MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 18/1992

Gruppentheorie (endliche p-Gruppen)
26.4. bis 2.5.1992

Organisatoren der Tagung waren Otto H. Kegel (Freiburg), Wilhelm Plesken (Aachen) und Gernod Stroth (Berlin).

Ein Schwerpunkt der Tagung war die Klassifikation endlicher p-Gruppen nach ihrer Koklasse. Im Zusammenhang damit standen auch pro-p-Gruppen endlicher Koklasse, sowie die Klassifikation nach Weite. Hier ordnen sich die Vorträge über die 'Nottingham' Gruppe ein. Weitere Schwerpunkte bildeten potenzreiche p-Gruppen und die Liealgebrenmethoden zur Untersuchung von p-Gruppen, z.B. im Zusammenhang mit Burnsidegruppen. Struktureigenschaften von p-Gruppen wurden in Vorträgen über Normalteiler- und Untergruppenverbände, Darstellungstheorie, minimale Relationenzahl, Automorphismengruppen und andere Themen untersucht. Benachbarte Gebiete waren vertreten in Vorträgen über torsionsfreie endlich erzeugte nilpotente Gruppen, Engelbedingungen, Galoisgruppen, p-Untergruppen in gewissen endlichen und lokal-endlichen Gruppen, etc.

Die sehr anregende 'Problemsession' am Donnerstagabend läßt auf eine Fortsetzung der lebhaften Entwicklung in der Theorie der p-Gruppen hoffen. Zum Gelingen der Tagung trugen die offene Atmosphäre und sehr gute technische Ausstattung des Instituts (Bibliothek und Computer) wesentlich bei. Alle Vorträge wurden in englischer Sprache gehalten.

Vortragsauszüge

C. Baginski: Finite p-groups with p-automorphisms of large order

In 1970 V.G.Berkovich classified all finite p-groups of order p^n with automorphisms of order p^{n-1} (Algebra & Logic). Here we give a complete classification of all finite p-groups G with automorphisms of order $\frac{|G|}{p^3}$.
Main results:
Theorem 1. If $p > 2$ and $|G| = p^n$, $n > 4$ then G has an automorphism of order p^{n-2} $\Leftrightarrow$ G contains a cyclic subgroup of index p.
Theorem 2. A group G of order 2^n, $n > 5$ has an automorphism of order 2^{n-2} $\Leftrightarrow$ one of the following holds:

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a) G is cyclic.

b) G is of maximal class.

c) $G = A \times B$, where A is dihedral or generalized quaternion and $|B| = 2$.

d) $G = A \curlyvee B$ (central product), where A is dihedral and B cyclic of order 4.

W. Bannuscher: Eine Verallgemeinerung des Regularitätsbegriffes bei p-Gruppen

In Verallgemeinerung des HALLschen Regularitätsbegriffes bei p-Gruppen werden sogenannte k-reguläre p-Gruppen betrachtet: Eine p-Gruppe G heißt k-regulär, falls für alle $x, y \in G$ stets

$$(xy)^{p^k} = x^{p^k} y^{p^k} \prod_i d_i^{p^k}$$

gilt mit geeigneten $d_i \in \langle x, y \rangle'$. Viele Sätze über reguläre p-Gruppen ($k = 1$) lassen sich auf k-reguläre p-Gruppen verallgemeinern.

B. Baumann: Some p-groups with given groups of automorphisms

Some p-groups were constructed which are related to the classical finite simple groups: the general linear groups $GL_m(p^h)$ and the symplectic groups $Sp_m(p^h)$ are related to the automorphism group of a certain factor group of $F_n/F_n^p[F_n', F_n]$ for odd primes p; the orthogonal and unitary groups $O_m(p^h)$ and $U_m(p^h)$ are related to the automorphism group of a certain factor group of $F_n/F_n^p[F_n', F_n][F_n', F_n, F_n]$ for primes $p > 3$. Here $n = m \cdot h$ and F_n is the free group fo rank n. In all cases the representation on the Frattini factor module is the natural one for these groups.

S. Blackburn: Enumeration of finite p-groups: Isoclinism

Define $V_\Phi(p^m)$:= the number of (isomorphism classes of) groups in an isoclinism class Φ of order p^m.
What can we say about this function:

1) When m is fixed and Φ varies?

2) When Φ is fixed and m varies?

R. Brandl: p-groups with 'few' subgroups

The set $\mathcal{C}(G)$ of all conjugacy classes $[H]$ of subgroups H of a group G admits a natural partial order, defined by $[H_1] \leq [H_2]$ if and only if $H_1 \leq H_2^g$ for some $g \in G$.
Theorem 1. Let G and H be finite groups and let H be a noncyclic p-group. If $\mathcal{C}(G) \cong \mathcal{C}(H)$, then

a) $|G| = |H|$, $expG = expH$ and $d(G) = d(H)$.

b) If H is abelian or metacyclic, then $G \cong H$.

Let $w_c(G)$ be the greatest length of an antichain in $\mathcal{C}(G)$. It is easy to see that a p-group G has $w_c(G) = 3$ if and only if G is a 2-group of maximal class. For larger values, however, we have the following result:
Theorem 2. Let $n \geq 4$. Then there exist only finitely many p-groups G with $w_c(G) = n$.

G. Busetto: Some results on permutable p-subgroups and projectivities of groups

We construct a finitely generated infinite perfect p-group G with the following properties:

1) G contains a permutable subgroup H such that H/H_G is infinite.

2) There is an autoprojectivity of G sending a normal subgroup to a non normal one.

A. Caranti: Subgroups of p-groups inducing the same permutation character

The main topic of this talk is a characterization of pairs of p-groups H, K with the property:
There exists a p-group $G \geq H, K$ such that $1_H^G = 1_K^G$.
The same question for arbitrary (finite) groups H, K, G has a nice answer in terms of orders of elements in H and K.
For p-groups, the situation is subtler: We provide some results and examples.

R. Dentzer: p-groups as Galois groups

In the first part a survey is given of realizations of finite p-groups as Galois groups. As base fields occur number fields, rational function fields $\mathbb{F}_q(t)$ and $\mathbb{Q}_{ab,nil}$ for regular realizations. In the second part a class of soluble groups is introduced that occur as Galois groups over a greater number of base fields. This class is examined further.

M. Hartl: On the fourth integer dimension subgroup

We construct a broad class of groups of class 3 but of dimension ≥ 4, containing the known examples of Rips and Tahara as simplest cases. Under certain restrictions completeness of the list can be shown.

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The method relies on a near constructive approach which starts with the description of 'relative' dimension subgroups in terms of a polynomial 'co'-approximation of the Schur multiplier H_2G. The approximation $H_2G \twoheadrightarrow P_2H_2G$ for groups of class 2 which enters in the description of $D_4(G)$ is computed, namely by a determination of the first two filtration quotients of a canonical filtration of H_2G and of the corresponding extension problem as well.

B. Hartley: Locally finite groups with finite Sylow subgroups

I should like to report on the work of my former student S.D. Bell.
Theorem 1. Let G be a countable locally finite group with finite Sylow-p-subgroup for all p. Then G has a proper subgroup isomorphic to itself $\Leftrightarrow$ G is not hyperfinite. Under these circumstances G can be embedded in an uncountable locally finite group with the same Sylow subgroups as G. It is then shown that this can be done in $2^{\aleph_0}$ ways.

H. Heineken: Normal embedding of p-groups into p-groups

For a given finite p-group M to be a normal subgroup of G which is contained in $w(G)$, where $w(G)$ is a word subgroup of the finite p-group G, it is necessary that $Inn(M)$ is contained in $w(S)$, where S is a p-Sylow subgroup of $Aut(M)$. On the other hand, if this condition on $Inn(M)$ is satisfied, a suitable p-group G can be constructed, making this condition also sufficient.

L. Héthelyi: On uniserially embedded subgroups of a p-group

Let G be a p-group. A subgroup A of G is *uniserially embedded* if the subgroups of G containing A form a chain.

If A is a maximal abelian subgroup of G such that $|N_G(A) : A| = p$ then A is uniserially embedded. Such a subgroup of G is called *soft*. First I would like to mention some properties of a p-group G having a soft subgroup. Then I consider the case when a maximal elementary abelian subgroup A of G is uniserially embedded and $|N_G(A) : A| \geq p^2$. Then the unique maximal subgroup M of G containing A is contained in the normalizer of A and if A is an indecomposable $N_G(A)$-module then A is normal in G. If A is not normal in G then all uniserially embedded maximal abelian subgroups of G are conjugate and there are exactly p such subgroups in G. G contains a uniserially embedded abelian subgroup not contained in M iff G contains a soft subgroup. A contains a great number of uniserially embedded subgroups of G and these are maximal in their normalizers. Finally let B be a cyclic subgroup of a p-group G such that $|N_G(B) : B| = p$. Then B is uniserially embedded. If $p \geq 5$ and if a p-group G contains a uniserially embedded cyclic subgroup C such that $|N_G(C) : C| \geq p^2$ then G is metacyclic.

B. Huppert: Character-degrees of some p-groups

If G is a p-group with exactly $k(G)$ different character-degrees (over $\mathbb{C}$), then by Taketa $dl(G) \leq k(G)$. Is asymptotically a better result true like $dl(G) \leq C \cdot log(k(G))$? Two series of p-groups are considered:

1) Let $S(n,p) = \underbrace{Z_p \wr \ldots \wr Z_p}_{n}$ (wreath-product). Then $dlS(n,p) = n$.

a) If $p > 2$, then $cdS(n,p) = \{p^j \mid 0 \leq j \leq 1 + p + \ldots + p^{n-2}\}$.
b) If $p = 2$, then $cdS(n,2) = \{2^j \mid 0 \leq j \leq 2^{n-2} + 2^{n-3} - 1\}$.

2) Let $G = G(n,p)$ be the group of unitriangular matrices of degree (n,n) over $GF(p)$. Then $dl(g) = s$, where $2^s \geq n > 2^{s-1}$. Now $cdG(n,p) = \{p^j \mid 0 \leq j \leq f(n)\}$, where $f(n) = \begin{cases} \frac{n(n-2)}{4} & \text{if } 2|n \\ \frac{(n-1)^2}{4} & \text{else} \end{cases}$. (The method does not work for unitriangular groups over $GF(q)$, where $q = p^f$.)

P. Igodt: Classifying almost Bieberbach groups up to dimension 4

Based on an algebraic set-up known in literature as the 'Seifert Fiber Space Construction' we can describe and construct faithful affine representations for torsion free, finitely generated nilpotent (and virtually finitely generated nilpotent) groups N. This approach might be of particular interest in relation to a conjecture of John Milnor (1977). The technique used is based on iteration from the upper central series of the nilpotent groups. The representations obtained are called 'of canonical type' and are unique up to conjugacy in a well determined group. Moreover they are also valid on the Lie group level.

Using information obtained from this type of representations, we could obtain an algebraic classification of all 3-dimensional almost Bieberbach groups. In particular cases we also remark that information on the polynomiality of $H^2(N, \mathbb{Z})$ might be deduced and that computation of these groups $H^2(N, \mathbb{Z})$ might be possible.
(joint work with K. Dekimpe)

D.L. Johnson: Recurrence relations in finite p-groups

Define $N_2(p,n) :=$ the 2-generator relatively free group in the variety of exponent-p-groups of class n

$$F(2,n) := \langle x_1, \ldots, x_n \mid x_1x_2 = x_3, \ldots, x_nx_1 = x_2 \rangle$$

$R(2,5) :=$ restricted Burnside group on 2 generators of exponent 5

Theorem 1. (Smith-Aydin, 1991) If $F(2,n) \twoheadrightarrow N_2(p,1)$ then $F(2,n) \twoheadrightarrow N_2(p,4)$.
Theorem 2. (Smith-Aydin-Cayley, 1991) $F(2,20) \twoheadrightarrow R(2,5)$.
As a very special case of current joint work with RWK Odoni, we know exactly when the condition of Theorem 1 holds. I also discuss Fibonacci identities mod p.

L.-C. Kappe: The nonabelian tensor square of a 2-generator p-group of class 2

The nonabelian tensor square of a group G, denoted by $G \otimes G$, is the group generated by the symbols $u \otimes v,\ u, v \in G$ and defined by the relations

$$uv \otimes w = ({}^u v \otimes {}^u w)(u \otimes w) \text{ and } u \otimes vw = (u \otimes v)({}^v u \otimes {}^v w),$$

where ${}^v u = vuv^{-1}$. Explicit computations of such groups have been given so far only in a few special cases.

The nonabelian tensor square of a finite nilpotent group G is the direct product of the nonabelian tensor squares of the p-components. If G is nilpotent of class 2 then $G \otimes G$ is abelian and the defining relations reduce to $uv \otimes w = (u \otimes w)(v \otimes w)([u,v] \otimes w)(v \otimes [u,w])$ and
$u \otimes vw = (u \otimes v)(u \otimes w)(u \otimes [v,w])([v,u] \otimes w)$. We explicitely compute the nonabelian tensor square for 2-generator p-groups of class 2, p odd.

E.I. Khurkhro: The restricted Burnside Problem for varieties of groups with operators

Suppose Ω is a finite group (of operators) and let $\mathcal{M}$ be a variety of groups with operators Ω given by Ω-identities $\{v_\alpha\}$. Let $\overline{\mathcal{M}}$ denote the (ordinary) variety of groups given by $\{\overline{v}_\alpha\}$, where $\overline{v}_\alpha$ is obtained from v_α by replacing all operators by 1.
Theorem. Suppose there is a 'multilinear' positive solution of RBP for $\overline{\mathcal{M}}$ in the sense that the associated Lie ring of a free group in $\overline{\mathcal{M}}$ satisfies a system of multilinear identities which define a locally nilpotent variety of Lie rings with function $f(d)$ bounding the class of a d-generator Lie ring. Then if $G\lambda\Omega$ is locally nilpotent for $G \in \mathcal{M}$, then G belongs to a locally nilpotent variety with function $f(d \cdot |\Omega|^{|\Omega|})$.
An example shows that 'multilinear' is essential.
Illustration: $\overline{\mathcal{M}}$ given by $x^p = 1$, $\mathcal{M}$ given by $xx^{\varphi} \ldots x^{\varphi^{p-1}} = 1$, $|\varphi| = p$, G a finite p-group $\in \mathcal{M}$.

E.I. Khurkhro: New identities in $\mathcal{L}(B(n,p))$ and the problems of Hughes and of Blackburn and Espuelas

I. Blackburn and Espuelas proved that if in a metabelian p-group P, $|\mho_1(P)| = p$, then $|P : \Omega_1(P)| \leq p^p$ and conjectured that here 'metabelian' may be omitted (there were also examples of G.E. Wall with $|P : \Omega_1(P)| = p^p$).

Theorem 1. Suppose that for each $k = 1, 2, \ldots, r$ the (Vaughan-Lee's) multilinear identity of degree $1 + k(p-1)$ of $\mathcal{L}(B(n,p))$ is not a consequence of those of smaller degree. Then there is a finite p-group P with $|\mho_1(P)| = p$ and $|P : \Omega_1(P)| \geq p^{1+r(p-1)}$ and $\Omega_1(P) = \Phi(P)$ ('secretive').
The condition is known to hold for $p = 5, 7$ and $k = 2$ (Wall, Cannon & computer).

II. Under the condition of Theorem 1 Wall proved that there is a finite p-group P with $|P : H_p(P)| \geq p^3$ and $H_p(P) \neq 1$. In the opposite, positive direction we prove
Theorem 2. Suppose that $\mathcal{L}(B(n,p))$ is a free (relatively) Lie algebra over $GF(p)$ all of whose identities follow from its multilinear ones of degrees $\leq 1 + r(p-1)$. Then, in any finite group G, $|G : H_p(G)| \leq p^2$ if $H_p(G) \neq 1$.

III. Of special interest is a 2-generator anti-Hughes group, as it gives answer to several other questions.
Theorem 3. There exists a 2-generator counterexample to Hughes conjecture (for $p = 7$ with the aid of computer).

W. Kimmerle: Class sums of p-elements

Let G be a finite group, $\mathbb{Z}G$ its integral group ring. A subgroup H of the group of units of augmentation 1 in $\mathbb{Z}G$ is called a group basis, if $|H| = |G|$. The object of the talk is the following p-power variation of a conjecture of Zassenhaus.
p-power Variation: Let H be a group basis of $\mathbb{Z}G$. Then there exists an isomorphism $\sigma : H \to G$ such that σ fixes the class sums of all prime power elements.
Note that, if this variation holds, it gives a strong positive answer to the isomorphism problem. One of the results is the following
Theorem. Suppose that $G/F(G)$ is nilpotent. Assume that each conjugacy class preserving automorphism of $G/F(G)$ and of its quotient is inner. Then the p-power variation holds. In particular this is the case when $[G, G]$ is nilpotent.

A.I. Kostrikin: On finite sandwich p-groups

Let $C_m = Lie(c_1, \ldots, c_m)$ be a sandwich Lie algebra embedded in $A_m = Ass(c_1, \ldots, c_m)$ so that $c_i^2 = 0 = c_i x c_i$ for all $x \in C_m$. In accordance with M. Vaughan-Lee, E.I. Zelmanov the upper bound of the nilpotency class $cl(C_m)$ of C_m is $cl(C_m) \leq T(m, 20)$, where

$$T(m,n) = m^{T(m,n-1)}.$$

Perhaps, $cl(C_m) \leq m^2$, but it is obvious only for $m \leq 4$.
Proposition 1. $cl(C_5) \leq 16$.
Now let $P_m = Gr(a_1, \ldots, a_m) \subset U(A_m)$ be a finite sandwich p-group, where $a_i = 1 + c_i$ and $U(A_m)$ is the group of invertible elements of the algebra A_m over $GF(p)$.
Proposition 2.

(i) $cl(P_m) = cl(C_m)$;

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(ii) $exp(P_m) = p$ for $p \gg 1$.

In particular, we have $exp(P_4) = p^4$ for $p = 2$; p^2 for $p = 3$ or 5; p for $p > 5$.

C. Leedham-Green: The coclass conjectures

If G is a group of order p^n, p a prime, and nilpotency class c, the coclass of G is defined to be $n - c$. The coclass conjectures published by M.F. Newman and myself have now all been proved in a collaborative efford by a large number of mathematicians. The conjectures, or theorems as they should now be called, are in descending order of strength.
Conjecture A. There is a function $f(p,r)$ such that every p-group of coclass r has a normal subgroup of index $\leq f(p,r)$ and class 2. If $p = 2$, this subgroup can be taken to be abelian.
Conjecture B. There is a bound to the derived length of p-groups of coclass r.
Conjecture C. Every pro-p-group of finite coclass is soluble.
Conjecture D. There are only finitely many pro-p-groups of coclass r.
Conjecture E. There are only finitely many soluble pro-p-groups of coclass r.
In fact a stronger theorem than Conjecture A has been proved. Define a 'constructible' p-group to be one obtained by taking a finite quotient of a p-adic space group of coclass r and (if $p > 2$) 'twisting' the translation group to make it of class 2. Then every p-group P of coclass r has a normal subgroup N of order bounded in terms of p and r such that G/N is constructible.

F. Leinen: Unipotent finitary linear groups

Let V be a K-vector space. A subgroup G of $GL(V)$ is said to be finitary linear, if the degree $dim[V,g]$ is finite for every $g \in G$.
Theorem 1. Every countable unipotent finitary linear group has a faithful finitary linear representation by unitriangular matrices A with the property, that $A - E$ has only finitely many non-zero entries.
Theorem 2. McLain groups are not existentially closed (e.c.) in the class $\mathcal{L}_K$ of all unipotent finitary linear groups with fixed local degree function.
However, as direct limit of full unitriangular matrix groups with respect to certain canonical embeddings, we can construct an e.c. $\mathcal{L}_K$-group L_K, which has properties similar to those of McLain's group $M(\mathbb{Q}, K)$. If K is algebraically closed, then L_K is even e.c. in the class

$$\mathcal{L}_p = \bigcup_{char K = p} \mathcal{L}_K.$$

(joint work with O. Puglisi)

A. Mann: Powerful p-groups and the uncovered Conjecture A

A weaker form of the Conjecture A (cf C. Leedham-Green) is that a p-group of coclass r centers a 'nice' subgroup of small index. We prove the following result of that type

Theorem 1. A group of coclass 2 ($p > 2$) contains a powerful subgroup $N \trianglelefteq G$, such that $d(N) \leq p^2$ and $|G : N| \leq p^{p^2+1}$.

The proof applies some results about powerful groups and about uniserial modules. The result is helpful in providing shorter proofs for Conjecture E and C. It also helps in providing a proof with explicit bounds for Conjecture A in the case of uncovered groups, i.e. groups containing an element s that does not centralize any factor G_i/G_{i+2}. This special case of Conjecture A is proved by reducing it to the Lie case of so-called CF-groups, which were treated already by S. McKay.

S. McKay: The correct bound to the coclass of space groups

In 1986 Leedham-Green, McKay and Plesken proved that the coclass of a uniserial p-adic space group of dimension $p^{\rho-1}(p-1)$ is at least $\rho - 1$. In that paper we conjectured that the correct bound is ρ rather than $\rho - 1$. McKay has now proved that the coclass is at least ρ. We discuss the basic ideas of the proof.

M. Newell: On Engel elements of length three

Let G be a group and denote by $R_3(G)$ the set $\{a \in G \mid [a, x, x, x] = 1 \text{ for all } x \in G\}$. We prove the following

Theorem. Let $a \in R_3(G)$ and $x \in G$. Then the group $H := \langle a, x \rangle$ is nilpotent of class at most 6 and $\Gamma_4 H$ is a finite group of order dividing 2^6.

Corollary. The set $R_3 G$ is a subgroup of G whenever
1) it consists of periodic elements of odd order
or 2) G has no non-trivial elements of order 2.

M.F. Newman: Groups of prime-power order via coclass II

This talk continued the report of joint work with E.A. O'Brien. The results of the use of a p-group generation program to obtain information about the isomorphism types of 2-groups of coclass 3 were outlined. These results were used, together with some of the results arising in the proofs of Theorem A (see Leedham-Green's abstract), to state some shorter conjectures about 2-groups.

$F(2, r)$: The number $e(n)$ of isomorphism types of 2-groups of coclass r and order 2^n is virtually periodic in n with period 2^{n-1}.

A p-group P is *cofinitely descendant periodic* if it has a proper descendant Q such that the descendant tree J_Q of Q is isomorphic to J_P and the difference $J_P - J_Q$

is finite. Let P be a cofinitely descendant periodic p-group and let Q be the least proper descendant of P with $J_Q \cong J_P$; then $log_p(\frac{|Q|}{|P|})$ is the period of P. (Let π_P be the isomorphism.)

$DD(2,r)$: Every infinite pro-2-group of coclass r has a finite quotient which is cofinitely descendant periodic with period dividing 2^{r-1}.

A *one-parameter family* of p-groups of coclass r is a set of descendants of a cofinitely descendant periodic p-group P of coclass r which is monogenic under π_P. A 2-group of coclass r which does not lie in a one-parameter family is *sporadic*.

$AA(2,r)$: The 2-groups of coclass r can be divided into finitely many one-parameter families and finitely many sporadics.

M.F. Newman: Some computations

The Golod-Šafarevič Theorem can be viewed as saying that a finite pro-p-presentation with d generators and r relations defines a finite pro-p-group only if $r > \frac{d^2}{4}$. Wisliceny in his lecture reported on the existence of pro-p-presentations with d generators and $\frac{d^2}{4} + \frac{d}{2} - \frac{7+(-1)^d}{8}$ relations which define finite pro-p-groups. This leaves a small gap. The first case is 5 generators and 7 relations. Wisliceny gave a presentation of this kind which might define a finite pro-p-group. A computer program for computing p-quotients of such presentations shows that Wisliceny's presentation does define a finite pro-p-group for $p = 3, 5, 7, 11$ and that its order is p^{55} for $p = 5, 7, 11$. Moreover it can be used to give an outline of a proof for every $p \geq 7$. We also found a presentation with 7 generators and 14 relations which defines a finite pro-p-group and one with 6 generators and 10 relations.

E.A. O'Brien: Groups of prime-power order via coclass I

This is the first of two reports of work in progress in association with M.F. Newman. In this talk I describe the use of directed graphs to present information about p-groups of a particular coclass. I present sample directed graphs for the 2- and 3-groups of coclass 1. With each infinite branch of one of these directed graphs, we may associate an inverse limit which is a p-adic pre-space group. Hence, the classification of the groups of a particular coclass is closely related to the classification of these pre-space groups. The primary computational tool used in our classification project is the p-group generation algorithm. Given a p-group G, it can be used to construct certain extensions of G. I present a summary of the theory of this algorithm.

P.P. Pálfy: On the lattice of normal subgroups of finite p-groups

In 1877 Dedekind discovered the modular law, namely that for arbitrary subgroups X, Y, Z of the additive group of complex numbers $(X + Y) \cap (X + Z) = X + (Y \cap$

$(X + Z))$ holds. This identity is however valid in a more general setting, in the lattice of normal subgroups of an arbitrary group. The same is true for the so-called Arguesian law - a lattice theoretic equivalent of Desargue's Theorem from projective geometry. When Bjarni Jáusson introduced this identity in 1953 he asked whether the class of lattices embeddable into the lattice of normal subgroups of some groups is strictly larger than those embeddable into the subgroup lattice of some Abelian group. In a joint work with Csaba Szabó we answered this question: The identity $x_1 \wedge \{y_1 \vee [(x_2 \vee y_2) \wedge (x_3 \vee y_3)]\} \leq (\{[(P_{12} \vee P_{34}) \wedge (P_{13} \wedge P_{24})] \vee P_{23}\} \wedge (x_4 \vee y_1)) \vee y_4$ where $P_{ij} = (x_i \vee y_j) \wedge (x_j \vee y_i)$ holds in the subgroup lattice of any Abelian group but fails in the lattice of normal subgroups of a certain group of order 2^{20}. The question for p-groups with $p > 2$ is still open.

G. Pazderski: Finite p-groups and their conjugacy classes

We refer to a formula due to P. Hall on the number $k(G)$ of classes of conjugate elements of a group of order $p^n = p^{e+2m}$ $(e = 0, 1)$, namely $k(G) = p^e + (m + k_o(p - 1))(p^2 - 1)$ where $k_o = k_o(G)$ is a non-negative integer. Our main topic is the connection between the possible values of $k_o(G)$ and the structure of G. It is looked at from two points of view. On the one hand general results are stated. For instance:
(1) If $N \trianglelefteq G$ then $k_o(G/N) \leq k_o(G)$.
(2) If $N \trianglelefteq G$ and $|N| = p$ and $k_o(G/N) = k_o(G)$ then G/N and G have the same coclass or the same cobreadth.
(3) If G/N has maximal class and $k_o(G/N) = k_o(G)$ then G has maximal class.
On the other hand particularly groups G with $k_o(G) = 0$ are investigated. They have maximal class and are characterized by certain commutator conditions between the members of the central series. For these groups the characters are given explicitly and it is shown that their character table do coincide to a great extend.

W. Plesken: Extensions of p-adic analytic groups by p'-groups

Discussing these extensions leads to representation theoretic problems of finite groups G acting on $\mathbb{Q}_p$-Liealgebras. Evidence is given that for fixed G there are only finitely many such Liealgebras without nontrivial G-invariant subalgebras. On the group theoretic side the extensions can be studied by linearizing the problems to lattice problems as B. Souvignier pointed out. If an extension is of finite p-coclass then its associated Liealgebra is of the above type, but not conversely. In any case the p-adic groups showing up are of bounded width.

M.P.F. duSautoy: Zeta functions associated with groups

Let R be a number field and G a linear algebraic group over R. Let Γ be an S-arithmetic group in G. We define

$$c_n(\Gamma) = card\{H \leq \Gamma \mid |\Gamma : H| = n \text{ and } H \text{ is a congruence subgroup }\}$$

and $\zeta^c_{\Gamma,p}(s) := \sum_{n=0}^{\infty} c_{p^n}(\Gamma)p^{-ns}$. We prove the following:
Theorem. Suppose that G has strong approximation. Then
a) if $p \geq 3$ then $\zeta^c_{\Gamma,p}(s)$ is rational in p^{-s};
b) if $p = 2$ and SL_2 does not occur as a factor of G then $\zeta^c_{\Gamma,p}(s)$ is rational in p^{-s};
b) if $p = 2$ and SL_2 does occur as a factor of G then, if there are finitely many Mersenne primes $\zeta^c_{\Gamma,p}(s)$ is rational in p^{-s}.
The proof involves Guralnick's classification of subgroups of p-power index in simple groups, Shoney and Tidjeman's finiteness theorems for exponential differential equations together with my result that $\zeta_{G,p}(s)$ is rational if G is a compact p-adic analytic group.
Conjecture. If $\Gamma \leq_f SL_2(\mathbb{Z})$ and there are infinitely many Mersenne primes then $\zeta^c_{\Gamma,2}(s)$ is irrational.

C.M. Scoppola: Lattice of normal subgroups of p-groups and pro-p-groups

(report on joint work with R. Brandl and A. Caranti)
Let p be a prime, $p > 3$. Let $\{G_i\}$ be the lower central series of the group G. Let G be a p-group or a pro-p-group such that $|G : G_i| < \infty$.

G is a DN-group if $G_3 \neq 1$ and (DN) $N \trianglelefteq G, |G : N| = |G : G_i| \Rightarrow N = G_i$. It is easy to show that $|G_i : G_{i+1}| \leq |G : G_2|$ and G_i/G_{i+1} is elementary abelian. Thus G is of bounded width. Furthermore
Proposition 1. (with C. Bonmassar)
Let G be a DN-group, and $p^d = |G : \Phi(G)|$. Then the following are equivalent:

(i) d is even.

(ii) G/G_3 has exponent p.

(iii) There exists i such that $G_{i+1} \neq 1$ and G_i/G_{i+2} has exponent p.

(iv) G is not powerful.

No examples of DN-groups are known in which d is odd. No such examples can exist for $d = 3$. Johnson and York 'Nottingham groups' $N(q)$, where q is a power of p, are DN-groups (Johnson and York). We say that G is *thin* if G is a 2-generated DN-group and G is not of maximal class. We have

Proposition 2.

(i) If G is metabelian thin, then $|G| = p^{2n}$, $cl(G) \leq p+1$.

(ii) If G is metabelian, G is thin $\Leftrightarrow$ G/G_5 is thin.

(iii) An explicit construction of metabelian thin p-groups is given.

A. Shalev: Nonsingular derivations and a proof of Conjecture A

We prove the following quantitative version of Conjecture A of Leedham-Green and Newman:
Theorem. Let G be a finite p-group of coclass r, and let $\{G_i\}$ be its lower central series. Then

(i) If $p \neq 2$ then $G_{2(p^r - p^{r-1} - 1)}$ has class at most 2.

(ii) If $p = 2$ and the order of G is at least $2^{2^{2r+5}}$ then $G_{7 \cdot 2^r - 2}$ is abelian.

A.Shalev: Almost fixed-point-free automorphisms

Let G be a finite p-group admitting an automorphism of order p^k and p^m fixed points. We show that the derived length of G is bounded in terms of p^k and p^m. This extends a theorem of Alperin dealing with the case $k = 1$ and gives rise to the following
Theorem. The derived length of a finite p-group G is bounded in terms of the minimal order of a centralizer $C_G(x)$ in G.
Corollary. A locally finite p-group with an element of finite centralizer is soluble.

G. Tiedt: Über die Existenz eines normalen p-Komplements

Es wird folgende Frage von Asaad (1981) beantwortet:
Sei P eine p-Sylowgruppe der endlichen Gruppe G. Angenommen $N_G(P)$ ist p-nilpotent und es gilt $|\Omega_1(P) \cap \Omega_1(P^x)| \leq p^{p-1}$ für alle $x \in G - N_G(P)$. Ist G p-nilpotent?
Im Falle $p = 2$ ist die Gruppe $GL(2,3)$ ein Gegenbeispiel. Für ungerades p gilt folgender
Satz. Sei P eine p-Sylogruppe von G, $N_G(P)$ p-nilpotent und $|\Omega_1(P \cap P^x)| \leq p^{p-1}$. Dann ist G ebenfalls p-nilpotent.

A. Vera-López: New bounds for the degree of commutativity of a p-group of maximal class

No doubt, the most important invariant related with a p-group of maximal class G is its degree of commutativity $c(G)$. Since the defining relations of G are much simpler when $c(G)$ is large, this raises the problem of finding lower bounds for $c(G)$. If $|G| = p^m$ and the index of the maximal abelian normal subgroup of G is p^a ($a \geq 3$), Blackburn showed that $c(G) \geq m - p - 2a + 4$. Shephard, Leedham-Green and McKay proved that $c(G) \geq [\frac{m-3p+7}{2}]$, a good bound when working with fixed p and large m. Nevertheless, in our study of conjugacy classes in these groups we work with m small and p arbitrary. This has led us to work with the invariant $b := max\{i \mid \exists j \geq i+1 \text{ with } \alpha_{ij} \neq 0\}$, where we follow Blackburn's notation for the α_{ij}'s. Among some other interesting bounds, for $b \geq 3$ we prove that $c(G) \geq m - p - 2b + 4$, generalizing Blackburn's result (since $b \leq a$) and that $c(G) \geq \frac{m-3b+5}{2}$. Since $b \leq \frac{m}{2}$, this last inequality yields good bounds for all primes p. Also, we precise when we can substitute 4 by a greater integer in Blackburn's type bound: if $4 \leq k \leq p$ and $m \leq 2(p-k)+b+4$ then $c(G) \geq m-p-2b+k$.

A. Weiss: Finite subgroups of the 'Nottingham' group

The Nottingham group is the group of automorphisms σ of the ring $\mathbb{F}_p[[t]]$ (of formal Taylor series over $\mathbb{F}_p$) which satisfy $\sigma t \equiv t \bmod t^2$. Using methods of Galois theory one can show that the Nottingham group contains, up to isomorphism,

1) every finite p-group

2) an infinite abelian group of exponent p

3) etc.

V. Welker: On the set of p-subgroups of $PGL_n(q)$ for $p \mid q-1$

Let p be a prime. We denote by $S_p(G)$ the partially ordered set $\{P \mid 1 \neq |P| = p^i\}$ of all nontrivial p-subgroups. To $S_p(G)$ we associate the simplicial complex $CS_p(G) = \{P_1 < \ldots < P_k \mid P_i \in S_p(G)\}$ of all chains in $S_p(G)$. This complex has been firstly studied by Guilleu and has gained considerable interest recently. We derive the dimension of the unique nonvanishing homology group of $CS_p(G)$ for $G = PGL_n(q)$ and $p \mid q-1$.

J. Wisliceny: Minimale Anzahl von Relationen bei der Präsentation algebraischer Strukturen

Die vorzustellenden Untersuchungen zur Präsentation algebraischer Strukturen mit möglichst wenig Relationen knüpfen an eine Optimalitätsaussage zum Satz von Golod-Šavarevič an, nach der eine Folge endlicher p-Gruppen existiert, für die der Quotient $\frac{r}{d^2}$ gegen $\frac{1}{4}$ konvergiert (d Erzeugendenrang, r Relationenrang).

Folgende Ergebnisse seien genannt:
Es gibt Pro-p-Gruppen G mit $d(G) = 5$ und $r(G) = 7$, so daß $g^{p^{10}} = e$ und

$(\dots(g_1, g_2), g_3), \dots, g_{15})^{p^2} = e$ für beliebige $g_i \in G$ gelten. Es existieren analytische Pro-p-Gruppen G mit $d(G) = 5$ und $r(G) = 7$ (ebenfalls z.B. mit $d(G) = 7$ und $r(G) = 14$).
In der Varietät der metabelschen Liealgebren gibt es zu jedem $n \geq 2$ nilpotente Liealgebren mit n Erzeugenden und $2n-3$ Relationen. In der Varietät der metabelschen Pro-p-Gruppen existieren endliche Pro-p-Gruppen mit n Erzeugenden und $2(n-1)$ Relationen.

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