MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1992

Singularitäten

31.5. bis 6.6.1992

Die Tagung fand unter der Leitung von G.-M. Greuel (Kaiserslautern), J. Kollár (Salt Lake City) und J.H.M. Steenbrink (Nijmegen) statt. Aufgrund der sich zu einem gewissen Grad überschneidenden Interessengebiete war es möglich, einen Teil der Vorträge gemeinsam mit der Paralleltagung „Free resolutions in algebraic geometry and representation theory" stattfinden zu lassen. (Dieser Bericht bezieht sich auf alle Vorträge von Teilnehmern, die formal der Tagung über Singularitäten angehörten.)

Bei den Vorträgen und Diskussionen standen Untersuchungen von Singularitäten komplex-analytischer Abbildungen und Varietäten im Mittelpunkt des Interesses. Einen breiten Raum nahmen dabei Fragen ein, die die Topologie und Monodromie von Milnor-Fasern betreffen. Andere Vorträge befaßten sich vornehmlich mit Äquisingularitätsfragen. Das Studium von Deformationen und Modulräumen bildete einen weiteren Schwerpunkt, während in einer Reihe von Vorträgen Auflösungen die grundlegende Rolle spielten. Ein Resultat zur Klassifikation von Singularitäten war durch die Klassifikationstheorie der algebraischen 3-Mannigfaltigkeiten motiviert. In einzelnen Beiträgen wurde auch ein mehr globaler Standpunkt eingenommen, in anderen wurden reelle Aspekte untersucht.

Im Rahmen der Tagung hielt E. Brieskorn einen Sondervortrag mit dem Titel „Felix Hausdorff — elements of a biography".

Vortragsauszüge

K. ALTMANN:

Deformations of affine toric varieties

As a generalization of the case of cyclic quotient singularities, we regard the deformation theory of affine toric varieties $X_\sigma = Spec\,\mathbb{C}[\check{\sigma} \cap \mathbb{Z}^n]$.

1. The tangent space T^1 of the base space S of the miniversal deformation is $\mathbb{Z}^n$–graded. For a fixed $R \in \mathbb{Z}^n$ it is possible to compute the graded piece $T^1(R)$ as a vector space that is related to the linear dependences between certain (minimal) generators of $\check{\sigma} \cap \mathbb{Z}^n$ as a semigroup.
2. In the case of CQS, the total spaces over the irreducible components of S_{red} are toric varietes again. Motivated by this, a polyhedral description of the case when a toric variety is contained as a relative complete intersection in another one, is given.
3. In many examples, these topics are related to the splitting of certain polytopes into a Minkowski sum.

R. BLACHE:

Moishezon surfaces with log–canonical singularities

We study examples of the influence of the presence of log–canonical singularities on the theory of Moishezon surfaces. We use the minimal model theory of F. Sakai. The results include the following:

Let X be a minimal log–canonical Moishezon surface with $kodX = 0$. Then for the canonical covering Y of X there are exactly the following possibilities.

1. Y is a K3–surface (possibly with RDP's), or
2. Y is a torus, or
3. $\#Sing_{elliptic} Y = 1$, or
4. $\#Sing_{elliptic} Y = 2$.

One has $0 \leq \chi(X) \leq 2$ and $\#Sing_{non-l.c.} X \leq 2$. If the index satisfies $I(X) = 1$ then the global index is $G(X) \in \{1, 2, 3, 4, 6\}$. If $I(X) \neq 1$ then $I(X) = G(X) \leq 66$.

Our results are obtained by three methods: minimal resolution, canonical covering, RDP-resolution (i.e. you blow down each exceptional (-2)-curve of the minimal resolution again).

W. EBELING:

On Coxeter-Dynkin diagrams of singularities

Let $f : (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ be an isolated hypersurface singularity. The intersection matrix S with respect to a strongly distinguished basis of vanishing cycles of f is represented by a graph D, called a Coxeter-Dynkin diagram of f. This is a graph with edges weighted by $+1$ and -1 and with a numbering of the vertices. A closed path in D is called a monotone cycle if in traversing the path the numbering of the vertices is increasing (except for the last step).

G.G. Il'yuta has conjectured that the modality of f is equal to the minimum over all diagrams of f of the smallest number of entries of S that have to be put equal to zero in order to delete all monotone cycles of length ≥ 3, and also of the number of negative entries of S.

We give counter examples to these conjectures. We also show that the set $\mathcal{M}(D)$ of monotone cycles of the graph D has in many cases the structure of an abstract simplicial or polytope complex. We also indicate diagrams for the unimodal singularities with this property which satisfy Il'yuta's minimality conditions.

We conjecture that if f has at least one diagram D with $\mathcal{M}(D)$ a polytope complex then the minimum over all diagrams D with $\mathcal{M}(D)$ a polytope complex of the minimal embedding dimension of $\mathcal{M}(D)$ is equal to the corank of f.

S. GUSEIN-ZADE:

Calculation of Dynkin diagrams of some complete intersections via real morsifications

The matrix of intersections of elements of a distinguished basis (or the Dynkin diagram) is an important topological characteristic of a singularity. One method to calculate Dynkin diagrams of isolated hypersurface singularities (due to A'Campo and Gusein-Zade) uses real morsifications of a special type and can be applied to functions of two variables. This method can be formulated for one dimensional complete intersections, too. In this case the Dynkin diagram can be read from a real immersed curve with ordinary self-intersections on a real surface (in case there exist real perturbations of the equations of a special sort). In particular, effective results can be obtained for complete intersections in $(\mathbb{C}^3, 0)$ defined by equations of the form $x^2 + y^2 + z^2 = 0$, $f(x,y) = 0$. Recently there was elaborated a version of the method which can be applied to a number of complete intersections in $(\mathbb{C}^3, 0)$ of the form $g(x,y) + z^2 = 0$, $f(x,y) = 0$.

S. KATZ:

Threefold singularities arising from contraction of curves

A curve $\mathbb{P}^1 \cong C \subset X$; X a smooth threefold, is exceptional if there exists an analytic contraction $f : X \to Y$, $f(C) = p \in Y$, $f : X - C \approx Y - p$. If $C \cdot K_X = 0$, Miles Reid proved that the resulting Y has a cDV singularity, that is, its general hyperplane section through p is a rational double point. Let $l(C) = \text{length}_{\mathcal{O}_{X,C}} f^{-1}(p)$. A couple of years ago, D. Morrison and I described which RDP arise for each $l(C)$ between 1 and 6.

To generalize this to $C \cdot K = n \geq 0$, one more notion is needed. $C \subset X$ has a rational formal neighbourhood (RFN) if $H^1(\widehat{X}, \mathcal{O}_{\widehat{X}}) = 0$, $\widehat{X}$ = formal completion of X along C. Assuming this, a classification is given for arbitrary n.

If C does not have a RFN, weaker results can still be given. These are illustrated for the case for which $N_{C/X} \cong \mathcal{O}_C(2) \oplus \mathcal{O}_C(-5)$. A recent example of T. Ando's is found on this list.

These considerations can also be used to describe some flips.

J. KOLLÁR:

Flatness criteria

Let $f : X \to Y$ be a morphism of schemes. Assume that f "looks flat" in the sense that say all fibres have the same dimension and X has no embedded points. We want to find conditions on the fibres of f which imply that f is flat.

The best known result is the so-called Hironaka lemma: if Y is regular and $f^{-1}(0)$ is generically reduced and *red* $f^{-1}(0)$ is normal then f is flat. I am developing more refined versions of this result, some of them conjectured at the moment.

LÊ DŨNG TRÁNG:

Singularities at infinity of affine curves

Let $f : \mathbb{C}^2 \to \mathbb{C}$ be a complex polynomial function. Let F be the corresponding homogenized polynomial. We blow up $\mathbb{P}^2$ at the points $F = z^d = 0$ to obtain a non-singular space Z and a map $\varphi : Z \to \mathbb{P}^1$ which compactify $\mathbb{P}^2 - \{F = z^d = 0\} \xrightarrow{(F:z^d)} \mathbb{P}^1$.

The blowing-up is obtained by resolving the general curve of the linear system $\mu F - \lambda z^d = 0$ and by blowing up points in a simple way. The components of the exceptional divisors are either dicritical, i.e. the restriction of φ is not constant, or φ is constant on them.

We show that the family $F - \lambda z^d = 0$ is not equisingular at $F = z^d = 0$ iff there are dicritical components of a special type.

J. LIPMAN:

Differential invariants of an embedding of a complex manifold in a complex space

Let W be a submanifold of a complex analytic space V, with normal cone $C = C(V, W)$. Let (V', W') be a second such pair, and let $f : (V, W) \to (V', W')$ be a C^1 map. Then (theorem) f naturally induces a continuous map from $C(V, W)$ to $C(V', W')$, compatible with the action of $\mathbb{R}$ on the fibres. There is a "simultaneous complexification" $\tilde{C}(V, W)$ which is a complex cone over W whose fibres are the complexifications of those of C, considered as real-analytic cones. (Actually, one needs first to remove a complex subvariety from W.) This $\tilde{C}$, together with the natural $\mathbb{C}$–action on the fibres is also C^1–functorial; and consequently *its Segre classes are differential invariants of* (V, W). In particular, if V itself is a manifold, then the total Segre class of $\tilde{C}$ is essentially the inverse Pontryagin class of the vector bundle $C(V, W)$. The differential invariance of the multiplicity of W in V — a weak, previously known special case of Zariski's notorious multiplicity conjecture — is a particular instance.

D. MOND:

How good are real pictures?

Background: Theorem (Damon, Mond): Let $f : \mathbb{C}^n, S \to \mathbb{C}^p, 0 (n \geq p-1, S$ finite) be of finite A_e–codimension. Then the discriminant of a stable perturbation $f_t : U_t \to \mathbb{C}^p$ has the homotopy type of a wedge of $(p-1)$–spheres. Call the number of these μ_Δ ("discriminant Milnor number"). It is an invariant of f. To what extent can one see this over $\mathbb{R}$?

Proposition: If, in the above situation, f and f_t are real, and $f_{\mathbf{R},t} = f_t|_{U_t \cap \mathbf{R}^n}$, then if rank $H_{p-1}(D(f_{\mathbf{R},t})) = \mu_\Delta$ then the inclusion $D(f_{\mathbf{R},t}) \hookrightarrow D(f_t)$ induces an isomorphism on H_{p-1}.

Call a perturbation f_t with this property a good real perturbation. We conjecture that such perturbations always exist when A_e–codimension $(f) = 1$.

Theorem 1: This conjecture holds when

(i) $|S| = 1,\ n \geq p,$ corank $df(0) = 1$

(ii) $(n, p) = (2, 3)$

((i) includes all mono germs with $p \leq 3$).

Theorem 2: The germs $\mathbb{C}^2, 0 \to \mathbb{C}^3, 0$ with good real perturbations are the members of the series $H_k : (x, y) \to (x, y^3, xy + y^{3k-1})$.

Proposition: If $f : \mathbb{C}^n, S \to \mathbb{C}^p, 0 (n \geq p-1, (n, p)$ nice) is real, quasi-homogeneous of A_e–codimension 1, and if f_t is any real stable perturbation then $D(f_{t,\mathbf{R}})$ is homotopy equivalent to S^k for some $0 \leq k \leq p-1$.

Proposition: Monodromy of $D(f_t) = (-1)^{(\text{change in } k \text{ as } t \text{ crosses } 0 \text{ in } \mathbf{R})}$

A. NÉMETHI:

Injective analytic maps (a conjecture of Lê – Hironaka)

We prove that the injectivity of $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^{n+1}, 0)$ implies that the image $X = imf$ is either an equisingular family of plane curve singularities or a special singularity with a very precise analytic property.

By definition, a germ of a hypersurface singularity $F : (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ (or a space-germ $(X, 0) = (F^{-1}(0), 0))$ is "good" if there exists a coordinate system $\{w_1, \ldots, w_{n+1}\}$ in $(\mathbb{C}^{n+1}, 0)$ such that

$$(\frac{\partial F}{\partial w_1}, \ldots, \frac{\partial F}{\partial w_{n-1}})\mathcal{O}_{n+1} \not\subset (w_1, \ldots, w_{n-1}, \frac{\partial F}{\partial w_n}, \frac{\partial F}{\partial w_{n+1}})\mathcal{O}_{n+1}.$$

Theorem: If $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^{n+1}, 0)$ is a "good" injective germ, then rank $df(0) \geq n - 1$. Moreover, imf is an equisingular family of plane curve singularities over a smooth base-space.

This gives a quasi-answer to the conjecture of Lê (about injective map germs $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$; conjecturally their rank $[df(0)]$ is nonzero).

G. PFISTER:

Moduli for singularities

The aim of this talk is to give a survey on recent results about moduli spaces for curve singularities, special hypersurface singularities and for modules over the local ring of a fixed curve singularity. A general method for constructing moduli spaces is the following:

1. One starts with an algebraic family $X \to T$ with finite dimensional base T which contains all isomorphism classes of objects to be classified. This is usually a versal deformation of the "worst" object.

2. In general, T will contain analytically trivial subfamilies and one tries to interpret these as orbits of the action of a Lie algebra or an algebraic group on T. This is often the kernel of the Kodaira–Spencer map of the family $X \to T$. In the cases we are going to consider we are able to reduce this to an action of a finite dimensional Lie–algebra L such that the orbits of L are the isomorphism classes of an object.

3. Understand the structure of the orbit space $M = T/L$. Usually one needs a stratification $T = \cup T_\alpha$ such that T_α/L is a geometric quotient (in the sense of Mumford). The stratification will be defined by fixing certain invariants of the objects to be classified. $\amalg T_\alpha/L$ is a coarse moduli space for the objects we started to classify.

Let $A = \mathbb{C}[[x_1, \ldots, x_n]]/f$ be the complete local ring of a quasi-homogeneous hypersurface singularity with weights $w_1, \ldots, w_n$. If the moduli stratum of $A_0 = \mathbb{C}[[x]]/f_0$ (f_0 the principal part of f) has dimension 0 then A_0 is uniquely determined by the weights. Let H^i be the ideal generated by all quasihomogeneous polynomials of degree $\geq iw$, $w = min\{w_1, \ldots, w_n\}$. This filtration defines a Hilbert function on the Tjurina algebra of A by $\tau^i(A) := \dim_{\mathbb{C}} \mathbb{C}[[x]]/(f, \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}, H^i)$. Assume now that $f_0 = x_1^{m_1} + \ldots + x_n^{m_n}$, $gcd(m_i, m_j) = 1$ for $i \neq j$. Then the moduli stratum is zero dimensional.

Theorem There exists a coarse moduli space $\mathcal{M}_{\underline{w},\underline{\tau}}$ for all semiquasihomogeneous singularities with weight $\underline{w}$ and Hilbert function $\underline{\tau}$ fixed. $\mathcal{M}_{\underline{w},\underline{\tau}}$ is an algebraic variety, locally closed in a weighted projective space.

A similar result is true for torsion free modules of rank one over a local ring R of an irreducible curve singularity.

A.J. PARAMESWARAN:

Toplogical equisingularity for isolated complete intersection singularities (icis)

For any isolated complete intersection singularity (X_0, x) we define:

(i) $\mu(X_0, x) =$ the Milnor number of (X, x)

(ii) $\nu(X_0, x) =$ the multiplicity of the discriminant in a versal deformation

(iii) $\mu_i(X_0, x) = \inf\{\mu(X_i, x) | f^i : (X_i, x) \rightarrow (\mathbb{C}^i, 0)$ is a deformation of $(X_0, x)\}$.

We define the monodromy fibration of an icis as the fibration obtained from any μ_1-minimal embedding. The topological type of an icis is defined to be the homeomorphism type of the nested sequence $(X_0, x) \subset (X_1, x) \subset \ldots \subset (X_k, x)$, with $k =$ embedding codimension of (X_0, x) and $\mu_i(X_0, x) = \mu(X_i, x)$.

Theorem 1 All fibres of a ν-constant deformation have isomorphic monodromy fibrations ($\dim X_s \neq 2$).

Theorem 2 In a $\mu_* := (\mu_0, \mu_1, \ldots)$-constant deformation of an icis the topological type is constant.

A. RÖHR:

Formate rationaler Flächensingularitäten

Eine Singularität Y heiße **Format** einer gegebenen Singularität $(X, 0) \subset (\mathbb{C}^n, 0)$, wenn ein vollständiger Durchschnitt $Y \rightarrow Z$ existiert, und wenn für jeden vollständigen Durchschnitt $Y \rightarrow Z$ gilt: $Z \simeq Y \times \mathbb{C}^k$. Totalräume von Deformationen über glatten Glättungskomponenten von X sind bis auf einen glatten Faktor Formate von X; sie sind nämlich starr.

Sei X eine rationale Flächensingularität der Multiplizität $n > 2$ und $F(X)$ ihr zur Artinkomponente assoziiertes Format. Sei $\widehat{X} \to X$ die RDP–Auflösung von X und $\widehat{Z}$ die exzeptionelle Faser. Sei $C(X)$ der Kegel ber $\widehat{Z} \hookrightarrow \mathbb{P}^{n-2}$, eingebettet mit der sehr amplen invertierbaren Garbe $\omega_{\widehat{X}} \otimes \mathcal{O}_{\widehat{Z}}$. Sei X' eine andere rationale Flächensingularität.

Vermutung: Die Aussagen (1) $F(X') \simeq F(X)$, (2) es existiert ein vollständiger Durchschnitt $X' \to F(X)$, (3) $C(X') \simeq C(X)$, sind äquivalent. Unsere partiellen Ergebnisse lauten: Es gilt (1) $\Rightarrow$ (2) $\Rightarrow$ (3). Ist X straff und $\widehat{X}$ glatt, so gilt (2) $\Rightarrow$ (1). Ist $\widehat{Z}$ eine „Kette" reduzierter rationaler Kurven, so gilt (3) $\Rightarrow$ (1), und X' ist „quasideterminantal".

Diese Resultate implizieren insbesondere: Quasideterminantale rationale Flächensingularitäten erkennt man an ihren Auflösungsgraphen; ihre Deformation über der Artinkomponente ist ebenfalls quasideterminantal.

D. SIERSMA:

Functions on singular spaces: a bouquet theorem for the Milnor fibre

We consider a holomorphic map germ $f : (X, x) \to (\mathbb{C}, 0)$ where both X and f have an *isolated singularity* at $x \in X$. Let F^0 be the complex link of (X, x), i.e. the Milnor fibre of a generic linear map $l : (X, x) \to (\mathbb{C}, 0)$ for some $(X, x) \hookrightarrow (\mathbb{C}^N, 0)$.

Theorem: The Milnor fibre F of f is homotopy equivalent to a bouquet:

$$F^0 \vee S^n \vee \ldots \vee S^n$$

where $\dim X = n + 1$. The number of these spheres is equal to the number of A_1–points (Morse) in a generic perturbation of f.

J. STEVENS:

Base spaces for deformations

Let X be an isolated singularity and let $\mathcal{X} \to S$ be its miniversal deformation. The structure of the base S for X a rational quadruple point on a surface was found by de Jong and van Straten and formulated in terms of divisibility conditions on one variable polynomials. We derive these conditions by regarding X as a deformation of $X_0 \times \mathbb{C}$, where X_0 is a general hyperplane section. Openness of versality and explicit knowledge of the equations of the infinite dimensional base of $X_0 \times \mathbb{C}$ allows us to describe equations for the base of X, up to a smooth factor. The same technique works for Gorenstein multiplicity 6 singularities. For 3–fold singularities with minimally elliptic singularities as general hyperplane sections there are similar equations for the base as for rational quadruple points. I pose the problem of finding a similar beautiful explanation of the components for these 3–folds, as Kollár's for rational surface singularities.

D. VAN STRATEN:

Deformations of rational surface singularities

Consider configurations in the plane consisting of a (fixed) number of lines, and on each line a (fixed) number ≥ 2 of points, "belonging to the line", subject to the condition that "on the intersection point of any two lines there has to be a point belonging to each of them". Such configurations are classified by a certain moduli space $\mathcal{M}(\underline{\varphi}, \underline{S})$.

This object, which is defined in an elementary way, has a very direct relation to the *base space* B of the semi-universal deformation of a certain class of rational surface singularities with reduced fundamental cycle. More precisely:

Theorem Let X be defined by equations

$$y_i z_i = S_i(x),\ y_i - y_j = \varphi_i(x) - \varphi_j(x) \subset \mathbb{C}^{2m-1}$$

Then the base space B_x at X is isomorphic to $\mathcal{M}(\underline{\varphi}, \underline{S})$.

In this way we obtain, in particular, an elementary geometric interpretation of various well-known examples including the Pinkham example ($\overset{-4}{\bullet} = cone(\mathbb{P}^1 \subset \mathbb{P}^4) \subset \mathbb{C}^5$). (Collaboration with T. de Jong)

M. TIBÁR:

Lefschetz number and cyclic quotient singularities

If $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ is a holomorphic germ, then the Lefschetz number of f, denoted by $\Lambda(f)$, can take only two values:

(i) $\Lambda(f) = 0$, iff $f \in m^2 \subset \mathcal{O}_n$ (A'Campo)

(ii) $\Lambda(f) = 1$, iff $f \in m \backslash m^2$ (obvious)

If $f : (X, x) \to (\mathbb{C}, 0)$, and (X, x) is a singular space, then: if $f \in m$ then $\Lambda(f) = 0$.

Proof: Slight generalization of Lê's proof in the smooth case.

We stick to the case $(X, x) \cong (\mathbb{C}^n/G, 0)$ where G is a finite cyclic group such that $(\mathbb{C}^n/G, 0)$ is an isolated singularity. We give an explicit procedure to get the Lefschetz number and the zeta–function of any $f : (\mathbb{C}^n/G, 0) \to (\mathbb{C}, 0)$. We construct an algebraic space Y together with a diagram:

$$\begin{array}{ccc} Y & \xrightarrow{\tilde{\pi}} & \mathbb{C}^n \\ \downarrow & & \downarrow \\ X' & \xrightarrow{\pi} & X \cong \mathbb{C}^n/G \end{array}$$

such that a certain group G' acts on Y, $\tilde{\pi}$ is an algebraic morphism which is equivariant with respect to a morphism of groups $\rho : G' \to G$. We finally get $X' \cong Y/G$ which resolves X. We use this diagram to get an explicit description of $\Lambda(f)$ and $\zeta_f(t)$, especially when f is a generic function. We compare the zeta–function of f to the zeta–function of the G–invariant correspondent $\tilde{f}$.

J. WAHL:

Chern classes of rank two vector bundles on resolutions of surface singularities

Let $(\tilde{X}, E) \to (X, 0)$ be a resolution of a germ of a complex normal surface singularity. X should be contractible, and the boundary $\partial X = \partial\tilde{X}$ is the link of $(X,0)$. A line bundle $L \in Pic\tilde{X}$ has a first Chern class $c_1(L) \in H^2(\tilde{X}, \mathbb{Z})$. Since $H^2(\tilde{X}, \partial\tilde{X}) \hookrightarrow H^2(\tilde{X})$ has finite cokernel, one may lift $c_1(L) \in H^2(\tilde{X}, \partial\tilde{X}, \mathbb{Q})$. This allows one to describe Chern numbers $L \cdot M \in H^4(\tilde{X}, \partial\tilde{X}, \mathbb{Q}) \cong \mathbb{Q}$.

Now suppose F is a rank 2 vector bundle. In our talk we defined a $c_2(F) \in H^4(\tilde{X}, \partial\tilde{X}, \mathbb{R}) \cong \mathbb{R}$ which is natural with respect to proper, generically finite $f : (\tilde{Y}, F) \to (\tilde{X}, E)$. It also satisfies $c_2(L \oplus M) = L \cdot M$; however, it is not multiplicative with respect to non-split extensions.

Theorem: Let $(X, 0) = (\mathbb{C}^2/G, 0)$ be a quotient singularity, $F = \Omega^1_{\tilde{X}}(\log E)$ on a resolution $(\tilde{X}, E) \to (X, 0)$. Then $c_2(F) = -1/|G|$.

This shows the non-triviality of c_2, which is generally hard to compute. For the appropriate Euler characteristic $\chi'(F) = \dim H^0(\tilde{X} - E, F)/H^0(\tilde{X}, F) + \dim H^1(\tilde{X}, F)$, there should be a Riemann–Roch theorem:

$$(*) \quad \chi'(S^n F) = -\frac{n^3}{6}(c_1(F)^2 - c_2(F)) + O(n^2).$$

Theorem: $(*)$ is true for $(X, 0) = (\mathbb{C}^2/G, 0)$ a quotient singularity, $F = \Omega^1_X(\log E)$.

Our work is motivated by the log-Miyaoka inequality.

C.T.C. WALL:

Topological triviality of unfoldings (joint work with A. du Plessis)

If $f_0 : (\mathbb{C}^s, 0) \to (\mathbb{C}^t, 0)(s > t)$ is a holomorphic map-germ of finite singularity type with a versal unfolding $\tilde{F}$ and a partial unfolding F, we want to know when $\tilde{F}$ is topologically trivial as unfolding of F. To attack this, our strategy is to determine the set $V(F)$ in the target $\mathbb{C}^p$ of F over which some germ of F is not C^∞–stable. Triviality follows outside $V(F)$, and may be decided in many cases by detailed examination of the strata $V(F)$.

In this talk I concentrate on determining $V(F)$. Begin with $M(F) = \operatorname{Coker} tf : \Theta_n \to \Theta(F)$ (the jacobian map), considered as $\mathcal{O}_p$–module. $M(F)$ admits a presentation

$$0 \to \mathcal{O}_p^k \xrightarrow{A} \mathcal{O}_p^k \to M(F) \to 0.$$

The matrix A can be explicitly computed (effectively in the weighted homogeneous case). The instability locus $V(F)$ is the support of Coker $\omega F : \Theta_p \to M(F)$. In practice, $\Theta_p = \mathcal{O}_p^p$ is a factor of $\mathcal{O}_p^k$, so $V(F)$ is the support of the cokernel of the matrix defined by the final $mv = k - p$ columns of A. If $mv = 1$, we just have an ideal of A, and can simplify the set of generators to a useful form. In the case ($t = 1$) when f_0 defines a hypersurface singularity it was shown by Mond & Pellikaan using Gorenstein properties that the matrix A can be taken to be symmetric. Thus here it suffices to compute the first mv rows of A.

A detailed study of the case of plane triple point singularities gives a complete classification by topological type of the stable unfolding, thus answering a question raised by Pham and Briançon in 1970-71.

K. WIRTHMÜLLER:

Connectedness of conflict strata (joint work with T. Gaffney)

Let $f : X = (\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0) = Y$ be a holomorphic map germ which is stable (or sufficiently generic) in the sense of Mather. If $\Sigma \subset X$ denotes the set of singular points at f then define the conflict stratum $C \subset Y$ by $C = \overline{\{y \in Y | \#(f^{-1}y \cap \sum) \geq 2\}}$ and the swallow-tail stratum $S \subset Y$ by $S = \overline{\{y \in Y | f^{-1}y \text{ contains an } A_3\text{-point}\}}$. We show the following:

Theorem The inclusion $S \subset C$ induces a surjection between sets of irreducible components.

Theorem If $n \leq p$ then S (hence C) is irreducible unless f has type $f^{-1}(0) \cong$ *Spec* $\mathbb{C}\{s,t\}/(s^2, t^2)$.

Theorem If $n = 3, p = 2$ then S (hence C) is irreducible for all f not from a finite list of exceptions.

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