MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 14/1992

Topologische Methoden in der Gruppentheorie

29.03. bis 04.04.1992

Die Tagung fand unter Leitung von R.Bieri (Frankfurt) und K.S.Brown (Ithaca) statt. Ein Hauptakzent lag nach wie vor auf der schnell wachsenden Theorie verallgemeinerter hyperbolischer Gruppen, bzw. auf neuen Methoden, die sich daraus entwickelt haben. Daß daneben auch bei "klassischen" Problemen substantielle Fortschritte erzielt wurden, sei durch Hervorheben der folgenden zwei Resultate dokumentiert:
1) Helmut Behr hat die mehr als 15 Jahre offene Vermutung bewiesen, daß sich die endliche Präsentierbarkeit von reduktiven arithmetischen Gruppen im Funktionenkörperfall durch eine einfache Rangformel ausdrücken läßt.
2) Martin Dunwoody hat die seit mehr als 20 Jahren offene Vermutung von C.T.C.Wall widerlegt, indem er eine endlich erzeugte Gruppe G konstruierte, aus der unendlich oft ein amalgamierter Faktor H, $G = G_1 *_E H$ mit $|E| < \infty$ abgespalten werden kann.
Um einen Eindruck über die Thematik der 32 Vorträge zu vermitteln seinen die folgenden Stichworte aufgeführt:
Hyperbolizität (Semi-hyperbolische Gruppen, "combings", Hyperbolisierung von Polyedern, Automorphismen von freien und hyperbolischen Gruppen),
Operation auf Bäumen, Gebäuden und CW-Komplexen (Accessibilität, arithmetische Gruppen, Arithmetik von Baumoperationen, Komplexe von Gruppen),
Gruppen von Homöomorphismen von $\mathbb{R}$ (Wachstum, "amenability", Endlichkeitseigenschaften),
Kombinatorische Gruppentheorie (1-Relator-Produkte, 2-dimensionale Komplexe),
Gruppen(Co)homologie (Cohomologie mit beschränkten Coketten, Mackey Funktoren, Homologie von frei auflösbaren Gruppen).

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Vortragsauszüge

H.ABELS

Properly discontinuous groups of affine transformations.

The motivating questions are: What is the structure of properly discontinuous subgroups Γ of the affine group $\mathrm{Aff}(\mathbb{R}^n)$ (Milnor), respectively of those Γ with compact orbit space $\Gamma \backslash \mathbb{R}^n$ (Auslander)? Equivalently, what is the fundamental group Γ of a complete flat affine manifold like; is it virtually solvable or does it contain a free subgroup?
In the talk the geometry behind some of Margulis' results on the subject are explained, namely the dynamics of linear and affine maps and Margulis' key notion of the sign of an affine map. This will make clear why there are free subgroups of $\mathrm{SO}(n, n+1) \ltimes \mathbb{R}^{2n+1} \subset \mathrm{Aff}(\mathbb{R}^{2n+1})$ acting properly discontinuously on $\mathbb{R}^{2n+1}$ for odd n but not for even n. Also work in progress (A., Margulis, Soifer) is mentioned toward the following result believed to be true: There is no properly dicontinuous subgroup Γ of $\mathrm{Aff}(\mathbb{R}^{2n+1})$ with $\Gamma \backslash \mathbb{R}^{2n+1}$ compact and whose linear part $\lambda(\Gamma)$ is Zariski dense in $\mathrm{SO}(n+1, n)$.

PETER ABRAMENKO

Buildings and Finiteness properties of Chevalley groups over $\mathbb{F}_q[t]$.

If G is a reductive group, defined over a number field k, then a famous theorem of Borel and Serre states (among other things) that every S-arithmetic subgroup Γ of G enjoys the homological finiteness property F_∞.
By contrast, not much is known concerning higher finiteness properties (meaning F_r with $r > 2$) of S-arithmetic subgroups of reductive groups over global function fields. The talk discussed the following

> *Conjecture*: If G is an almost simple Chevalley group of rank n, then $\Gamma = G(\mathbb{F}_q[t])$ is of type F_{n-1} but not of type F_n.

This conjecture is proved now in "most cases", especially if G is of type A_n, B_n, C_n or D_n and $q \geq 2^{2n-1}$. The proof, which was sketched during the talk, uses the action of Γ on the Bruhat-Tits building X of the locally compact group $G(\mathbb{F}_q((\frac{1}{t})))$. Additionally, the local structure of X had to be studied, looking to questions concerning the homotopy type of certain subcomplexes of spherical building.

JUAN M. ALONSO

Semihyperbolic groups.
Joint work with Martin R. Bridson

We define semihyperbolicity, a condition which describes non positive curvature in the large for an arbitrary metric space. This property is invariant under quasi-isometry. A finitely generated group is said to be weakly semihyperbolic if when endowed with the word metric associated to some finite generating set it is a semihyperbolic metric space. Such groups are finitely presented, of type FP_∞ and satisfy a quadratic isoperimetric inequality. We define a group to be semihyperbolic if it satisfies a stronger (equivariant) condition. We prove that this class of groups has strong closure properties. Word-hyperbolic groups and biautomatic groups are semihyperbolic. So too is any group which acts properly and cocompactly on a space of nonpositive curvature. A discrete group of isometries of a 3-dimensional geometry is not semihyperbolic if and only if the geometry is *Nil* or *Sol* and the quotient orbifold is compact. We give necessary and sufficient conditions for a split extension of an abelian group to be semihyperbolic; we give sufficient conditions for more general extensions. Semihyperbolic groups have solvable word and conjugacy problems. We prove an algebraic version of the flat torus theorem; this includes a proof that a polycyclic group is a subgroup of a semihyperbolic group if and only if it is virtually abelian. We answer a question of Gersten and Short concerning rational structures on $\mathbb{Z}^n$.

HELMUT BEHR

Arithmetic groups over function fields: A complete characterization of finitely presented arithmetic subgroups of reductive algebraic groups.

I. *Theorem:* Let F be a function field, i.e. $[F : \mathbb{F}_q(t)] < \infty, S$ a finite set of primes of $F, s := \#S > 0, F_{v_i}$ completion of F with respect to $v_i \in S; G$ an absolutely almost simple alg. group, defined over F, r the F-rank of G, r_i the F_{v_i}-rank of G and Γ a S-arithmetic subgroup of G.

Then:

1) Γ is not finitely generated $\Leftrightarrow r = s = r_i = 1$

$\Leftrightarrow \sum_{i=1}^{s} r_i = 1$ and $r > 0$

2) Γ is fin.gen., but not finitely presented $\Leftrightarrow \sum_{i=1}^{s} r_i = 2$ and $r > 0$.

Remarks: 1) The case of reductive groups can be reduced to the theorem above.
2) Many special cases of the theorem were known before.

II. *Methods* for the proof:
1) Action on Bruhat-Tits-buildings $X = \prod_{v \in s} X_v$, $X_v \leftrightarrow G(F_v)$.
2) Γ-invariant filtration of X by means of reduction theory for arithmetic groups.
3) K.Brown's criterion for actions on simplicial complexes with filtration.
4) Equivalence between group actions on simply connected complexes and presentations of groups as an amalgam of stabilizers.
5) Construction of amalgams type-by-type.
These methods were discussed only for some *examples* e.g. $SL_2(\mathcal{O}_S)$ with $s = 1, 2, 3$ and groups of type B_2.

MARTIN R.BRIDSON

Nonpositive curvature and bicombings of groups.

The two main classes of semihyperbolic groups are biautomatic groups and the class of groups which act properly and cocompactly on nonpositively curved ("CAT(0)") spaces. We distinguish between these classes of groups by giving examples of biautomatic groups which do not act properly and cocompactly on any CAT(0) space. One such example is the fundamental group of the unit tangent bundle of a closed hyperbolic surface.

Theorem 1 (J.Alonso, B.): Let X be one of the 8 3-dimensional geometries. If a finitely gen. group acts discretely by isometries on X then it is *not* semihyperbolic if and only if [X = *Nil* or *Sol* and X/Γ is compact].

Theorem 2 (B.): Let X be one of the eight 3-dimensional geometries. If Γ acts properly and cocompactly on X then it acts in the same way on a 1-

connected CAT(0) space if and only if X is one of $\mathbb{E}^3$, $\mathbb{H}^3$, $\mathbb{H}^2 \times \mathbb{R}$, S^3, $S^2 \times \mathbb{R}$.

MIKE DAVIS

Strict hyperbolization of polyhedra.
Joint work with Ruth Charney

Gromov has described several procedures for "hyperbolizing" a cell complex. This means a functorial process for converting an abstract simplicial or cubical complex into a piecewise Euclidean polyhedron of curvature ≤ 0. On practice this polyhedron consists of regular Euclidean cubes glued together. Gromov claimed that in at least two of these procedures the resulting metrics could be perturbed to have curvature strictly less than 0. This is false since in dimensions ≥ 4 the hyperbolized object has a $\mathbb{Z} \times \mathbb{Z}$ in its fundamental group which hence cannot be word hyperbolic. The situation can be remedied by replacing each regular Euclidean n-cube by a certain hyperbolic n-manifold with corners in which the "faces" intersect orthogonally.

T.DELZANT

Subgroups and quotients of hyperbolic groups.

One defines small cancellation conditions for hyperbolic groups; a set R satisfying such a condition generates a free normal subgroup and the quotient is shown to be hyperbolic.

M.DUNWOODY

Inaccessible groups and graphs.

An example of a f.g. group is given which is not accessible. Applications to infinite graph theory are discussed.

ROSS GEOGHEGAN

Higher Euler characteristics of groups.

Let G be a group for which there exists a finite $K(G,1)$ complex. Gottlieb's theorem says that if $\chi(G) \neq 0$ then the center of G is trivial. I will discuss a "higher" Euler characteristic $\chi_1(G)$ whose non-vanishing implies, at least in certain cases, that the center of G is infinite cyclic.

R.GRIGORCHUK

On ergodic and amenability properties of groups acting on the reals.

Amenable properties of some important topological groups such as the group of measure preserving automorphisms of Lebesgues space, the group Aut([0,1]) and others and its discrete subgroups (for instance R. Thompson's group) will be discussed. The example of a group of intermediate growth acting by homeomorphisms on the reals will be given.

KARL GRUENBERG

Homotopy classes of truncated projective resolution.

The aim of the talk was to present a new treatment of work of W.Browning concerning the homotopy classification of (G,m)-complexes. Two (G,m)-complexes X, Y are homotopically equivalent if and only if there exists an isomorphism $f_1 : \pi_1(X) \xrightarrow{\sim} \pi_1(Y)$ under which $C(\tilde{X})$ and $C(\tilde{Y})$ are equivariantly homotopically equivalent (MacLane – JHC.Whitehead 1950).
Let G be a finite group and $P(\ell,m)$ the set of equivalence classes of chain homotopically equivalent m-truncated $\mathbb{Z}G$-projective resolutions of $\mathbb{Z}$ of Euler characteristic ℓ. The set of G-linked homotopy classes of (G,m)-complexes of Euler characteristic ℓ is bijective with a subset (the whole set if $m \geq 3$) of the free elements in $P(m,\ell)$. If $[m,Q] \in P(m,\ell)$ and $M = \ker(Q_m \to Q_{m-1})$, then (provided M satisfies a weak cancellation condition) $P(\ell,m)$ is bijective with the group $\mathrm{Cl}(M,\Phi) = T_0(M)/A(\Phi)$, where $\Phi = H^{m+1}(G,-), T_0(M)$ is the Grothendieck group on all simple $\mathbb{Z}G$-module

images of M of order prime to $|G|$ and the subgroup $A(\Phi)$ is generated by all $[U]$ where $0 \to M \xrightarrow{f} M \to U \to 0$ with $\Phi(f) =$ id. There is a natural homomorphism: $\mathrm{Cl}(M,\Phi) \to \mathrm{Cl}(\mathbb{Z}G)$ and its kernel W is bijective with the set of free elements in $P(\ell,m)$. Finally, if $B = \langle \varphi \in \mathrm{Aut}(Q_m)_{(G)} \mid \varphi\delta = \delta \rangle$, then there is a natural homomorphism $\beta : B \times K_1(\mathbb{Z}G) \to K_1(\mathbb{Z}_{(G)}G)$ and W is the torsion subgroup of the cokernel of β. This provides a framework for explicit calculations.

A.HAEFLIGER

Complexes of groups and homological algebra.

Complexes of groups $G(X)$ on an ordered simplical complex X are generalizations of graph of groups (Bass-Serre) and triangle of groups (Gersten-Stallings). They describe orbihedra structures on X as well as stratified spaces Y over X, strata being Eilenberg-MacLane spaces.
In this talk we observe that $G(X)$ can be considered as a small category $CG(X)$; the fundamental group of $G(X)$ is the fundamental group of the geometric realization $BG(X)$ of the nerve of $CG(X)$. Left and right $G(X)$-modules can be defined (they correspond to some kind of sheaves on $BG(X)$), as well as homology and cohomology of $G(X)$ with coefficients in those modules.
For instance, isomorphism classes of extensions of $G(X)$ by an abelian kernel A correspond to elements of $H^2(G(X),A)$.
The notion of Poincaré-duality complexes of groups is also defined; they arise naturally from a stratification of a manifold Y by submanifolds which are Eilenberg-MacLane spaces.

C.HOG-ANGELONI

Andrews-Curtis equivalence of group presentations.

Joint work with W. Metzler

That complexes with isomorphic fundamental group and same Euler characteristic fall into the same restricted simple homotopy type (i.e. are related by a sequence of expansions and collapses of dimension ≤ 3) after wedging on 2-spheres is well known. We show that the standard complex of suitably many copies of $\mathbb{Z}_2 \times \mathbb{Z}_4$ likewise converts simple homotopy equivalence (but NOT: homotopy equivalence) to restricted simple homotopy equivalence. This phenomenon can be understood by a description of the simple resp. the restricted simple homotopy type in terms of the algebra of the underlying presentation. It concerns commutators of the relator subgroup of the involved presentations and is part of a program to study whether simple homotopy type and restricted simple homotopy type coincide or not.

JAMES HOWIE

Some generalizations of the Freiheitssatz.

Let $G = \frac{A*B}{N(r)}$ where A, B are groups and $N(r)$ is the normal closure of $r \in A * B$, a cyclically reduced word of length at least two. It is conjectured that the natural maps $A \rightarrow G \leftarrow B$ are injective in either (1) r is a proper power s^m for some $m \geq 2$, $s \in A * B$; or (2) A, B are torsion free. I shall report on some partial results in the direction of conjecture (1) (joint work with Andrew Duncan) and conjecture (2) (joint work with Sergei Brodskii).

T.JANUSZKIEWICZ

Relative hyperbolization.

Joint work with M.Bestvina

Relative hyperbolization is a variation of hyperbolization construction, which starting from a pair of simplical complexes (K, L) produces a pair (K_h, L) with nonpositive curvature in the complement of L. Moreover if L was non positively curved, K_h is non positively curved and L is convex in K_h.
Applying this construction to pairs (M^n, T^{n-2}) with M a manifold and T^{n-2} a torus, embedded in not locally flat way into M, one obtains examples of $K \leq 0$ PL manifolds which do not carry C^∞ $K \leq 0$ riemannian metrics.

A.JUHASZ

An application of small cancellation theory to one-relator products.

Theorem: Let A and B be torsion free groups and let W be a cyclically reduced word in $A * B$ of length ≥ 2. Let G be the quotient of $A * B$ by the normal closure of W^2. Then the natural maps $A \to G$, $B \to G$ are embeddings.

The theorem confirms a special case of a conjecture of J.Howie. The proof of the theorem is based on a certain construction which for a given simply connected van Kampen diagram with connected interior associates a tesselation of it by connected and simply connected subdiagrams which have the following properties:
(a) Each subdiagram which doesn't touch the boundary has at least 6 neighbouring subdiagams.
(b) Every such subdiagram is itself a small cancellation diagram in terms of the original regions.
The theorem now follows by standard arguments from small cancellation theory.

D.G.KHRAMTZOV

Finite graphs of groups with isomorphic fundamental groups.

The construction of fundamental group of graph of groups was introduced by Bass, Tits and Serre in the theory of groups acting on trees. It generalizes the constructions of amalgamated free product and HNN-extension. Natural question arises: when do non-isomorphic graphs of groups have isomorphic fundamental groups? It turns out that under some natural assumption such a graph can be transformed into each other by a finite sequence of elementary transformations of inverting the edge, slide of one edge along another one, slide of edge along the vertex group of its end.

P.H.KROPHOLLER

Right orderable groups and bounded cohomology.

The following theorem is joint work with Ian Chiswell: If G is a soluble group then G is right orderable if and only if G is locally indicable. This is proved by reducing

first to the case of RO-simple groups (-right orderable groups with no non-trivial proper right orderable quotients). The fact that soluble groups are amenable and that bounded cohomology vanishes for such groups plays an important rôle. There is a marked contrast with Bergman's example $\langle x, y, z | x^2 = y^3 = z^7 = xyz \rangle$ of a finitely generated perfect right orderable group.
A related theory can be developed for amenable cyclically ordered groups.
It is an interesting question whether every amenable right orderable group must be locally indicable. This can be proved for *supramenable* (in particular, exponentially bounded) groups.

YURI KUZMIN

Homology of group extensions with abelian kernels.

Let Φ_s be a free (non-cyclic) soluble group of length s. We study the homology groups $H_n(\Phi_s, -)$ with trivial coefficients $\mathbb{Q}, \mathbb{Z}[\frac{1}{2}], \mathbb{Z}, \mathbb{Z}_p = \mathbb{Z} / p\mathbb{Z}$. Here is one of the results (unpublished).

Theorem. Let $\Phi_s^{(k)}$ be the k-th term of the derived series of $\Phi_s(\Phi_s^{(s-m)}$ is free soluble of length m). Fix a prime number $p \neq 2$ and consider the image $H_n^{(k)}$ of the homomorphism $H_n(\Phi_s^{(k)}, \mathbb{Z}_p) \to H_n(\Phi_s, \mathbb{Z}_p)$ induced by the embedding $\Phi_s^{(k)} \to \Phi_s$. Then

(i) if $n \not\equiv 1 \bmod p$ then $H_n(\Phi_s, \mathbb{Z}) = H_n^{(s-1)}$;

(ii) if $n = p + 1$ then $H_n(\Phi_s, \mathbb{Z}) = H_n^{(s-2)}$;

(iii) if $n = p^2 + 1$ then $H_n(\Phi_s, \mathbb{Z}) = H_n^{(s-3)} (p \neq 2, 3, 5)$;

(iv) for $s > 2$ and any $n \geq p^3 + 1$ such that $n = 1 \bmod p$ all inclusions $H_n^{(s-1)} \subseteq H_n^{(s-2)} \subseteq \ldots \subseteq H_n^{(1)} \subseteq H_n(\Phi_s, Z_p)$ are proper.

Other properties of $H_n(\Phi_s, -)$, including a description of the torsion, can be found in Yu.V.Kuzmin,Commun. Algebra, 16 (1988), 2447-2533; and L.G.Kovács, Yu.V.Kuzmin, Ralph Stoehr, Mat.Sb.182(1991),526-542.

M.PAUL LATIOLAIS

Generators for the relation module and the browning obstruction group.

Given a group G and a presentation, $G = \frac{F\{a_i\}}{N\{R_j\}} = \langle a_i | R_j \rangle$, it is well known that the relation module $N/[N,N]$ is a G-module, with the image of the $\{R_j\}$ as generators. A standard question is, "When is a set of generators for the relation module the image of a set of defining relators for the group." (We are fixing the generating set).
The above question is related to and can be phrased in terms of the realizability of Browning obstruction elements of the Browning obstruction group. The Browning obstruction group is the complete obstruction to two 2-complexes being homotopy equivalent. The fundamental group of these 2-complexes must be finite Eichler. Given a 2-complex K and an obstruction element θ, a natural question is, "Is there a 2-complex L whose obstruction to being homotopy equivalent to K is θ?"
In the right context an affirmative answer to the first question implies yes to the second, and with add conditions yes for the second question implies yes for the first.

ALEX LUBOTZKY

Trees and discrete subgroups of Lie groups.

If G is a rank one Lie group over a local field e.g. $SL_2(\mathbb{Q}_p)$ or $SL_2(\mathbb{F}_p((t)))$, we study the lattices of G. We construct a lot of non-arithmetic lattices, proving Serre's conjecture that arithmetic lattices fail to satisfy the congruence subgroup property and more. *All methods* are based on the action of G on the associated tree and Bass-Serre theory.

If X is a tree and $G = \mathrm{Aut}(X)$, one wants to develop a theory of lattices in G analog to the one that exists for Lie groups G. Some works some does not. Bass and Kulkarni studied cocompact lattices. I gave a survey of their work and added some more.

Cyclic subgroups of exponential growth.

If Γ is a group of exponential word growth and C a cyclic subgroup, it is possible that C will have exponential growth w.r.t. the generators of Γ. The talk discusses it and gives a characterization of such cyclic subgroups of lattices of semi-simple Lie groups.

MARTIN LUSTIG

Pseudo-Anosov automorphisms of free groups and invariant actions on $\mathbb{R}$-trees.

Pseudo-Anosov automorphisms of a free group F_n are defined by means of invariant combinatorial train tracks. Special cases are automorphisms which are induced by pseudo-Anosov homeomorphisms of bounded surfaces, and Bestvina-Handel's irreducible automorphisms with irreducible powers.

> *Theorem 1:* For every pseudo-Anosov automorphism $\varphi \in \mathrm{Out}(F_n)$ there is a well defined φ-invariant small action of F_n on some $\mathbb{R}$-tree $\mathcal{T}_\varphi$, and every free simplicial F_n-action on any $\mathbb{R}$-tree $\mathcal{T}$ converges under the induced action of φ on Culler-Vogtmann space towards $\mathcal{T}_\varphi$.

The presented methods extend to more general automorphisms of F_n:

> *Theorem 2:* There exist indecomposable automorphisms $\varphi \in \mathrm{Out}(F_n)$ with an arbitrary number of fixed points on the boundary of Culler-Vogtmann space.

An extension of the presented methods to word-hyperbolic rather than free groups concerns work in progress.

MICHAEL L.MIHALIK

qsf groups and tame combings.

We consider two geometric properties for finitely presented groups, the "quasi-simply-filtered" property and the "tame 1-combable" property. If G is the fundamental group of a closed irreducible 3-manifold M, and G has either of these properties, then the universal cover of M is $\mathbb{R}^3$.

> *Theorem [Brick-Mihalik].* If G is a 1-relator group or G is simply connected at infinity, then G is quasi-simply-filtered.
>
> *Theorem [Brick-Mihalik].* If $1 \to A \to G \to B \to 1$ is a short exact sequence of infinite finitely presented groups, then G is quasi-simply-filtered.
>
> *Theorem [Tschantz-Mihalik].* If G has a bounded combing by quasi-geodesics (e.g. automatic groups, semi hyperbolic groups, Coxeter groups, small cancellation groups ...) or G is a asynchronously automatic, then G has a tame 1-combing.

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S.PRIDE

Generators of the second homotopy module of group presentations.

A knowledge of the generators of the second homotopy module π_2 of a group presentation is useful for various reasons. I will first discuss those ones (computation of group (co)homology, investigation of the Cockroft and related properties, computation of the second Fox ideal , determination of 2-complexes of minimal Euler characteristic). I will then discuss some types of presentations where generators of π_2 have been computed, and will give some consequences of these computations.

MARK RONAN

Twin Buildings.

The purpose of this talk is, firstly to explain what a twin building is, using the special case of twin trees, particularly those arising from the arithmetic group $SL_2(k[t,t^{-1}])$. We shall then consider how twin buildings arise naturally from Kac-Moody groups and discuss the important special cases: spherical, affine, hyperbolic. Finally I shall explain how, in joint work with J.Tits, we hope to classify twin buildings; what we can prove already, and what remains to be done. This will give a classification of groups "of Kac-Moody type", similar to the classification of algebraic groups over an arbitrary field. It also offers some promise of a deeper understanding of hyperbolic buildings and of hyperbolic Kac-Moody groups.

ZLIL SELA

On the automorphism group of a (torsion-free) hyperbolic group.

We study the algebraic structure of the automorphism group and the dynamics of individual automorphisms of a (torsion-free) freely indecomposable hyperbolic group.

M.STEIN

Groups of PL-homeomorphisms.

R.J.Thompson discovered the first examples of finitely presented infinite simple groups in 1965. Thompson's groups may be described as groups of piecewise linear homeomorphisms of the unit interval and the circle. We consider a class of PL homeomorphism groups which generalize the original Thompson's group, and show that they all are either simple and have large simple subgroups. We then construct, for many examples, classifying spaces of finite type, illustrating the finiteness properties of these groups. From these spaces we can compute the homology and cohomology, and obtain finite presentations.

RALPH STÖHR

Homology of groups with coefficients in Lie powers.

Let G be a group given by a free presentation $G = F/R$, and consider the quotients

$$F/[\gamma_n R, F], \quad F/[\gamma_n F, R]R'',$$

where $\gamma_n R$ denotes the n-th term of the lower central series of R, R'' denotes its second commutator group, and $n \geq 2$. A peculiar feature of these groups is that they can contain elements of finite order, even if G is torsion-free. Work of Kuzmin, Hartley, Hannebauer and the speaker provided a satisfactory description of this torsion in case when n is a prime. However, practically nothing was known in case when n is a composite number. In the talk I will present some recent results for the case $n = 4$: If G has no 2-torsion, then the torsion subgroup of $F/[\gamma_4 R, F]$ is isomorphic to $H_6(G, \mathbb{Z}_2)$ and the torsion subgroup of $F/[\gamma_4 R, F]R''$ is isomorphic to $H_7(G, \mathbb{Z}_2) \oplus H_6(G, \mathbb{Z}_4) \oplus H_4(G, \mathbb{Z}_2)$, where $\mathbb{Z}_2$ and $\mathbb{Z}_4$ denote the integers mod 2 and mod 4, respecitively, regarded as trivial G-modules. These are, in fact, consequences of some more general results on the homology of G with coefficients in Lie powers of relation modules, which will also be discussed in the talk.

G.A.SWARUP

Combination theorems for hyperbolic groups.

This is an exposition of some of Maskit's combination theorems with emphasis on theorems that are used in Thurston's uniformization theorem. Two points of interest in this approach are (a) we dispense with fundamental domains and Koebe's theorem, and (b) use Nielsen convex hulls in the discussion of geometric finiteness which makes the discussion somewhat transparent. A special case of Maskit's first combination theorem is:

> *Theorem.* Let G_1, G_2 be geometrically finite hyperbolic groups of dimension n, J a subgroup of $G_1 \cap G_2$ with $[G_i; J] \geq 3$ for at least one i. Suppose that W, the limit set of J is connected and divides S^{n-1} into two open sets B_1, and B_2, both of which are connected and
>
> $hB_i = B_i$, if $h \in J$,
>
> $gB_1 \subseteq B_2$ if $g \in G_1 - J$ and $gB_2 \subseteq B_1$ if $g \in G_2 - J$.
>
> Then the subgroup G of *Iso* $\mathbb{H}^n$ generated by G_1, G_2 is naturally isomorphic to $G_1 \underset{J}{*} G_2$ and G, J are geometrically finite.

One can give a description of the domain of discontinuity of G, the parabolics in G etc. and there is a similar theorem which gives rise to HNN extensions.

JACQUES THEVENAZ

On the equivariant K-theory of Brown's complex.

Let G be a finite group and let p be a prime. The main result is that a deep conjecture of J.Alperin in the modular representation theory of G in characteristic p is equivalent to a conjecture about the Euler characteristic of the equivariant K-theory of Brown's complex (= the complex of non-trivial p-subgroup of G). Let Δ be Brown's complex, let $K_G^*(\Delta)$ be the equivariant K-theory and let $\chi_G(\Delta) = \text{rank } K_G^1(\Delta) - \text{rank} K_G^0(\Delta)$. The conjecture is that $\chi_G(\Delta) = k(G) - z_p(G)$ where $k(G)$ is the number of irreducible representations of G over $\mathbb{C}$, and $z_p(G)$ is the number of those representations which have dimension multiple of $|G|_p$ (where $|G|_p = p$-part of the order of G).

KAREN VOGTMANN

Automorphisms of free-by-finite groups.
Joint work with S. Krstić

Let E be a finite extension of a finitely-generated free group F. The group $\mathrm{Out}(E)$ of outer automorphisms of E is commensurable with the centralizer of a finite subgroup G of outer automorphisms of F. The group $\mathrm{Out}(F)$ acts on a contractible simplicial complex K with finite stabilizers and finite quotient, so the centralizer $C(G)$ acts on the subcomplex K_G of K fixed by G. We find a $C(G)$-equivariant deformation L_G of K_G on which $C(G)$ acts with finite stabilizer and finite quotient, show that L_G is contractible and compute its dimension. This gives an upper bound to the virtual cohomological dimension of $C(G)$, and hence of $\mathrm{Out}(E)$. Under mild restrictions we show that this bound is equal to the v.c.d..

PETER WEBB

The application of Mackey functors to cohomology.

By considering the simple and projective objects in certain categories of Mackey functors one obtains a new computational method in group cohomology and also a proof of the theorem of Benson-Feshbach and Martino-Priddy on the stable splitting of classifying spaces.

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