The results are illustrated by numerical computations for a hexagonal elastic rod systems with 21 degrees of freedom where the dihedral group D_6 is involved.

S.R. WIGGINS:

Transport in Two-Dimensional, Quasiperiodic Vector Fields: Application to Molecular Photodissociation

In this talk I will discuss the notion of phase space transport in two-dimensional, time-quasiperiodic vector fields. The dynamics is generated by a bi-infinite sequence of two-dimensional maps and I will show how from this one gets "time-dependent" phase space structure. In particular I will focus on the role played by the geometrical structure of homoclinic and heteroclinic tangles. The ideas will be illustrated in the context of a single example: the quasiperiodically excited Morse oscillator. This models the dynamics of the "bond stretch" in a diatomic molecule subjected to multi-frequency electromagnetic radiation. I will show how questions such as dissociation rates, probability of dissociation of a state of specified energy, and chaotic bond length oscillations can be naturally formulated as "phase space transport" problems which are mediated by the geometrical structure associated with a homoclinic tangle.

J.A. YORKE:

Prevalence: A translation invariant "Almost Every" on infinite dimensional space
(joint work with Brian Hunt and Tim Sauer)

Let X be a Banach Space or C^∞. We present a measure-theoretic condition for a property to hold "almost everywhere" on X. It is defined in terms of the class of all Borel probability measures with compact support on X. The difference between "generic" behavior and "prevalent" behavior is discussed.

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