MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 7/1992

Funktionentheorie
16.2. bis 22.2.1992

Die Tagung stand unter der Leitung von G. FRANK (TU Berlin), ST. RUSCHEWEYH (Würzburg) und K. STREBEL (Zürich). Von den insgesamt 46 Teilnehmern wurden 31 Vorträge gehalten, von jeweils 30–45 Minuten Dauer.

Das Schwergewicht der behandelten Themen der diesjährigen Tagung lag bei der Geometrischen Funktionentheorie und der Theorie der Quasikonformen Abbildung. Es wurden aber auch Beiträge aus anderen Bereichen der Funktionentheorie geliefert.

Es ist ein besonderes Verdienst der Oberwolfacher Funktionentheorie-Tagung, durch eine nicht zu ausgeprägte Spezialisierung der Themen, Wissenschaftler verschiedener Richtungen innerhalb des weiten Feldes der Funktionentheorie zusammenzubringen und so neue und fruchtbare Kooperationen zu ermöglichen. Die Teilnehmer waren sich einig in dem Wunsch, daß dieses Konzept auch in Zukunft erhalten bleiben möge.

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Vortragsauszüge

L.V. Ahlfors:

Quasicomplex Algebra

For a given integer $n \geq 1$ we consider the product space $\mathbf{R} \times \mathbf{R}^n$ with elements (x, y), $x \in \mathbf{R}$, $y \in \mathbf{R}^n$. We study an algebra QC^n on $\mathbf{R} \times \mathbf{R}^n$ which for $n = 1$ is the algebra of complex numbers. In this case the algebra is isomorphic to the algebra of the matrices $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$.

For $n > 1$ we imitate the complex notation $z = x + iy$ and write $z = \begin{pmatrix} x \\ y \end{pmatrix}$ as a vertical vector with the horizontal transpose $z^T = (x,\ y^T)$. I have taken the liberty of calling such "numbers" quasicomplex. Another common choice will be $n - \begin{pmatrix} u \\ v \end{pmatrix}$.

The counter part of $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ will be $\begin{pmatrix} x & -y^T \\ y & xI_n \end{pmatrix}$ considered as a <u>block matrix</u> in the obvious manner given that I_n is the unit matrix in n dimensions. The mapping from z to the corresponding block will be denoted by L. By linearity

$$L(z) = xI_{n+1} + \begin{pmatrix} 0 & -y^T \\ y & 0 \end{pmatrix}.$$

At least in a formal way the multiplication takes the form

$$L(z)L(w) = \begin{pmatrix} xu - y^T v & -(xv + uy)^T \\ xv + uy & (xu - yv^T)I_n \end{pmatrix}.$$

Here $y^T v$ is to be interpreted as the matrix product of a horizontal and a vertical vector in that order. In the opposite order yv^T stands for a square matrix $||y_i v_j||$.

It is not true that $L(z)L(w) = L(w)L(z)$, but

$$L(z)L(w) - L(w)L(z) = \begin{pmatrix} 0 & 0 \\ 0 & vy^T - yv^T \end{pmatrix} = \Delta.$$

The mapping from z to $L(z)$ has an inverse, namely

$$L(z) \mapsto L(z) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

One sees that

$$L(z)L(w) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = L(w)L(z) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{or } L(z)L(w) \equiv L(w)L(z) \bmod \Delta.$$

In view of this property one defines the multiplication in QC^n by

$$z \times w = L(w)L(z) \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

This product is commutative but non-associative as soon as $n > 1$.

The additive structure of QC^n is obvious from the linearity of L. The use of matrix multiplication is a welcome safeguard against unwarranted use of the associative law.

A. Baernstein:

The area problem in quasiconformal mapping

Let f be a K - qc map of the unit disk $\mathbf{D}$ onto $\mathbf{D}$ with $f(0) = 0$.
Area conjecture (AC): $\forall E \subset \mathbf{D}$, we have $|f(E)| \leq C(K)|E|^{1/K}$, when $|E|$ denotes the area of E.

It is known that AC holds when $1/K$ is replaced by K^{-a}, where $a = 7.29$ (T.Iwaniec - G.Martin, 1991). The radial stretch map $f_K(z) = z|z|^{1/K-1}$ shows that $1/K$ can be replaced by no smaller exponent.

AC is equivalent to many other conjectures. I state in my lecture a theorem which asserts its equivalence with 5 other ones. These involve the maximal p for which f belongs to the Sobolev space $W^{1,P}$, integrals of rational functions of the form $\phi(z) = \frac{1}{n}\sum_{j=1}^{n}(z - z_j)^{-2}$, sharp bounds for the 'Ahlfors-Beurling' singular integral operator and 'stretch inequalities' for solutions of Teichmüller-Beltrami equations $\overline{\partial}f = -k\frac{\overline{\phi}}{|\phi|}\partial f$.

R.W. Barnard:

Ruscheweyh's conjecture with application to Krzyż's conjecture on bounded non-vanishing functions

Let $\mathbf{D} = \{z : |z| < 1\}$,

$$\begin{aligned} \mathcal{P} &= \{p : p \text{ analytic and } \operatorname{Re} p(z) > 0 \text{ in } \mathbf{D}\}, p(0) = 1\}, \\ \mathbf{B} &= \{f : f \text{ analytic and } 0 < |f(z)| < 1, \ z \in \mathbf{D}\}. \end{aligned}$$

The author and K. Pearce have verified S. Ruscheweyh's conjecture that

$$\frac{1}{2\pi}\int_0^{2\pi} \lambda \operatorname{Re} p(re^{i\theta})e^{-\lambda \operatorname{Re} p(re^{i\theta})}d\theta$$

maximizes overall p in $\mathcal{P}$ by $p(z) = (1 + z^n)/(1 - z^n)$. This, as Ruscheweyh has shown, proves for $A_n = \sup_{f\in B} |a_n|$ that $A_n < 0.81960\ldots$ for all $n \geq 1$, towards the Krzyż conjecture that $A_n = 2/e$.

M. Bonk:

An extremal property of the Ahlfors-Grunsky function

A function f is called a Bloch function, if it is analytic in the unit disk $\mathbf{D}$ and if

$$||f||_{\mathcal{B}} := |f(0)| + \sup_{z\in\mathbf{D}}(1 - |z|^2)|f'(z)| < \infty.$$

We denote the space of all Bloch functions by $\mathcal{B}$. Then $||\cdot||_{\mathcal{B}}$ is a norm on $\mathcal{B}$.

It is possible to show: the Ahlfors-Grunsky function (cf. Math. Z. **42** (1937), 671-673) is an extreme point of the unit ball $\{f \in \mathcal{B} : ||f||_{\mathcal{B}} \leq 1\}$ of $\mathcal{B}$.

M. Brandt:

Typisch reelle Polynome

Für die Klasse T_n der in der Einheitskreisscheibe D typisch reellen Polynome mit der Normierung

$$f(z) = z + a_2 z^2 + \cdots + a_n z^n \qquad (1)$$

gelten Parameterdarstellungen, die für $n \to \infty$ in die bekannte Integraldarstellung typisch reeller Funktionen übergehen. Aus diesen Parameterdarstellungen ergeben sich die Lösungen zahlreicher Extremalprobleme bez. T_n.
Da T_n die Klasse Q_n der in D schlichten durch (1) normierten Polynome mit reellen Koeffizienten enthält, folgen auch Abschätzungen für Q_n; z.B. gilt bei fixiertem $k = 2$

$$\max_{f \in Q_n} |a_k| = k - \frac{2\pi^2}{3} \frac{k^3 - k}{n^2} + O(\frac{1}{n^3}).$$

J. Clunie:

The Grace-Heawood theorem

Theorem *Let $f(z)$ be entire (non-constant) of order 1 and minimal type with $f(-1) = f(1)$. Then each of the closed half-planes $\{Re\ z \geq 0\}$ and $\{Rez \leq 0\}$ contains zeros of $f'(z)$. This result is best possible.*

The result for polynomials $f(z)$ is due to Grace and is one form of the Grace-Heawood theorem.

A. Córdova:

A Convergence Theorem for Continued Fractions

Lorentzen und Ruscheweyh stated the following conjecture: Let V be a domain $\subsetneq \mathbb{C}$, and $E := \{c \in \mathbb{C} : \phi_c(V) \subsetneq V\}$, where $\phi_c(z) := c/(1+z)$. Then for every sequence $\{c_n\}$ in a compact subset of E the sequence of approximants $S_n(z)$ defined by $S_1 := \phi_{c_1}$ and $S_n := S_{n-1} \circ \phi_{c_n}$ converges to a constant, for every $z \in V$.
We present some partial results which support this conjecture, and prove it in a special, but interesting case: a case where ∂V consists of fixed points of the parabolic transformations which belong to the semigroup generated by ϕ_c, $c \in E$.

P.L. Duren:

Bergman spaces, harmonic mappings, extremal problems for univalent functions

The Bergman space A^p $(0 < p < \infty)$ consists of all functions f analytic in the unit disk $\mathbb{D}$ with

$$\|f\|_p = \left\{ \int\!\!\int_{\mathbb{D}} |f(z)|^p \, d\sigma \right\}^{1/p} < \infty,$$

where $d\sigma = \frac{1}{\pi} dxdy$ is the normalized area measure. For $1 \leq p < \infty$, a canonical (and unique) contractive zero-divisor is constructed in A^p. This is a function G of norm $\|G\|_p = 1$, vanishing precisely on any given A^p zero-set, such that $\|f/G\|_p \leq \|f\|_p$ for all $f \in A^p$ having (at least) the prescribed zeros. G is constructed by an extremal problem whose analogue for H^p spaces leads to Blaschke products. For a finite zero-set it is shown that G has an analytic continuation to a larger disk. The theory for $p = 2$ was discovered by H. Hedenmalm (1990), but the extension to other A^p spaces requires new techniques.

[Joint work with D. Khavinson, H.S. Shapiro, and C. Sundberg]

J. Fernández:

The Hayman-Wu Theorem

To understand the geometry of its statement one would like to know whether $(\star) \sum' \text{diam}\,(f(L_i)) < +\infty$ if Ω is simply connected, f conformal from Ω into $\mathbb{D}$ and L a K-quasiline, where L_i are the components of $\Omega \cap L$. Väisälä has shown that the sum is finite if the diameters are raised to exponent $1 + \epsilon$, for any $\epsilon > 0$. Recently, K. Astala, S. Rohde and myself have shown that if L is a quasiline and if the harmonic measures of the two complementary components are mutually singular then there exists always Ω and f for which the sum is infinity; while if these two harmonic measures are absolutely continuous w.r. to each other in a uniform way (i.e. the derivative of the conformal welding belongs to A_∞) then $(\star)$ holds.

In the talk I will also discuss a recent proof by Øyma of the Hayman-Wu Theorem, and the geometric content of the theorem.

C. FitzGerald:

Marty equalities for mappings in several complex variables

Some of the geometric function theory of one complex variable is generalized to several variables. We consider biholomorphic mappings of the unit ball in several variables onto convex sets. These mappings are normalized so that the origin is carried to the origin and at the origin the complex Jacobian of each mapping is the identity matrix. Extremal problems for this class are examined. With elementary variations, an array of Marty-type equations is found. Combining these results with previous work, we find infinitely many restrictions on the extremals. In a case of interest, these conditions determine the mapping on various cross sections of the unit ball.

[Joint work with Carolyn Thomas]

R. Fournier:

Some remarks on a convolution conjecture

Let $\mathcal{S}$ denote the class of univalent functions on the unit disk E with the usual normalization and $\mathcal{D}$ the subclass of $\mathcal{S}$ defined by

$$\mathcal{D} = \{F \in \mathcal{S} \mid |F''(z)| \leq \operatorname{Re} F'(z), z \in E\}.$$

We shall discuss some aspects of the following conjecture about the Hadamard product $*$ of univalent functions, namely that

$$\operatorname{Re}\left(\frac{(F * f * g)(z)}{z}\right) > 0, \quad z \in E,$$

if $F \in \mathcal{D}$ and $f, g \in \mathcal{S}$.

D. Gaier:

Abraham Plessner, his work and his life

Plessner's work on trigonometric series and on the boundary behavior of meromorphic functions is known to many mathematicians. Less known are the difficult circumstances under which he was working.

Plessner (1900-1961) came to Giessen in 1919 to study under Schlesinger and Engel. He stayed there with interruptions in Göttingen (Landau, Courant, Noether) and Berlin (Bieberbach, von Mises, Schur). In 1922 he got his doctorate at Giessen, and thereafter collaborated with Schlesinger and Hensel (Marburg). In 1929 he tried to become Privatdozent at the University of Giessen, without success because of lack of German citizenship. From 1932 he worked in the University of Moscow and in the Steklov Institute. His interest shifted towards functional analysis where he did fundamental work. He is considered as one of the founders of the Moscow school of functional analysis.

In 1949 he was dismissed from his post and worked, in spite of failing health, on a book 'Spectral theory of linear operators' which appeared in 1965 in Russian and in 1969 in English translation.

F. W. Gehring:

Discrete Möbius group

Roughly speaking a Möbius group G is discrete if the two generator group $< f,g >$ is discrete for each pair $f,g \in G$. The group $< f,g >$ is determined uniquely by the complex numbers

$$x = \mathrm{tr}^2(A) - 4, \; y = \mathrm{tr}^2(B) - 4, \; z = \mathrm{tr}[A,B] - 2$$

where A and B are 2×2 matrices representing f and g.

This talk describes how to get necessary conditions on the pairs (x,y) and (y,z) in order that $< f,g >$ be discrete. The method involves complex iteration of a new family of polynomials with curious Mandelbrotsets. It yields sharp bounds for the distance between axes of elliptics and new information on hyperbolic orbifolds.

D. Hamilton:

Conformal welding and applications

We develop the theory of homeomorphisms $\phi : S \to S$ proving:

H1: If ϕ is 'regular' (i.e. $\forall E \subset S$, $\dim E = 0 \Rightarrow \lambda\phi E = \lambda\phi^{-1}E = 0$) then there exist conformal maps $\alpha : \mathbb{D} \to A, \beta : \overline{\mathbb{D}}^c \to B$, $A \cap B = \emptyset, \partial A = \partial B : \beta \circ \phi = \alpha$ (a.e. on S). This is applied to prove:

H2: If R and R^* are homeomorphic then there exists a Kleinian group Γ with disjoint invariant domains A, B such that $(A|\Gamma, B|\Gamma)$ is conformally equivalent to (R, R^*).

Such groups we call AB. Also we describe a rigidity result for conjugations of Blaschke products.

W.K. Hayman:

An inequality for bounded Fourier series, related to an isoperimetric problem

Let $\mathcal{F}$ be the class of functions

$$f(x) \sim \sum_{-\infty}^{+\infty} c_n \, e^{inx}$$

satisfying $-1 \leq f(x) \leq 1$ on $[-\pi, \ \pi]$. We obtain sufficient conditions on the sequence λ_n so that

$$\sum_{n=1}^{\infty} \lambda_n |c_n|^2$$

is maximized in $\mathcal{F}$ by $f(x) = \operatorname{sgn}(\sin(kx))$ for some positive integer k. When $k = 2$ a special case is the inequality

$$\sum_{n=2}^{\infty} \frac{|c_n|^2}{n^2 \ - 1} \leq \frac{4 - \pi}{\pi}.$$

This was conjectured in an earlier paper by A. Weitsman and the authors and leads to an asymptotically sharp form of an inequality relating the perimeter, area and Fraenkel asymmetry of a bounded plane set.

When $k = 1$ results are simple consequences of subordination. For $k > 1$ some Fourier series techniques also seem to be necessary.

[Joint work with R.R. Hall]

W. Hengartner:

Univalent harmonic mappings

In 1926, H. Kneser has proved the following result which was proposed by. T. Radó:

Theorem A: *Let f^* be a complex-valued homeomorphism from the unit circle ∂U onto a convex Jordan curve G in $\mathbb{C}$. Then, the Poisson integral $f = Pf^*$ is a univalent harmonic mapping from the unit disk U onto the bounded component $\tilde{G}$ of $\mathbb{C} \setminus G$.*

Theorem A fails whenever G is not a convex curve. However, Kneser has shown that the Poisson integral $f = Pf^*$ of any homeomorphism from ∂U onto a Jordan curve G is univalent if and only if $f(U) = \tilde{G}$. In 1945, G. Choquet gave a second proof for Theorem A and J. Jost, R. Schoen and S.T. Yau (1978, 1982) have given generalizations to harmonic mappings on Riemannian manifolds. There is no anlogous result for multiply-connected domains.

However we have:

Theorem B: *Let $f^*(e^{it})$ be a complex-valued homeomorphism from the unit circle ∂U onto a convex Jordan curve G, such that $\int_{[0,2\pi)} f^*(e^{it})dt = a$. Then, the solution f of the Dirichlet Problem $f^*(e^{it}) \equiv f(e^{it})$and $f(re^{it}) \equiv a$ is a univalent harmonic mapping from the annulus $A(r,1) = \{z : r < |z| < 1\}$ onto $\tilde{G} \setminus \{a\}$, where $\tilde{G}$ is the bounded component of $\mathbb{C} \setminus G$.*

We shall use Theorem B to give counterexamples to a question posed by J.C.C. Nitsche (1975). Furthermore we give a characterization of univalent harmonic mappings vanishing on ∂U.

A. Hinkkanen:

On Julia sets of polynomials

It is proved that the Julia set $J(P)$ of a polynomial P of degree at least two is uniformly perfect. This means that there is a number c between 0 and 1 such that $J(P)$ intersects the annulus with centre z and radii cr and r whenever z belongs to $J(P)$ and the positive number r does not exceed the Euclidean diameter of $J(P)$. This answers a special case of the question of Pommerenke if Julia sets of rational functions are uniformly perfect. The proof is based on considering the Green's function of the unbounded component of the set of normality of P, and the hyperbolic metric of suitable domains.

J. G. Krzyż:

Generalized Neumann - Poincaré operator, chord - arc curves and Fredholm eigenvalues

Suppose D is a quasidisk and φ maps D conformally onto the unit disk. If

$$l(z,t) = (1/\pi)\ [\varphi'(z)\varphi'(t)(\varphi(z)-\varphi(t))^{-2} - (z-t)^{-2}]$$

then there exists $\kappa \in (0,1)$ such that the antilinear operator L :

$$(Lw)(t) = \iint_D l(z,t)\ \overline{w(z)}\ dxdy$$

is bounded on

$$\mathcal{A}(D) = \{f \text{ holomorphic in } D : ||f||^2 = \iint_D |f|^2 dxdy < +\infty\}$$

and $||L|| \leq \kappa$.

Moreover, there exists a CON system (p_n) in $\mathcal{A}(D)$ and a sequence (d_n), $0 \leq d_n \leq \kappa < 1$, such that

$$l(z,t) = \sum_0^\infty d_n\ p_n\ (z)\ p_n(t).$$

All d_n vanish if D is a disk. If D is a Lavrentiev domain in the finite plane, i.e. if $\partial D = \Gamma$ is a chord - arc then

$$(B\,w)(z) := (2\pi i)^{-1} \int_\Gamma \overline{w(\zeta)}(\zeta - z)^{-1}\overline{d\zeta}$$

is bounded on the Hardy space $H^2(D)$, $H^2(D) \subset \mathcal{A}(\mathcal{D})$ and $Bw = Lw$ on $H^2(D)$. The numbers d_n are obviously eigenvalues of B and L, and also eigenvalues of the generalized Neumann-Poincaré operator C_1^Γ with real-valued , absolutely continuous eigenfunctions on Γ. We define $(C_1^\Gamma h)(z), z \in \Gamma$, as

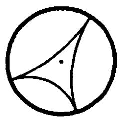
$$\text{Re } \{(\pi i)^{-1} \text{ P.V.} \int_\Gamma h(\zeta)(\zeta - z)^{-1} d\zeta\}$$

for real-valued h.

R. Kühnau:

Zum konformen Radius

Unter allen Kreisbogendreiecken $\ni 0$, deren drei Randbögen innerhalb des Einheitskreises liegen und senkrecht auf diesen auftreffen, wird nach denjenigen gefragt, für die der konforme Radius in 0 maximal ausfällt. Es wird bewiesen, daß die Lösung dieser (von J. Hersch gestellten) Aufgabe gegeben wird durch das zugehörige reguläre Kreisbogendreieck (mit also drei gleichlangen Kreisbögen).



K. Menke:

On a class of functions univalent in an annulus

Let $\mathcal{F}_\rho$ be the family of all functions f univalent in the annulus $R = \{\rho < |w| < 1\}$ mapping R onto the unit disk minus some continuum E with $0 \in E$ and satisfying $|f(w)| = 1$ on $|w| = 1$. The class $\mathcal{F}_\rho$ is important in conformal mapping theory (oscillation method). Here (sharp) distortion theorems for $f \in \mathcal{F}_\rho$ and the Schwarzian derivative of f are presented. In addition some estimates for the coefficients of the Laurent expansions of $f(w)$ and of $\log f(w)/w$ for starlike f are given.

D. Minda:

Two-point comparison theorems for Euclidean and hyperbolic geometry

In 1978 Christian Blatter established a two-point distortion theorem for univalent functions defined on the unit disk. That is also sufficient for univalence. A generalization of this result is established which provides a connection with an invariant form of a classical distortion theorem of Koebe. All of these distortion theorems can be reformulated as two-point comparison theorems between Euclidean and hyperbolic geometry on simply connected regions, but none of these characterize simply connectivity. For convex regions the situation is simpler: there are two-point comparison theorems that characterize convexity. There are similar comparison theorems that characterize uniformly perfect regions.

[Part of this work is joint with Seong-A Kim]

R. Mortini:

Interpolating Blaschke products and division in Douglas algebras

Interpolating Blaschke products resp. Blaschke products whose associated measure is a Carleson measure play an important role in the study of uniform algebras on the unit circle and the disk. This is best demonstrated by Chang-Marshall's theorem. Only recently [3] a characterization of those Douglas algebras on the circle has been given for which all invertible inner functions are Blaschke products of Carleson type. It turned out that the generators of those algebras are just those Blaschke products which appeared in a paper of Gorkin and Izuchi [1] on the structure of Douglas algebras on the disk. In the first part we investigate these Blaschke products in detail. In the second part we study the role played by interpolating Blaschke products in division problems in Douglas algebras. In particular, a solution to a problem of Guillory, Izuchi and Sarason is given. This result appeared in a joint work with P. Gorkin [2].

References

[1] P. Gorkin, K. Izuchi, *Some counterexamples in subalgebras of* $L^\infty(\mathrm{D})$. Indiana Univ. Math. J. **40** (1991), 1301-1313.

[2] P. Gorkin, R. Mortini, *Interpolating Blaschke products and factorization in Douglas algebras.* Michigan Math. J. bf 38 (1991), 147-160.

[3] V. Tolokonnikov, *Carleson-Blaschke products and Douglas algebras.* Algebra i Analyz **3** (1991), 185-196 (Russian).

E. Reich:

An integral transform and its applications to some extension problems

We will discuss the integral transform

$$(T\beta)(z) = \frac{(1-|z|^2)^3}{2\pi i} \int_{|\zeta|=1} \frac{\beta(\zeta)}{(1-\bar{z}\zeta)^3(\zeta - z)} d\zeta, \ (z \in D = \{|z| < 1\}),$$

as follows:

1. Determination of T on the basis of axioms [Earle, Reich].
2. Some applications:
   (a) Extensions and extremal extensions of boundary functions to functions F for which $\bar{\partial}F \in L^\infty(D)$.
   The Zygmund classes [Gardiner, Reich, Sullivan].
   Theorem of Fehlmann-type.
   (b) The parametric representation for qc mappings.
   Representation of close-to-extremal qc mappings.

H.M. Reimann:

Beltrami equations

The Heisenberg group H^n is identified with the boundary ∂D of the domain

$$D = \{\rho > 0\} \subset \mathbb{C}^{n+1}, \ \rho = \operatorname{Im} z_{n+1} - \sum_1^n |z_j|^2.$$

A quasiconformal mapping $f = (f_1, \ldots, f_{n+1}) : \partial D \to \partial D$ is a homeomorphism, which is absolutely continuous on lines, a.e. differentiable in the sense of Pansu and satisfies the Beltrami equation.

$$\overline{z_j} f_k = \sum \mu_{ij} z_i f_k, \quad k = 1, \ldots, n+1,$$

with $\mu = \mu^{tr}$ and $||\mu||_\infty < 1$.

Vectorfields $v \in C_c(H^n)$ which are of the form

$$v = pT + \frac{i}{2} \sum_{j=1}^{n} ((\overline{z_j} p) z_j - (z_p) \overline{z_j})$$

generate a flow f_s of quasiconformal mappings, if $||z_i z_j p||_\infty$ is bounded for all i, j. Such flows extend to flows in D which are symplectic with respect to the $SU(n+1,1)$ invariant Kähler form in D.

H. Renelt:

Verallgemeinerte Potenzen bei elliptischen Differentialgleichungssystemen

Gegeben sei ein gleichmäßig elliptisches System $v_y = au_x + bu_y$, $-v_x = cu_x + du_y$. Nach dem Darstellungstheorem hat jede Lösung $w(z) = u(z) + iv(z)$, $z = x + iy$, die Gestalt $w(z) = f \circ h(z)$, wobei $h(z)$ eine quasikonforme Abbildung und f eine analytische Funktion ist. Für derartige Systeme können verallgemeinerte Potenzen konstruiert werden, wobei asymptotische Entwicklungen eine wesentliche Rolle spielen. Überraschenderweise gelten für diese verallgemeinerten Potenzen analoge Integralrelationen wie für gewöhnliche Potenzen $(z - z_0)^n$, insbesondere gilt eine verallgemeinerte Cauchysche Integralformel für die Ableitung $w_z(z)$.

G. Schmieder:

Fusion rationaler Funktionen und Randeigenschaften

Ein Paar kompakter Mengen $K_1, K_2 \subset \mathbf{C}$ hat die Fusionseigenschaft, wenn zu jedem Paar rationaler Funktionen r_1, r_2 und zu jedem Kompaktum $K \subset \mathbf{C}$ eine rationale Funktion r existiert mit

$$|r(z) - r_j(z)| \leq \alpha \, \|r_1 - r_2\|_{K \cup (K_1 \cap K_2)} \quad (z \in K_j \cup K, j = 1, 2),$$

wobei $\alpha > 0$ nur von K_1 und K_2 abhängt.

Nach dem Fusionslemma von A. Roth hat jedes Paar disjunkter Kompakta die Fusionseigenschaft.

Es wird die Frage untersucht, für welche K_1, K_2 mit nichtleerem Schnitt die Fusionseigenschaft vorliegt. Unter bestimmten, im 'Normalfall' erfüllten, topologischen Annahmen kann eine Charakterisierung solcher Paare gegeben werden.

T. Sheil-Small:

3 unsolved problems on polynomials

1. The Jacobian conjecture

   Let $P(x,y) = Q(x,y) + iR(x,y)$ be a real analytic polynomial whose Jacobian determinant

   $$J(P) = \frac{\partial Q}{\partial x}\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial y}\frac{\partial R}{\partial x}$$

   is non-zero on $\mathbf{R}^2$. Is it true that the mapping $P : R^2 \to R^2$ is a global homeomorphism? True for $n = 1, 2, 3$, (4 probably).

2. Number of zeros of a harmonic polynomial

Let $P(re^{i\theta}) = \sum_{k=-n}^{n} a_k r^{|k|} e^{ik\theta}$ be a harmonic polynomial of degree n such that $|a_n| \neq |a_{-n}|$. The number of zeros is at least n (counted according to 'multiplicity'). Is it true that it is at most n^2 ? True $n = 1, 2$. Otherwise, as the number is certainly finite, find an upper bound dependent only on n.

3. Smale's conjecture

Let $P(z)$ be a polynomial of degree $n \geq 2$ of the form $P(z) = z + a_2 z^2 + \cdots + a_n z^n$. Is it true that there exists a zero ζ of P' such that $|P(\zeta)| \leq |\zeta|$, or even $|P(\zeta)| \leq \frac{n-1}{n}|\zeta|$. The last conjecture is true for $n = 2, 3, 4$. Smale showed $|P(\zeta)| \leq 4|\zeta|$ for suitable critical points ζ.

T.J. Suffridge:

Holomorphic mappings on the unit ball of $\mathbb{C}^n$

Geometric properties such as starlikeness or convexity of mappings defined on the unit ball of $\mathbb{C}^n$ have not been studied in much detail. In fact, relatively few such mappings are known. The ideas from problems in the plane carry over to some extent but there are many intriguing differences. We will discuss some of these differences and give some examples.

P. Tukia:

Maps of limit sets inducing an isomorphism of two Möbius groups

Let G and H be two discrete Möbius groups of the open unit disk B^n and let θ be an isomorphism of G into H. There have been important situations where the existence of a homeomorphism of the limit sets inducing θ has played a decisive role like Mostows rigidity theorem. We call such a map f a limit map and study this question in a general setting. Usually we cannot require that f is defined everywhere but we can establish that if G is geometrically finite, f can be defined almost everywhere with respect to a natural measure.

J. Väisälä:

Coarse length and free quasiconformality

(1) Coarseness. For $h \geq 0$, the h-coarse length of an arc α in a metric space is the supremum of the sums $|x_0 - x_1| + \ldots + |x_{k-1} - x_k|$ over all finite sequences of successive points $x_0, \ldots, x_k$ of α such that $|x_{j-1} - x_j| \geq h$ for all j.

(2) The quasihyperbolic metric of a domain G in a Banach space E is defined by $k_G(a,b) = \inf_{\alpha} \int_{\alpha} \frac{|dx|}{d(x,\partial G)}$ over all rectifiable arcs joining the points a and b in G.

(3) Free quasiconformality is an extension of the n-dimensional theory of quasiconformal maps to arbitrary Banach spaces. Its definition is based on (2) and many proofs on (1).

N. Dudley Ward:

Atomic decompositions of continuous or integrable functions

An atomic decomposition for a Banach space X establishes that every element may be represented in the form

$$f = \sum_{k=1}^{\infty} \lambda_k u_k,$$

where $\lambda = < \lambda_k >$ belongs to a suitable sequence space, and the $< u_k >$ belongs to a subset E of X (E is called a set of atoms for X). In a series of papers by Bonsall, Bonsall and Walsh and Hayman and Lyons, the authors prove characterizations of E, where E consists of the Poisson kernels for the disk, and the spaces considered are the positive continuous functions and the integrable functions over the unit circle. In this talk, I shall outline some more recent results, where E consists of a general kernel, and shall emphasize some of the constructive aspects of the theory.

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