MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 10/1992

Mathematische Stochastik

8.3. bis 14.3.1992

Die Tagung fand unter der Leitung von G. Kersting (Frankfurt) und R. Grübel (Delft) statt. Ihr Konzept, sowohl Wahrscheinlichkeitstheoretiker als auch mathematische Statistiker zu gemeinsamer wissenschaftlicher Diskussion einzuladen, hat sich erneut bewährt. Thematische Schwerpunkte waren

— stochastische Analysis, zufällige Bäume

— inverse Probleme, empirische Prozesse und bootstrap.

Die weiteren Vorträge behandelten ein weites Spektrum von Einzelthemen und reichte von großen Abweichungen, Erzeugung von Zufallszahlen in Kryptographie und Statistik bis zur Statistik von dünnbesetzten Daten. Sie stießen auf breites Interesse.

An der Tagung nahmen insgesamt 42 Wissenschaftler aus 12 Ländern teil.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

VORTRAGSAUSZÜGE

D. Aldous

Random Trees

The "Continuum Random Tree" is an interesting object which arises in 4 ways

(1) Take a Galton–Watson branching process, condition on total population size $= n$, draw the family tree with edge–lengths $= \frac{1}{\sqrt{n}}$, then let $n \to \infty$.

(2) Via a direct construction from Brownian excursion of duration 1.

(3) Via a direct construction involving cutting the interval $[0, \infty)$ into random segments, and reassembling the segments into a tree.

(4) The particular distribution of this tree can be specified by giving (for each k) the distribution of the spanning subtree on k randomly–chosen points. The distribution has a simple form: the topological shape of the tree is independent of the edge–lengths $(L_1, \ldots, L_{2k-1})$, and their density is

$$f(\ell_1, \ldots, \ell_{2k-1}) = c_k \, s e^{-s^2/2}, \quad s = \sum \ell_i.$$

T. Arak

Polygonal Markov fields and their simulation

The first examples of polygonal Markov fields on $\mathbb{R}^2$ were given in [1], and the notion in whole generality was introduced in [2]. There exists a subclass of polygonal fields called "consistent". The fields of this subclass are Markovian not only in two–dimensional sense but also in one–dimensional sense on every straight line $\ell \subset \mathbb{R}^2$. Some of their probabilistic characteristics can be expressed by explicit formulae. Consistent fields can be described in terms of evolution of a system of particles on $\mathbb{R}^1$, and this fact makes them easy to simulate on the computer.

The simulation of a nonconsistent field can be based on its description as a stationary measure of some special Markov process, defined on the space of all realizations (of polygonal fields). Two algorithms of such kind are described.

[1] Arak, T.: *On Markovian random fields with finite number of values*, 4th USSR–Japan Symposium on Prob. Theory and Math. Stat. Abstracts of communications. Tbilisi 1982.

[2] Arak, T., Surgailis, D.: *Markov fields with polygonal realizations*, Probab. Th. Rel. Fields **80**, 543–579 (1989).

S. Asmussen

Wiener–Hopf identities and maxima for some dependent processes

We consider the problem of computing the distribution of the maximum M and the ascending ladder height of a Markov additive process $\{S_t\}_{t\geq 0}$ defined on a Markov process $\{J_t\}_{t\geq 0}$ with continuous time and a finite state space E. Important tools are time reversal, occupation measures and an auxiliary Markov process $\{m_x\}_{x\geq 0}$ obtained by observing $\{J_t\}$ only when $\{S_t\}$ is at a minimum at $-x$. Solutions are obtained subset to some weak conditions on the jumps in one direction, say that the paths are downwards skip-free as is the case in the MAP's arising in queues and fluid flow models. The crucial step in the algorithm is the evaluation of the intensity matrix Q of $\{m_x\}$ and is performed by solving a fixpoint problem $Q = \psi(Q)$ iteratively.

V. Bentkus

On smoothness conditions and convergence rates in the CLT in Banach spaces

In Banach spaces estimates $O(n^{-1/2})$ of the rate of convergence in the Central Limit Theorem are known. These estimates are obtained for the probability of a set provided the boundary of the set is thrice differentiable. We show that in general one can not weaken this condition. Thus the situation in the infinite dimensional spaces differs completely from that in finite dimensional spaces, where the convexity of sets is sufficient for the rate $O(n^{-1/2})$. Similar results hold for the expectation of smooth functionals, where the rate $O(n^{-1/2})$ can be proved only under the condition that the functional is three times differentiable.
(joint work with F. Götze, Bielefeld)

N.H. Bingham

Applications of Large Deviations

[Professor Bingham was prevented at a late stage from attending the meeting, and his talk was presented by C.M. Goldie].
The talk presents (I) a large–deviation theorem, (II) applications to branching processes, (III) applications to number theory. I and II are joint work with J.D. Biggins.
In I the theorem is a variant of Ellis' Large Deviation Theorem, giving convergence of $-\log P(Y_n \geq a_n x)/a_n$ to a limit $k^*(x)$ when the Y_n are random variables for which $a_n^{-1} \log Ee^{sY_n} \to k(s)$ $(n \to \infty)$ for a sequence of constants $a_n \to \infty$.
In II some results are deduced for the almost–sure limit random variable W of a supercritical Bienaymé–Galton–Watson process, suitably normed. When the maximum family size is $d < \infty$ the right–hand tail of W is proved to decay exponentially. When the minimum family size is $m \geq 2$ a similar result is established for the left–hand tail.
In III divisor functions are treated. It is proved that $\log \frac{1}{n} \sum_1^n 1\{\omega(m) > (<)\, a \log\log n\} \sim -(\log\log n)(a \log a - a + 1)$ as $n \to \infty$, for $a > 1$, $(0 < a < 1)$. A similar result is given for Ω, for $0 < a < 2$.

L. BIRGÉ

Estimation of integral functionals of quadratic type of a density

From n i.i.d. observations of an unknown density f, belonging to some smoothness class $\mathcal{F}_{s,A}$ defined on $[0,1]$ by

$$\mathcal{F}_{s,A} = \left\{ f \mid |f^{(m)}(x) - f^{(m)}(y)| \leq A|x-y|^\alpha \right\} \quad s = m + \alpha$$

we want to estimate $\int \varphi(f)$ where φ is a smooth function. This can be done at rate $1/\sqrt{n}$ when $s \geq 1/4$. For $s < 1/4$ one cannot estimate at a rate better than $n^{-\frac{4s}{1+4s}}$. (joint work with P. MASSART, University of Paris-Sud).

E. BOLTHAUSEN

Critial large deviations in a Gaussian model with a continuous symmetry

Critical large deviations for the Ising model have recently attracted much attention, e.g. in works by R. Schonman, Föllmer–Ort, Dobrushin–Kolecky–Shlosman. These results show that within the phase transition region, the appropriate large deviation principle estimates probabilities in terms of the surface and not the volume of a box: A simple Gaussian model is presented where large deviations have probabilities decaying still at a smaller order, namely $\exp(-n^{d-2}I)$, where n^d is the volume of the box.
(joint work with J.D. DEUSCHEL, ETH Zürich)

R. BUCKDAHN

Linear Skorohod stochastic differential equations

Let σ and b be bounded processes on the Wiener space $(\Omega, \mathcal{F}, P)$, $\Omega = C([0,1])$, which are possible anticipating the Brownian motion $W_t(\omega) = \omega(t)$, and let G be a bounded random variable. Then there is a unique solution X of the linear S.D.E. with Skorohod integral

$$X_t = G + \int_0^t \sigma_s(X_s)dW_s + \int_0^t b_s(X_s)ds\ , \quad 0 \leq t \leq 1\ , \tag{1}$$

under rather weak assumptions on σ and no additional requirements on b and G. The description of the solution X requires to study the family of $\{T_t, 0 \leq t \leq 1\}$ of possibly anticipating transformations T_t of Ω into itself associated to (1) by the equation

$$T_t\omega = \omega + \int_0^{t\wedge \cdot} \sigma_s(T_s\omega)ds\ , \quad \omega \in \Omega\ , \quad 0 \leq t \leq 1\ . \tag{2}$$

If σ is nonanticipating, the solution X of (1) has the form

$$X_t = G\left(\omega - \int_0^{t\wedge\cdot} \sigma_s(\omega)ds\right)$$

$$\exp\left\{\int_0^t \sigma_s(\omega)dW_s(\omega) - \frac{1}{2}\int_0^t \sigma_s^2(\omega)ds + \int_0^t b_s\left(\omega - \int_{s\wedge\cdot}^{t\wedge\cdot} \sigma_r(\omega)dr\right)ds\right\} \quad (3)$$

Choosing $G(\omega) = \text{sign}(\omega_1)$, $\sigma_s(\omega) \equiv 1$, $b_s(\omega) \equiv 0$, we see that the solution process

$$X_t = \text{sign}(W_1 - t)\exp\{W_t - t/2\}\ , \quad 0 \le t \le 1\ , \quad (4)$$

jumps and changes the sign on $\{0 < W_1 < 1\}$ at time $t = W_1$. This shows that, in difference to the nonanticipating case, the Skorohod integral process can have jumps. Reference: BUCKDAHN, R.: *Linear Skorohod stochastic differential equations*, Probab. Th. Relat. Fields **90**, 223–240 (1991). Please, find further references there.

I. CUCULESCU

The infinite tensor product of conditional mean values in noncommutative probability

The conditional mean values are mappings from a von Neumann algebra to a von Neumann subalgebra, in the sense of Umegaki. We show the existence of the infinite tensor product of such conditional mean values. The finite case was established by F. Combes and C. Delaroche in Bull. Soc. Math. France 103 (1975) Nr. 4.

R. DAHLHAUS

Modelling of locally stationary processes

Locally stationary processes are defined as processes that have a time varying spectral representation. We model those processes e.g. by autoregressive processes with time varying coefficients. As a goodness of fit measure the Kullback–Leibler distance is used. We study the asymptotic behaviour of parameter estimates without assuming that the true process lies in the model class. Furthermore, the distance of the fitted model to the true unknown process is studied. As an example we show the analysis of sound data.

L. DAVIES

Unimodal models

Let $\mathcal{X}$ be a sample space and x a generic point of $\mathcal{X}$, $\mathcal{P}$ the model space whose elements $\mathbb{P}$ are distributions over $\mathcal{X}$. A data feature $\mathcal{F}$ is a mapping of $X \times \mathcal{P}$ into $\{0,1\}$. We denote by $X(\mathbb{P})$ a random element of $\mathcal{X}$ with distribution $\mathbb{P}$. $\mathcal{F}$ is called an α–feature if $\mathbb{P}(\{x : \mathcal{F}(x,\mathbb{P}) = 1\}) \ge \alpha$ for all $\mathbb{P} \in \mathcal{P}$. It is clear that if $\mathcal{F}_j$ is

an α_j-feature, $1 \le j \le k$, then $\mathcal{F} = \prod_1^k \mathcal{F}_j$ is an α-feature with $\alpha = 1 - \sum_1^k (1 - \alpha_j)$. Given a feature $\mathcal{F}$ we define the adequacy region $\mathcal{A}(x, \mathcal{F})$ of the data point x by $\mathcal{A}(x, \mathcal{F}) = \{\mathbb{P} : \mathcal{F}(x, \mathbb{P}) = 1\}$.
Using features based on (i) the Kuiper metric (ii) the behaviour of the extreme statistics and (iii) a modified dip statistic we investigate several data sets from the statistical literature. A similar analysis is performed using unimodal models $\mathbb{P}$ with at most 3 dips and bumps.

H. DEHLING

Log-density limit theorems

Our research originates from the question whether one can observe the CLT along a single path of the normalized random walk $S_k/\sqrt{k}$. Brosamler, Fisher and Schatte proved independently in 1988 that the logarithmic density of the set of integers for which $S_k/\sqrt{k} \le x$ equals $\Phi(x/\sigma)$. Where Φ is the standard normal d.f. (Pathwise CLT).
For a function $f : \mathbb{N} \to \mathbb{R}$ we define the log-average by $\mu_L(f) = \lim_{N\to\infty} \frac{1}{\log N} \sum_{k=1}^N \frac{1}{k} f(k)$.
For a set H the log-density $\mu_L(H)$ is defined by $\mu_L(1_H)$.
Theorem : Let $X_1, X_2, \ldots$ be independent r.v's, $a_n > 0$ and define $S_n = \sum_{k=1}^n X_k$. Suppose $a_\ell / a_k \ge (\ell/k)^\gamma$ $(\ell \ge k)$ for some $\gamma > 0$ and that $\mathbb{E}\,|S_n/a_n|^p \le e^{(\log n)^{1-\varepsilon}}$, $\varepsilon > 0$. If G is a d.f., then the following are equivalent

(I) $\mu_L(\{k : \frac{S_k}{a_k} \le x\}) = G(x) \quad \forall \quad x \in C_G$ (a.s.)
(II) $\mu_L(P(\frac{S_k}{a_k} \le x)) = G(x)$ "

As corollaries we obtain extensions of the pathwise CLT to stable convergence, conditions for the Law of Large Numbers in log-density, as well as strong approximations by Wiener and stable processes in log-density.
(joint work with I. BERKES, Budapest)

W. EHM

A class of quasi-Bayesian procedures for sparse discrete data

Statistical models for sparse discrete data typically involve unbalanced designs and/or many parameters, which tend to be "close to the boundary". For such cases, and for use with a class of linear exponential family models, we propose an approach in which the asymptotics is driven by "critical quantities". These quantities reflect properties of leading terms and are used to control remainder terms in stochastic expansions of statistics of interest.
It is shown that for certain likelihood type procedures confidence statements remain valid under more general conditions if the likelihood function is replaced by the posterior density w.r.t. specifically chosen priors which are modifications of the well-known Jeffreys' prior. In particular, we discuss properties of the Jeffreys' posterior mode estimator and the "posterior ratio test" within the extended asymptotic setting indicated above.

H.J. Einmahl

Local processes (Empirical and quantile processes in a shrinking neighbourhood of a point)

The most simple example of the local processes we consider is the so called tail empirical process

$$w_n(t) = (n/a_n)^{1/2}(\Gamma_n(ta_n) - ta_n),\ 0 \leq t \leq 1\ ,$$

where $\{a_n\}_{n=1}^{\infty}$ is a sequence of numbers in $(0,1)$ which converges sufficient slowly to 0 and Γ_n the uniform empirical distribution function at stage n.
More general we will consider weak convergence of empirical and quantile processes in the neighbourhood of a point in $\mathbb{R}^d$. Dependent on the choice of $\{a_n\}_{n=1}^{\infty}$ (and a weight function) the limiting process will be Gaussian or Poisson.
The material generalizes recent results in the literature and finds application in local statistical procedures like statistics of extreme values and density estimation.

R. Gill

Cryptography, Statistics and the Propagation of Randomness

We survey the approach to random number generation developed in the last ten years by workers in cryptography (Bhem, Micali, Yao, Levin, ...), and argue its relevance to statistics and probability. In fact the approach measures the quality of random number generators in a way exactly appropriate for their use in large statistical simulation experiments. The approach also uses some nice probabilistic techniques (coupling, time reversal, ...). From the statistical point of view we propose some areas for future research (cryptographic security of classical generators, improvement by combination of 'independent' generators, ...):
Reference: S. Brands, The cryptographic approach to pseudo-random bit generation, Cryptography group, CWI Amsterdam.

C.M. Goldie

Tail decay of stationary solutions of tree-structured random recurrences

A theorem is presented that gives the tail decay of the stationary solution, assumed to exist, of a class of random recurrence relations. It is specialized to the case of the 'linear multiplicative transform', leading to a result extending those of Durrett and Liggett, and Guivarc'h.

P. Groeneboom

Inverse problems

In statistics one usually denotes by "inverse problems" the situation where one wants to estimate (in a non- or semiparametric way) a probability distribution on a basis of indirect or censored data and where the usual machinery of $\sqrt{n}$-asymptotics and asymptotic normality

is not available. A typical example is the "Wicksell problem", where one tries to estimate the distribution of radii of balls on the basis of observations of sections of these balls. Wicksell derived in 1925 an (Abel–type) integral equation, describing the relation between the distribution of the radii of the balls and the distribution of the radii of the sections of these balls. This integral equation is a well-known example of an inverse (or "ill-posed") problem in numerical analysis.
Other examples of inverse problems are: *interval censoring* and *deconvolution*. These problems have recently received much attention in the context of AIDS research, where on tries to make predictions using both interval censored data and data which are only indirectly observable via convolutions. We discuss nonparametric maximum likelihood estimators (NPMLE's) in this context and formulate some conjectures about their properties.

R. GRÜBEL

A piece of statistical archaeology

We introduce a new multivariate version of the median that shares the equivariance properties of the spatial median, but seems to behave better for heavy–tailed distributions. Its asymptotic properties are derived, and the procedure is illustrated by two numerical examples. The original motivation for such procedures seems to have been the determination of population centers (US at about 1900, Italy in the 1930's).

H.G. KELLERER

Ergodic theorems for affinely-recursive stochastic sequences

The affine recursion $X_n = Y_n X_{n-1} + Z_n$, $n \in \mathbb{N}$, with state space $\mathbb{R}_+$ is classified according to its Markovian properties (transience, null recurrence, positive recurrence) in the topological framework. Main result in the recurrent case is the existence and uniqueness of a locally finite invariant measure, which yields mean and pointwise ergodic theorems. Moreover, under weak boundedness conditions on Z with respect to Y one obtains a bijection between the asymptotic behaviour of the associated random walk $S_n = \sum_{1 \leq m \leq n} \log Y_m$ and the Markovian behaviour of the sequence $(X_n)_{n \geq 0}$.

U. KRENGEL

Speed limits on the circle

The motivation of the above topic is to study a class of examples of "generalized" Markov operators. In the Markov case, particles move with transition probabilities depending only on their location. If one wants to admit interaction, one is led to operators in L_1^+ (an integr. function is a mass distribution) which preserve the order and integrals. There is now a deep generalization of the Birkhoff ergodic theorem for such operators, proved by R. Wittmann. An interesting class of such operators is provided by generalized measure preserving transformations. Here we study mainly another class defined on the line or on

the circle. It is determined by a speed limit function φ taking only finitely many values, and piecewise constant. Mass moves according to the following rules: 1) The total mass is preserved; 2) The particles move in a fixed direction as fast as possible but never faster than the speed limit allows; 3) There is never an increase of potential energy. It is shown that there is a well defined semigroup of order preserving, positively homogeneous operators in L_1^+, say $\{T_t,\ t \geq 0\}$. On the circle, if there are at least 2 speeds, $T_t f$ converges exponentially fast to a constant (for bounded f). On the line, for $f \in L_1^+ \cap L_\infty$, there is uniform convergence of $T_t f$ to 0, provided the intervals of constancy of the speed limit are bounded below and above.
(joint work with B. Fernow)

H.R. LERCHE

Prediction prisms

Two geometric bodies were shown. The first describes a random prediction procedure for data taking values in three categories. The second is a puzzle arising from the first by some more cuts. The procedure has the property that for i.i.d. random data it predicts asymptotically as good as if the probabilities of the outcomes were known beforehand. For all other data the procedure does a least as good as in the i.i.d. case. The prediction procedure is a natural generalization of Blackwell's for two categories.

J. MICHALEK

Detection of changes in the behaviour of random sequences

In the contribution the author wants to present a method detecting changes in the behaviour of a locally stationary random sequence when a possible change can occur in a mean value jump or in a spectral characteristic, too. The method is based on an approximation of an observed sequence by a suitable autoregressive model and a statistic yielding the information on a possible change is derived from the asymptotic I-divergence rate between Gaussian probability measures. The presented method will be documented by practical results on PC's.

D.W. MÜLLER

Minimax Correlation Methods

A method is proposed to assess "goodness of fit" to "estimates" of a (conditional) α-quantile regression curve $(0 < \alpha < 1)$. Let P be an absolutely continuous probability measure on $\mathbb{R}^k \times \mathbb{R}^1$, and let g_α be its conditional α-quantile curve, $P[y < g_\alpha(x) \mid x] = \alpha$. We treat the case where g_α is totally unknown. The method consists in fixing two entities

(1) a class $\mathcal{H}$ of curves ℓ (to work with)

(2) a family $\mathcal{C}$ of ("forbidden") residual patterns C
($\equiv$ measurable subsets of the design space $\mathbb{R}^k$.)

These entities are related by the condition that $C^* = \{x : \ell(x) > g_\alpha(x)\} \in \mathcal{C}$. As **a measure of fit**, one considers the quantity $E(\alpha, \ell) \equiv P[y$ is between $\ell(x)$ and $g_\alpha(x)]$. This quantity can be estimated in a natural way from an i.i.d. sample $(x_1, y_1), \ldots, (x_n, y_n)$ from P: i.e. by $E_n(\alpha, \ell)$. For a hypothetical curve ℓ, the value $E_n(\alpha, \ell)$ is then compared to the distribution of $D_n \equiv \min_{\ell \in \mathcal{H}} E_n(\alpha, \ell)$, where $g_\alpha \in \mathcal{H}$ (this is the case where the best fit occurs). In many models, this distribution depends only slightly on the nuisance parameters (e.g. the form of marginal distributions of P). This is shown by extensive simulations. An asymptotic theory explaining part of this phenomenon is developed.
The main technical problems of the proposed procedure consist in the construction of fast (enumerative or stochastic) search algorithms for D_n.

W. Näther

Probabilistic interpretation of some notions in fuzzy theory

Essential notions of fuzzy set theory can be expressed in terms of random sets. For example, the membership function of a fuzzy set A can be considered as one–point–coverage probability of a random set (symb. $A \sim S$). Fuzziness measures of A appear as "variance" terms of S. The specification of set–theoretic operations between fuzzy sets $A \sim S_1$, $B \sim S_2$ (e.g. the intersection) can be thought as specification of a correlation structure between S_1 and S_2. Also for essential classes of fuzzy measures a probabilistic interpretation is possible: Belief– and plausibility measures are capacities and, using Choquet's theorem, can be considered as hit– or miss–probabilities of random sets. Decomposable fuzzy measures in the Archimedean case are "distorted" additive measures (in the special case "distorted" probabilities).

M. Nussbaum

Gaussian Approximation of some Curve Estimation Problems

We are concerned with asymptotic decision theory in some non–$\sqrt{n}$–consistent problems like density and nonparametric regression estimation. It is known that in such problems, which may also be described as inverse ones, the standard approach of *localizing* around a fixed point θ_0 in the parameter space according to $\theta = \theta_0 + n^{-1/2}h$ does not lead to a sensible Gaussian limit experiment, and hence does not yield nontrivial risk bounds. A natural approach then would be to try to approximate the original nonlocalized experiment by a suitable *accompanying Gaussian shift* – the signal recovery problem in white noise, which is in fact known for some time as the model problem for nonparametric curve estimation. An approximation by LeCam's deficiency distance Δ would be desirable, but turns out to be beyond reach at this stage. However, we are nevertheless able to establish risk approximation by the signal recovery problem in some interesting special cases, where the experiment is generated by i.i.d. observations. In this case LeCam's result on approximation in Δ by the *poissonized* version can be utilized, which yields an observed Poisson process on the sample space. Due to the property of independent increments, we may then treat the estimation problem separately for each one of small intervals subdividing the sample space. Furthermore, when the size of these tends to zero, we can employ the result of Low (1989) on *strong* convergence to a Gaussian limit on each interval, when reparametrization is done in a nonstandard fashion by asymptotically delta-shaped alternatives. Thus, for some i.i.d. models and integral-type loss functions which behave additively when the function is broken into pieces, we can automatically transfer asymptotic risk bounds (optimal rates and constants) from the accompanying Gaussian shift experiments.

L. PARTZSCH

Some contributions to the problem of uniformly conditional ergodicity for Markov processes with finite life-time

For "positive R-recurrent" Markov processes $(X_t)_{t\geq 0}$ with topological state space E and a finite life-time ζ conditions are considered, under which we have the convergence to the quasi-stationary distribution μ (i.e. $\lim_{t\to\infty} P_x(X_t \in A|\zeta > t) = \mu(A)$) and to the transition function $\widetilde{P}(t,x,A)$ of the corresponding conditional process (i.e. $\lim_{s\to\infty} P_x(X_t \in A|\zeta > t+s)$) uniformly with respect to $x \in E$.
On the one hand, we use the spectral theory of positive operators, where a Doeblin condition "with upper bound" for the normalized kernel $Q(t,x,A) = e^{\gamma_0 t}\cdot P(t,x,A)$ is involved. On the other hand, classical proofs of J.L. Doob are transfered to substochastic kernels using an appropriately defined ergodic coefficient.

S.M. PITTS

Nonparametric estimation of the stationary waiting time distribution function for the $GI|G|1$ queue

The $GI|G|1$ queueing model is regarded as a functional that maps the service and interarrival time distribution functions onto the stationary wating time distribution function. When random samples from the service and interarrival time distributions are available, a nonparametric estimator of the stationary waiting time distribution function is obtained by applying the functional to nonparametric estimators of the input distributions. Using appropriate continuity and differentiability properties of the functional, we show that strong consistency and asymptotic normality of the input estimators carry over to corresponding properties for the output estimator.

A. RAČKAUSKAS

On the bootstrap for some statistics from empirical measures

We prove some results on estimates of the accuracy of the bootstrap approximation for the

Euclidean norm of empirical processes indexed by finite but increasing number of functions. As a consequence we obtain the bootstrap approximation of the quadratic measure of deviations for the orthogonal series density estimate.
(joint work with F. GÖTZE, Bielefeld)

P. RESSEL

Non-homogeneous DeFinetti-theorems

The classical results of DeFinetti and Schoenberg are closely related to an abstract integral representation theorem for positive definite functions on semigroups. We introduce the new notion of a "positivity forcing" function and use an amended version of the mentioned abstract result for a derivation of a number of non-homogeneous DeFinetti type theorems, generalizing slightly some recent results of Diaconis, Eaton and Lauritzen on mixtures of certain linear models. Finally we show an application to row–column–exchangeability.

L.C.G. ROGERS

The joint law of the maximum and terminal value of a martingale

If $(M_t)_{t\geq 0}$ is a uniformly integrable martingale, $\overline{M}_t \equiv \sup_{u\leq t} M_u$, and $S \equiv \overline{M}_\infty$, $Y \equiv \overline{M}_\infty - M_\infty$, what are the possible joint laws μ of the pair (S, Y) ? It is clear that the conditions

(1) $\int\int |s-y|\mu(ds,dy) \equiv E|M_\infty| < \infty$

(2) $c(s) \geq s$

(3) $c(\cdot)$ is increasing

are necessary conditions (where the function c is defined by $c(s) \equiv E[M_\infty | \overline{M}_\infty > s]$). (The necessity of (3) is a simple proposition.) The main result discussed in this talk is to prove the converse, namely that if a probability μ on $\mathbb{R} \times \mathbb{R}^+$ satisfies (1), (2) and (3), then there exists a UI martingale such that $(S, Y) \sim \mu$. [Note that the conditions (1), (2), (3) can be expressed solely in terms of μ.] The method of proof is a variant of the Skorohod embedding technique of Azéma & Yor. A recent result of R.P. Kertz & U. Rösler (Isr. J. Math. 69) follows as a corollary. The possible joint laws when the martingale M is restricted to be continuous can also be computed, by quite different methods. In the uniformly integrable case, this characterization was first established by P. Vallois.

L. RÜSCHENDORF

On the rate of convergence in the CLT

Two different methods for proving stable limit theorems for sums of independent random variables with values in separable Banach spaces are presented. The first method is based

on an inequality of Woyzinski and is applicable in Banach spaces of type p. The second method is based on convolution metrics and does not need a restriction on the space. Both results are formulated for the Kantorovich metric, which implies rates of convergence results also for the (full) Prohorov metric. The conditions on the domain of attraction are given in metric form. Based on the notion of dependence metrics an extension is given for the stable limit theorems for martingales with values in separable Banach spaces.
(joint work with S.P. Rachev).

M. Scheutzow

An integral inequality

We show that the inequality $\int_{-\infty}^{\infty}\int_{0}^{\infty} g\left(\int_{x}^{x+s} b(v)dv\right) dsdx \le \int_{-\infty}^{\infty} \frac{1}{b(v)}\, dv \int_{0}^{\infty} g(s)ds$ holds true for $g : [0,\infty] \to [0,\infty]$, $b : \mathbb{R} \to [0,\infty]$ both measurable and $g(\infty) = 0$. An application to $g(s) := \exp\left(-\frac{2}{\sigma^2}\, s\right)$, $b > 0$, shows that if the solutions of $\dot{X}(t) = b(X(t))$ explode in finite time, i.e. $\int_0^{\infty} \frac{1}{b(v)}\, dv < \infty$, then so do those of the stochastically perturbed equation $dX(t) = b(X(t))dt + \sigma dW(t)$, where $\sigma > 0$ and $W(t)$, $t \ge 0$, is Brownian motion (using Feller's test of explosion). The key is to show that the function $F(y) := \lambda\left(\left\{(x,s) \in \mathbb{R} \times \mathbb{R}^+ : \int_x^{x+s} b(v)dv \le y\right\}\right)$ has a density w.r.t. Lebesgue measure on $\mathbb{R}^+$ which is bounded above by $\int_{-\infty}^{\infty} \frac{1}{b(v)}\, dv$. We also formulate an integral inequality in a more general set-up.

W. Stute

The SLLN under random censorship

Let $\widehat{F}_n$ denote the sample size n Kaplan–Meier estimator, and let φ be integrable w.r.t. the true survival function F. We show that $\int \varphi d\widehat{F}_n$ converges with probability one and in the mean, under no additional assumptions whatsoever. A necessary and sufficient condition is derived under which the limit is $\int \varphi dF$, the quantity of interest. The result is a key tool for proving consistency of many estimators, as is the classical SLLN for completely observable data.
(joint work with Jane–Ling–Wang, the Univ. of California at Davis).

A. Wakolbinger

Recurrence and transience in time–stationary branching particle systems

In typical examples of time–stationary branching particle systems (TSBPS) like, e.g., critical binary random walk on $\mathbb{Z}^d$, $d \geq 3$, the lines of descent of the individual particles "enter from infinity". We give examples of TSBPS's whose lines of descent are all recurrent.

(1) in the presence of critical branching (and with finite expected particle number at any site)

(2) in the presence of even subcritical branching (and with infinite expected particle number per site)

Consider now the following three properties of a family tree in a TSBPS:

(FT): all rooted forward subtrees of T are transient

(BT): the line of descent of any individual in T is transient

(R): T grows out of a "backbone" which is the path of a recurrent Markov chain

As a general result there holds that each family tree of a TSBPS either is (FT and BT), or (FT and R), or (R), and that each of these three subsystems constitutes a TSBPS for itself. Finally, another aspect of recurrence on the level of family trees is discussed. An example (3) of a TSBPS is given which consists of only one (FT and BT) family tree, which hits any site with a positive frequency in the course of time. In a time stationary critical binary branching Brownian particle system, any family tree hits each fixed ball over an infinite time horizon in dimensions 3 and 4, and over a finite time horizon in dimensions ≥ 5. (joint work with K. MATTHES and R. SIEGMUND–SCHULTZE; in particular, examples (2) and (3) are due to the latter).

H. v. WEIZSÄCKER

Jeffreys' prior as a Hausdorff measure

The following result of W. Doster is reported. It is motivated by the desire to measure the 'size' of a distribution family, e.g. by the number of Hellinger balls need to cover it.

Theorem: Let $(p_\vartheta)_{\vartheta\in\Theta}$, $\Theta \subset \mathbb{R}^k$ open be a family of probability distributions on some observation space Ω. Suppose that the identity $(\Theta$, euclidean metric$) \to (\Theta$, Hellinger metric$)$ is locally Lipschitz. Then the Fisher information matrix $I(\vartheta)$ exists Lebesgue a.e. and the k-dimensional Hausdorff measure on Θ constructed from the Hellinger metric is given by

$$d\mathcal{H}^k_{\text{Hellinger}}(\vartheta) = \sqrt{\det I(\vartheta)}\, d\vartheta\ .$$

This measure does not depend on the parametrization. It is Jeffreys' 'noninformative prior'. The proof of the theorem is based on a **coarea formula** for Lipschitz maps with values in Banach spaces. This technique may be used to construct many similar invariant priors.

J.A. WELLNER

Multiplier CLT's and alternative bootstraps

A general exchangeably weighted "bootstrap" can be described as follows:
Let $X_1, X_2, \ldots, X_n$ be i.i.d. P on $(A, \mathcal{A})$ with empirical measure $\mathbb{P}_n$.
Let $\underline{W}_n = (W_{n1}, \ldots, W_{nn})$ be a random weight vector satisfying

A.1 $\underline{W}_n$ is exchangeable for each n,

A.2 $W_{ni} \geq 0$, $\sum_1^n W_{ni} = n$.

Then for fixed $X_1(w), \ldots, X_n(w)$ the general exchangeably weighted bootstrap empirical measure is $\mathbb{P}_n^W = \frac{1}{n}\sum_1^n W_{ni}\delta_{X_i(w)}$. When $\underline{W}_n = \underline{M}_n \sim$ Multinomial$_n$ $(n, (\frac{1}{n}, \ldots, \frac{1}{n}))$, $\mathbb{P}_n^W$ is Efron's bootstrap. When $\underline{W}_n = (Y_1, \ldots, Y_n)/\overline{Y}_n$ with $Y_1, Y_2, \ldots$ i.i.d. nonnegative, $\mathbb{P}_n^W$ is an "i.i.d. weighted" bootstrap, and, in particular, if the Y's are exponential (1), $\mathbb{P}_n^W$ is Rubin's "Bayesian bootstrap". Many other bootstrap resampling schemes are also included in this formulation.
To validate the general exchangeably weighted bootstrap asymptotically, suppose that the weights also satisfy

A.3 $\lim_{\lambda\to\infty} \limsup_{n\to\infty} \|W_{n1} 1_{[W_{n1}\geq\lambda]}\|_{2,1} = 0$, and

A.4 $\frac{1}{n}\sum_1^n (W_{ni} - 1)^2 \to_p c^2$

where $\|Y\|_{2,1} := \int_0^\infty \sqrt{Pr(|Y| > t)}dt < \infty$.

The following theorem of Jens Praestgaard generalizes results of Giné and Zinn for Efron's bootstrap:
Theorem 1: Suppose that $\mathcal{F} \subset L_2(P)$ satisfies $\mathcal{F} \in M(P)$ (measurability), W_n satisfies A.1 — A.4

A. If $\mathcal{F} \in CLT(P)$ and $P(F^2) < \infty$, then $\sqrt{n}(\mathbb{P}_n^W - \mathbb{P}_n^\omega) \Rightarrow cG_p$ P^∞–a.s.

B. If $\mathcal{F} \in CLT(P)$, then $\sqrt{n}(\mathbb{P}_n^W - \mathbb{P}_n^\omega) \Rightarrow cG_p$ in P^∞–prob. in $\ell^\infty(\mathcal{F})$.

The methods used to prove this theorem yield the following result for Efron's bootstrap with bootstrap sample size $m \neq n$: let $\mathbb{P}_{m,n}^E := \frac{1}{m}\sum_1^n M_{ni}\delta_{x_i(\omega)}$ where $\underline{M}_n \sim$ Multinomial$_n(m, (\frac{1}{n}, \ldots, \frac{1}{n}))$.
Theorem 2: Suppose that $\mathcal{F} \in M(P)$, $\mathcal{F} \in CLT(P)$ and $P(F^2) < \infty$.
Then $\sqrt{m}(\mathbb{P}_{m,n}^E - \mathbb{P}_n^\omega) \Rightarrow G_p$ as $m \wedge n \to \infty$ P^∞–a.s.

Berichterstatter: J. Geiger

Tagungsteilnehmer

Prof.Dr. Vidmantas J. Bentkus
Department of Mathematics
University of Vilnius
Naugarduko 24

232006 Vilnius
LITHUANIA

Prof.Dr. David Aldous
Department of Statistics
University of California
367 Evans Hall

Berkeley, CA 94720
USA

Prof.Dr. Lucien Birgé
L.S.T.A.
45/55 - 3e - Boîte 158
Université Paris VI

F-75252 Paris Cedex 05

Dr. Gerold Alsmeyer
Mathematisches Seminar
Universität Kiel
Ludewig-Meyn-Str. 4

W-2300 Kiel 1
GERMANY

Prof.Dr. Erwin Bolthausen
Institut für Angewandte Mathematik
Universität Zürich
Rämistr. 74

CH-8001 Zürich

Dr. Taivo Arak
Dept. of Mathematics
Chalmers University of Technology
and University of Göteborg

S-412 96 Göteborg

Dr. Rainer Buckdahn
Institut für Stochastik
Fachbereich Mathematik
Humboldt-Universität Berlin
Unter den Linden 6

O-1086 Berlin
GERMANY

Prof.Dr. Soren Asmussen
Institut for elektroniske systemer
Aalborg Universitetscenter
Fredrik Bajers Vej 7

DK-9220 Aalborg

Prof.Dr. Ioan Cuculescu
Dept. of Mathematics
Str. Academiei 14

70109 Bucharest
ROMANIA

Prof.Dr. Rainer Dahlhaus
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

W-6900 Heidelberg 1
GERMANY

Dr. John H.J. Einmahl
Dept. of Mathematics and
Computing Science
Technical University Eindhoven
Postbus 513

NL-5600 MB Eindhoven

Prof.Dr. P. Laurie Davies
FB 6 - Mathematik
Universität-GH Essen
Postfach 10 37 64

W-4300 Essen 1
GERMANY

Jochen Geiger
Fachbereich Mathematik
Universität Frankfurt
Postfach 11 19 32

W-6000 Frankfurt 1
GERMANY

Dr. Herold Dehling
Vakgroep Wiskunde
Rijksuniversiteit Groningen
Postbus 800

NL-9700 AV Groningen

Prof.Dr. Richard D. Gill
Mathematical Institute
University of Utrecht
6 Budapestlaan

NL-3584 DC Utrecht

Prof.Dr. Hermann Dinges
Mathematisches Seminar
Fachbereich Mathematik
Universität Frankfurt
Postfach 11 19 32

W-6000 Frankfurt 1
GERMANY

Dr. Charles M. Goldie
School of Mathematical Sciences
Queen Mary & Westfield College
Mile End Road

GB- London E1 4NS

Dr. Werner Ehm
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Friedrich Götze
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

W-4800 Bielefeld 1
GERMANY

Prof.Dr. Piet Groeneboom
Dept. of Mathematics and
Computer Science
Delft University of Technology
P. O. Box 356

NL-2600 AJ Delft

Prof.Dr. Hans-Rudolf Lerche
Institut für Mathematische
Stochastik
Universität Freiburg
Hebelstr. 27

W-7800 Freiburg
GERMANY

Prof.Dr. Rudolf Grübel
Faculty of Mathematics and
Informatics
Delft University of Technology
Mekelweg 4

NL-2628 CD Delft

Dr. Jiri Michálek
Inst. of Information Theory and
Automation
Czechoslovak Academy of Sciences
Pod Vodarenskou vezi 4

182 08 Prague 8
CZECHOSLOVAKIA

Prof.Dr. Hans G. Kellerer
Mathematisches Institut
Universität München
Theresienstr. 39

W-8000 München 2
GERMANY

Prof.Dr. Dietrich Werner Müller
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Götz-Dietrich Kersting
Mathematisches Seminar
Fachbereich Mathematik
Universität Frankfurt
Postfach 11 19 32

W-6000 Frankfurt 1
GERMANY

Dr. Wolfgang Näther
Fachbereich Mathematik
Bergakademie Freiberg
Bernhard-von-Cotta-Str. 2

O-9200 Freiberg
GERMANY

Prof.Dr. Ulrich Krengel
Institut für Mathematische
Stochastik
Universität Göttingen
Lotzestr. 13

W-3400 Göttingen
GERMANY

Dr. Michael Nussbaum
Karl-Weierstraß-Institut für
Mathematik
Postfach 1304
Mohrenstr. 39

O-1086 Berlin
GERMANY

Dr. Lothar Partzsch
Fachbereich Mathematik
Technische Universität Dresden
Mommsenstr. 13

O-8027 Dresden
GERMANY

Prof.Dr. Hermann Rost
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Susan M. Pitts
Dept. of Statistical Sciences
University College
Gower Street

GB- London WC1E 6BT

Prof.Dr. Ludger Rüschendorf
Institut für Mathematische
Statistik
Universität Münster
Einsteinstr. 62

W-4400 Münster
GERMANY

Dr. Alfredas J. Račkauskas
Dept. of Mathematics
Vilnius University
Naugarduko 24

2006 Vilnius
LITHUANIA

Prof.Dr. Michael Scheutzow
Fachbereich Mathematik, MA 7-5
Technische Universität Berlin
Straße des 17. Juni 136

W-1000 Berlin 12
GERMANY

Prof.Dr. Paul Ressel
Mathematisch-Geographische Fakultät
Universität Eichstätt
Ostenstr. 26-28

W-8078 Eichstätt
GERMANY

Prof.Dr. Winfried Stute
Mathematisches Institut
Universität Giessen
Arndtstr. 2

W-6300 Gießen
GERMANY

Prof.Dr. Leonard C.G. Rogers
School of Mathematical Sciences
Queen Mary and Westfield College
University of London
Mile End Road

GB- London , E1 4NS

Prof.Dr. Anton Wakolbinger
Institut für Mathematik
Universität Linz
Altenbergerstr. 69

A-4040 Linz

Prof.Dr. Heinrich von Weizsäcker
Fachbereich Mathematik
Universität Kaiserslautern
Postfach 3049

W-6750 Kaiserslautern
GERMANY

Prof.Dr. Jon A. Wellner
Department of Statistics
University of Washington

Seattle , WA 98195
USA