MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 4/1992

APPLIED AND COMPUTATIONAL CONVEXITY

26.1. bis 1.2. 1992

Die Tagung fand unter der Leitung von P. Gritzmann (Trier), V. Klee (Seattle) und P. Kleinschmidt (Passau) statt. Sie hatte 39 Teilnehmer, von denen 37 Vorträge hielten.

Die Tagung spiegelte die interessanten Entwicklungen im Bereich der Applied and Computational Convexity wider, die dieser jungen mathematischen Disziplin ihr Profil verleihen. Die Wurzeln des Gebietes finden sich im Bereich der Geometrie, der mathematischen Optimierung sowie der Informatik. Die Fragestellungen sind algorithmischer Natur, die studierten Objekte sind geometrischer Art, wobei der Begriff der Konvexität eine besondere Bedeutung hat, und motiviert sind die Fragen oft durch praktische Anwendungen innerhalb der mathematischen Programmierung und der Informatik.

Entsprechend dem Konzept der Tagung kamen die Teilnehmer aus vier verschiedenen Disziplinen, der klassischen Konvexitätstheorie, der mathematischen Optimierung, der algorithmischen Geometrie und der Informatik. Aufgrund der günstigen Zusammensetzung der Tagung fand eine rege Kommunikation zwischen den verschiedenen Gruppen statt, und es kristallisierten sich neue Forschungsansätze um das zentrale Konzept der Tagung heraus.

Die Vorträge behandelten vielfältige Themen aus dem weiten Spektrum der Applied and Computational Convexity. So gab es eine Reihe von Vorträgen zum Bereich der Polyedergeometrie und der Kombinatorik, in der Methoden der Polyedergeometrie benutzt werden, um kombinatorische Optimierungsprobleme zu lösen. Probleme der linearen Optimierung wurden sowohl im Hinblick auf das durchschnittliche Verhalten von Algorithmen, als auch auf die Entwicklung randomisierter Verfahren und Verfahren mit möglichst kleiner Parallelkomplexität vorgestellt.

Geometrische Aspekte nichtlinearer Optimierungsaufgaben wurden ebenso behandelt wie

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Fragen aus dem Bereich der Gitterpunktlehre, zum Teil vom Gesichtspunkt der ganzzahligen Optimierung aus gesehen. Ein weiterer Bereich deckte Phänomene der klassischen Konvexität ab, insbesondere auch gemischte Volumina konvexer Körper, und stellte diese in Bezug zu Anwendungen etwa im Bereich der Computer Algebra oder zu Fragen der mathematischen Programmierung, etwa im Mixture Management.

Weitere Vorträge beschäftigten sich mit Problemen des Geometric Probing, die insbesondere im Zusammenhang mit verschiedenen Fragen der Numerik und der Computertomographie gesehen wurden. Hierbei spielte natürlicherweise auch die algorithmische Theorie konvexer Körper eine große Rolle. Eine Reihe von konkreten Anwendungen wurde besprochen; hierzu zählten insbesondere auch Fragen der Klassifikation von Chromosomen. Ferner wurden algorithmische Ansätze zum Studium von Tilings untersucht, die im Zusammenhang mit der Untersuchung von Quasikristallen an Bedeutung gewonnen haben. Verschiedene offene Probleme gaben Anlaß zu weitreichender Diskussion.

Die Tagung zeigte, daß trotz der Zuordnung der Teilnehmer zu verschiedenen Arbeitsgebieten, die mit unterschiedlichen Methoden und Ideen an Probleme herangehen, eine tiefe enge Verwandtschaft vorhanden ist, die am zentralen Begriff der Konvexität festzumachen ist, und deren weiteres Studium zu einer fruchtbaren Weiterentwicklung des Bereichs der Applied and Computational Convexity führen wird.

Vortragsauszüge

Imre Bárány

On the number of convex lattice polygons

(Joint work with J. Pach and A. Vershik)

Two convex lattice polygons are equivalent if there is a lattice preserving affine transformation mapping one to the other. This is an equivalence relation. Write $N(A)$ for the number of different convex lattice polygons of area A. V.I. Arnold proved in 1980 that $c_1A^{1/3} \leq \log N(A) \leq c_2A^{1/3}\log A$. We improve upon this by showing that $\log N(A) \leq c_3A^{1/3}$. We also show that the convex lattice polygons that lie in the box $C_n = [-n,n] \times [-n,n] \subset \mathbb{R}^2$ have a limit shape as $n \to \infty$.

Louis J. Billera

Fiber polytopes and transportation polytopes

(Joint work partly with B. Sturmfels and A. Sarangarajan)

For $\pi : P \to Q$ a surjective affine map of convex polytopes,we define the *fiber polytope* $\Sigma(P,Q)$ by the Minkowski integral

$$\Sigma(P,Q) := \int_Q \pi^{-1}(x)\, dx.$$

$\Sigma(P,Q)$ is again a convex polytope of dimension $\dim P - \dim Q$, whose face lattice is isomorphic to the lattice of all coherent subdivisions of Q. We consider the case $P = \Delta_n$ (the n-simplex), in which case $\Sigma(P,Q) = \Sigma(Q)$, the *secondary polytope* of Q. In particular we discuss the problem of determining $\Sigma(\Delta_{m-1} \times \Delta_{n-1})$, which corresponds to the polytope map whose fibers are all the $m \times n$ transportation polytopes.

Jürgen Bokowski

Spatial polyhedra without diagonals

(Joint work with Amos Altshuler and Peter Schuchert)

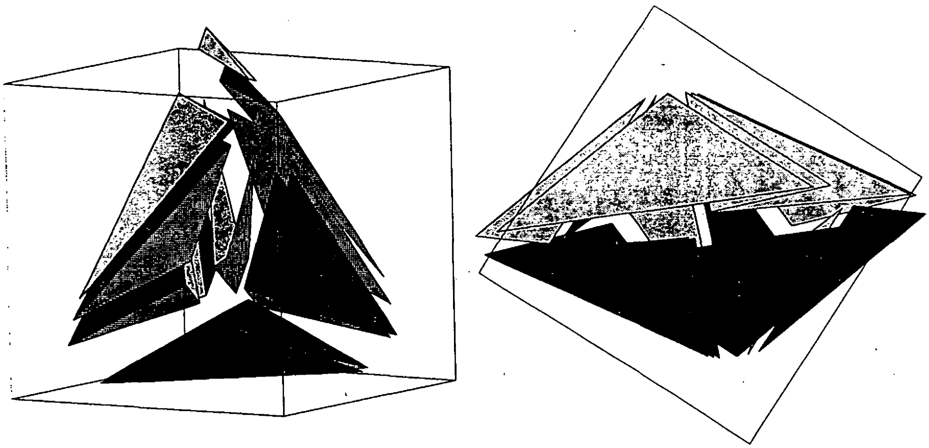
Our investigation was motivated by the following:

- Find a triangulated 2- manifold which can not be embedded in 3-space (longstanding, open)!

- Apply, test and improve algorithms for realizing oriented matroids or finding final polynomials (NP- hard)!

Our results can be summarized as follows:

Theorem: There are altogether five orientable neighborly 2-pseudo-manifolds with 9 vertices. Each of them is geometrically embeddable in IR. The following example is depicted as a exploded view:



Vladimir G. Boltyanski

The Helly dimension of convex bodies

The problem is to find the Helly dimension him$T(M)$ of the family $T(M)$ of all translates of a convex body M in n-dimensional Euclidean space. Two new results will be formulated in the report. The first one contains a description of all n-dimensional convex bodies M (generally, non centrally symmetric), that satisfy the condition him$T(M) = 2$.
The second result contains a counterexample M' to one Kinszes's theorem (that gives a description of all centrally symmetric n-dimensional convex bodies M with him$T(M) = 3$). This counterexample M' is the polar set for the convex polytope conv$H$, where H consists of all the vectors $\pm e_1 \pm e_i, i = 2, .., n$ (all combinations of signs).

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Karl Heinz Borgwardt

Improvements in the average-case analysis of the simplex-method-based on geometrical properties of randomly generated polyhedra

For linear programming problems of the type max $v^T x$ s. t. $Ax \leq \mathbb{1}$ with $A \in \mathbb{R}^{(m,n)}; v, x \in \mathbb{R}^n$ the author had proven that – under the so called rotation-symmetry-model – the average number of pivot steps of a certain variant of the simplex method is not greater than $m^{1/n-1} n^3 \cdot const$, when $m \gg n$.

This result refers to the solution of $n-1$ successive optimization problems of increasing dimension until finally the original problem has been solved. The upper bound mentioned above is based on a run along a complete circle in the dual polyhedron.

Now we show that in each stage first we have to correct the irregularity of the polyhedron (qualitative difference to a ball) and that then only an arc of length $O(1/\sqrt{k})$ (k = number of stage) has to be traversed. Inclusion of this insight into our consideration leads to an improved bound of $m^{1/n-1} n^{5/2}$ for $m \gg n$.

L. Danzer

Strategies for the generation of PENROSE-tilings with defects, which (hopefully) will not lead into dead ends

To explain the growth of quasicrystals one would like to have appropriate local matching rules (LMR). Unfortunately all known LMRs leading – together with a finite set of prototiles – to a species **S** of quasiperiodic tilings are unsatisfactory since they are "non-local" in the following sense:

(NL): For every real r there is a patch $\mathcal{A}$ in some member of **S**, a tile T, and a patch $\mathcal{B} \supset T + r\mathbb{B}^d$, such that $\mathcal{A}$ is extendable to the whole of space, while $\mathcal{A} \dot{\cup} \{T\}$ is not, but nevertheless $\mathcal{A} \cup \mathcal{B}$ satisfies the LMR.

Therefore I suggested to allow additional matchings in order to avoid (NL) (i.e. allow $\mathcal{A} \cup \mathcal{B}$ to be extended) and provide all permitted matchings with a preference order. In the case of the PENROSE-tilings it seems to be sufficient to allow some vertexstars where opposite arrows match. This leads to strictly locally defined strategies for the creation of large patches by adding one vertexstar at every step. So far they did not run into a dead end. The maximal number of tiles in a patch so obtained is now about 430 000. Probably the method can be carried over to $\mathbb{E}^3$.

Klaus Donner

Best L^2-approximation with order convex and cone star-shaped sets in MR-tomographic images

In digital image processing we often wish to localize graphical objects and to estimate their contours from rather noisy image data. A typical example is the reconstruction of cross sections of vessels in magnetic resonance images in order to detect dangerous narrowings of veins or arteries. Modeling assumptions about the type of set used for the approximation are essential for good reconstruction results. In many cases star-shaped fitting systems are appropriate but digital analogues of star-shaped sets are inefficient in algorithms. We therefore introduce the notion of cone star-shaped sets and characterize local L^2- fitting optima in this context. The main problem how to choose significant local set-approximations can be transformed into a problem of efficiently computing upper concave envelopes of real-valued functions. This leads to rather good reconstruction results and can be extended to other set types.

Martin Dyer

Random walks and unimodular linear programs

(Joint work with A. Frieze)

We examine three problems:

1. Generating an (almost) uniformly distributed random basis of a $m \times n$ unimodular matrix A in polynomial time.

2. Solving a linear program $\max\{c \cdot x : Ax = b, x \geq 0\}$ where A is unimodular in strongly polynomial time.

3. Placing a polynomial bound on the combinatorial diameter of the polyhedron underlying the linear program in 2.

In each case the algorithms are based on random walk techniques, which are analyzed using conductance and a geometric isoperimetric inequality. The result of 2. is also not new, but is of interest since it is close to the dual simplex method. The result of 3. is, as far as we knew, new.

Günter Ewald

Projections of polytopes onto k-spaces

Given a polytope P in $\mathbb{R}^n$, $\dim P = n$, we discuss the maximal volume of projections of P onto k-dimensional subspaces, $1 \leq k \leq n-1$ (k fixed). For $k = 1$ and $k = n-1$ the problem of finding a good algorithmic solution has been solved. For $1 < k < n-1$ partial results have been found by Filliman. We introduce a numerical algorithm for finding the maximum provided a sharp shadow boundary is given such that those directions in the Graßmann-manifold to which the shadow-boundary is assigned must contain the maximal direction.

Miroslav Fiedler

An application of simplex geometry to graphs and resistive electrical circuits

In [1], I proved that to a connected resistive electrical circuit C with nodes $1, \ldots, n$ and total resistances R_{ik} between the nodes i and $k, i \neq k$, an $(n-1)$-simplex Σ with vertices $A_1, \ldots, A_n$ in a Euclidean $(n-1)$-space can be assigned for which $R_{ik} = \rho^2(A_i, A_k)$, where ρ denotes the Euclidean distance. Σ does not have any obtuse dihedral interior angle (between $(n-2)$-dimensional faces). Conversely, every $(n-1)$-simplex without obtuse dihedral interior angles can be realized in this way by a resistive electrical circuit with n nodes. Using algebraic and graph-theoretical equivalent models, we shall describe relations between some geometric properties of Σ and properties of C and formulate problems of interpretation of some geometric invariants of Σ in the electrical model.

[1] M. Fiedler, Aggregation in graphs. In: Combinatorics (A. Hajnal, V.T. Sós, Editors), Coll. Math. Soc. J. Bolyai 18, 1976, 315-330.

Richard J. Gardner

Determination of convex polytopes by X - rays

(Joint work with Peter Gritzmann)

Theorem 1: Given an convex polytope P in $\mathbb{E}^d$, (almost any) set of $n = \left[\frac{d}{d-k}\right] + 1$ k-dimensional X-rays parallel to the chosen k-dimensional subspaces will distinguish P from any other convex polytope.

(A k-dimensional X-ray parallel to a k-dimensional subspace S gives the k-dimensional volume of each section of the polytope by a k- dimensional plane parallel to S). Zonotopes can be constructed which show that the number n is best possible; the theorem is false for non-convex polytopes.

Theorem 2: A convex polytope in $\mathbb{E}^3$ can be successively determined by (almost any set of) 2 ordinary (1- dimensional) X-rays.
This means that the first X-ray taken in an arbitrary direction can be consulted to decide in which direction the second X-ray should be taken; the two together determine the polytope.

The methods include a volume formula of Lawrence and a duality formula for the spherical Radon transform.

Peter Gritzmann

Polytope containment and determination by linear probes

(Joint work with Victor Klee and John Westwater)

We discuss the following algorithmic problem which arose from a question in abstract numerical analysis: suppose we are faced with a convex body C in $\mathbb{R}^d$ with $0 \in \text{int}C$ that is accessible only by means of its ray-oracle $\mathcal{O}_C$ – when presented with any ray R issuing from 0, $\mathcal{O}_C$ returns the point at which R intersects C's boundary; and suppose we want to use $\mathcal{O}_C$ to find a polytope that contains P. It is assumed that the usual arithmetic operations in $\mathbb{R}^d$ are available at no cost, so the problem's difficulty is measured solely in terms of the number of calls to the ray-oracle.

A main result shows that, if $d \geq 3$, then even when C is known to be symmetric about the origin and to be rotund, or when C is known to be a polytope that is symmetric about the origin, no finite number of calls to the ray-oracle is sufficient: a polytope containment algorithm based solely on information supplied by the ray-oracle simply does not exist. And this is the case even when much more powerful oracles are available.

Contrasting this result we give various algorithms which solve relaxations and variations of the containment problem. In particular, we show that by means of a finite number of calls to the ray-oracle we can determine whether C's condition number exceeds a given bound and, if this is not the case, construct a containing polytope. Further, we show how a polytope P can be reconstructed by means of a finite number of calls to its ray-oracle; this number, however, depends on the combinatorial complexity of P.

Martin Henk

Approximating the volume of convex bodies

(Joint work with U. Betke)

It is a well-known fact that every deterministic polynomial time algorithm which gives an upper bound $\overline{V}(K)$ and a lower bound $\underline{V}(K)$ for the volume of a convex set in the d-dimensional Euclidean space, the ratio $\overline{V}(K)/\underline{V}(K)$ is at least $(cd \log d)^d$, where c is a constant. Here we describe an algorithm which gives for every $\epsilon > 0$ in polynomial time an upper bound and lower bound with the property $\overline{V}(K)/\underline{V}(K) \leq d!(1+\epsilon)^d$.

Reiner Horst

Global optimization and the geometric complementarity problem

Let D, C be convex sets in $\mathbb{R}^n$. Then the problem of finding a point $x \in D \backslash C$ (or else establishing that $D \subset C$) is called Geometric Complementary Problem (GCP). It is shown that, for large classes of optimization problems which include concave minimization, reverse convex programming, DC optimization and Lipschitz programming, the fundamental problem of transcending stationarity (in the sense of finding a better feasible solution than a given one which is usually a stationary point or local minimum) can be reduced to a geometric complementarity problem.

We also discuss solution methods for solving a (GCP), and show that in certain interesting cases, a dual (GCP) of considerably reduced dimension can be solved by an outer approximation technique.

Alexander Hufnagel

(Joint work with Martin Dyer and Peter Gritzmann)

On the complexity of computing the volume of a zonotope

Given Vectors $x_1, \ldots, x_r \in \mathbb{R}^n$, the *zonotope* generated by these vectors is the set of all points which can written in the form $z = \sum_{i=1}^{r} \lambda_i x_i$ with $0 \leq \lambda_i \leq 1$. Zonotopes are special convex polytopes whose number of vertices and facets may be exponential in the input size. The volume of a zonotope is equal to

$$\sum_{I \subseteq \{1,\ldots,r\}, |I|=n} |\det(x_i),\ i \in I|.$$

It is shown that computing the volume of a zonotope is $\#P$-hard. From this it can be deduced that computing the number of integer points in an integer zonotope is $\#P$- complete. There are also other interesting applications in mixture management problems .

Gil Kalai

The diameter of graphs of convex polyhedra and a randomized simplex algorithm

Let P be a convex d-dimensional polyhedron with n facets and let ϕ be a linear functional. We prove that from every vertex v of P there is a monotone path from v to a top vertex of P (or unbounded ray) of length $\leq \binom{d+\log n}{\log n}$. We describe a randomized simplex algorithm which requires an expected subexponential number of arithmetic operations. A simple version of the algorithm is: given a vertex v, choose a facet containing v at random, apply the algorithm recursively to each top of the facet and repeat.

Victor Klee

Three unsolved problems concerning cubes

The following three problems are discussed, partly for their intrinsic interest and partly for their computational aspects:

(1) If $\mathbb{R}^d$ is tiled by congruent cubes, must some cubes share a common facet? (Affirmative answer by Perron for $d \leq 6$. There are purely graph-theoretical formulations due to Lawrence and to Covádi and Szabó, that turn the problem into one that is (in theory) finitely computable).

(2) What is the mininum number of hyperplanes needed to cut all edges of a d- cube? (Obviously min $\leq d$. Emamy-K. proved min $= d$ for $d \leq 4$. Paterson showed min $= d - 1$ for $d \geq 6$. O'Neil gave a sharp upper bound on the number of edges cut by a single hyperplane.)

(3) What is the minimum number $T(d)$ of simplices for triangulating a d- cube? (Min is known only for $d = 2, 3, 4, 5$ and is resp. $2, 5, 16, 67$. Best asymptotic bound is given by the inequality $\left(\frac{T(d)}{d!}\right)^{1/d} \leq 0.870$ for all $d \geq 8$, obtained by combining general results of Haiman with Sallee's triangulations of the 8-cube.)

Peter Kleinschmidt

Methods of automated chromosome classification

(Joint work with Ilse Mittereiter, Christian Rank)

The automated chromosome classification of human chromosomes (Karyotyping) is a useful tool for prenatal diagnostics. Karyotyping systems use methods of computational geometry for image processing and combinatorial optimization for the classification process.
New approaches for classification and segmentation of chromosomes are presented. In classification, a transportation model based on the Mahalanobis distance yields good accuracy. For segmentation, a non-bipartite matching model based on geometric data seems promising on first attempts.

Jeffrey C. Lagarias

The spectral radius of a set of matrices and matrix norms

(Joint work with Yang Wang, Georgia Tech)

The *spectral radius* of an $n \times n$ matrix A is $\sigma(A) = \max\{|\lambda|, \lambda \text{ eigenvalue of} A\}$.
The *generalized spectral radius* $\bar{\sigma}(\Sigma)$ of a set $\Sigma = \{A_1, \ldots, A_m\}$ of matrices is $\bar{\sigma}(\Sigma) = \limsup_{k\to\infty} \bar{\sigma}_k(\Sigma)^1$, where $\bar{\sigma}_k(\Sigma) = \max\{\sigma(A_{i_1} \cdots A_{i_k}) : A_{i_j} \in \Sigma\}$.

Finiteness conjecture: For any finite set Σ there exists a finite product $A_{i_1} \cdots A_{i_k}$ with $\bar{\alpha}(\Sigma) = \sigma(A_{i_1} \cdots A_{i_k})^{1/k}$. Here k depends on Σ, and examples show that k can be arbitrarily large, even when Σ consists of two 2×2 matrices. This conjecture is shown equivalent to the following conjecture for *all* norms $\|.\|$ on $\mathbb{R}^n$.

Normed finiteness conjecture for $\|.\|$:. Let $\|.\|_{op}$ be the operator norm on $n \times n$ matrices induced from $\|.\|$. Suppose $\Sigma = \{A_1, \ldots, A_m\}$ has all $\|A_j\|_{op} \leq 1$.
Let $T_\Sigma = \{A_{i_1} \cdots A_{i_k} : k \geq 1, \|A_{i_1} \cdots A_{i_k}\|_{op} = 1\}$. Then exactly one of the following holds. (a) T_Σ is finite (b) There is a finite product $A_{i_1} \cdots A_{i_k}$ with $\sigma(A_{i_1} \cdots A_{i_k}) = 1$.
L. Gurits showed that the normed finiteness conjecture is true for polytopal norms (unit ball = polytope). We prove it for norms whose unit ball in $\mathbb{R}^n$ is contained in the zero-set of the holomorphic function on a subset of C^n containing 0 (piecewise analytic norms). This includes polytopal norms and the Euclidean norm. We also derive a bound for k in the normed finiteness conjecture depending on n and $m = |\Sigma|$ only for the Euclidean norm.

D. G. Larman

A Ramsey theorem for convex sets in the plane

Suppose we have a set of n convex bodies $\mathcal{S}$ in the plane. We wish to pick a large subset S of $\mathcal{S}$ with the following property: Either two members of S are disjoint or every two members of S intersect.
Using the usual Ramsey theorem we can always find a set S with more than $\log n$ Elements. Here, using ideas of Tõrõscik, modified by myself and Janos Pach, we prove that we can always find a set S with more than $n^{1/9}$ elements. Simple examples show that an upper bound is $n^{1/2}$. Since every graph can be represented as the intersection graph of convex sets in $\mathbb{R}^4$, extensions to higher dimensions are somewhat limited.

Jim Lawrence

Transversals and the Euler characteristic

A *convex tranversal* of a collection of sets is a convex set which has nonempty intersection with each member of the collection. When do two finite collections of closed convex polyhedra have precisely the same convex transversals? We describe an arithmetical condition which determines this.

Carl Lee

Generalized stress and rigidity

The notions of stress and rigidity of a bar and joint framework are extended to higher dimensional faces of simplicial complexes. This provides a geometrical interpretation of the face ring of a complex. From this one obtains a new inductive proof that p.l.-spheres are Cohen-Macaulay, and can see how the hard Lefschetz theorem for toric varieties associated with simplicial convex polytopes is related to volume polynomials, Minkowski's theorem, and the Brunn-Minkowski theorem.

Horst Martini

The generalized Fermat-Torricelli problem

Let $P_n = \{p_1, \ldots, p_n\}$ be a finite point set in $\mathbb{R}^d$ $(d \geq 2)$ and $\{w_1, \ldots, w_n\}$ the set of corresponding positive weights. If $\mathbb{F}^k$ is the set of all $k-$dimensional flats in $\mathbb{R}^d k \in \{0, \ldots, d-1\}$, we consider the problem

$$\min_{F \in \mathbb{F}^k} \sum_i w_i \text{ Dist } (p_i, F),$$

with Dist $(\cdot, \cdot)$ denoting the Euclidean distance function. The following results are presented:

$k = d - 1$:	(MINSUM HYPERPLANE PROBLEM) solvable in $0(n^d)$ time; joint work with N.M. Korneenko),
$k \in \{1, \ldots, d-2\}$:	at least as difficult as the classical Fermat/Torricelli problem in $\mathbb{R}^{d-k}$ (joint work with P. Gritzmann),
$k = 0$	(classical FERMAT-TORRICELLI-PROBLEM) Already for $n = 5, d = 2$, the solution cannot be expressed in radicals over the field of rationals in terms of the input points (Bajaj 1988).

Nimrod Megiddo

Parallel complexity of linear programming

For any fixed d the LP-problem in d variables and n inequalities can be solved on n processors in $O((\log\log n)^d)$ time. Probabilistically the problem can be solved almost surely in constant time.

Günter Meisinger

On the face and flag numbers of convex polytopes

I developed a computer program, that proves theorems about face and flag numbers of general (not necessarily simplicial) polytopes and computes all known linear relations between face and flag numbers. These relations are the generalized Dehn-Sommerville equations and the g-numbers and their convolutions.
Using the program I was able to prove among others the following result: Every rational d-polytope ($d \geq 9$) has "small" 3-faces (i. e. one with less than 150 vertices).
The idea of the program is to take all known linear relations and the relation corresponding to the negation of the theorem's claim as input of a linear programming problem. An infeasibility of the LP implies the correctness of the theorem.

Shmuel Onn

Permutation polytopes

Each group G of permutation matrices gives rise to a *permutation polytope* $P(G) = conv(G) \subset \mathbb{R}^{d\times d}$, and, for any $x \in \mathbb{R}^d$, an *orbit polytope* $P(G,x) = \text{conv}(G\cdot x)$. A special subclass is formed by the *Young permutation polytopes*, which correspond bijectively to partitions $\lambda = (\lambda_1,\ldots,\lambda_k) \vdash n, n \in \mathbb{N}$.

Young polytopes, such as the *traveling salesman polytope*, arise naturally in polyhedral combinatorics, and many algorithmic combinatorial problems, such as deciding hypergraph isomorphism, reduce to optimizing linear functionals over such polytopes.

Second, the *assignment polytope* $P((n-1,1))$ is studied. Large stable sets in its 1-skeleton are exhibited, and it is shown that its stability number $\alpha(n)$ is $2^{\Omega(\sqrt{n\ \log n})}$.

Next, letting l be the largest integer for which $P(\lambda)$ is l-neighborly, under some restrictions on λ it is shown that $[\frac{k^2}{2}] \leq l < \frac{1}{2}(k+1)!$.

Finally, it is shown that, unlike the orbit polytope $P(S_n, x)$ which, for every generic point x is combinatorially the same (the *permutohedron*), in general there may exist generic points having nonisomorphic orbit polytopes $P(G,x) \not\cong P(G,y)$. This settles a question raised by D. Kozen.

Panos Pardalos

Minimization of separable convex functions subject to equality and box constraints

(Joint work with N. Kovoov)

We consider the problem of minimizing a separable differentiable strictly convex function on $\mathbb{R}^n$ subject to m equality constraints and upper and lower bounds (box constraints). We provide parametric characterization in $\mathbb{R}^m$ of the family of solutions to this problem, thereby showing equivalence with a problem of search in an arrangement of hyperplanes in $\mathbb{R}^n$. We use this characterization to develop an exact algorithm for the problem. For the special case of the least distance problem we obtain a strongly polynomial algorithm running in time $\Theta(n^m)$ for each fixed dimension m.

Richard Pollack

Arrangements, spreads and topological projective planes

(Joint work with J. E. Goodman, R. Wenger, and T. Zamfirescu)

We prove the following theorems which resolve (affirmatively) several conjectures stated by B. Grünbaum in "Arrangements and Spreads":

1. Every arrangement of pseudolines can be extended to a topological projective plane.
2. There exists a universal topological projective plane T; i.e. given any arrangement of pseudolines $\mathcal{A}$ there is an arrangement $\mathcal{A}'$ of *lines* of T which is isomorphic to $\mathcal{A}$.
3. There are uncountably many non-isomorphic universal topological projective planes.

A corollary of the first result is the theorem that any arrangement can be extended to a spread, which we had proved by a more complicated argument a year ago.

A corollary of the third theorem is that there exist non-isomorphic topological planes T and T' such that every arrangement in T has an isomorphic copy in T' and vice versa. This resolves another conjecture of Grünbaum.

Bill Pulleyblank

On splittable sets

(Joint work with F. B. Shephard and B. A. Reed)

Let S be a finite set of points in the plane. A set $X \subseteq S$ is splittable if there exists a line l in the plane such that all members of X lie on one side of l and all members of $S \setminus X$ lie on the other side. Our goal is to obtain a linear system $Ax \leq b$ such that the extreme solutions are precisely the incidence vectors of the splittable sets.
In the case that the members of S form the vertices of a convex polygon, we show that the minimal such system is the following:

$$0 \leq x_v \leq 1 \text{ for all } v \in S$$

$$\sum_{v \in R^+} x_v - \sum_{v \in R^-} x_v \leq 1 \text{ for all suitable } R^+, R^- \subseteq S$$

Here R^+ and R^- are disjoint equicardinal subsets of S, such that each pair of members of R^+ is separated on the polygon by a member of R^-, and conversely.
This we show by projecting network flow polyhedra. We also describe some additional essential inequalities for the general case.

Alexander Schrijver

The stable set and odd path polytopes

(Joint work with P. D. Seymour)

We consider the stable set and odd path polytopes. Let $G = (V, E)$ be an undirected graph. The stable set polytope $S(G)$ of G is the convex hull of the stable set incidence vectors. As the stable set problem is NP-complete, the facets of $S(G)$ are NP-hard to recognize. We describe a new approximation for $S(G)$ that is polynomially optimizable and that gives $S(G)$ exactly for a class of graphs including all perfect graphs (joint work with L. Lovász). We also show that the facets of the odd path polytope (= convex hull of incidence vectors of odd $s-t$ paths, for fixed s, t) all have $0, 1/2, 1$ coefficients and are of a special type.

Ron Shamir

Unimodal separable minimization subject to partial order constraints

(Joint work with Endre Boros, Rutgers.)

We describe a combinatorial, network flow based algorithm for the minimization of a separable function subject to partial order constraints, under certain unimodality assumptions on the objective function. Special cases of the problem include convex separable objective and isotonic regression. The results generalize and simplify previous results of Maxwell and Muckstadt and of Picard and Queyranne.

György Sonnevend

Analytic centers for semiinfinite sets of convex inequalities

Let $P = \{x \mid f(\alpha, x) \geq 0, \alpha \in A\}$ be the description of a convex set – intersection of "elementary" ones –, where $f(\alpha, .)$ is concave (e.g. linear or quadratic) and $f(.,.)$ is analytic (algebraically simple) in its variables, $A = [0,1]^m$. We study the problem of picking up one "central" element of P, $x(f, A)$ so that it be an analytic function of the "data" (f, A) and shares invariance properties under the class of affine transformations and scaling. As a generalization of the "analytic centre" for polyhedra (i.e. where $\text{card}(A) = m$) we propose affine invariant central points, which provide also optimal order ellipsoidal approximations for the case $A = [0,1]^m$, $m = 1$ and $m = n - 1$ and are easily computable (updatable): they are generalization of the classical maximum entropy solutions of Nevanlinna-Pick type moment problems.

Josef Stoer

On the complexity of continuation methods following an infeasible path

To get around the existence and knowledge of strictly feasible solutions of a dual pair $(P_0), (D_0)$ of linear programs, it is common to embed them into a family of perturbed linear programs

$$(P_r) \quad \begin{array}{l} \min(c + r\tilde{c})^T x \\ x : A^T x \leq b + r\tilde{b} \end{array} \qquad\qquad (D_r) \quad \begin{array}{l} \min(b + r\tilde{b})^T y \\ y : Ay + c + r\tilde{c} = 0, y \geq 0, \end{array}$$

where A is a $n \times m$-matrix of rank n. This gives rise to a central path $x = x(r), y = y(r), s = s(r) = b + r\tilde{b} - A^T x(r), r \downarrow 0$, of solutions x, y, s to the perturbed system

$$A^T x + s = b + r\tilde{b}, \quad Ay + c + r\tilde{c} = 0$$

$$y_i s_i = r, \quad i = 1, \ldots, m.$$

Path following methods compute approximations to the central path at parameters $R = r_0 > r_1 > \ldots$, and their complexity is measured by the number $N = N(r, R)$ of steps needed to reduce R to an $r > 0$, $R = r_0 > r_1 > \ldots > r_{N-1} \geq r_N$. It is shown for a standard method that, as in the unperturbed case, $N(r,R)$ is bounded above by a curvature integral

$$N(r,R) \quad \le \quad c_1 \int_r^R \left[\sum_{i=1}^{m} \left(\frac{\tilde{s}_i(r)}{s_i(r)} \right)^2 + \sum_{i=1}^{m} \left(\frac{\tilde{y}_i(r)}{y_i(r)} \right)^2 \right]^{1/2} dr \; + \; c_2 \log \frac{R}{r} \,,$$

leading for small perturbations $\tilde{c}, \tilde{b}$ to $N(r,R) = O(\sqrt{m}\ \log \frac{R}{r})$.

Bernd Sturmfels

Product formulas for sparse resultants

(joint work with Paul Pedersen).

This work concerns an application of convexity to computational algebra. The sparse resultant of a system of polynomial equations

$$f_i(x) = \sum_{j=0}^{n_i} c_{ij} x^{a_{ij}} = 0, \qquad i = 0, .., k \qquad (*)$$

in k variables $x = (x_1, \ldots, x_k)$ is an irreducible polynomial $R(c_{00} \ldots, c_{kn_k})$ which vanishes whenever $(*)$ has a solution in $(\mathbb{C}^*)^k$. The degree of R in $c_{i0}, c_{i1}, \ldots, c_{in_i}$ equals Minkowski's mixed volume $V(P_0, \ldots, \hat{P}_i, \ldots, P_k)$, where $P_l = conv\{a_{l0}, a_{l1}, \ldots, a_{ln_l}\} \subseteq \mathbb{R}^k$ are the Newton polytopes of $(*)$. Our main result is a formula of Poisson type,

$$R(c_{00}, \ldots, c_{kn_k}) = \prod_{\gamma \in \Omega} f_0(\gamma) \cdot R'(c_{i0}, \cdots, c_{kn_k})$$

where Ω is the zero set of $f_1, .., f_k$, and R' is a certain rational function in the coefficients of $f_1, .., f_k$. The irreducible factors of R' are indexed by the facets of the Minkowski sum $P_1 + .. + P_k$.

Emo Welzl

A randomized LP-algorithm with a subexponential number of arithmetic operations

(joint work with Jirka Matoušek and Micha Sharir; see also talk by Gil Kalai).

We present a randomized algorithm which solves linear programs with n constraints and d variables in expected $O\left(nde^{4 \cdot \sqrt{d \ln(n+1)}}\right)$ time in the unit cost model (when we count the number of arithmetic operations on the numbers in the input). The expectation is over the internal randomization performed by the algorithm, and holds for any input. The algorithm can be presented in an abstract frame work, which faciliates its applications to several other related optimization problems (e.g. smallest enclosing ball of n points in $\mathbb{R}^d$, smallest volume ellipsoid containing n points in $\mathbb{R}^d$, largest ball (ellipsoid) in a convex polytope in $\mathbb{R}^d$ with n facets).

J.M. Wills

A lattice point problem

Let $K \subset E^d$ be a convex body with (normalized) Minkowski's quermassintegrals $V_i, i = 0,1,\ldots,d$ ($V_d = V$ volume, $V_{d-1} = \frac{1}{2}F$ surface area, $V_0 = 1$). Further let $L \subset E^d$ be a lattice and $D_i(L) = \min\{|det L_i| : L_i$ i-dim. sublattice of $L\}, i = 0,1,\ldots,d$. In particular $D_0(L) = 1,\ D_d(L) = D(L) = \det L$. Then for centrally symmetric K

$$\lambda_{i+1}(K,L)\cdots\lambda_d(K,L)V(K)/D(L) \leq i!\ 2^{d-i}V_i(K)/D_i(L); \quad i = 0,1,\ldots,d-1 \qquad (1)$$

and for general convex K

$$\frac{V(K)}{D(L)} - d^{3/2}\frac{V_{d-1}(K)}{D_{d-1}(L)} \leq G(K,L) \leq \sum_{i=0}^{d} i!\frac{V_i(K)}{D_i(L)}. \qquad (2)$$

(1) is tight for $i = 0$ and $i = 1$, and for $i = 0$ it is Minkowski's main theorem on successive minima. For $i \geq 2$, (1) is far from being best possible, as well as the right-hand side of (2) is. We discuss improvements for special cases by Henk, Schnell and the author.

Günter M. Ziegler

Subspace arrangements and their homotopy types

(Joint work with Rade T. Živaljević)

We prove combinatorial formulas for the homotopy type of the union of the subspaces in an (affine, compactified affine, spherical or projective) subspace arrangement. For example, let $\mathcal{A}$ be a set of affine subspaces in $\mathbb{R}^n$, let P be the poset of all non-empty intersections of subspaces in $\mathcal{A}$, ordered by reversed inclusion, and let $d : P \longrightarrow \mathbb{N}_0$ be the dimension function. Then the one-point compactification of the union $\bigcup\mathcal{A}$ is homotopy equivalent to a wedge of suspended order complexes $\bigvee_{p\in P} \Sigma^{d(p)+1}\Delta(P_{<p})$. From this one immediately gets the formula of Goresky & MacPherson for the cohomology of the complement of $\mathcal{A}$.

Our method consists in interpreting the union of an arrangement as the direct limit of a diagram of spaces over the intersection poset, which is homotopy equivalent to the homotopy direct limit. We construct a combinatorial model diagram over the same poset, whose homotopy limit can be compared to the original one by usual homotopy comparison results for diagrams of spaces.

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