

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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p-adische Analysis und Anwendungen

23.-29.2.1992

Die Tagung fand unter der Leitung von S. Bosch (Münster), L. Gerritzen (Bochum) und A. Ogus (Berkeley) statt.

The use of p-adic methods is spreading more and more, most notably in algebraic geometry and number theory, but also in various other fields, including theoretical Physics. It was the aim of the conference to bring together researchers from different disciplines, who all are experiencing the need of applying p-adic methods in their work. The subjects which were covered include L-functions, p-adic differential equations, p-adic Physics, motives, De Rham cohomology, rigid cohomology, rigid uniformization, periods of abelian varieties, p-adic symmetric spaces, Drinfeld modular forms, stable reduction of curves as well as various other topics from algebraic and arithmetic algebraic geometry.

Vortragsauszüge

M. Rapoport:

Variation of F-isocrystals

Let G be a connected reductive group over \mathbb{Q}_p . Kottwitz introduces the set B(G) and a map

 $\nu: B(G) \to \mathcal{M}(G)^{\Gamma}_{\mathbb{Q}}$.

Here $\mathcal{M}(G)^{\Gamma}_{\mathbb{Q}}$ denotes the set of conjugacy classes defined over \mathbb{Q}_p of homomorphisms $\nu: \mathbb{D} \to G$, where \mathbb{D} is the diagonalizable pro-algebraic group



with character group \mathbb{Q} . In case $G = \operatorname{GL}_n$, the set B(G) is the set of isomorphism classes of F-isocrystals of rank n on an algebraically closed field of characteristic p, and the map ν associates to an F-isocrystal its slopes. We introduce an order on $\mathcal{M}(G)^{\mathbb{Q}}_{\mathbb{Q}}$, which in the case of GL_n reduces to the order on the set of Newton polygons. Using it one can, in certain cases, generalize to this context the theorem of Grothendieck on the behaviour of Newton polygons in a family of F-isocrystals.

A.A. Panchishkin:

A general criterium for the existence of p-adic L-functions

For a motive M over a totally real field F with coefficients in another number field T p-adic properties are studied of the corresponding L-function $L(M,s)=\prod\limits_{p}L_{p}(M,Np^{-s})$, where $L_{p}(M,X)^{-1}$ is the characteristic polynomial of the Frobenius element $\operatorname{Fr}_{p}\in G_{F}=\operatorname{Gal}(\overline{F}/F)$ in the corresponding Galois representation $r_{\lambda,M}:G_{F}\to\operatorname{GL}(M_{\lambda})$ (M_{λ} is the λ -realization of M, λ a finite place of T). A local p-invariant $h=h_{M}(p)=(h_{\sigma})_{\sigma}$ is introduced, which generalizes the Hasse invariant of an elliptic curve E/\mathbb{Q} , whose components h_{σ} are indexed by imbeddings $\sigma:F\to\overline{\mathbb{Q}}$ of the ground field F, and are defined as the differences $h_{\sigma}:=P_{\operatorname{Nwt},\sigma}(d^{+})-P_{\operatorname{Hdg},\sigma}(d^{+})$ for $d^{+}=\dim_{T}M_{\sigma}^{+}$, where $P_{\operatorname{Nwt},\sigma}$ and $P_{\operatorname{Hdg},\sigma}$ are the Newton polygon and the Hodge polygon of M. Suppose that there exist a Hecke character χ of finite order and an integer $m\in\mathbb{Z}$ such that the twist $M(\chi)(m)$ is critical in the sense of Deligne.

A general criterium is established for the existence of a (bounded) p-adic interpolation of the algebraic numbers $\Lambda_{(p,\infty)}(M(\chi)(m),0)/\Omega(\varepsilon)$, where $\Lambda_{(p,\infty)}$ denotes the modified L-function of J. Coates and B. Perrin-Riou, obtained by a canonical replacement of local factors above p and ∞ , and $\Omega(\varepsilon)$ is a certain period of M, which depends only on the sign $\varepsilon = (\varepsilon_{\sigma})_{\sigma}, \varepsilon_{\sigma} = \chi_{\sigma}(-1)(-1)^{m}$. This criterium says that this p-adic interpolation exists if and only if the local invariant vanishes: $h = (h_{\sigma})_{\sigma} = 0$. In the general case a more elaborated technique of admissible measures can be applied in order to obtain certain growing p-adic L-functions.

Examples are given for the L-functions of Hilbert modular forms, of Siegel

modular forms, and of Hecke characters of CM-type.

J-Y. Etesse:

L-functions in rigid cohomology: unit root zeroes and poles

Dwork proved in 1971 that the unit root zeta function $\zeta_u(t)$ of the ordinary part of the Legendre family of elliptic curves $X \xrightarrow{f} S = \operatorname{Spec} \mathbb{F}_p[\lambda, \frac{1}{\lambda(\lambda-1)H(\lambda)}]$ is meromorphic in \mathbb{C}_p ; this ζ_u can be described as the *L*-function: $\zeta_u(t) = L(S, R^1f_*\mathbb{Z}_p, t)$ associated to the lisse \mathbb{Z}_p -sheaf $R^1f_*\mathbb{Z}_p$. The aim of the talk is to extend this result to more general *L*-functions to get conjectures of Katz in his Bourbaki talk ("Travaux de Dwork", 1972).

In the case S is proper and smooth over \mathbb{F}_q and E is an F-crystal, L(S, E, t) is a rational function (using crystalline cohomology). We get a functional equation and if E is a unit root F-crystal we can single out the zeroes and poles of L(S, E, t) with q-order an integer r (cf. "Annales de l'Institut Fourier" t. 38, fasc. 4, 1988, p. 33-92).

When S is separated of finite type over \mathbb{F}_q , we consider a field K of characteristic 0, complete under a discrete valuation, with perfect residue field $k \supset \mathbb{F}_q$, and an overconvergent F-isocrystal E on S/K. Then L(S, E, t) is proved to be a meromorphic function of t, using rigid cohomology with compact supports. Applying this to the structure sheaf $E = \mathcal{O}_{S/K}$ one can show that

$$L(S,t) \left/ \prod_{i=0}^{2\dim S} \left[\det(1-tF|H^i_{\mathrm{\acute{e}t,c}}(\overline{S},\mathbb{Z}_p) \otimes \mathbb{Q}_p \right]^{(-1)^{i+1}} \right. ,$$

 $\overline{S} := S \times_{\mathbb{F}_q} \overline{\mathbb{F}}_q$, has neither zeroes nor poles on the annulus $|t|_p = 1$. This generalizes to sheaves E with finite monodromy.

Y. André:

Q-realization of p-adic motives

General problem: Let $\sum a_n z^n$, $a_n \in \mathbb{Q}$, be a formal power series. We want to compare the real and p-adic evaluations for $z \in \mathbb{Q}$ in case of conver-



gence to rational or algebraic numbers.

Conjecture: Let $(y_1(z), \ldots, y_n(z))^t \in \mathbb{Q}[[z]]^n$ be a vector-solution to a Picard-Fuchs differential system. If the ∞ -adic evaluations of $y_1(\xi), \ldots, y_n(\xi), \xi \in \mathbb{Q}$, are $\overline{\mathbb{Q}}$ -linearly dependent, so are the *p*-adic evaluations for every prime *p* where they make sense.

Indeed, the Grothendieck conjecture implies that the relation between $y_1(\xi)_{\infty}, \ldots, y_n(\xi)_{\infty}$ should come from an algebraic cycle, i.e. an endomorphism of a certain motive $M_{/\mathbb{Q}}$.

We define a $\overline{\mathbb{Q}}$ -realization of $M_{/\mathbb{Q}_p}$, at least in the case of good reduction, stable under End $M_{/\mathbb{Q}_p}$, say $H_B(M_{/\mathbb{Q}_p}, \overline{\mathbb{Q}})$, such that $H_B(M_{/\mathbb{Q}_p}, \overline{\mathbb{Q}}) \otimes_{\overline{\mathbb{Q}}} \overline{\mathbb{Q}}_p \cong H_{\mathrm{DR}}(M) \otimes \overline{\mathbb{Q}}_p$. Moreover, there is a relative version $H_B(\underline{M}_{/S}, \overline{\mathbb{Q}})$, and for constant reduction $H_B(\underline{M}_{/S}, \overline{\mathbb{Q}}) \otimes_{\overline{\mathbb{Q}}} \overline{\mathbb{Q}}_p \cong H_{\mathrm{DR}}(\underline{M}_{/S})^{\mathrm{an} \, \nabla} \otimes \overline{\mathbb{Q}}_p$. We explicit $H_B(X, \overline{\mathbb{Q}})$ for any elliptic curve $X_{/\mathbb{C}_p}$. We can prove:

Theorem. The above conjecture is true if the dependence relation between $y_1(\zeta)_{\infty}, \ldots, y_n(\zeta)_{\infty}$ comes (according to Grothendieck's conjecture) from an algebraic cycle.

M. Candilera:

Periods on p-adic Barsotti-Tate groups via Witt realization

Let k be a perfect field of char p > 0, A = W(K), A' the ring of integers of a totally ramified extension K' of Frac A with ramification index e < p-1.

If R is the affine algebra of a Barsotti-Tate group over k and $\sigma: {}^{\#}R \to R$ is a lifting to A', then ${}^{\#}R$ can be realized as a sub-bialgebra of $W(\mathfrak{R})$, where \mathfrak{R} is the completion of $\lim_{n \to \infty} (R \xrightarrow{p_i} R \xrightarrow{p_i} R \dots)$.

In this way one has a representative for any class of isomorphism of liftings of R inside $A' \otimes W(\mathfrak{R})$.

Using Witt bivectors, one can realize the Dieudonné module M(R) inside biv \mathfrak{R} . In particular, via Witt realization, the integrals of first kind of ${}^{\sharp}R$ are embedded into $A' \otimes M(R)$. This doesn't happen for the integrals of second kind, but for any such integral h there is a unique element $\eta_h \in A' \otimes M(R)$ such that $h \equiv \eta_h \mod \mathfrak{m} \otimes W(\mathfrak{R})$.

Using this fact and the identification between Tate spaces over A' and over k, one can define, for any P in the Tate space and any integral of second

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kind h, the pairing $\int_{\overline{P}} \partial h = [1_{A'} \otimes \text{biv}(\overline{P})] \eta_h$ with values in bivectors.

Using this definition and the properties of Witt realization, one can compare different approaches to the periods.

E.-U. Gekeler:

Shimura-Taniyama-Weil for function fields

Let K be a function field over a finite field \mathbb{F}_q , and fix a place ∞ of K (principal example: $K = \mathbb{F}_q(T)$, " ∞ " the usual place at infinity).

We consider elliptic curves E over K that satisfy

(*) E has split multiplicative reduction at ∞ .

(Equivalent: E is a Tate curve over the completion K_{∞}). It forces that the conductor cond(E) is $\infty \cdot f$ with some "finite" divisor f of K.

By results of Grothendieck and Jacquet-Langlands, E corresponds to a certain automorphic representation π_E for $GL(2,\mathfrak{A}_K)$ (\mathfrak{A}_K the adèle ring of K). Using Drinfeld's theory of elliptic (or Drinfeld) modules, π_E gives rise to a one-dimensional factor of the Jacobian $J_0(f)$ of a Drinfeld modular curve $M_0(f)$. Taken together, this establishes bijections between the sets of

$$\left\{ \begin{array}{l} \text{isogeny classes of} \\ E \text{ that satisfy (*)} \\ \text{and cond}(E) = \\ \infty \cdot f \end{array} \right\} , \quad \left\{ \begin{array}{l} \text{one-dimensional} \\ \text{factors of } J_0^{\text{new}}(f) \end{array} \right\} \quad \text{and} \quad \left\{ \begin{array}{l} \text{automorphic new-forms of a certain} \\ \text{type with rational} \\ \text{eigenvalues} \end{array} \right\} ,$$

respectively.

Besides the fact that it is proved, it is the analogue of the conjecture of Shimura, Taniyama, and Weil, the coefficient field \mathbb{Q} being replaced by K. The assumption (*) is not a serious one since ∞ is at our disposal; if (*) cannot be achieved for a place of K, E will be a twisted constant curve. But the above correspondence is not explicit; so far, it is a pure existence statement without constructing the objects under consideration.

In the talk, we (joint work with M. Reversat/Toulouse) described how to explicitly obtain a curve $E=E_{\varphi}$ as a Tate curve from an automorphic newform φ . The construction depends on

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- results of M. van der Put on holomorphic functions on the Drinfeld upper half-plane;
- Teitelbaum's description of Drinfeld modular forms as certain measures on the Bruhat-Tits tree of $GL(2, K_{\infty})$;
- the existence of liftings of Drinfeld modular forms (char p-valued) to automorphic forms (char 0-valued);
- the description of the Jacobians of Drinfeld modular curves.

P. Schneider:

The p-adic holomorphic discrete series of SL_{d+1}

Let X be Drinfeld's d-dimensional upper half space over a local field K/\mathbb{Q}_p . For any cocompact discrete (torsionfree) subgroup $\Gamma \subset \mathrm{SL}_{d+1}(K)$ the quotient $X_{\Gamma} := \Gamma \backslash X$ is a projective smooth algebraic variety over K. Any finite-dimenensional $K[\Gamma]$ -module M gives rise to a local system on X_{Γ}^{an} . Its de Rham cohomology $H^*_{DR}(X_{\Gamma}, M)$ by Kiehl's GAGA-theorem actually is the de Rham cohomology of an algebraic vector bundle with connection on X_{Γ} . Based on joint work with U. Stuhler the dimensions of these cohomology groups can be computed explicitly in terms of the covolume of the group Γ . In particular the only interesting cohomology group turns out to be $H_{\mathrm{DR}}^d(X_{\Gamma}, M)$. It remains to understand the de Rham filtration on this group. Consider the case where M comes from an irreducible K-rational representation of SL_{d+1} . In this case the Hodge-de Rham spectral sequence usually will not degenerate which makes it difficult to analyze the induced filtration. It was shown that for such an M the de Rham complex for M is naturally quasi-isomorphic to a much simpler "reduced" de Rham complex. We hope that the associated "reduced" Hodge-de Rham spectral sequence always degenerates. This reduced complex is built out of the holomorphic discrete series representations in the title (for d=1 this notion was studied by Morita and Murase); it is a p-adic analog of the Bernstein-Gelfand-Gelfand resolution in the theory of Verma modules. Its construction is based upon Lie algebra cohomology computations of Kostant.

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H. Voskuil:

Non-archimedean flag domains

Part of the talk is joint work with M. van der Put.

Non-archimedean flag domains Y for G(K) are open analytical subspaces of X = G/P, that are G(K)-invariant and have the property that Y/Γ is a compact analytical space for any $\Gamma \subset G(K)$. Here K is a non-archimedean local field, G a (simply) connected, absolutely almost simple, semisimple linear algebraic group defined over K, $P \subset G$ a parabolic subgroup, and X = G/P a projective homogeneous variety, defined over K.

The description of a flag domain is as follows:

Let \mathcal{L} be an ample line bundle on X, defined over K, which means that the embedding of $X \hookrightarrow \mathbb{P}^n$ determined by \mathcal{L} is defined over K (i.e. is an embedding in \mathbb{P}^n_K). Let $S \subset G$ be a K-split torus of maximal rank. Take a G-linearization of \mathcal{L} . This induces an S-linearization of \mathcal{L} . Now we have a set of stable points, denoted by $X^s(S, \mathcal{L})$, and a set of semi-stable points, denoted by $X^{ss}(S, \mathcal{L})$, for this S-linearization of \mathcal{L} . Take $Y := \bigcap_{g \in G(K)} g \cdot X^s(S, \mathcal{L})$.

Then $Y \subset X$ is a flag domain if $X^{s}(S, \mathcal{L}) = X^{ss}(S, \mathcal{L})$.

To find X such that X contains flag domains, we only have to study sets of (semi-)stable points. We have the following result:

Theorem. The projective homogeneous variety X = G/P, defined over K, has a line bundle defined over K, such that $X^{s}(S, \mathcal{L}) = X^{ss}(S, \mathcal{L})$ if and only if:

- 1) $G = \operatorname{SL}_n(D)$, D/K a skew field of dimension d^2 , $X = G/P_I$, $\gcd(i \in I, n) = 1$, $I \subset \{1, \ldots, nd-1\}$. Here $P_I = \bigcap_{i \in I} P_i$ and P_i are maximal parabolics containing some fixed B such that $G \otimes K_s/P_i \otimes K_s = \operatorname{Gr}(i, nd)$ where $K_s \supset K$ is a maximal separable closure of K
- 2) G =the non-split form of the symplectic group, X = G/B and X = G/P for an extra parabolic subgroup (B is the Borel group, i.e. the minimal parabolic subgroup)
- 3) $G \neq \text{outer form of type } A_l$, quasi-split form of type D_l or E_6 , X = G/B, where B is the Borel group.

The quotients Y/Γ for $\Gamma \subset G(K)$ a discrete co-compact subgroup are in



general non-algebraic. In fact they have only constant meromorphic functions except when:

 $G=\operatorname{SL}_n(K)$ and we have a G-equivariant projection $\varphi:X\to \mathbb{P}^{n-1}$, such that $Y=\varphi^{-1}(\Omega_{n-1})$. Here Ω_{n-1} is Drinfeld's symmetric space $\mathbb{P}_K^{n-1}-\{K\text{-rational hyperplanes}\}$. In these exceptional cases Y/Γ is an algebraic variety.

M. van der Put:

Uniformization of Jacobi varieties and deformation of curves

This is joint work with Jean Fresnel (Univ. de Bordeaux).

The problem. Let Z/K be a non-singular curve (projective) over a field K which is complete with respect to a non-archimedean valuation. Suppose that Z has stable reduction X_0/k where k is the residue field of K. Then the Jacobian variety of Z has the following uniformization (Raynaud, Bosch, Lütkebohmert, Fresnel, van der Put, ...)

with T a torus of dimension h, A(Z) an abelian variety with principal polarization of dimension g, and Λ a lattice.

Is A(Z) again a Jacobian variety?

Answer: "In general" No!

Two methods:

(A) Suppose X_0/k is a "general stable curve", then there exists a \mathbb{Z}/K with stable reduction X_0 such that $A(\mathbb{Z})$ is not a product of Jacobian varieties.

Proof: Deformation theory of curves and a special version of Petri's theorem of canonical embedded curves.

(B) Let Z denote an étale extension of degree 2 of a Mumford curve. If the

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Mumford curve is general enough, then A(Z) is not a product of Jacobian varieties.

Proof: Prym-varieties, Kodaira-Spencer maps, work of F. Oort and J. Steenbrink on the hyperelliptic locus in \mathcal{M}_g and the Torelli map. A preprint is available!

B. Le Stum:

The geometrical meaning of monodromy and Frobenius over p-adic fields

Fix a smooth proper curve C over \mathbb{C}_p . Using Coleman's integration of differential forms on basic wide open spaces, one can define a map

$$H^1_{\mathrm{DR}}(C) \cong H^1_{\mathrm{DR}}(C^{\mathrm{an}}) \longrightarrow H^1(C^{\mathrm{an}}, \mathbb{C}_p)$$
.

Setting $H_1(C^{an}) := H^1(C^{an}, \mathbb{Z})^{dual}$, one can see this map as a pairing

$$H_1(C^{\mathrm{an}})\otimes_{\mathbf{Z}}\mathbb{C}_p imes H^1_{\mathrm{DR}}(C)\longrightarrow \mathbb{C}_p\;,\; (\gamma\otimes lpha,\omega)\longmapsto: lpha\int_{\gamma}\omega\;,$$

the period pairing.

One can show that $H^1(C^{an}, \mathbb{Z})$ injects in $H^1_{DR}(C)$, that the period pairing induces the natural pairing between $H_1(C^{an})$ and $H^1(C^{an}, \mathbb{Z})$, and that $H^1(C^{an}, \mathbb{Z})$ is orthogonal to itself with respect to the Poincaré pairing. It formally follows that there is a natural isomorphism

$$H^1_{\mathrm{DR}} \cong H^1(C^{\mathrm{an}}, \mathbb{C}_p) \oplus \tilde{H} \oplus H_1(C^{\mathrm{an}}) \otimes_{\mathbb{Z}} \mathbb{C}_p$$

with $\tilde{H} := H^1(C^{an}, \mathbb{C}_p)^{\perp}/H^1(C^{an}, \mathbb{C}_p)$. Then one can show that if X is any reduction of C with only ordinary multiple points (with normal tangents) as singularities, there is a natural isomorphism

$$H^1_{\mathrm{cris}}(\tilde{X}/W) \otimes_W \mathbb{C}_p \cong \tilde{H}$$
,

where \tilde{X} is the normalisation of X and W the ring of Witt vectors of \mathbb{F}_p . Thus, if we set

$$D:=H^1(C^{\mathrm{an}},W)\oplus H^1_{\mathrm{cris}}(\tilde{X}/W)\oplus H_1(C^{\mathrm{an}})\otimes_{\mathbf{Z}}W\ ,$$



we get a natural isomorphism $D \otimes_W \mathbb{C}_p \cong H^1_{\mathrm{DR}}(C^{\mathrm{an}})$. Moreover, one can endow D with a Frobenius and a monodromy operator as follows. One defines φ on $H^1(C^{\mathrm{an}}, W)$ as σ when σ is the Frobenius of W, φ is the usual Frobenius on $H^1_{\mathrm{cris}}(\bar{X}/W)$ and $\varphi = p \otimes \sigma$ on the last part. Actually, monodromy factors through an isogeny

$$H_1(C^{\mathrm{an}}) \otimes_{\mathbf{Z}} W \longrightarrow H^1(C^{\mathrm{an}}, W)$$

that comes from a scalar product

$$H_1(C^{\mathrm{an}}) \times H_1(C^{\mathrm{an}}) \longrightarrow \mathbb{Q} , (\gamma, \gamma') \longmapsto \gamma \cdot \gamma' .$$

This product is just Grothendieck's monodromy pairing of SGA 7. Also, the above description of Frobenius is mainly due to Coleman.

A. Ogus:

F-crystals on the Punctured Point

Let V be a finite extension of \mathbb{Z}_p , $K = V \otimes \mathbb{Q}$, k = the residue field of V, and W = the Witt ring of k. Suppose Y/V is projective, with semi-stable reduction. Then Hyodo and Kato have endowed the De Rham cohomology $H(Y_K/K)$ with "hidden structure", as suggested by Jannsen:

Given a uniformizer π of V, there exists a W-module H_0 endowed with - an isomorphism $\rho_{\pi}: H_0 \otimes_W K \xrightarrow{\sim} H_{DR}(Y_K/K)$

- an F_W -linear injection $\Phi: H_0 \to H_0$
- a nilpotent endomorphism $N: H_0 \to H_0$, such that $p\Phi N = N\Phi$.

The main idea of the proof is to endow Y and $S = \operatorname{Spec} V$ with log structures, so that we obtain a smooth proper map of log schemes Y^\times/S^\times , this map reduces modulo p to a "perfectly smooth" Y_e^\times/ξ_e^\times , where ξ_e^\times is $\operatorname{Spec} V/pV$ with log structure with chart $\mathbb{N} \to V/pV$, $1 \mapsto (\operatorname{class of } \pi)$. The theory of log crystalline cohomology then furnishes an F-crystal on ξ_e^\times whose value at S^\times is $H_{\mathrm{DR}}(Y^\times/S^\times)$. We attempt to explain the hidden structure alluded to above by studying abstract F-crystals on ξ_e^\times . This is supposed to be analogous to the approach of W. Schmidt and Cornalba in the study of limit mixed Hodge structures and the approach of Grothendieck to the monodromy theorem in l-adic $(l \neq p)$ étale cohomology.



D. Bertrand:

Auto-dual 1-motives and hypergeometric equations

In [J. Number Th., 25, 1987, 152-161], K. Ribet constructed 1-motives whose l-adic realizations satisfy a strong degeneration property.

We construct in a similar fashion reducible differential modules whose differential Galois groups are "as small as possible". More precisely, let (K, ∂) be a differential field and write K_M (resp. $\operatorname{Gal}_{\partial}(K_M/K)$) for the Picard-Vessiot extension of K defined by $M \in K[\partial]$ (resp. for its differential Galois group). Consider an irreducible L in $K[\partial]$, isomorphic to its adjoint L^{\vee} , and two elements P, Q in $K[\partial]$, of order 1, such that P is isomorphic to Q^{\vee} .

Theorem. Let $M = PLQ \in K[\partial]$, and let X be the relative differential Galois group $Gal_{\partial}(K_M/K_{PL} \cdot K_Q)$. Then:

i) If the autoduality L ≅ L[∨] is antisymmetric, X is always isomorphic to C.
ii) If the autoduality L ≅ L[∨] is symmetric, and if M is isomorphic to M[∨], X is trivial. Otherwise, it is isomorphic to C.

This theorem can be applied to the study of $Gal(K_M/K)$ when M is a generalized hypergeometric operator. By the work of K. Boussel, each of the cases above can be expressed by congruences and sign conditions on the parameters.

Time permitting (and time didn't permit), we also explain how the de Rham/Betti realizations of Ribet's motives can be viewed as an extended Riemann-Legendre type of relations. It is worthwhile pointing out that their period matrix involves logarithms of Lamé functions.

R. Crew:

Finiteness questions in rigid cohomology

Suppose X/k is separated of finite type, where k is a perfect field of characteristic p>0. Drawing on the work of Dwork, Washnitzer-Monsky, and others, Berthelot has constructed a "rigid" cohomology theory H(X), as well as a theory $H_c(X)$ of "rigid cohomology with compact supports". It



is not known to be finite-dimensional. Grothendieck's proof of the finiteness of l-adic cohomology suggests that a basic case to study is that of the $H^{i}(X, M), H^{i}_{c}(X, M)$ where X is a smooth curve, and M is a "local coefficient system" - in this case, an overconvergent isocrystal in Berthelot's sense. Now in fact the H'(X, M) are not always finite-dimensional, and so the question arises of finding sufficient conditions for this. We define the notion of a "strict" isocrystal, and sketch a proof that if M is a strict isocrystal on a smooth curve X/k, then the $H'(X,M), H_c(X,M)$ are finite-dimensional, and the duality pairing $H^i(M) \times H^{2-i}(M) \to K$ is perfect. We show also that a "locally quasi-unipotent" isocrystal, i.e. one which satisfies an analogue of Grothendieck's local monodromy theorem, is strict. The key idea of the proof is that the $H(X,M), H_c(X,M)$ carry natural topologies, and the duality theorem holds in a topological sense, without knowing finiteness. This duality, and a simple functional-analytic argument involving the duality of the "middle extension" $\operatorname{Im}(H_c^1(M) \to H^1(M))$ shows that $H^1(M)$ and $H_c^1(M)$ are finite-dimensional.

P. Colmez:

A product formula for periods of CM abelian varieties

We gave a construction of the period pairing $H^1_{DR}(X) \times T_p(X) \to B^+_{DR}$, where X is an abelian variety defined over a finite extension of \mathbb{Q}_p and B^+_{DR} is the ring of p-adic periods constructed by Fontaine and which is the completion of $\overline{\mathbb{Q}}_p$ for a certain topology. This construction is quite analogous to the complex case and uses a theory of integration on X.

The second theorem was a (conjectural) product formula for periods of CM abelian varieties. This formula gives rise to a conjecture relating derivative of Artin-L-functions and periods of CM abelian varieties which generalizes the well-known formula $\zeta'(0)/\zeta(0) = \log 2\pi$ which can be seen as the product formula for $2\pi i$ (using the convention $\zeta'(1)/\zeta(1) = -\zeta'(0)/\zeta(0)$ and the formula $\nu_p(2\pi i) = 1/(p-1)$).

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I.V. Volovich:

p-adic analysis and quantum groups

Motivations for application of p-adic analysis in mathematical physics were discussed including a non-archimedean geometry at the Planck length and p-adic analysis as an instrument for consideration of fractal-like behavior of some systems. Then classical mechanics over \mathbb{Q}_p , and quantum mechanics as a triplet $(L_2(\mathbb{Q}_p), W(z), U(t))$ were discussed with $L_2(\mathbb{Q}_p)$ a Hilbert space of complex valued functions, W(z) a representation of the Heisenberg-Weyl group, and U(t) a unitary representation of an additive subgroup \mathbb{Q}_p . Further topics were: quantum field theory, p-adic gravity, quantum groups and q-analysis; strings. In particular the following formula takes place:

$$\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} + \frac{\Gamma(b)\Gamma(c)}{\Gamma(b+c)} + \frac{\Gamma(c)\Gamma(a)}{\Gamma(c+a)} = \frac{\zeta(1-a)}{\zeta(a)} \frac{\zeta(1-b)}{\zeta(b)} \frac{\zeta(1-c)}{\zeta(c)} \; , \; a+b+c=1 \; ,$$

noted by I. Aref'eva, B. Dragovič and I.V.

W. Lütkebohmert:

The structure of proper p-adic groups

The fundamental example of a proper smooth connected p-adic group G_K of dimension g is obtained in the following way:

Let $B/\operatorname{Spf}(R)$ be a formal abelian scheme of dimension (g-r) and let E_1, \ldots, E_r be translation invariant line bundles over $B \otimes_R K$. Then $E_K = E_1^* \times_B \ldots \times_B E_r^*$ is a commutative smooth p-adic group due to the theorem of the square. Since any E_ρ extends to a formal line bundle on B, there exists a well-defined absolute value on E_ρ . So one gets a map

$$-\log: E_K(\overline{K}) \to \mathbb{R}^r, (x_i) \mapsto (-\log|x_i|)$$
.

A lattice M_K of E_K is a closed étale subgroup of E_K such that $-\log$ maps $M_K(\overline{K})$ bijectively to a lattice of \mathbb{R}^r of rank r. Then E_K/M_K is a proper smooth connected p-adic group.



The main results are:

Theorem. Let R be a complete discrete valuation ring with field of fractions K. Let G_K be a smooth proper connected p-adic group over the field K. Then there exists a finite separable field extension K'/K such that $G_K \otimes K'$ is of type $E_{K'}/M_{K'}$ as in the example.

Corollary. For any G_K as above there exists a finite separable field extension K'/K such that $G_K \otimes K'$ has a stable and, hence, semi-abelian formal Néron model.

Corollary. Any G_K as above has a dual; i.e. $\operatorname{Pic}_{G_K/K}^r$ is representable by a smooth proper connected p-adic group.

J. Teitelbaum:

Modular Symbols for $\mathbb{F}_q(T)$

I describe a theory of modular symbols for automorphic forms on $\mathrm{GL}_2(\mathbb{F}_q(T))$ modelled closely on Manin's classical theory. If T= the Bruhat Tits tree of $\mathrm{PGL}_2(\mathbb{F}_q((\frac{1}{T})))$, a "modular symbol" is a path in T joining two rational ends. There is a natural "integration" pairing between such symbols and cuspidal harmonic functions on T which are invariant by an arithmetic subgroup $\Gamma \subset \mathrm{GL}_2(\mathbb{F}_q[T])$. This pairing is equivariant with respect to the action of the Hecke algebra.

I describe explicit generators and relations for the space of modular symbols modulo the Γ -action, and explain various applications to the arithmetic of Drinfeld modular curves, elliptic curves over $\mathbb{F}_q(T)$, and automorphic forms.

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V.S. Vladimirov:

Spectral theory of some pseudo-differential operators on the field of p-adic numbers

Let G be a clopen set in \mathbb{Q}_p^n . The operator A of the form

$$(A\Psi)(x)=\int\limits_{\mathbb{Q}^n_n}a(\xi,x)\chi_p(-(\xi,x)) ilde{\Psi}(\xi)d\xi\;,\;x\in G,\Psi\in\mathcal{L}^2(G)\;,$$

is called a pseudo-differential operator in the set G with symbol $a(\xi, x), \xi \in \mathbb{Q}_p^n, x \in G$. Here the functions $\Psi(x), x \in G$ are complex valued of p-adic arguments; $\tilde{\Psi}(\xi)$ is their Fourier transform

$$ilde{\Psi}(\xi) = \int\limits_{\mathbb{Q}_p^n} \Psi(x) \chi_p((x,\xi)) dx \; , \; \Psi \in \mathcal{L}^2(\mathbb{Q}_p^n) \; , \; \mathrm{supp} \; \Psi \subset G \; ;$$

 $\chi_p(x)$ is the standard additive character of \mathbb{Q}_p , $(x,\xi) = x_1\xi_1 + \ldots + x_n\xi_n$. The aim is to study the spectral theory (including explicit calculations of

eigen-values and eigen-functions) for the following operators: 1) $A = D^{\alpha}$, $\alpha > 0$, symbol $|\xi|_{p}^{\alpha}$, a) $G = \mathbb{Q}_{p}$, b) $G = [|x|_{p} \leq 1]$, c) $G = [|x|_{p} \leq 1]$

- 1) $A = D^{\alpha}$, $\alpha > 0$, symbol $|\xi|_{p}^{\alpha}$, a) $G = \mathbb{Q}_{p}$, b) $G = [|x|_{p} \le 1]$, c) $G = \{|x|_{p} = 1\}$.
- 2) a*+V(x), symbol $\tilde{a}(\xi)+V(x), 0 \leq \tilde{a}(\xi) \to +\infty, |\xi|_p \to \infty, 0 \leq V(x) \to +\infty, |x|_p \to \infty, x \in G$. In particular $A=D^\alpha+V(|x|_p)$.
- 3) The evolution operator $A=U(T), |t|_p \leq \frac{1}{p} \ (p \neq 2)$ for p-adic harmonic oscillator with symbol $a_t(\xi,x)=\chi_p(x^2 \lg t+\xi^2 \frac{\lg t}{4}+x\xi \frac{1}{\cos t}), \ U(t)U(t')=U(t+t')$. Connections with a group representation.

References: V.S. Vladimirov, I.V. Volovich, and I.E. Zelenov, Spectral theory in p-adic quantum mechanics, and representation theory, Math. USSR Izvestia, v. 36(1991); N2. p-adic Analysis and Mathematical Physics, World Scientific, Singapore, 1992 (in press).



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P. Ullrich:

The direct image theorem in formal and rigid analytic geometry

According to an idea of Raynaud, one should develop the theory of rigid analytic spaces over a complete non-archimedean valued field via the corresponding results on formal schemes over the valuation ring of the field. The direct image theorem for formal schemes, however, is documented in the literature (EGA III, §3.4) only for the noetherian case, i.e., in the present situation, only for fields resp. rings with a discrete valuation.

The talk gives a proof of the direct image theorem for formal schemes that also works in the situation over the valuation ring of a general nonarchimedean valued field.

Q. Liu:

Stable reduction of the curves of genus two

Let R be a discrete valuation ring, $K = \operatorname{Frac}(R)$, C a projective smooth, geometrically connected curve over K, g(C) = 2. Then C is defined by an equation

$$y^2 = a_0 x^6 + a_1 x^5 + \ldots + a_6 \in K[x]$$
.

Our aim is to determine the stable reduction of C (after some extension of K) in function of the coefficients a_0, a_1, \ldots, a_6 . We have succeeded by using Igusa's invariants J_2, J_4, \ldots, J_{10} of $a_0x^6 + a_1x^5 + \ldots + a_6$ and two "new invariants" I_4, I_{12} . This is the analogue of the case of elliptic curves whose stable reduction is determined by the modular invariant j.

A further application of this result is the determination of the minimal model of C over K (without extension!).

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M. Matignon:

Extensions of a finite morphism between formal fibres

Let R be a non-archimedean valued ring, $K = \operatorname{Frac}(R), k = R/m$. Let C_1, C_2 be two projective smooth geometrical connected curves over R. Assume there are C_1, C_2 models over R with reduced special fibres and $f: C_1 \to C_2$ a finite morphism. Let $p \in (C_1)_s$ (special fibre of C_1). We compare the singularities of the special fibres at p and f(p) = q.

This problem appears in a result of H. Lange (C_1 has good reduction $\Rightarrow C_2$ has potentially good reduction); case C_1 is smooth.

We see this problem in the category of analytic spaces over K (assume now complete and algebraically closed). Let us denote by r (indistinctly from C_1 and C_2) the reduction map. Then $r^{-1}(q)$ (formal fiber) is a K-analytic space and we can compactify $r^{-1}(q)$ in an algebraic curve C_q by "adding" balls from \mathbb{P}^1_K (Bosch-Lütkebohmert, v.d. Put); we define the genus $g(r^{-1}(q)) := g(C_q)$. Then we have the following result:

Theorem (T. Yousseffi). Under the hypothesis above we have the following inequalities: $g(r^{-1}(p)) \geq g(r^{-1}(q))$; $m_p \geq m_q$ where $m_q = \operatorname{card} \operatorname{Spm} \mathcal{O}_q$, the number of branches at q.

Denote by $\{q_i\} = \operatorname{Spm} \mathcal{O}_q$ and p_{ij} denote points in \mathcal{O}_p above q_i ; and $e_{i,j} = e(p_{i,j}|q_i)$ the ramification index.

Theorem. If char $k = p \nmid e_{ij}$ for all i, j then one can prolongate the finite morphism $f_p : r^{-1}(p) \to r^{-1}(q)$ between the formal fibres in a morphism $\tilde{f}_p : C_p \to C_q$ for suitable curves compactifications as above mentioned.

Corollary.
$$g(r^{-1}(p)) - 1 \ge (\deg f_p)(g(r^{-1}(q)) - 1)$$
.

Now suppose p,q are regular points. Then $f_p: r^{-1}(p) \to r^{-1}(q)$ is a finite morphism between balls. Then f_p can be prolongated to $\mathbb{P}^1_K \cong C_p \to C_q \cong \mathbb{P}^1_K$ iff the valuation of the different $D_{\mathcal{O}_p/\mathcal{O}_q} \geq 2 \deg f_p - 2$.

If we have big wild ramification we prove that it is possible to prolongate to $X \to \mathbb{P}^1_K \cong C_q$, where X has good reduction. This is possible due to the following theorem.

Theorem (lifting of morphisms). Let k be algebraically closed, $\overline{g}: C \to \mathbb{P}^1_k$



a finite morphism with C projective smooth connected. Let W(k) be the Witt ring; then one can lift \overline{g} to $g: C \to \mathbb{P}^1_{W(k)}$, where C is smooth /W(k).

Question. Find the minimal genus for the compactification X. It comes from the preceding theorem, it is the same to find a covering of the affine line A_k^1 with small ramification and imposed ramification at ∞ .

F. Baldassari:

Dwork's theory via toric varieties.

We recall the Morita p-adic gamma function and its role in interpolation of Gauß sums, via the Gross-Koblitz formula. We are interested in generalized Jacobi sums of the type

$$S_{a,b} = \frac{1}{q-1} \sum_{x \in \mathbb{F}_a^{n+1}} \prod_{i=0}^n \chi_{a_i}(x_i) \prod_{j=1}^r \chi_{-b_j}(f^{(j)}(x))$$

where χ_a , for $a \in \frac{1}{q-1}\mathbb{Z}$, denotes the multiplicative character of \mathbb{F}_q , $\chi_a = \text{Teich}^{(1-q)a}$ ($\chi_a(0) = 0$), and where $f^{(j)}(x)$ is a homogeneous form of degree d_j : $\sum a_i = \sum d_j b_j$.

It is well-known, e.g. in the context of Dwork's theory or of rigid cohomology, how to associate to the previous sums overconvergent F-crystals $\xi_{a,b}$, such that $S_{a,b}$ may be recovered via a trace formula for the action of Frobenius on $H_{\text{rig}}(\xi_{a,b})$. But we are interested in variation with respect to (a,b): there are difficulties in trying to extend $\xi_{a,b}$ into a continuous family of overconvergent crystals with finite dimensional cohomology. Dwork's dual theory provides such a possibility: Frobenius essentially operates on algebraic dual cohomology and satisfies the Boyarsky Principle. That means: "When cohomology is parametrized rationally by a character, the Frobenius operation will vary continuously (loc. analytically) with the character."

We interpret Dwork's theory in the framework of toroidal embeddings.

Berichterstatter: K. Schlöter, P. Ullrich

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