MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Klassifizierende Räume und Anwendungen der Steenrod-Algebra

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The meeting was organized by Hans-Werner Henn (Heidelberg), Haynes Miller (Cambridge, U.S.A.) and Dieter Puppe (Heidelberg). About 45 participants from Asia, Europe and North America attended the conference, exchanged ideas and reported on recent progress in their research areas. Most of the 20 talks were concerned with different aspects in the theory of classifying spaces, questions involving the Steenrod algebra and periodicity in homotopy theory.

Vortragsauszüge

ALEJANDRO ADEM:
Invariant Theory and Splittings in Group Cohomology

In this lecture we describe rings of invariants which appear in certain decompositions of the cohomology of a finite group (at a prime p) derived from local methods. Particular applications include $H^*(PSL_3(\mathbb{F}_4), \mathbb{F}_2)$ and $H^*(M_{22}, \mathbb{F}_2)$. (joint work with J. Milgram)

JAUME AGUADÉ:
Some Spaces with Interesting Cohomology

Consider the following unstable algebras over the Steenrod algebra:

$$\mathbb{A}_r = (\mathbb{F}_p[x_2] \otimes E(y_3))^{\mathbb{Z}/r},\ \beta x = y,\ P^1 y = x^{p-1}y,\ r|p-1$$

$$\mathbb{B}_{i,r} = (\mathbb{F}_p[x_{2p^i}] \otimes E(y_{2p^i+1}))^{\mathbb{Z}/r},\ \beta x = y,\ P^k y = 0,\ k > 0,\ r|p-1$$

(here subscripts denote degree). $\mathbb{A}_r$ and $\mathbb{B}_{i,r}$ are well defined (i.e. the Adem relations are satisfied) and moreover, if $A = \mathbb{F}_p[x_{2n}] \otimes E(\beta x)$ is an unstable algebra over the Steenrod algebra then A is isomorphic to either $\mathbb{A}_r$ or $\mathbb{B}_{i,r}$. For each of these algebras there is a non-trivial homomorphism of algebras $f : A \longrightarrow H^*(B\mathbb{Z}_p; \mathbb{F}_p)$. Let T_f denote the component of the T-functor corresponding to f. Then:

$$T_f \mathbb{B}_{i,r} \cong \mathbb{B}_{i,1}$$

$$T_f \mathbb{A}_r \cong \mathbb{F}_p[x_2] \otimes E(z_1),\ \beta x \neq 0$$

In 1976 G. Cooke conjectured that the algebras $\mathbb{B}_{i,r}$, $i > 1$ can not be the cohomology of a space. On the other side, $\mathbb{B}_{0,r}$ and $\mathbb{B}_{1,r}$ are realizable. For instance, $\mathbb{B}_{1,r} \cong H^*(S^3 < 3 >; \mathbb{F}_p)$.

Theorem If $H^*(X; \mathbb{F}_p) \cong \mathbb{B}_{i,r}$, then $i = 0, 1$.

Theorem $\mathbb{A}_r$ is realizable. If $H^*(X; \mathbb{F}_p) \cong H^*(Y; \mathbb{F}_p) \cong \mathbb{A}_r$ (in $\mathcal{K}$),then $X_p^\wedge \simeq Y_p^\wedge$ $(p > 2)$.

However there is no homotopy uniqeness for $\mathbb{B}_{i,r}$:

Theorem Let $p > 2$. There is a family of spaces $X_{i,r}(n)$ with $i \in \{0, 1\}$ and $n | p - 1$, $n = 1, 2, \ldots, \infty$, such that if $H^*(X; \mathbb{F}_p) \cong \mathbb{B}_{i,r}$ then $X_p^\wedge$ is homotopy equivalent to exactly one space $X_{i,r}(n)$.

In particular, there are infinitely many "fake" $S^3 < 3 >$.
(joint work with C. Broto and D. Notbohm)

FRED R. COHEN:
On the Homotopy Theory of Double Suspensions

Let $\alpha : \Sigma^2 B \longrightarrow A$ be an element of order q in the abelian group $[\Sigma^2 B, A]$.

Theorem 1 If $q = 2$, then $4\Omega^2(\alpha) = 0$ in the abelian group $[\Omega^2\Sigma^2 B, \Omega^2 A]$.

Remark 2 It seems likely that the techniques used in the proof of Theorem 1 will give that $q^2(\Omega^2(\alpha)) = 0$ if $q(\alpha) = 0$. If the order of the identity for $\Sigma^2 B$ is q, then this last result implies that $q^2(\pi_*\Sigma^2 B) = 0$.

Remark 3 The techniques involve studying a homomorphism

$$G_N \longrightarrow [X^N, \Omega\Sigma X]$$

where $X = \Sigma(X')$ and G_N has the following presentation:

Generators: $x_1, \ldots, x_N$
Relations: (1) $[x_{i1}^{\epsilon_1}, \ldots, x_{ik}^{\epsilon_k}] = [x_{i1}, \ldots, x_{ik}]^\epsilon$ where $\epsilon_i = \pm 1$ and $\epsilon = \epsilon_1 \cdots \epsilon_k$
(2) $[x_{i1}, \ldots, x_{ik}] = 1$ if some $x_{ij} = x_{il}$, $j < l$, and $[x_{i1}, \ldots, x_{ik}]$ is a commutator of weight k in the x_i.

ALBRECHT DOLD:
Lusternik-Schnirelman Category of B_kG, $1 \leq k \leq \infty$

By definition, a space B has $cat(B) \leq k$ iff B admits a numerable covering $B = \bigcup_{i=1}^{k} V_i$ such that $V_i \hookrightarrow B$ is nullhomotopic.
A principal G-bundle $E \to B$ has $genus(E) \leq k$ iff it admits a numerable covering $\{V_i\}_{1 \leq i \leq k}$ which trivializes E iff E admits a G-map $E \to E_kG$ into the Milnor bundle $E_kG \to B_kG$. The following are familiar:

$$genus(E) \leq cat(B), \quad cat(B_kG) \leq k$$

We prove:

(i) If G has elements of finite order $\neq 1$ and $dim(G) < \infty$ then $cat(B_kG) = genus(E_kG) = k$ for all k.

(ii) If $dim(G) < \infty$ and $cat(BG) = \infty$ then $cat(B_kG) = genus(E_kG) = k$ for all k.

(iii) If $genus(EG) < \infty$ then $cat(BG) = genus(BG)$ and $cat(B_kG) \leq cat(BG) + 1$ for all k.

The assumption $dim(G) < \infty$ may be redundant in (ii) but not in (i).

EMMANUEL DROR FARJOUN:
Periodic Homotopy Theory

We gave a survey of some typical recent results relating K-Theory and v_1-localization. We work in the framework of general homotopical localization W/r to a general map $f : A \longrightarrow B$ denoted by L_f. When $f : W \longrightarrow *$ we write $L_f = P_W$. Recent progress by Bousfield and Thompson:

Theorem (Bousfield-Thompson) A map $f : X \longrightarrow Y$ induces $K_{(p)*}$-isomorphisms on all $\Omega^i f$ for 3-connected spaces X, Y iff $v_1^{-1}\pi_*(f, \mathbb{Z}/p)$ and $\mathbb{Q} \otimes \pi_* f$ are isomorphisms.

Bousfield reported privately on similar progress for higher v_n and $K < n >$.
We strive to understand v_1- and K-localization on spaces are special cases of L_f. We are especially interested in preservation of fibrations and the fibre of $X \longrightarrow L_f X$.

Theorem (with J. Smith after Bousfield for X with p-torsion) Let $X = \Omega Y$ be any 1-connected space. Let $A \xrightarrow{f} B \xrightarrow{g} C$ be a cofibration of connected spaces. The fibre of $P_W \Omega X \longrightarrow \Omega P_W X$ is a product of Eilenberg-MacLane spaces (GEM).

Corollary Let $X = \Omega^2 Y$ and L_K the $K_{\mathbb{C}}$-localization of Bousfield. Then the fibre of $L_K \Omega X \longrightarrow \Omega L_K X$ has only two non trivial π_* in dimension 1 and 2.

This is based on a key lemma of Bousfield together with a "calculus" of L_K:

Theorem (1) $P_W(\text{fibre of } X \longrightarrow P_W X) \simeq *$
(2) If $P_W \Omega X \simeq *$, then $P_W \Omega^2 X$ is GEM.
(3) $L_f(hocolim\underline{X}) = L_f(hocolim L_f \underline{X})$.
(4) P_W preserves fibrations with W-local base.

Corollary If X is 5-connected and $K_{\mathbb{C}} \Omega^5 X = 0$, then X is made by cofibrations out of $V(1)$.

VINCENT FRANJOU:

MacLane Cohomology via the De Rham Complex and the Koszul Complex

Let p be a prime and R the finite field $\mathbb{F}_p$. Let $\mathcal{F}$ be the category of covariant functors from the category of finite vector spaces to the category of vector spaces over R. MacLane cohomology with coefficients in a functor F is defined by

$$H^i_{ML}(R, F) = Ext^i_{\mathcal{F}}(I, F), \quad i \in \mathbb{N}$$

where I is the inclusion functor.
We compute it when F is any of the functor I, exterior power $\Lambda^n, (n \in \mathbb{N})$ or homogenous polynomials of degree n, S_n where $S_n V = (V^{\otimes n})_{\sigma_n}$. In particular:

$$Ext^i_{\mathcal{F}}(I, I) = \begin{cases} \mathbb{F}_p & : \ i \text{ even} \\ 0 & : \ i \text{ odd} \end{cases}$$

To perform the computation, we use as input:
(1) $Ext^i_{\mathcal{F}}(I, S_{p^k}) = 0$ for $k \gg i$, $i \neq 0$.
(2) $Ext^i_{\mathcal{F}}(I, F \otimes G) = 0$ for all i, if $F(0) = G(0) = 0$.
and we work with the De Rham complex, and a subcomplex of it, built as the kernel of the Koszul differential $\mathcal{K}$ (the latter is a subcomplex because of the Euler formula: $d\mathcal{K} + \mathcal{K}d$ is multiplication by the degree). The related hypercohomology spectral sequences allow us to perform a downward induction on k, starting with (1).
(joint work with J. Lannes and L. Schwartz)

PAUL G. GOERSS:

Bousfield Local Homotopy Theory is Determined by Cochains

Let X be a space and $S^* X$ the simplicial cochains over a field $\mathbb{F}$. We ask to what extent X is determined by $S^* X$.
We recover first a result similar to that of I. Kriz and V.Smirnov.

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Theorem 1 Suppose X, Y are spaces of finite type and there is a map $f : S^\bullet X \longrightarrow S^\bullet Y$ of cosimplicial algebras, such that $H^\bullet f : H^\bullet X \longrightarrow H^\bullet Y$ is an isomorphism. Then there is a weak equivalence of Bousfield localizations

$$L_{\mathbb{F}} X \simeq L_{\mathbb{F}} Y$$

This is obvious if f is a continous map, but this is not assumed. Theorem 1 is a corollary of examining $S^\bullet X$ as a cosimplicial complete algebra.

Theorem 2 Let $c\mathcal{A}$ be the category of cosimplicial complete algebras. There is a closed model category structure on $c\mathcal{A}$, such that if $B \longrightarrow S^\bullet X$ is a cofibrant cover of $S^\bullet X$, then

$$Sp(B) = Hom_{\mathcal{A}}(B, \mathbb{F}) = L_{\mathbb{F}} X$$

Then one places an appropriate closed model category structure on $c\mathcal{A}$ without the topologies.

JOHN PATRICK C. GREENLEES:

The Local Cohomology Theorem and Rationality of the Tate Spectrum

For any finite group G we have the augmentation ideal $I = (\alpha_1, \ldots, \alpha_n)$ of the Burnside ring $A(G)$. One may then construct the G-spectrum $H_I(S^0) = S^{-1}/\alpha_1^\infty \wedge \cdots \wedge S^{-1}/\alpha_n^\infty$, and it is easy to show there are short exact sequences (for an equivariant cohomology theory $F_G^*(\cdot)$)

$$0 \longrightarrow L_1^I F_G^{n+1} \longrightarrow F_G^n(H_I(S^0)) \longrightarrow L_0^I F_G^n \longrightarrow 0$$

(where $L_\bullet^I(\cdot)$ are left derived functors of I-completion) and

$$0 \longrightarrow H_I^1(F_{n+1}^G) \longrightarrow F_n^G(H_I(S^0)) \longrightarrow H_I^0(F_n^G) \longrightarrow 0$$

(where $H_I^\bullet(\cdot)$ are Grothendieck's local cohomology groups).
The essential fact is that $A(G)$ is of Krull dimension 1.
It is also elementary to construct a map

$$c : EG_+ \longrightarrow H_I(S^0)$$

and hence we have an improved version of the completion conjecture and a homological analogue.

Completion statement for $\mathbf{F}_G^\bullet(\cdot)$: $F_G^\bullet(c)$ is an isomorphism.
Local cohomology statement for $\mathbf{F}_\bullet^G(\cdot)$: $F_\bullet^G(c)$ is an isomorphism.

If true these calculate $F_G^*(EG_+)$ and $F_*^G(EG_+)$ as algebraic functors of F_*^G. Of course the Atiyah-Segal theorem is the completion statement for equivariant K-theory and the Segal conjecture is the completion statement for stable cohomotopy. The local cohomology statement is false for stable cohomotopy.

Theorem 1 The local cohomology statement holds for equivariant K-theory.

This is a reformulation of a deduction of G. Wilson, but it admits a short direct proof. Analogues exist for any compact Lie group, but are only proved in certain cases.

Theorem 2 If the local cohomology statement holds for $F_*^G(\cdot)$ then the Tate spectrum $t(F)$ is rational.

There is some evidence for a converse of theorem 2: if $t(F)$ is rational then there is an F' with $t(F) \simeq t(F')$ such that the local cohomology statement holds for ${F'}_*^G(\cdot)$.

JEAN LANNES:
Algebraic Smith Theory

Let p be a prime and V be an elementary abelian p-group. One sets $H^*V = H^*(BV;\mathbb{F}_p)$ and $c_V = \prod_{u \in H^1V-\{0\}} u$ for $p = 2$ and $c_V = \prod_{u \in H^1V-\{0\}} \beta u$ for $p > 2$.

Let $H^*V - \mathcal{U}$ denote the category of unstable $H^*V - \mathcal{A}$-modules (i.e. unstable modules M over the Steenrod algebra $\mathcal{A}$ which are H^*V-modules such that the structure map $H^*V \otimes M \longrightarrow M$ is $\mathcal{A}$-linear). Let $Fix : H^*V - \mathcal{U} \longrightarrow \mathcal{U}$ be the left adjoint of the functor $\mathcal{U} \longrightarrow H^*V - \mathcal{U}, N \longmapsto H^*V \otimes N$.
The main technical result is:

Proposition 1 Let M be an object in $H^*V - \mathcal{U}$. Then the following conditions are equivalent:
(i) $M[c_V^{-1}] = 0$;
(ii) $Fix(M) = 0$.

Corollary 2 (compare with Dwyer and Wilkerson) Assume M finitely generated as an H^*V-module. Then $Fix(M)$ is finite and for $dim V = 1$ the kernel and cokernel of the natural map $M \longrightarrow H^*V \otimes Fix(M)$ are finite too.

Corollary 3 (Dwyer-Wilkerson) Let X be a p-complete space equipped with an action of V. Assume that $H^*(X;\mathbb{F}_p)$ is finite and that X^{hV} is p-good. Then $\chi_{\mathbb{F}_p}(X^{hV}) \equiv \chi_{\mathbb{F}_p}(X) \bmod p$.

Proposition 1 leads also to the determination of all the injective objects in the category $H^*V - \mathcal{U}$.
(joint work with S. Zarati)

JOHN MARTINO:
A Classification of the Stable Type of BG

We determined a necessary and sufficient condition for the p-completions of two finite groups G, G' to be stably homotopy equivalent.

Theorem For two finite groups G, G' the following are equivalent:
(1) $BG_p^\wedge$ and $BG'^\wedge_p$ are stably homotopy equivalent.
(2) For every p-group Q,

$$\mathbb{F}_p Rep(Q, G) \cong \mathbb{F}_p Rep(Q, G')$$

as $Out(Q)$-modules. $Rep(Q, G) = Hom(Q, G)/G$ with G acting by conjugation.
(3) For every p-group Q,

$$\mathbb{F}_p Inj(Q, G) \cong \mathbb{F}_p Inj(Q, G')$$

as $Out(Q)$-modules. $Inj(Q, G) \subseteq Rep(Q, G)$ consists of conjugacy classes of monomorphisms.

We also gave an unstable analogue of this theorem. We applied the theorem to the special case of G, G' having normal Sylow p-subgroups. We used the special case to analyse the Minami-Webb formula, which express BG in terms of cyclic mod-p groups.
(joint work with S. Priddy)

HAYNES MILLER:
Higher Real K-Theory

This is joint work in progress with Mike Hopkins. We construct analogues of KO, by forming homotopy fixed point spectra of actions on Landweber theories E_n with $\pi_* E_n = W(\mathbb{F}_{p^n})[u^{\pm 1}][[v_1, \ldots, v_{n-1}]]$, where $u^{p^n - 1} = v_n$. This requires the construction of a topological (closed model) category of A_∞-ring spectra. If $n = p = 2$, the binary tetrahedral group acts, and there results a theory in many ways analogous to $KO_2^\wedge$.

DIETRICH NOTBOHM:
Homotopy Uniqueness of Classifying Spaces of Compact Connected Lie Groups

Let p be an odd prime. Let G be a compact connected Lie group, T_G the maximal torus and W_G the Weyl group. We say that a p-complete space X has the *mod-p type* of BG, if $H^*(BG; \mathbb{Z}/p) \cong H^*(X; \mathbb{Z}/p)$ as algebras over the

Steenrod algebra.
Under these conditions, Dwyer, Miller and Wilkerson constructed a maximal torus of X. That is a map $BT_X{}_p^\wedge \longrightarrow X$ of the classifying space of a torus T_X, such that the homotopy fibre has finite mod-p cohomology and that $rank(T_X) = rank(G)$. $BT_X{}_p^\wedge$ denotes the p-adic completion. Moreover they got an action of W_G on BT_X and a map $BT_X{}_p^\wedge \longrightarrow BT_G{}_p^\wedge$, such that $BT_X \longrightarrow X$ is W_G-equivariant (up to homotopy) and such that the following diagram commutes and is equivariant.

$$\begin{array}{ccc} H^*(X;\mathbb{Z}/p) & \longrightarrow & H^*(BG;\mathbb{Z}/p) \\ \downarrow & & \downarrow \\ H^*(BT_X;\mathbb{Z}/p) & \longrightarrow & H^*(BT_G;\mathbb{Z}/p) \end{array}$$

We say, that a space X with the mod-p type of BG has the *p-adic type* of BG, if $BT_X{}_p^\wedge \simeq BT_G{}_p^\wedge$ as W_G-spaces; i.e. there exist a W_G-equivariant map $BT_G \longrightarrow X$ such that the following diagram commutes

$$\begin{array}{ccc} & H^*(BTG;\mathbb{Z}_p^\wedge) & \\ \nearrow & & \nwarrow \\ H^*(BG;\mathbb{Z}_p^\wedge) & \longrightarrow & H^*(X;\mathbb{Z}_p^\wedge) \end{array}$$

Theorem 1 Let G be a compact connected Lie group, such that $H^*(BG;\mathbb{Z})$ is p-torsion free, and let X be a p-complete space with the mod-p type of BG.
(1) There exists a compact connected Lie group H, such that X has the p-adic type of BH.
(2) If in addition G is simply connected or a product of unitary groups, then X has the p-adic type of BG.

Theorem 2 Let G be a compact connected Lie group, such that $H^*(BG;\mathbb{Z})$ is p-torsion free. If X has the p-adic type of BG, then X and $BG_p^\wedge$ are homotopy equivalent.

Theorem 1 and 2 together imply the following corollary.

Corollary 3 Let G be simply connected and $H^*(BG;\mathbb{Z})$ be p-torsion free, or let G be a product of unitary groups. If X has the mod-p type of BG, then X and $BG_p^\wedge$ are homotopy equivalent.

BOB OLIVER:
Higher Limits via Steinberg Representations

This dealt with higher derived functors of inverse limits of functors on certain categories based on elementary abelian p-groups:

$\mathcal{A}_p(G)$, for a compact Lie group G, is the category whose objects are elementary abelian p-subgroups $E \subseteq G$, and where morphisms between objects are those morphisms induced by inclusions and conjugation.

$\mathcal{A}_p(X)$, for a space X such that $H^*(X;\mathbb{Z}/p)$ is noetherian, is the category whose objects are homotopy classes of maps $\psi : BE \longrightarrow X$ such that $H^*(BE)$ is finitely generated as an $H^*(X)$-module; and where morphisms are monomorphisms between groups which induce triangles commutative up to homotopy.

Theorem Fix $\mathcal{A} = \mathcal{A}_p(G)$ or $\mathcal{A}_p(X)$.
(1) If $F : \mathcal{A} \longrightarrow \mathbb{Z}_{(p)} - mod$ is a covariant functor which vanishes except on the isomorphism class of one object (E,ψ), where $E \cong (\mathbb{Z}/p)^k$, then

$$\varprojlim{}^i(F) \cong \begin{cases} Hom_{Aut_{\mathcal{A}}(E,\psi)}(St_E, F(E)) & : \quad i = k+1 \\ 0 & : \quad i \neq k+1 \end{cases}$$

Here, St_E denotes the Steinberg representation of $GL(E)$.
(2) If $F : \mathcal{A} \longrightarrow \mathbb{Z}_{(p)} - mod$ is any covariant functor, then $\varprojlim{}^*(F)$ is the homology of a cochain complex

$$0 \longrightarrow \prod_{(E,\psi)_{\mathrm{rank}1}} Hom_{Aut_{\mathcal{A}}(E,\psi)}(St_E, F(E)) \longrightarrow$$

$$\longrightarrow \prod_{(E,\psi)_{\mathrm{rank}2}} Hom_{Aut_{\mathcal{A}}(E,\psi)}(St_E, F(E)) \longrightarrow \ldots$$

In particular, $\varprojlim{}^i(F) = 0$ for $i \geq p - rk(G)$ (if $\mathcal{A} = \mathcal{A}_p(G)$) or for $i \geq dim(H^*(X;\mathbb{Z}/p))$ (if $\mathcal{A} = \mathcal{A}_p(X)$).

STEWART PRIDDY:
A Generalization of Swan's Theorem for Group Cohomology

Let P be a finite p-group. We say P satisfies condition (A) if

(A) $[P, \Omega_{k+1}P] \leq \Omega_k P$ for all $k \geq 0$.

Here $\Omega_k P$ is the subgroup of P generated by all elements of order $\leq p^k$.

Theorem A If P satisfies condition (A) and G is any finite group with P as p-Sylow subgroup, then

$$H^*(BG;\mathbb{Z}/p) \cong H^*(BP;\mathbb{Z}/p)^W$$

where W is the Weyl group $W = N_G(P)/P$ and $N_G(P)$ is the normalizer of P in G.

Swan proved the same result under the hypothesis that P is abelian (1960).

Theorem B Almost all p-groups satisfy condition (A).

Theorem A uses the Segal conjecture to show the transfer is a weak map. Then $H^*(BP;\mathbb{Z}/p)$ is only influenced by the automorphisms of P, in particular the Weyl group. Theorem B is related to the following theorem which is actually the main part of our work.

Theorem C BP is stably indecomposable for almost all finite p-groups P.

(joint work with H. W. Henn)

YULY RUDYAK:
On the Mahowald Conjecture that the Spectrum k is Not a Thom Spectrum

Main Theorem The spectrum kO, as well as the spectrum k, is not a Thom spectrum of any stable sperical fibration, i.e. of any map $X \longrightarrow BG$.

Let p be an odd prime, and let $Q_0 = \beta$, $Q_1 = \beta P^1 - P^1\beta$ are the elements of the Steenrod algebra $\mathcal{A} = \mathcal{A}_p$. The proof of the Main Theorem is based on the following

Basic Theorem Let l be any connected $\mathbb{Z}_{(p)}$-local spectrum of finite $\mathbb{Z}_{(p)}$-type such that $\pi_0(l) = \mathbb{Z}_{(p)}$ and $H^*(l;\mathbb{Z}/p) = \mathcal{A}/\mathcal{A}(Q_0,Q_1)$. There is no morphism $f : l \longrightarrow TBSPL_{(p)}$ such that

$$f^* : H^0(TBSPL;\mathbb{Z}_{(p)}) \longrightarrow H^0(l;\mathbb{Z}_{(p)})$$

is epic.

HAL SADOFSKY:
A Chromatic Version of Lin's Theorem

Let X be a finite CW-complex. A theorem of W. H. Lin implies the statement $holim(\mathbb{R}P^{\infty}_{-k} \wedge X) = \Sigma^{-1}X^{\hat{}}_2$. Here $\mathbb{R}P^{\infty}_{-k} = (\mathbb{R}P^{\infty})^{-k\cdot\xi}$ is the Thom spectrum of $-k$ times the tautological bundle over $\mathbb{R}P^{\infty}$. At odd primes there is a generalization (due to Gunawardena): $holim((B\Sigma_p)_{(p)-k} \wedge X) = \Sigma^{-1}X^{\hat{}}_p$ for $(B\Sigma_p)_{(p)-k}$ defined appropriate.
Let L_n be Bousfield localization with respect to $E(n)$, where $E(n)$ is given by $E(n)_* = \mathbb{Z}_{(p)}[v_1,\ldots,v_{n-1},v_1,v_n{}^{-1}]$. Let P_{-k} be $\mathbb{R}P^{\infty}_{-k}$ or $(B\Sigma_p)_{(p)-k}$ as p is 2

or odd. Hopkins and Mahowald conjecture:

$$\lim_{\leftarrow}(P_{-k} \wedge L_n X) = \Sigma^{-1} L_{n-1}(X_p^{\hat{}}) \vee \Sigma^{-2} L_{n-1}(X_p^{\hat{}})$$

This should be considered a version of the slogan "the Mahowald invariant should convert v_i-periodicity to v_{i+1}-periodicity".
It is easy to verify this conjecture in the following cases: $n = 0, n = 1, X$ such that $L_{n-1}X$. We show it is true for $n = 2, p \geq 5$. Our technique is to show (where $M(p)$ is the mod p Moore spectrum):

Proposition $\lim_{\leftarrow}(P_{-k} \wedge M(p) = \Sigma^{-1} L_1 M(p) \vee \Sigma^{-2} L_1 M(p)$.

Our methods use a comparison to Shimomura's v_i-Bockstein spectral sequence calculation.
(joint work with M. Hopkins)

KATSUMI SHIMOMURA:
Relations between a Mahowald Spectral Sequence and the Universal Greek Letter Map

Considering the spectra BP and $E(n)$, we have the Mahowald spectral sequence in the sense of H. Miller:

$$E_2^{s,t} = Ext^s_{BP_*BP}(BP_*, Ext^t_{E(n)_*E(n)}(E(n)_*, E(n)_*(BP))) \Rightarrow Ext^{s+t}_{E(n)_*E(n)}(E(n)_*, E(n)_*)$$

On the E_2-term, we have:

Theorem A

$$Ext^t_{E(n)_*E(n)}(E(n)_*, E(n)_*(BP)) = \begin{cases} BP & : t = 0 \\ N_0^{n+1} & : t = n \\ 0 & : else \end{cases}$$

Here N_0^{n+1} is defined inductively by:
$N_0^0 = BP$, $0 \longrightarrow N_0^i \longrightarrow v_i^{-1} N_0^i \longrightarrow N_0^{i+1} \longrightarrow 0$ is exact.

This and the Mahowald spectral sequence give:

Theorem B
(1) $Ext^s_{BP_*BP}(BP_*, N_0^{n+1}) \cong Ext^{s+n+1}_{BP_*BP}(BP_*, BP_*)$ if $n < p-1$ and $s > n^2$.
(2) $Ext^s_{BP_*BP}(BP_*, BP_*) \cong Ext^s_{E(n)_*E(n)}(E(n)_*, E(n)_*)$ if $n > s$.

The first result (1) of theorem B is induced from the differential d_{n+1} of the Mahowald spectral sequence. Similar result is obtained by Miller, Ravenel and Wilson using the universal Greek letter map η. Here we get the relation between them:

Theorem C $d_{n+1} = (-1)^{n+1}\eta$.

These results are applied to the stable homotopy of a Bousfield localization of a spectrum.
(joint work with M. Hikida)

VLADIMIR A. SMIRNOV
The Cohomology of the Steenrod Algebra

Let S_∞-acyclic operad with generator $v_i \in S_\infty$ of dimension i, $dv_i = v_{i-1} + (-1)^i v_{i-1}T$, $T \in \Sigma_2$ and $\pi_i \in S_\infty(i+2)$, $d\pi_i = \Sigma(-1)^\epsilon \gamma(\pi_k \otimes 1 \otimes \ldots \otimes \pi_{i-k-1} \otimes \ldots \otimes 1)$. All relations between these operations follows from acyclic condition.

Theorem On the cohomology of the Steenrod algebra there exists the structure of S_∞-algebra and as S_∞-algebra it has one generator h_0 of dimension 1 and relations generated by the folowing:

$$\pi_i(h_0, h_1, \ldots, h_{i+1}) = 0, \quad h_{i+1} = h_i \cup_i h_i$$

JEFF SMITH:
An Approach to Constructing Morava K-Theories

Let B denote a sub Hopf algebra of the mod p Steenrod algebra. Let $A//B$ denote the quotient of A by the left ideal generated by B_+, the elements of positive degree. We hope to show that the geometric problem of constructing an A_∞- ring spectrum R with $H^*(R) = A//B$ is equivalent to an algebraic problem in the category of functors, $Funct(\mathcal{V}, \mathcal{V})$, where $\mathcal{V}$ denotes the category of $\mathbb{F}_p$-vector spaces. To solve the algebraic problem one must construct a simplicial comonad $T.$, satisfying:
(1) $T.$ is homotopy linear.
(2) $\pi T. = B_*$ as a module over the algebra of homotopy operations in the category of homotopy linear simplicial functors.

It is a separate computation, that the ring of operations is isomorphic to A_*, the dual Steenrod algebra.

NOBUAKI YAGITA:
Representations of the Steenrod Algebra

We construct a map of Hopf algebras

$$\theta : K[U_r] \longrightarrow P(n-2)^*$$

where $K[U_r]$ is the coordinate ring of r-th Frobenius kernel of the maximal unipotent subgroup U in GL_n and where $P(n-2)^*$ is the dual of the subalgebra of the Steenrod algebra generated by $P^1, \ldots, P^{p^{(n-2)}}$. Therefore we get a K-algebra embedding

$$P(n-2) \hookrightarrow Dist(U_n)$$

Using this we show if a weight λ is p-regular, then the Weyl module $\nabla(\lambda)$ is generated only by $P(n-2)$. Moreover we consider some embedding of $\nabla(\lambda)$ into products of $H^*(\mathbb{C}P^\infty)$, which are closely related to GL_n-simplicity of $\nabla(\lambda)$. (joint work with M. Kaneda, N. Shimada and M. Tezuka)

Berichterstatter: Frank Neumann

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Tagungsteilnehmer

Prof.Dr. Alejandro Adem
Dept. of Mathematics
University of Wisconsin-Madison
Van Vleck Hall
480 Lincoln Drive

Madison , WI 53706
USA

Prof.Dr. Frederick R. Cohen
Dept. of Mathematics
University of Rochester
Ray P. Hylan Building

Rochester , NY 14627
USA

Prof.Dr. Jaume Aguadé
Departamento de Matematicas
Universidad Autonoma de Barcelona
Campus Universitario

E-08193 Bellaterra Barcelona

Prof.Dr. Tammo tom Dieck
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

W-3400 Göttingen
GERMANY

Dr. Dominique Arlettaz
Institut de Mathématiques
Université de Lausanne

CH-1015 Lausanne

Prof.Dr.Dr.h.c. Albrecht Dold
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288/294

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Martin Bendersky
Mathematics, Graduate School
University Center
City University of New York
33 West 42nd Street

New York 10036
USA

Prof.Dr. Emmanuel Dror Farjoun
Institute of Mathematics and
Computer Science
The Hebrew University
Givat-Ram

91904 Jerusalem
ISRAEL

Prof.Dr. Carlos Broto
Departamento de Matematicas
Universidad Autonoma de Barcelona

E-08193 Bellaterra Barcelona

Dr. Giora Dula
Department of Mathematics
Bar-Ilan University

52900 Ramat Gan
ISRAEL

Dr. Mohamed Er-Rokh
Département de Mathématiques
Faculté des Sciences de Tunis
Le Campus Universitaire

1060 Tunis
TUNISIA

Prof.Dr. John Patrick C. Greenlees
Dept. of Mathematics
University of Sheffield
Hicks Building
Hounsfield Road

GB- Sheffield , S3 7RH

Prof.Dr. Mark Feshbach
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.

Minneapolis , MN 55455
USA

Dr. Hans-Werner Henn
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288/294

W-6900 Heidelberg 1
GERMANY

Dr. Vincent Franjou
Mathématiques
Université de Paris Sud (Paris XI)
Centre d'Orsay, Bâtiment 425

F-91405 Orsay Cedex

Prof.Dr. Steve Hutt
Dept. of Mathematics
University of Wisconsin-Madison
Van Vleck Hall
480 Lincoln Drive

Madison , WI 53706
USA

Prof.Dr. Paul G. Goerss
Dept. of Mathematics
University of Washington
C138 Padelford Hall, GN-50

Seattle , WA 98195
USA

Prof.Dr. Kenshi Ishiguro
Mathematisches Institut
Universität Göttingen
Sonderforschungsbereich 170
Bunsenstraße 3-5

W-3400 Göttingen
GERMANY

Prof.Dr. Brayton Gray
Dept. of Mathematics
University of Illinois at Chicago
Box 4348

Chicago , IL 60680
USA

Prof.Dr. Stefan Jackowski
Instytut Matematyki
Uniwersytet Warszawski
ul. Banacha 2

02-097 Warszawa
POLAND

Prof.Dr. Karlheinz Knapp
Fachbereich 7: Mathematik
U-GHS Wuppertal
Gaußstr. 20
Postfach 10 01 27

W-5600 Wuppertal 1
GERMANY

Prof.Dr. Joseph A. Neisendorfer
Dept. of Mathematics
University of Rochester
Ray P. Hylan Building

Rochester , NY 14627
USA

Prof.Dr. Jean Lannes
Centre de Mathématiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Frank Neumann
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

W-3400 Göttingen
GERMANY

Prof.Dr. John Martino
University of Virginia
Department of Mathematics
Madison Hall

Charlottesville VA 22903
USA

Dr. Dietrich Notbohm
Mathematisches Institut
SFB 170
Universität Göttingen
Bunsenstr. 3-5

W-3400 Göttingen
GERMANY

Prof.Dr. Haynes Miller
Dept. of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139
USA

Prof.Dr. Robert Oliver
Matematisk Institut
Aarhus Universitet
Ny Munkegade
Universitetsparken

DK-8000 Aarhus C

Prof. Dr. Jesper Moller
Mathematical Institute
University of Copenhagen
Universitetsparken 5

DK-2100 Copenhagen

Prof.Dr. Erich Ossa
Fachbereich 7: Mathematik
U-GHS Wuppertal
Gaußstr. 20
Postfach 10 01 27

W-5600 Wuppertal 1
GERMANY

Prof.Dr. Franklin P. Peterson
Dept. of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139
USA

Prof.Dr. Hal Sadofsky
Dept. of Mathematics
Lunt Hall
Northwestern University
2033 Sheridan Road

Evanston , IL 60208-2730
USA

Prof.Dr. Mihail M. Postnikov
Steklov Mathematical Institute
MIAN
Academy of Sciences
42, Vavilova str.

Moscow 117 966 GSP-1
RUSSIA

Prof.Dr. Katsumi Shimomura
Dept. of Mathematics
Faculty of Education
Tottori University

Tottori 680
JAPAN

Prof.Dr. Stewart B. Priddy
Dept. of Mathematics
Lunt Hall
Northwestern University
2033 Sheridan Road

Evanston , IL 60208-2730
USA

Prof.Dr. William M. Singer
Department of Mathematics
Fordham University

Bronx , NY 10458
USA

Prof.Dr. Dieter Puppe
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288/294

W-6900 Heidelberg 1
GERMANY

Dr. Vladimir A. Smirnov
Jaroslavskoye Chaussee 55-77

Moscow 129337
RUSSIA

Prof.Dr. July B. Rudjak
Universität Heidelberg
Mathematisches Institut
Im Neuenheimer Feld 288

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Jeff Smith
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907
USA

Dr. Stephan Stolz
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

W-5300 Bonn 3
GERMANY

Prof.Dr. Zdzislaw Wojtkowiak
Université de Nice
Mathématiques
Parc Valrose

F-06034 Nice Cedex 1

Prof.Dr. Vladimir V. Vershinin
Institute of Mathematics
Siberian Branch of the Academy of Sciences
Universitetskiy Prospect N4

Novosibirsk 630090
RUSSIA

Prof.Dr. Nobuaki Yagita
Musashi Institute of Technology
Department of Mathematics
Tamazutsumi Setagaya

Tokyo 158
JAPAN

Prof.Dr. Friedhelm Waldhausen
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

W-4800 Bielefeld 1
GERMANY

Prof.Dr. Said Zarati
Département de Mathématiques
Faculté des Sciences de Tunis
Le Campus Universitaire

1060 Tunis
TUNISIA