MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3/1992

MODELLTHEORIE

19.1. bis 25.1.1992

Die Tagung fand unter der Leitung von D. Lascar (Paris), A. Prestel (Konstanz) und M. Ziegler (Freiburg) statt. Der Schwerpunkt lag auf dem Thema "Geometrische Stabilitätstheorie". Hierzu fand vormittags eine Vortragsreihe statt, in der S. Buechler über seinen Beweis der Vaught'schen Vermutung im unidimensionalen Fall referierte. Weitere Hauptvorträge dienten teilweise der Erg"anzung und der Vermittlung der bei diesem Beweis benutzten Methoden und Ergebnisse, teilweise der Darstellung von wichtigen neueren Ergebnissen in diesem Gebiet. Nachmittags fanden halbstündige Kurzvorträge zu einer Vielzahl von Themen zumeist aus der Stabilitätstheorie statt.

Vortragsauszüge: Hauptvorträge

S. Buechler: *Vaught's Conjecture for Unidimensional Theories*

This is an exposition of the proof that a unidimensional theory has $2^{\aleph_0}$ many or countably many countable models. Let T be unidimensional, not ω-stable, with $< 2^{\aleph_0}$ many countable models. First we prove for T what is called the Tree Theorem. This allows us to view the model as being almost atomic over a tree of elements where each node realises a rank 1 type over its predecessor. Using results about groups in 1-based theories we eventually prove a Structure Theorem for the countable models of the Theory. This Theorem says that a model is prime over an independent set of elements which is the union of finitely many sets of indiscernibles. It follows that there are countably many countable models.

E. Bouscaren: *Defining Groups in Stable Theories*

Definable Groups arise in a natural and unavoidable way in the study of "interactions" in the context of stable theories. The recognition of this and its use to get back information about the theories has been one major new feature of Model Theory in recent years.
In order to give an illustration of this, we explain the construction involved in the following result, a particular case of which is used by S. Buechler in the proof of Vaught's Conjecture for superstable unidimensional theories.
This generalises earlier constructions of Zil'ber for locally modular strongly minimal sets, and of Hrushovski for locally modular regular types.
Theorem [B-Hrushovski]: Let T be a stable one-based theory.
1) Let a_1, a_2, a_3 be pairwise independent (over $\mathrm{acl}(\emptyset)$) and such that $a_i \in \mathrm{acl}(a_j a_k)$. Then there exists e, $e \perp \{a_1 a_2 a_3\}$, and a_1', a_2', a_3' such that for all i, $\mathrm{acl}(ea_i) = \mathrm{acl}(ea_i')$ and $a_i' \in \mathrm{dcl}(a_j a_k)$.
2) Given a_1, a_2, a_3 pairwise independent (over $\mathrm{acl}(\emptyset)$) such that $a_i \in \mathrm{dcl}(a_j a_k)$, let P (resp. Q, R) denote $\mathrm{tp}(a_1/\mathrm{acl}(\emptyset))$ (resp. a_2, a_3). Then there is an $\bigwedge$-definable group G, connected abelian, such that:
— there is a definable regular generic action of G on P, Q and R,
— there is an $\bigwedge$-definable subgroup H of G^3, with generic of the form $(g_1 + g_2, g_1, g_2)$ for g_1, g_2 independent generics of G, and a definable regular generic action of H on $V = \mathrm{tp}(a_1 a_2 a_3/\mathrm{acl}(\emptyset))$.

C. Herrmann: *Finite Geometries*

The purpose of the talk is to give some background from combinatorial geometry resp. geometric lattices and alternative views on Zil'ber's theorem on homogeneous locally finite geometries of infinite dimension. E.g. local modularity can be derived from any localisation of corank at least 3, using results of Doyen-Hubaut and Cameron. Groh's theorem on topological geometric lattices is suggested as a method for establishing modularity in finite corank. Also, the finitary case is discussed based on Kantor's classification of 2-transitive 2-designs. Only a special case of this is needed: homogeneous rank 4 geometries which are the linear closure of a basis (and of order ≥ 3). These are indeed projective or affine and one might avoid Kantor's heavy use of group theory. Such proofs, due to Evans and Zil'ber, exist for rank ≥ 7.

A. Pillay: *Unimodular Theories*

We present the following result due to Hrushovski:
Theorem: Let D be a *unimodular* strongly minimal set. Then D is locally modular.
D is said to be unimodular if whenever $d_1, \ldots, d_n$ are independent in D, $e_1, \ldots, e_n$ are independent in D and $\bar{d}$ and $\bar{e}$ are interalgebraic, then mult($\bar{d}/\bar{e}$) = mult($\bar{e}/\bar{d}$).
The proof proceeds by first associating to any definable set X in D^{eq} a rational number $f(X)$. An intersection formula is developed for "curves" in D^2 with canonical based of dimension ≥ 2. It is deduced that D is "2-pseudo-modular" (any family of curves in D^2 has dimension ≤ 2). The group configuration is used to deduce local modularity.
The proof both generalises and explains Zil'ber's proof in the $\aleph_0$-categorical case.

B. Poizat: *Udi's Amalgamation*

Theorem: Let D_i, $i^2 - 3i + 2 = 0$ be two countable saturated strongly minimal structures in disjoint languages L_1 and L_2 (i.e. $L_1 \cap L_2 = \{=\}$), and in which Morley degree of formulae is definable. Then there exists a strongly minimal structure D in the language $L = L_1 \cup L_2$, whose reduct to L_i is the corresponding D_i.
If X is a set with two pregeometries G_1 and G_2 and $A \subseteq X$ finite, we define $\delta(A) = d_1(A) + d_2(A) - \text{card}(A)$; the pregeometries form a *positive amalgam* if δ is always nonnegative. We then set $d(A) = \min\{\delta(B) : A \subseteq B\}$ and declare A self-sufficient in B if $d(A) = \delta(A)$. For any A there exists a minimal self-sufficient set containing A, called claut(A).
If A is self-sufficient in two extensions B and C, we define step by step (it is not unique!) a free amalgam $B \oplus_A C$ of B and C over A; the idea is to take B and C independent over A in both languages, if we can. The obstruction may be a $b \in B - A$ which is alg$_1$ but trans$_2$ over A, in which case Ab is self-sufficient in B. If there is $b' \in C$ with the same type$_1$ over A, it must be trans$_2$ over A by self-sufficiency and we identify b and b'. Otherwise we make b trans$_2$ over C.
Amalgamation Lemma: If A is self-sufficient in B and C, then any free amal-

gam $B \oplus_A C$ is positive, and A, B, C are self-sufficient in it.
Corollary: There is a unique (up to permutation) countable D such that if A is self-sufficient in D and in some finite B, then B can be embedded in D over A self-sufficiently. The restrictions of D to L_i is D_i.
Now consider the theory T in the language $L = L_1 \cup L_2$, which contains $T_1 \cup T_2$, $\delta \geq 0$ and says for every partial description of some self-sufficient extension B over A, that if A is sufficiently self-sufficient, then B can be sufficiently self-sufficiently embedded.
Theorem: T is consistent and D is a model of T.
Corollary: D is saturated.
Now except in some special cases, D has Morley rank omega, and the amalgam geometry is the geometry of its unique type of rank omega. In D, all types of finite rank are trivial, and D has elimination of imaginaries, provided D_1 and D_2 have.
How now to produce a structure of rank one? We must work in a class of finite structures where the realisations of unlimited minimal types (i.e. $B = A \cup \{b\}$ and b is alg_1 over A and trans_2 (or vice versa)) will be bounded, so that in D these types will have rank zero instead of finite. However, we have a problem of canonical basis. Still, we have no choice for the amalgamation: If b is not already realised, then we must takke the free amalgam. Apparently, when we choose the bound big relatively to the transcendence degree of the canonical bases, this works. My understanding of the situation is weak, and I don't see what is the use of the crucial hypothesis of definability of Morley degree. Is it really necessary when both D_1 and D_2 are $\aleph_0$-categorical?

M. Prest: *Representation Theory*

Baldwin and McKenzie proved that a countable ring with fewer than continuum many countable modules (up to isomorphism) has countably many such modules (indeed, is of finite representation type). This has been used in connection with Vaught's conjecture for varieties and in Buechler's proof of Vaught's conjecture for unidimensional theories. A proof of Baldwin and McKenzie's result is presented along with associated ideas which are used in Buechler's proof.

Kurzvorträge

J. T. Baldwin: *Some Problems on Generic Models*

Hrushovski's constructions of counterexamples to the Zil'ber conjecture and solving Lachlan's problem on $\aleph_0$-categorical stable theories involve the notion of a $(K, \leq)$-generic model. Different examples can be obtained by varying the language, the class K, the notion of strong substructure $(\leq)$ and in several other ways. By changing the notion of $\leq$, the author constructed an $\aleph_1$-categorical non-Desarguesian projective plane. Further varying the class K he constructs a plane Π that is $\aleph_1$-categorical and of Lenz-Barlotti class I. In particular, Π is in the definable closure of any line.
Can one classify the geometries or theories that arise in a fixed language and with a fixed notion of $\leq$?
For M a countable model of a strongly minimal theory, let $L(M)$ be the lattice of algebraically closed submodels of the countably saturated model which contain M. K. Holland showed that if T is modular, for each M, M' we have $L(M) \cong L(M')$, while for T any theory of algebraically closed fields $L(M) \cong L(M')$ implies $M \cong M'$. Does either ot these conditions hold for the strongly minimal sets constructed as $(K, \leq)$-generic models?

A. Baudisch: *A New $\aleph_1$-categorical Group*

By constructing a strongly minimal ($\aleph_1$-categorical) theory with a non-locally modular geometry that does not allow the interpretation of a group Udi Hrushovski gave a negative answer to Zil'ber's conjecture.
Theorem: There is an $\aleph_1$-categorical theory of a pure group with a non-locally modular geometry, which does not allow the interpretation of a field.

O. V. Belegradek: *Some Model Theory of Unitriangular Groups*

We discuss the question of C. R. Videla whether, for an associative ring with unit R, $n \geq 3$ and an infinite cardinal κ, $I(\kappa, R) = I(\kappa, UT(n, R))$.

1) In general, the class of groups of the form $UT(n,R)$ is not elementarily closed. However, if the reduced part of the additive group of R is bounded (in particular if Th(R) is stable or small), any group elementarily equivalent to $UT(n,R)$ is of the form $UT(n,S)$ for some S. (An example shows that in general one cannot choose $S \equiv R$.)
2) If R and S are domains or commutative, $UT(n,R) \cong UT(n,S)$ iff $R \cong S$ or $R \cong S^{op}$. (In general, this fails.)
3) If R is commutative, then $I(\kappa, R) = I(\kappa, UT(n,R))$ for every infinite κ.
4) Let R be a domain. Then $I(\kappa, R) = I(\kappa, UT(n,R))$, for every uncountable κ, and $I(\omega, R) \geq I(\omega, UT(n,R))$. I conjecture that indeed $I(\omega, R) = I(\omega, UT(n,R))$. (We show that R is a counterexample iff R is a skew field with $I(\omega, R) < \omega$, and, for some countable S, $S \equiv S^{op} \equiv R$ and $S \not\cong S^{op}$.)
5) There is R with $I(\omega_1, R) = 3$ and $I(\omega_1, UT(n,R)) = 2$.

L. Bröcker: *Reduction of Semialgebraic Sets and Limits*

We discussed the following result on the non-approximability of semialgebraic sets by algebraic ones:
Theorem: Let $U \subset \mathbf{R}^n$ be semialgebraic and open, $B \subset U$ semialgebraic and closed in U and let $S \subset \mathbf{R}^n \times \mathbf{R}^k$ be a semialgebraic family such that the following conditions hold:
— $\pi(S)$ is bounded,
— S_y is a p-chain for all $y \in \pi(S)$,
— $S_y \cap U$ is essentially algebraic in U for all $y \in \pi(S)$,
— $\dim(B) \leq p$.
Then there exists $\epsilon > 0$ such that
— for all δ with $0 < \delta < \epsilon$,
— for all q-chains $T \subset U \setminus B^q$ where $p + q = n$ and $\partial T \cap U_\delta(B) = \emptyset$,
— for all $y \in \pi(S)$ with $(S_y \cap U) \subset U_\delta(B)$
one has $T \circ_2 S = 0$.
Here "essentially algebraic" means algebraic up to smaller dimension, B^q is the essentially algebraic part of B in U and $T \circ_2 S$ is the intersection number of the chains T and S modulo 2.
The main ingredient of the proof is to translate the problem of approximation to the study of semialgebraic sets over valued real closed fields and their reduction.

L. van den Dries: $(\mathbf{R}, +, \cdot, exp)$ *is Model-complete*

Let T_{exp} be the elementary theory of $(\mathbf{R}, +, \cdot, \exp)$. Wilkie's proof that T_{exp} is model-complete consists of three main steps:
Step 1: Given models k and K of T_{exp} with $k \subseteq K$ and given $L_{exp}(k)$-terms $t_1(x_1, \ldots, x_n), \ldots, t_n(x_1, \ldots, x_n)$, it suffices to show that each *nonsingular* solution in K^n to the system (*) $t_1(x_1, \ldots, x_n) = \ldots = t_n(x_1, \ldots, x_n) = 0$ belongs to k^n.
Step 2: It suffices to show that each nonsingular solution to (*) in K^n is k-bounded.
Step 3: The proof that each nonsingular solution to (*) in K^n is k-bounded.
Step 2 makes essential use of Hovanskii's theorem saying that for $k = K = \mathbf{R}$ the nonsingular solutions to (*) are finite in number (with a uniform bound depending on the "complexity" of the system). (Step 1 follows from general facts on noetherian differential rings of C^{∞}-functions.) For step 3 Wilkie develops a bit of valuation theory for "smooth" o-minimal theories and exponential fields. I gave an outline of the proof of step 3.

D. Evans: *Finite Covers of* $\aleph_0$*-categorical Structures*

I report on some joint work with E. Hrushovski. We are concerned with identifying by how much a finite cover of a countable $\aleph_0$-categorical structure differs from a sequence of free covers. The main result shows that (in the best circumstances) this is measured by automorphism groups which are nilpotent-by-abelian. These results generalise results of Hrushovski obtained for the case where the base of the cover is a grassmannian of a disintegrated set. We show that if the base is a grassmannian of a projective space, then (in ranks > 1) "nilpotent-by-abelian" can be replaced by "nilpotent".

U. Felgner: *Solution of DeBruijn's Problem for Infinite Symmetric Groups*

N. G. DeBruijn posed the problem which homomorphic images of an infinite symmetric group G are embeddable into G. Let $S(\kappa)$ be the group of all permutations of a set Ω of cardinality κ (for simplicity take $\Omega = \kappa$). For

$\pi \in S(\kappa)$ let $\text{supp}(\pi) = \{\alpha \in \kappa : \pi(\alpha) \neq \alpha\}$ be the support of π. For cardinals κ and λ put $S_\lambda(\kappa) = \{\pi \in S(\kappa) : |\text{supp}(\pi)| < \lambda\}$. According to R. Baer, the alternating group $\text{Alt}(\kappa)$ and the groups $S_\lambda(\kappa)$ (for $\aleph_0 \leq \lambda$) are the only proper normal subgroups of $S(\kappa)$. We solve DeBruijn's problem as follows:
Theorem (ZF + V=L): For infinite cardinals $\lambda \leq \kappa$: $S(\kappa)/S_\lambda(\kappa)$ embeds into $S(\kappa)$ if and only if $\lambda < \text{cf}(\kappa)$.
The theorem cannot be proved in ZF + GCH, although the case $\text{cf}(\kappa) = \text{cf}(\lambda)$ can be proved even in ZFC using systems of almost disjoint sets.

U. Gropp: *Action of an ω-stable group on a set of Morley rank 2*

In an ω-stable theory let $\cdot : G \times X \to X$ be a definable, transitive and faithful action, where G is a connected group and X a set of Morley rank 2. Let $(a_i)_{i<\omega}$ be a Morley sequence of the unique generic type q of X. Consider the decreasing sequence $(\gamma_j)_{j<\omega}$, where γ_j is the Morley rank of the orbit $G_{a_1,\ldots,a_j} \cdot a_0$ of a_0 under the stabiliser of $a_1, \ldots, a_j$ in G.
Question: Is there a finite bound for the number of 2's in $(\gamma_j)_{j<\omega}$? (Equivalently, is there a finite bound for the multiplicity of generical transitivity?)
Partial Results:
— There are at most five 2's in $(\gamma_j)_{j<\omega}$ if G is sharply n–times generically transitive on X for some $n < \omega$ (i.e. $\forall \bar{c} \models q^n \ \forall \bar{d} \models q^n \ \exists! h \in G : h \cdot \bar{c} = \bar{d}$).
—There are at most three 2's in $(\gamma_j)_{j<\omega}$ if the action of G on X is ∞-imprimitive (i.e. has a definable and infinite domain of imprimitivity).
— If there are at least two 1's in $(\gamma_j)_{j<\omega}$, then the action of G on X is ∞-imprimitive.
— The question can be reduced to the one of the existence of a finite bound for the Morley rank of G in the case where G is simple.

I. Hodkinson: *The Small Index Property*

Let M be a countably infinite structure and let $G = \text{Aut}(M)$. For a finite tuple $\bar{a} \in M$ we write $G_{\bar{a}}$ for $\{g \in G : \bar{a}g = \bar{a}\}$. M is said to have the small index property if whenever H is a subgroup of G of index $< 2^\omega$, then there is $\bar{a} \in M$ such that $G_{\bar{a}} \leq H$. This approach is important in (e.g.)

reconstructing Th(M) from Aut(M).
I described joint work with W. Hodges, E. Hrushovski, D. Lascar and S. Shelah, showing that if M is ω-categorical and ω-stable, or if M is the random graph, then M has the small index property. The proof uses "generic" automorphisms of M.
Ref.: Hodges, Hodkinson, Lascar, Shelah, "The Small Index Property for ω-stable ω-categorical Structures and for the Random Graph", submitted to London Mathematical Society, 1991.

E. Hrushovski: *S_1-rank*

We introduce a generalisation of Morley rank, an integer-valued "dimension" function on definable sets with the following property: If $E \supseteq \bigcup_i E_i$, $E_i = E(b_i)$, where $\{E(b) : b \in P\}$ is a uniformly defined family of sets with moving parameter b, then $d(E_i) \geq n$, $d(E_i \cap E_j) < n$ $(i < j)$ implies $d(E) > n$. All the classical geometries over a finite field have dimension in this sense (whereas only the linear geometry is stable). This allows generalising Lachlan's theory of finite homogeneous structures with few 4-types. We prove the *independence theorem* for finite dimensional structures: If p_i, p_{ij} are types over an algebraically closed set, $p_{ij} = p_{ij}(x_i x_j) \supseteq p_i(x_i) \cup p_j(x_j)$ and $d(p_{ij}) = d(p_i) + d(p_j)$, then there exists $p_{ijk} \supseteq p_{ij} \cup p_{jk} \cup p_{ki}$, $d(p_{ijk}) = d(p_i) + d(p_j) + d(p_k)$. We discuss the usefulness of this as a replacement for uniqueness of nonforking extensions.

A. Ivanov: *Combinatorial Aspects of the Cover Problem for Totally Categorical Theories*

Let D be a strictly minimal set of projective or disintegrated type with ω-categorical theory. Let M_0 be a principal finite cover of D with $\emptyset$-definable surjection $f : M_0 \setminus D \to D$. Suppose for every $a \in D$ $|f^{-1}(a)| = p$, where p is prime. Covering binary expansions of M_0 (which are obtained by adding only binary relations) are described.

K. Kudaibergenov: *On Homogeneous Models of Unidimensional Theories*

A theory is called unidimensional if all nonalgebraic types are nonorthogonal. It is well-known that it is equivalent to the fact that any sufficiently saturated model is saturated. We prove the same thing for homogeneous models. Let $\lambda(T)$ be the minimal λ such that T is λ-stable.
Theorem: Let T be unidimensional and let λ be such that $\lambda > |T|$ or $\lambda(T) < 2^{\lambda}$. Then any λ-homogeneous model of T of cardinality $> \lambda(T)$ is homogeneous.

A. Lachlan: *Coordinatisations of Finitely Homogeneous, Stable Structures*

Let $\mathcal{M}$ be any structure which admits a finite relational language with respect to which it is homogeneous in the sense of Fraissé. Let G denote Aut($\mathcal{M}$). Let H be the canonical structure coordinatising $\mathcal{M}$ — H is $\emptyset$-definable and consists of the union of a finite number of infinite mutually indiscernible sets definable in $\mathcal{M}^{eq}$ over $\mathrm{acl}^{eq}(\emptyset)$, such that every infinite indiscernible set definable in $\mathcal{M}^{eq}$ over $\mathrm{acl}^{eq}(\emptyset)$ is equivalent to one of them. Let $A \subseteq M$ be finite and crd(A) denote $H \cap \mathrm{acl}^{eq}(A)$ (the set of coordinates of A).
Theorem: Let $K \leq \mathrm{Sym}(\mathrm{crd}(A))$ be the group induced by $G_{A \cup \mathrm{acl}^{eq}(\emptyset)}$. Then
i) K acts independently on its orbits, i.e. K is the product of the groups $K_{\mathrm{crd}(A)-O}$ (O an orbit of K).
ii) On each of its orbits K induces either the symmetric group or the action of $\mathbf{Z}_p$ for some prime p.

C. Laskowski: *Forcing Isomorphisms*

This is joint work with J. Baldwin and S. Shelah. We investigate how robust the notion of non-isomorphism between models of a theory is. Specifically, we are interested in when a first-order theory can have non-isomorphic models that become isomorphic in a generic extension of the universe. If we allow arbitrary forcings there are trivial examples, so we restrict our attention to forcings with the countable chain condition (ccc). Two models are called *potentially isomorphic* if they are non-isomorphic but there is a ccc forcing notion P and a P-generic filter G so that the models are isomorphic in $V[G]$. We show

1) there are potentially isomorphic suborderings of $(\mathbf{R}, <)$,
2) if T is unsuperstable or has DOP or has OTOP, then there are potentially isomorphic elementary substructures of $({}^{\omega}2, E_n)_{n\in\omega}$, where $E_n(b,c)$ iff $b|n = c|n$.

D. MacPherson: *C-minimal Groups and Fields*

In joint work with Haskell and Steinhorn, a variation of o-minimality is considered in which the role of the linear ordering is played by a ternary relation, the "C–relation" of Adeleke and Neumann. It is interpreted in a natural way on the set of maximal chains of a tree. The notion of *C-minimal structure* is introduced. There are versions of the cell-decomposition theorem and the monotonicity theorem of o-minimality. C-minimal fields are discussed (they are valued algebraically closed fields), together with some incomplete results on C-minimal groups.

A. Marcja: *Decidability for Modules over a Group Ring*

We deal with the decidability of theories of modules over $\mathbf{Z}/m\ G$, where m is a positive integer and G is a finite group. First we prove that this problem can be reduced to the case that, for some prime p, m is a power of p and p divides the order of G; then we discuss how to reduce the whole matter to the case that G is a p-group. In particular we prove that the theory of $\mathbf{Z}/p^k\ G$-modules is undecidable, if $k \geq 2$, p^2 divides the order of G and the Sylow-p-subgroup of G is normal in G.

D. Marker: *Integral Parts of Real Closed Exponential Fields*

Ressayre has recently given a novel proof of Wilkie's theorem on the model completenes of the reals with exponentiation. We say that a discrete ordered ring R is an integral part of an exponential field K if R^+ is closed under

$x \mapsto 2^x$ and every element of K is within distance at most one of an element of R. Ressayre proves that if $K_0, K_1 \models \mathrm{Th}(\mathbf{R}, +, \cdot, 2^x|[0,1])$ are real exponential fields which are saturated, then K_0 and K_1 have integral parts R_0 and R_1 such that $(K_0, +, \cdot, 2^x|[0,1], R_0, 2^x|R_0^+) \cong (K_1, +, \cdot, 2^x|[0,1], R_1, 2^x|R_1^+)$. We survey these results.

E. Palyutin: *Quantifier Elimination for B-Theories*

A complete theory T is called a B-theory if for some Freché filter F over an infinite set and a model $\mathcal{A}$ of T, the theory $\mathrm{Th}(\mathcal{A}^F)$ of the reduced power of $\mathcal{A}$ by F is weakly classifiable (i.e. stable with NDOP).
Theorem: If K is a class of structures such that for every $\mathcal{A} \in K$ the theory $\mathrm{Th}(\mathcal{A})$ is a B-theory, then $\mathrm{Th}(K)$ has quantifier elimination modulo positive primitive formulas and one-place formulas.

Y. Petersil: *Zil'ber Type Trichotomy for o-minimal Structures*

The trichotomy in question is between trivial, locally modular and nonlocally modular structures (the first two types are the "simple" cases). We formulate an analogue of that for o-minimal structures, where local modularity is replaced by the CF condition. Roughly spoken, a structure has the CF property if every definable family of functions can be locally written as a one-parameter family. The following theorems hold for o-minimal M:
1) If M has the CF property but is nontrivial, then an ordered abelian divisible group interval is defined in M.
2) (With J. Loveys) If M is a CF group interval, then $\mathrm{Th}(M)$ is a reduct of an interval in an ordered vector space.
3) If M is a reduct of $\langle \mathbf{R}, <, +, \cdot, e^x \rangle$ and not CF, then a real closed field is definable on an interval in M.

A. Pillay: *Locally Modular Regular Groups*

We prove:
Theorem: Let G be a superstable group such that G has locally modular regular generic, is not connected-by-finite, and for any a, b in G^{eq}, $\mathrm{stp}(a/b)$ is orthogonal to the generic of G iff $\mathrm{tp}(a/b)$ has Morley rank. Then $\mathrm{Th}(G)$ has $\aleph_0$ or $2^{\aleph_0}$ countable models.
The result generalises the proof of Vaught's conjecture for weakly minimal theories (or groups).
The theorem has some content due to the following fact: Let $\vartheta(x)$ be a formula of least ∞-rank α which does not have Morley rank in a superstable theory T with $< 2^{\aleph_0}$ countable models. Then there is some regular type p containing ϑ, with $R^{\infty}(p) = \alpha$, and p is domination equivalent to the generic type of a group as in the hypothesis of the above theorem.

F. Point: *Decidability of the Theory of Modules over the Gelfand and Ponomarev Algebra*

Let $B_{n,m} = k\langle x, y\rangle / (xy, yx, x^n, y^m)$, where $n+m \geq 5$, $n, m \geq 2$, be a Gelfand and Ponomarev algebra. The finitely generated indecomposable modules over $B_{n,m}$ have been described by Gelfand and Ponomarev. We describe a class of pure-injective indecomposable modules, i.e. those which contain a maximal element.
Then we prove that any module is elementarily equivalent to a direct sum of pure-injective indecomposable modules containing a maximal element. We get our decidability result by proving that one can distinguish between those particular indecomposables using pp-formulas over $k[x, x^{-1}]$ and very simple pp-formulas corresponding to finite words in $\{x, y, x^{-1}, y^{-1}\}$. We can generalise this decidability result to string algebras.

E. Rabinovich: *Reducts of Algebraically Closed Fields*

Conjecture (Zil'ber): For any non-locally modular structure M definable in an algebraically closed field K, the field is definable in the structure.

We prove two theorems.
Theorem 1: The conjecture is true for M with universe K.
The proof uses algebro-geometric intersection theory on the projective plane.
Theorem 2: The conjecture is true for M whose universe is an arbitrary algebraic curve.
We use a notion of indiscernible array introduced by Hrushovski to prove the definability of the multiplicative (additive in some special cases) group of the field. The definability of the field follows from the first theorem.

S. Starchenko: *Weakly Classifiable Varieties*

A theory T is called weakly classifiable if T is stable without DOP. Hart, Pillay and Starchenko have proved that a variety V is weakly classifiable iff $V = S \otimes A$, where A is an affine variety and S is equivalent to a multi-sorted linear unary variety.

F. Wagner: *Theories Without Dense Forking Chains*

Herwig, Loveys, Pillay, Tanovic and Wagner have proved that in a stable theory without dense forking chains (NDFC) one can define a dimension analogous to Krull dimension for modules, and a generalised Lascar rank. Furthermore, any type is domination equivalent to a finite product of regular ones.
We define a corresponding notion of dimension and rank for formulae in a stable theory, which is ordinal-valued if the theory is NDFC (but not conversely). We define a notion of hereditary p-weight and show, using methods of Hrushovski and Shelah, that for a nontrivial regular type this is definable and continuous inside some definable set. We apply this to deduce, following Buechler and Shelah, that in between any two models of a stable NDFC theory there lies the realisation of some regular type.

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