MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 19/1992

Wavelets

3.5. bis 9.5.1992

The meeting was organized by A. K. Louis (Saarbrücken) and Y. Meyer (Paris). The tone of the conference was set by an objective but controversial scientific exchange of ideas. All participants agreed that the results presented signify an important progress and that these ideas will stimulate their further research.
The lectures can be divided into three areas:

a) *Construction of wavelet bases, applications of wavelets in Analysis*

Within the theory of wavelets, the construction of a family of arbitrary smooth wavelet bases in $\mathbb{R}^2$ and on intervals is of most important interest. For $\mathbb{R}^2$, both the analytic and the geometric approach were considered. A complete solution of the problem seems to be only a matter of time.
Great attention was also directed to the investigation of Riemann's function, the classification of singularities of signals and the analysis of self-affine, fractal functions.

b) *Application of wavelets in Numerical Analysis*

In contrast to many papers on the subject, the numerical methods and algorithms shown at the meeting are based on very special properties of wavelets. These algorithms could not be done in the same way with other techniques. The abstract connection of multi-level methods (hierarchical bases), multi-grid methods and wavelet decompositions was clarified. Surely, the ultimate success in such a young field cannot be established quickly but the given approaches do justify optimism. The early euphoria about the tool 'wavelet' is changed into a realistic judgement.

c) *Application of wavelets in Signal and System Theory as well as Image Processing*

Most of the lectures were related to the 'classical' applications of wavelets. The spectrum ranged from more theoretical to very practical (commercial) work: investigation of frames of the affine group, irregular and efficient sampling of the wavelet transform, analysis of isotropic processes on the homogeneous tree, time-frequency representation of stochastic self-similar processes and bidimensional

Gefördert durch DFG Deutsche Forschungsgemeinschaft

signals. The lectures on image understanding, image compression and neural networks were more applied. The presentation of a commercially distributed software package for image processing, which only uses wavelet techniques, showed the possibilities and efficiency of such methods.

The organizers and participants thank the '*Mathematisches Forschunsinstitut Oberwolfach*' to make the conference possible in the usual comfortable and inspiring setting.

Abstracts:

Y. Meyer:

Wavelets, chirps and the Riemann series $\sum_1^\infty n^{-2}\sin(n^2x)$

B. Escudié and B. Torrésani studied asymptotic signals with the help of the wavelet transform. We are using wavelets for studying a class of chirps defined as follows. We consider functions $g(x)$ of the real variable x which are T-periodic ($T > 0$) and satisfy $\int_0^T g(z)dx = 0$ and $g \in C^r(\mathbb{R})$, the Hölder class. A (generalized) chirp is defined as $f(x) = |x|^\alpha g(|z|^\beta)$ where $\beta > 0$ and $\alpha \in \mathbb{R}$ (if $\alpha \leq -1, f(x)$ is distribution). The wavelet transform $\int f(x)\psi_{a,b}(x)dy$, $\psi_{a,b}(x) = \frac{1}{a}\psi(\frac{x-b}{a}), a > 0$, of a chirp attains its maximum on the 'ridge' (defined by $a = |b|^{\beta+1}$) and its restriction to the ridge is called the 'skeleton'. The skeleton is an improved chirp.

Applications are given to the behavior of the Riemann function near the regular points (where the derivative exists).

L. Auslander:

M-Expansions and Geometric Invariants

Let H be the Hilbertspace of functions on $\mathbb{R}^2$ satisfying the functional equations $F(x, y+1) = F(x, y); F(x+1, y) = e^{2\pi iy}F(x, y)$ and $\int_0^1\int_0^1 F(x, y)dxdy < \infty$. We described some of the continous functions in H and discussed the winding number of functions. That are continuous and in H. We will say that G has an M-expansion relative to F if $G = \sum_n \sum_m F(x, y)e^{2\pi i(nx+ny)}$. We showed that if F is continuous the M-expansion is never orthonormal. We defined the concept of approximate orthonormal bases and constructed examples of such. Some discussion of the use of approximate orthonormal basis in radar and soner was discussed.

S. Jaffard:

Wavelets and Hausdorff dimension of singularities of signals

The characterization of functional spaces on wavelet coefficients allows to obtain refined sobolev imbeddings. For instance, we prove, that if $F \in W^{s,p}$ and $s > \frac{n}{p}$, then, the Hausdorff dimension of the set where F is $C^\alpha(s-\frac{n}{p}<\alpha<s)$ is at most $n-(s-\alpha)p$ (f is C^α at x_0 if there exists a polynomial P such that $|f(x)-P(x-x_0)|\le c|x-x_0|^\alpha$). The characterization of pointwise regularity (the belonging to C^α at x_0) on the wavelet coefficients allowes to obtain other applications. For instance, a function belongs to $\Gamma^\alpha(x_0)$ if it is essentially C^α at x_0 and not better. The spectrum of singularities of f is the function which maps to α the Hausdorff dimension of the point x_0 when f belongs to $\Gamma^\alpha(x_0)$. Wavelets allow to construct a function f which has a prescribed spectrum of singularities.

A third example is given by analyzing by adapted wavelets a large class of gaussian processes that include fractional brownian motion, but also processes with non-stationary incements. The wavelet coefficients, for this paritucular choice of wavelets become i.i.d. gaussians. Then the local and global moduli of continuity of the process are obtained.

S. Dahlke:

Wavelets adapted to differential operators

We use wavelets as basis functions for a Galerkin approach for the numerical solution of partial differential equations in $\mathbb{R}^2$. Since the structure of the resulting stiffness matrix depends on the basis functions and on the differential operator, we adapt the wavelets to a given problem such that the stiffness matrix has a simple structure, and, in some cases, a uniformly bounded condition number.

One approach constructs wavelet bases directly from a generalized orthogonality condition induced by specific differential operators. To be able to treat a wider class of problems, we construct in a different approach an adapted biorthogonal wavelet basis.

W. Dahmen:

Multilevel preconditioning and wavelet computations

This talk is concerned with a general multilevel setting for preconditioning linear systems arising from Galerkin schemes for elliptic boundary value problems. The general results imply, in particular, that the BPX scheme (also for nonuniform adaptive refinements) as well as orthogonal and biorthogonal wavelet bases give rise to uniformly bounded condition numbers. The results are based on Besov space characterizations in terms of sequences of linear projectors onto nested approximation spaces satisfying appropriate Bernstein and Jackson estimates, which in case of scaling functions for wavelets already follow from their smoothness and refinability. It is also pointed out that refinability leads to a unified treatment of the basic computational tasks, such as computing stiffness matrices, in that these computations can be reduced to an eigenvector/moment problem whose size depends only on the support of the scaling functions and wavelets.

A. Rieder:

Semi-algebraic multi-level methods based on wavelet decompositions

For the iterative solution of a linear system we present a multi-level method based on a wavelet approximation of the successive errors of a classical iterative method. The resulting iteration is a hybrid between a pure algebraic multi-level technique and the usual multi-grid technique related to a discretization of an elliptic differential operator. The methods inherits its mean feature, i.e. the coarse grid correction term from the 'continous' multi-grid iteration but it despenses with geometric considerations to define the coarse-to-fine and the fine-to-coarse transfer. Furthermore the restriction and prolongation operators of the method can be adapted well to the smoothness of the error. Moreover, we are also able to solve linear equations arising from the discretization of integral operators of the first kind.

T. Yserentant:

Subspace decompositions from the view of partial differential equations

Stable subspace decompositions of finite element spaces (and wavelet-spaces) have developed into an important tool for the construction and analysis of fast solvers for elliptic partial diffenrential equations. For multigrid methods, this approach complements the classical interpretation, especially as it concerns nonuniformely refined meshes.

The talk gave an review to recent theories of fast solvers based on such subspace decompositions, its applications, the hierarchical basis decompositions and L_2-like decompositions (leading to classical multigrid methods) have been considered. The relations to approximation theory have been discussed.

A. Benveniste:

Multiscale system theory

Orthonormal wavelets are generated by filters $H(e^{i\theta}), G(e^{i\theta})$ satisfying the standard QMF property. Combining these operators with decimation, we get operators $\mathcal{H}, \mathcal{G}$: $l^2(2^n \mathbb{Z}) \rightarrow l^2(2^{n+1}\mathbb{Z})$ for each n , and it holds that $l^2(\mathbb{Z}) = Im\mathcal{H}^* \oplus Im\mathcal{G}^*$. So it is natural to consider $\mathcal{H}, \mathcal{G}$ as acting from $l^2(\mathcal{C})$ into itself, where $\mathcal{C}$ is the dyactic tree. We study the multiplicative algebra of linear combination of monomials into such 4-tuples $\{\mathcal{H}, \mathcal{G}, \mathcal{H}^*, \mathcal{G}^*\}$ where it holds that $l^2(\mathcal{C}) = Im\mathcal{H}^* \oplus Im\mathcal{G}^*$. Rationality of such systems if defined. Characterization of stationary systems (systems commuting with translations of $\mathcal{C}$) is given, and stationary gaussian random fields on $\mathcal{C}$ are studied. Such stochastic processes exhibit a behaviour of fractal type.

I. Daubechies:

Nonseparable two-dimensional wavelet bases

The standard construction of (orthonormal or biorthogonal) wavelet bases in 2 dimensions uses a tensor product structure, involving 1-dimensional wavelets and scaling functions. One can also make a 'genuinely' 2-dimensional construction, in which the dilation is given by a 2×2 matrix D. In general one then needs to find $(\det D)$-1 wavelets; the case $\det D = 2$ is therefore especially simple. Two possible choices of such D are $R = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ and $S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. They correspond to the same subsampling lattice $D\mathbb{Z}^2$, hence to the same equation for m_0, the trigonometric polynomial underlying the multiresolution analysis to be constructed. An infinite family of solutions for m_0 leads to arbitrarily high regularity for ϕ, ψ if S is used but not if $D = R$. Similar (but symmetric) constructions can be made for the biorthogonal case, where S and R give the same result. Nonseparable wavelet bases of this type are used in image compression.

A. Cohen:

Wavelet bases adapted to an interval and applications

Wavelet bases and multiresolution analysis are usually defined in the framework of functions defined on the whole real axis. In many practical situations, the function to be analyzed is known (or restricted) on an interval, say [0.1]. Wavelets have to be adapted at the edges if one wants to avoid 'border effects' in the algorithms.

In this talk, I reviewed some methods to construct wavelet bases on the inverval and finally describe a construction by I. Daubechies, P. Vial and myself (which was independently found by B. Jawerth) that preserves the main properties existing in $L^2(\mathbb{R})$ - localication, regularity, cancellation - and allow to derive a simple algorithm.

The applications presented are image coding (removed of the borders artefacts) and an inverse problem in geoseismic in which one wishes to isolate in the high scales, the discontinuities in the underground without being influenced by the limit of the image.

P. Maaß:

Families of orthogonal $2D$ wavelets

We consider wavelets related to two dimensional multiscale analysis with a dilation matrix A, $|det(A)| = 2$. The Fourier filters of orthogonal wavelets satisfy the orthognality relation

$$|H(\omega)|^2 + |H(\omega + \pi \begin{pmatrix} 1 \\ 1 \end{pmatrix}|^2 = 1 \quad (*).$$

We observe that $\mathcal{K}_{orth} = \{q | q = |H|^2 , \ H \ solves \ (*) \}$ has a flat subspace, namely the set of induced 1-dimensional filters $\mathcal{K}_1$. Moreover the extremal points of the convex set

Gefördert durch DFG Deutsche Forschungsgemeinschaft

$\mathcal{K}_1$ are known to be the Daubechies-wavelets D_N. The construction of 1-dimensional wavelets utilizes a factorization technique for positive triginometric polynomials. An equvialent technique is not known in higher dimensions. The starting point for our construction is an explicit description of the tangent space of $\mathcal{K}_{orth}$ at D_N. Starting at D_N we can trace $\mathcal{K}_{orth}$ along its tangents which gives a variety of families of orthogonal wavelets. Examples for the quinconx grid are given.

C.K. Chui:

On Frames of the Affine Group

This is a joint work with X.L. Shi. Under a very mild condition on the decay at infinity and either the assumption of piecewise Lip α, some $\alpha > 0$, continuity, or a very mild assumption on the control of the variation of ψ, where $\psi \in L^2(-\infty, \infty)$, we prove that $\{\psi_{b;j,k}\}$ is a Bessel family, if and only if ψ has zero mean. Here, $\psi_{b;j,k}(x) := a^{j/2}\psi(a^j x - kb)$, where $a > 1$ and $b > 0$. We also prove that the same conclusion can be made if $\hat{\psi}(w)$ is dominated by some $\Theta(|\omega|)$, where $\Theta(x)/x$ is integrable at 0 and $\Theta(x)\,x$ integrable at ∞. Hence, for $a = 2$ and $b = \frac{1}{n}$ where n is any positive integer, all the known wavelets give rise to frames. On the other hand, sufficient conditions for frames are also derived. We also study the problem of preservation of frame bounds in the oversampling process. In this direction, we have shown that if $a \geq 2$ is a positive integer and $\{\psi_{b;j,k}\}$ is a frame, then $\{n^{-1/2}\psi_{b/n;j,k}\}$ is again a frame with the same frame bounds, provided that $(a,n) = 1$. Consequently, under this condition, a tight frame remains tight in oversampling. However, this conclusion is not valid in general if $(a,n) \neq 1$. We also extend this result to preservation of frame series representations for frames with duals, again by oversampling.

A. Benveniste:

Wavelet networks

Functions of the form

$$f(x) = \sum_k w_k \psi(a_k x + b_k)$$

are studied, were ψ is a wavelet. Such f's can be used as universal approximants for functions, note that both w's, a's and b's are adjusted. Experiments of fitting such models from noisy data using stochastic gradient techniques are reported, very much likewise neural networks. Then both synthesis and decomposition formulae of continuous wavelet transform are used to derive tight bounds for both the approximation and fitting-frame-data problems. The idea is to draw translations and dilations at random from some 'probability' exhibited by the wavelet decomposition formula.

A. Grünbaum:

Local vs. global operators: translations, dilations and beyond...

I discuss the consequence of insisting that a certain natural family of integral operators should actually be given by differential operators. This problem makes sense for any manifold, like the sphere, where one does not have a nice theory of wavelets. Problem: Under what conditions on the differential operator L in $\mathbb{R}$ can one find nontrivial differential operators Θ such that $e^{-tL}\Theta e^{tL}$ is a differential operator in $\mathbb{R}$. Among the solutions one can find manifolds of potentials V with $L = D^2 + V$, such that translations and dilations, as well as a whole infinite hierarchy of other operators (including the KdV flow), act nicely.

T. Beth:

Fast algorithms for the feature transform

It is argued that wavelet transforms are special kinds of correlations of signals $\underline{s}$ with a test feature $\underline{\varphi}$ being moved, scaled etc. under a group G.
This group G is generally the holomorph

$$Hol(N) = Aut N \times N$$

of a translation group N, which, if it is locally compact and possibly abelian, allows direct comparisons between the wavelet transform

$$\forall (A,b) \in Hol(N) \quad T_s(A,b) = \int_{\mathbb{R}^n} s(\underline{x})\varphi(A\underline{x}+b)d\underline{x}$$

and the Fourier-Transform for which fast algorithms are available. Wavelet-transforms in this sense are feature transforms from finite energy signals to the group algebra

$$\Phi : L_2(\mathbb{R}^n) \to \mathbb{C}[G] .$$

In order to do proper computations, we only consider the representation on $L_2(N)$ of $\mathbb{C}[G]$.
Generalized Wedderbun-Direchlet-transforms give the equivalent to the Fourier-transform. It is shown that the WDT of a picture $g(\underline{x}) = \sum_{i=1} w_i\varphi(A_i\underline{x} + \underline{b})$ gives $\hat{g} = \hat{\varphi} \cdot \sum_{i=1}^{N} w_i P_i$ where $P_i = \rho(A_i, b_i)$ are the representations of the 'motions' (A_i, b_i).
Thus we have obtained a position descriptor in the representation domain, from which picture processing and understanding can be derived.
This talk is concluded with a suite of transparencies of the mechanisms applied to implement the motion transform using Mackey's theorem, (joint work with H. Hartl).

H.G. Feichtinger:

Wiener amalgam spaces with applications to (irreg.) sampling of Wavelet Transforms and band-limited functions

Given 2 suitable Banach spaces B, C, eg. $B = L^p, FL^r, B^s_{p,q}, C^0, M$ or $C = L^q_w$ or mixed norm weighted spaces, we define

$$W(B,C) = \{f | f \in B_{loc}, F : x \to \|(T_x k) \cdot f\|_B \text{ belongs to } C\}$$

is the Wiener amalgam space with local component B and global component C. There are natural results on duality, multiplication, convolution (just do it seperately for the local and global component). There are also Hausdorff-Young inequalities (cf. Oberwolfach, Butzer-Konf., 1980). Here we present appliction to the following cases:

i) L^P-convergence of usual Shannon-series for $L^P, 1 < p < \infty$:

$$(\star)\ f = \sum_{n \in \mathbb{Z}^m} f(\alpha n) T_{\alpha n} g \quad \text{for } g = \operatorname{sinc}.$$

ii) jitter error estimates, of $f(\alpha_n + \gamma_n)$ or $T_{\alpha_n + \delta_n} g$ is used.

iii) convergence of $(\star)$ in L^p_w, if $g \in L^1_w$ with compact spectrum, for $f \in L^p_w$ (band-limited).

iv) $(\sum_{i \in I} |f(x_i)|^p)^{1/p} \le C \|f\|_p$ for band-limited function, if (x_i) is relatively separated (= finite union of separated sets).

There is an analogy for wavelet transforms:
$F(x) := (\pi(x)g, f)$, where g is the analyzing wavelet, also satisfies a similar convolution relation: $F = F \star G$, with convolution on the $'ax + b'$-group (π being the natural representation on $L^2(\mathbb{R}^m)$. Therefore jitter errors (for example) can be estimated in a similar way.

K. Gröchenig:

Irregular Sampling of Wavelet Transforms

We discuss the problem of irregular sampling for wavelet transforms $W_g f(x,s) = \langle g_{x,s}, f\rangle$, where $g_{x,s}(t) = s^{-1} g\left(\frac{t-x}{s}\right)$, for $x \in \mathbb{R}$, $s > 0$.
If the wavelet g is band-limited and satisfies $\operatorname{supp} \hat{g} \subset [-\Omega, -\omega] \cup [\Omega, \omega]$ for some $0 < \omega < \Omega$ and $|\hat{g}(\xi)| \ge c > 0$ on a subinterval of the support, then any set of the form (x_{jk}, s_j) in the upper half plane is a set of sampling for $W_g f$ under the natural assumptions

$$\sup_{j \in \mathbb{Z}} \frac{s_{j+1}}{s_j} \le \gamma < \frac{\Omega}{\omega} \quad \text{and} \quad \sup_{j,k \in \mathbb{Z}} \frac{x_{j,k+1} - x - jk}{s_j} = \delta < \frac{\pi}{\Omega}.$$

This means that $\sum_{j,k\in\mathbb{Z}} \frac{1}{4}(x_{j+1,k} - x_{jk}) \ln \frac{s_{j+1}}{s_j} |W_g f(x_{jk}, s_j)|^2$ defines a norm equivalent to $\|f\|^2$. Here the occuring constants can be estimated explicitly. The occuring weights balance local variations of the sampling density and are a new feature of irregular sampling.

J. Byrnes:

A Low Crest Factor Complete Orthonormal Set

We construct a sequence of polynomials $\{P_{n,m}(z)\}, n \geq 0, 0 \leq m \leq 2^n$, each of which is a quadrature mirror filter, and each of which also represents an antenna array with crest factor $\sqrt{2}$. (The crest factor, im mathematical terms, is the ratio of the sup norm to the L^2 norm of a polynomial on yhe unit circle). Furthermore, by defining $Q_{n,m} = (1-z)\,P_{n,m}(z^2)$, and then taking the sequence of piecewise constant functions on $(0, 2pi)$ whose values are the ± 1 coefficients of the Qn, m's, there immediately results a complete orthonormal set (CONS) for the space $L^2(0, 2\pi)$ (and, in fact, for $C[0, 2\pi]$). Finally these 'Walsh-like' (but definitely not Walsh!) functions, when combined with wavelet-like dilations and translations, give a CONS for $L^2(\mathbb{R})$ which is optimal with respect to the uncertainty principle.

W. Hackbusch:

Panel Clustering

The boundary element method (BEM) in $\mathbb{R}^d$ leads to large full matrices and therefore to the disadvantages of a high amount of work for the assembling of the system matrix and for the matrix-vector multiplication. The panel clustering method involves an expansion of the kernel into Taylor's series of a certain length and a tree of clusters. It is essential that the surface can be represented as a disjoint union of $0(\log n)$ clusters where n is the number of panels. This leads to a work of $0(n \log^{d+1} n)$ operations per matrix-vector multiplications. Also the storage turns out to need $0(n \log^{d+1} n)$ data.

A. K. Louis:

Wavlets and Inverse Problems

In inverse and ill-posed problems typically the high frequency components are strongly affected by data noise. The standard methods for regularization of these problems which is a trade-off between accuracy and damping of the noise need a priori information and a selection of a parameter depending on the smoothness of the solution and the data error. The wavelets provide a tool to seperate the different frequencies in the solution. The decay rate of the wavelet coefficients give the possibility to insure the smoothness of the approximations. The wavelet decomposition is formulated as an iterative process with stopping rule leading to an order-optimal regularization method.

D. Walnut:

Wavelets and the Radon transform

Recently Holschneider, Kaizer, Streater, Louis, Donohue and others have pointed out a connection between wavelets and the Radon transform. We investigate this relationship and derive an inversion formula for the Radon transform based on the continuous wavelet transform. These formulas give direct reconstruction of f or $\Delta^{-1/2}f$ can be recovered from this data. We show that these formulas can be applied to the problem of local inversion of the Radon transform in even dimensions, as studied by Smith, Faridani, Natterer, Keinert and Ritman.

R. Schneider:

Wavelet Approximation Methods for Periodic Pseudodifferential and Calderòn Zygmund Operators

General Petrov Galerkin methods for the numerical solution of periodic pseudodifferential equations based on a multiresolution analysis for trial functions and admissible distributions as test functionals are investigated. A convergence theory is established in a framework of a symbolic calculus for the corresponding family of finite dimensional operators. Necessary and sufficient conditions for quasi optimal convergence are formulated. The perspective of a wavelet basis for efficient numerical calculation is discussed. In particular, preconditioning and compression of the arising stiffness matrices up to a fixed error bound or to obtain optimal convergence rates are investigated.

P. Flandrin:

Time-Scale Analysis of Stochastic Self-Similar Processes

Time-scal analyses are well-suited for analyzing nonstationary processes which exhibit self-similarity structures. Globally self-similar processes, such as fractional Brownian motions, can be approximately 'whitend' by wavelet (orthogonal) decompositions provided that a sufficient amount of moments is vanishing. In the case of locally self-similar processes (e.g. variations on fractional Brownian motions on the Weierstraßfunction with time dependent index), instantaneous scaling laws can be recovered from time-scal energy distributions generalizing in a bilinear way the classical continuous wavelet transforms.

B. Claus:

Isotropic processes on the homogeneous tree

The coefficients of the (1D) wavelet-transform live on the dyadic tree, while in image processing we encounter a pyramidal index-structure, i.e. a homogeneous tree of order 4. This is the motivation why we develop as a counterpart to the (deterministic)

wavelet-transform a theory of stochastic processes indexed by the nodes of a homogeneous tree of order q.
The simplest concept which comes to mind are isotropic processes, i.e. processes which covariance function depends only on the distance on the tree. We introduce generalized Levinson- and Schur-recursions, and we obtain thus two equivalent characterizations of the process: The covariance sequence r which admits a generalized Bochner-like representation (due to Arnaud), and the reflection coefficient sequence k, with $-1 \leq k_n \leq 1$ for n odd, $-1/q \leq k_n \leq 1$ for n even (recall that q is the order of the tree).
One defines AR(n)-processes by the condition $k_n \neq 0$, $k_m = 0 \; \forall m > n$. We adressed the following identification problem: Fix an AR(n)-model (on the tree) to a standard 1D-signal, which is supposed to be the restriction of the tree-process to one level of the tree. Via a maximum likelihood crition one computes the optimal position of the signal with respect to the indexing tree, and we obtain estimates of the even-indexed covariances $r(2i)$. If the process is supposed to be AR(n), then we have only two valid interpolations of the sequence $r(2i)$. Having thus obtained the convariance sequence r, we 'reconstruct' the process by calculating the conditional expectation of the process on the subtree above the level on which the signal was observed.

H.-G. Stark:

A remark on image understanding and invariance groups

An outline of knowledge based on computer vision, leading to a picture description as attributed relational graphs, is given. The nodes are labled by 'property values' of the image parts. Those properties actually are (realvalued) functionals in the image space. Position and scale invariance of the objects is related to invariance of the properties under the adjoint representation of IG(2) on the linear space of the functionals. It is argued that linear properties cannot be invariant in the above sense and examples of nonlinear invariant properties are given.

J. P. Antoine:

The scale-angle representation in image analysis with 2D wavelet transforms

The 2D continuous wavelet transform (CWT) of a signal s is given by $S(a,\vartheta,b) = \langle\psi_{a\vartheta b}, s\rangle$, i.e the L^2 inner product of the signal with the wavelet $\psi_{a\vartheta b}$, obtained from ψ through translation by $b \in \mathbb{R}^2$, dilation by $a > 0$ and rotation by angle ϑ: $\psi_{a\vartheta b}(x) = a^{-1}\psi\left(a^{-1}R^{-\vartheta}(x-b)\right)$, R^ϑ the usual 2×2 rotation matrix. Whereas the CWT is usually used in the position representation, i.e. for fixed a, ϑ, the scale-angle representation consits in analyzing the transform S as a function of (a,ϑ) for a fixed (observation) point b. The interest of this representation is illustrated on the following two problems:

- evaluating the performance of the wavelet ψ through its reproducing kernel, in particular its selectivity in scale and direction (as needed for discretizing the reconstruction formula), the standard examples treated being the mexican hat and the Morlet wavelet.

- determining all the parameters of a linear superposition of damped plane waves (this is the 3D analog of the detection of spectral lines).

Additional information on the angular selectivity of a given wavelet may be obtained by computing the CWT of a simple signal with an intrinsic orientation, e.g. a half infinte segment. We analyze the lattice with the two wavelets mentioned above. The result suggest a quantitative definition of the angular resolving power of a given wavelet.

Referee: Andreas Rieder

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Prof.Dr. Jean-Pierre Antoine
Institut de Physique Théorique
Université Catholique de Louvain

B-1348 Louvain-la-Neuve

Prof.Dr. Charles K. Chui
Dept. of Mathematics
Texas A & M University

College Station , TX 77843-3368
USA

Prof.Dr. Louis Auslander
Department of Mathematics
CUNY, Graduate School

New York NY 10036
USA

Bernhard Claus
IRISA/INRIA
Campus de Beaulieu

F-35042 Rennes Cedex

Dr. Albert Benveniste
IRISA/INRIA
Campus de Beaulieu

F-35042 Rennes Cedex

Prof.Dr. Albert Cohen
CEREMADE, Université de Paris
Dauphine (Université de Paris IX)
Place de Lattre de Tassigny

F-75775 Paris Cedex 16

Prof.Dr. Thomas Beth
Institut für Algorithmen und
Kognitive Systeme
Universität Karlsruhe
Am Fasanengarten 5, Geb. 5034

W-7500 Karlsruhe 1
GERMANY

Dr. Reiner Creutzburg
Fakultät für Informatik
Institut für Algorithmen und
Kognitive Systeme
Postfach 6980

W-7500 Karlsruhe 1
GERMANY

Prof.Dr. James Byrnes
Prometheus Inc.
21 Arnold Ave.

Newport, RI 02840
USA

Dr. Stephan Dahlke
Institut für Geometrie und
Praktische Mathematik
RWTH Aachen
Templergraben 55

W-5100 Aachen
GERMANY

Prof.Dr. Wolfgang Dahmen
Institut für Geometrie und
Praktische Mathematik
RWTH Aachen
Templergraben 55

W-5100 Aachen
GERMANY

Prof.Dr. Karlheinz Grochenig
Department of Mathematics U-9
University of Connecticut

Storrs , CT 06269-3009
USA

Prof.Dr. Ingrid Daubechies
AT & T Bell Laboratories
600 Mountain Avenue

Murray Hill , NJ 07974-2070
USA

Prof.Dr. F. Alberto Grünbaum
Dept. of Mathematics
University of California
881 Evans Hall

Berkeley , CA 94720
USA

Dr. Hans Georg Feichtinger
Institut für Mathematik
Universität Wien
Strudlhofgasse 4

A-1090 Wien

Prof.Dr. Wolfgang Hackbusch
Institut für Informatik und
Praktische Mathematik
Universität Kiel
Hermann-Rodewald-Str. 3/1

W-2300 Kiel 1
GERMANY

Dr. Patrick Flandrin
Laboratoire de Physique
Ecole Normale Supérieure de Lyon
46, Allée d' Italie

F-69364 Lyon Cedex 07

Prof.Dr. Michiel Hazewinkel
Centrum voor Wiskunde en
Informatica
Postbus 4079

NL-1009 AB Amsterdam

Dr. Reiner Fröhlich
Fachbereich Mathematik
Universität Kaiserslautern
Postfach 3049

W-6750 Kaiserslautern
GERMANY

Prof.Dr. Stephane Jaffard
CEREMADE
Université Paris-Dauphine
Place du Maréchal de Lattre de
Tassigny

F-75775 Paris Cedex 16

Prof.Dr. Björn Jawerth
Dept. of Mathematics
University of South Carolina

Columbia , SC 29208
USA

Prof.Dr. Frank Natterer
Institut für Numerische und
Instrumentelle Mathematik
Universität Münster
Einsteinstr. 62

W-4400 Münster
GERMANY

Prof.Dr. Alfred K. Louis
Fachbereich Mathematik - FB 9
Universität des Saarlandes
Bau 38

W-6600 Saarbrücken
GERMANY

Prof.Dr. E. Todd Quinto
Dept. of Mathematics
Tufts University

Medford MA 02155
USA

Prof.Dr. Peter Maaß
Fachbereich Mathematik
Bau 38
Universität des Saarlandes

W-6600 Saarbrücken
GERMANY

Andreas Rieder
Fachbereich Mathematik - FB 9
Universität des Saarlandes
Bau 38

W-6600 Saarbrücken
GERMANY

Prof.Dr. Wolodymyr R. Madych
Dept. of Mathematics
University of Connecticut
196, Auditorium Road

Storrs , CT 06268
USA

Reinhold Schneider
Fachbereich Mathematik
TH Darmstadt
Schloßgartenstr. 7

W-6100 Darmstadt
GERMANY

Prof.Dr. Yves Meyer
CEREMADE, Université de Paris
Dauphine (Université de Paris IX)
Place de Lattre de Tassigny

F-75775 Paris Cedex 16

Prof.Dr. Kristian Seip
Division of Mathematical Sciences
University of Trondheim

N-7034 Trondheim

Peter Singer
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2

W-8520 Erlangen
GERMANY

Dr. Hans-Georg Stark
TECMATH GmbH
Sauerwiesen 2

W-6750 Kaiserslautern
GERMANY

Prof.Dr. David F. Walnut
Department of Mathematics
George Mason University

Fairfax , VA 22030-4444
USA

Prof.Dr. Harry Yserentant
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

W-7400 Tübingen 1
GERMANY