MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 24/1992

Free resolutions in algebraic geometry and representation theory

31.05. bis 06.06.1992

Die Tagung fand unter der Leitung von W. Bruns (Vechta), D. Eisenbud (Waltham) und J. Herzog (Essen) statt.

In den Vorträgen und Diskussionen standen neue Ergebnisse und Methoden aus der kommutativen Algebra und der algebraischen Geometrie im Mittelpunkt, die insbesondere freie Auflösungen und die aus ihnen abgeleiteten Invarianten betreffen. Diskutiert wurden Querverbindungen zur Darstellungstheorie artinscher Ringe (Auslander, Reiten), zur Klassifikation projektiver Varietäten (Decker, Peskine, Stückrad, Ulrich, Valla) und zur Theorie determinantieller Ringe (Hashimoto, Kustin). Ein Vortrag (Roberts) beschäftigte sich mit Chernklassen von Matrizen in freien Auflösungen und ihrer Berechnung. Aspekte der Computeralgebra kamen ebenfalls zur Geltung (Decker, Roos). Aus zwei Vorträgen ergaben sich Bezüge zur Singularitätentheorie: aus der Berechnung der Hochschild-Homologie (Buchweitz) und aus der Charakteristik-p-Theorie lokaler Ringe (Watanabe). Bemerkenswert ist die Bestimmung determinantieller Formeln für Resultanten aus Komplexen von Vektorbündeln (Weyman). Weitere Themen: Liftungstheorie (Popescu), Beziehungen zwischen der Jacobischen Varietät der Durchschnitte zweier Quadriken, Clifford-Algebren und maximalen Cohen-Macaulay-Moduln (Schreyer), Rees-Algebren und assoziierte graduierte Ringe (Vasconcelos), der Syzygiensatz von Evans-Griffith aus Vektorbündel-Sicht (Zelewski).

Parallel fand eine Tagung über Singularitäten unter der Leitung von G.-M. Greuel, J. Kollar und J. Steenbrink statt. Die Tagungen ergänzten sich inhaltlich sehr gut. Zusammenarbeit und Abstimmung des Programms verliefen ausgezeichnet.

Vortragsauszüge

M. Auslander (Waltham)

Almost split sequences over regular local rings

The aim of this talk was to point out aspects of almost split sequences and other techniques developed in the theory of representations of artin algebras, which have been and might be in the future of interest in the study of commutative noetherian local rings $(R, \mathfrak{m})$. The talk centered on proving the existence of almost split sequences for such local rings $0 \to C \to E \to \operatorname{Tr}_R D(C) \to 0$, where C is an indecomposable noninjective module of finite length. A new proof was given when R is regular and it was shown how this could be used to derive the existence of such sequences for rings of the form $R/\mathfrak{a}$ for some ideal $\mathfrak{a}$. Also methods of calculating such sequences for R when $\operatorname{Soc} C$ is simple were described. In particular generators and relations were given for E and $\operatorname{Tr}_R D(C)$ when C is $R/\mathfrak{m}$, the residue class field. The lecture ended by discussing aspects of the theory of morphisms in $\operatorname{mod} R$, the category of finitely generated modules where R is artinian.

R.-O. Buchweitz (Toronto)

Hochschild (Co-)Homology of Complete Intersection Maps and the Developable of the Discriminant

We report and generalize results from the thesis of J. Mauji (Toronto). Let $\mathcal{F}\colon X \to S$ be a complete intersection map. For S a point, Wolffhardt (1971), Guccione-Guccione (1991), Lago-Rodicio (1992) and others have remarked that the Hochschild (co-)homology groups $\mathrm{HH}(\mathcal{F})$ can be obtained as the (co-)homology of a suitable Koszul complex. If X and S are smooth over some base Z, the cotangent complex of $\mathcal{F}$ is quasi-isomorphic to $0 \to \mathcal{F}^*\Omega^1_{S/Z} \xrightarrow{J\mathcal{F}} \Omega^1_{X/Z} \to 0$, where $J\mathcal{F}$ is the Jacobian matrix of $\mathcal{F}$. Set $T'_{\mathcal{F}} = \operatorname{Cok}(J\mathcal{F}^*)$. Then $\operatorname{Spec}(S^{\cdot}_X T'_{\mathcal{F}})$ is the affine developable of $\Delta(\mathcal{F})$, the discriminant of $\mathcal{F}$, and $\tilde{C} = \operatorname{Proj}(\mathbb{P}_X(T'_{\mathcal{F}}))$ is Teissier's development of the discriminant. The Hochschild (co-)homology is then obtained from the (projectivized) Koszul complex $\mathcal{K}(J\mathcal{F}^*\colon \Omega^1_{X/Z} \otimes_{\mathcal{O}_X} \mathcal{O}_{\mathbb{P}}(-1) \to \mathcal{O}_{\mathbb{P}}),\ \mathbb{P} = \mathbb{P}_X(\mathcal{F}^*\Omega^{1*}_{S/Z})$. In case that $\mathcal{F}$ has only isolated singularities in its fibers, this Koszul complex is exact and resolves $\mathcal{O}_{\tilde{C}}$ in $\mathbb{P}(\mathcal{F}^*\Omega^{1*}_{S/Z})$. It follows that one has split exact sequences

$$0 \to S^{\nu/2}T'_{\mathcal{F}} \to \mathrm{HH}^{\nu}(\mathcal{F}) \to \operatorname{Hom}(\Omega^{\nu}_{\mathcal{F}}, 0_X) \to 0.$$

and that $\mathrm{HH}_{\cdot}(\mathcal{F})$ equals $\oplus_{i;\nu\in\mathbb{Z}_{<0}} H^i(\tilde{C}, \mathcal{O}_{\tilde{C}}(\nu))$ modulo the direct summand $\bigwedge^{\cdot}(\Omega^1_{\mathcal{F}})$. These cohomology groups can be calculated explicitly in many cases, and one obtains precise information about the support of $\mathrm{HH}_{\cdot}(\mathcal{F})$.

W. Decker (Saarbrücken)

Bielliptic and abelian surfaces in $\mathbb{P}^4$

Every smooth algebraic surface X can be embedded in $\mathbb{P}^5$, but only few of them in $\mathbb{P}^4$. Moreover Ellingsrud and Peskine proved that there are only finitely many families of such surfaces $X \subset \mathbb{P}^4$ which are not of general type.

Problem: Classify these surfaces. The talk is concerned with the known bielliptic and abelian surfaces.

1. *The abstract point of view.* We recall how to obtain (minimal) abelian and bielliptic surfaces in $\mathbb{P}^4$ of degree 10 via linear systems. We describe a $\mathbb{P}^2$-bundel $\mathbb{P}^2_E$ over an elliptic curve E. $\mathbb{P}^2_E$ contains a pencil of abelian surfaces and 8 bielliptic surfaces. $\mathbb{P}^2_E$ may be mapped to $\mathbb{P}^4$ embedding the abelian and bielliptic surfaces. The image of $\mathbb{P}^2_E$ is the unique quintic hypersurface containing the bielliptics.

2. *The geometric point of view.* Nonminimal abelian surfaces of degree 15 can be obtained by linkage from the minimal ones in degree 10. Nonminimal bielliptic surfaces of degree 15 can be obtained by applying the quadro-cubic Cremona transformation of Semple to the minimal ones in degree 10.

3. *The syzygy point of view.* We explain how to construct all surfaces mentioned by constructing the finite length graded cohomology modules of their ideal sheaves first. As an upshot we present a new family of nonminimal abelian surfaces of degree 15.

This is joint work with A. Aure, K. Hulek, S. Popescu, and K. Ranestad.

H.-B. Foxby (Kobenhavn)

Structure of local homomorphisms

Let $\varphi\colon (R,\mathfrak{m},k) \to (S,\mathfrak{n},l)$ be a local homomorphism of local rings, and assume that S is complete (in the $\mathfrak{m}$-adic topology).

Existence Theorem. φ has a Cohen factorization i. e. $\varphi = \varphi'\varphi'', \varphi'\colon R' \to S, \varphi''\colon R \to R'$, where R' is a complete local ring, φ'' is a flat local homomorphism, $R'/\mathfrak{m}R'$ is a regular ring and φ' is surjective.

Comparison Theorem. Any two Cohen factorizations, $\varphi = \varphi_1'\,\varphi_1''$ and $\varphi = \varphi_2'\,\varphi_2''$ are reductions of a third, i.e. there exists a Cohen factorization $\varphi = \varphi_3'\varphi_3''$ and two surjections $\eta_1 : R_3' \to R_1', \eta_2 : R_3' \to R_2'$ such that $\varphi_1'\eta_1 = \varphi_3' = \varphi_2'\eta_2, \varphi_1'' = \eta_1\varphi_3'', \varphi_2'' = \eta_2\varphi_3''$.

This makes the following definition possible: $\dim\varphi = (\dim R' - \dim R) - \operatorname{codim}_{R'} S$, where $\operatorname{codim} = \operatorname{ht}\operatorname{Ann}$. Set $\operatorname{depth}\varphi = \operatorname{depth} S - \operatorname{depth} R$ and define the Cohen-Macaulay defekt $\operatorname{cmd}\varphi = \dim\varphi - \operatorname{depth}\varphi$. Note that $\operatorname{cmd}(\mathbb{Z}_{(p)} \to S) = \operatorname{cmd} S$, $p = \operatorname{char} l$. If the flatness dimension $\operatorname{fd}\varphi < \infty$ then the following hold:

(1) $\operatorname{cmd}\varphi \geq 0$, (2) $\operatorname{cmd}\varphi = \operatorname{cmd}(S/\mathfrak{m}S)$ if φ is flat, (3) $\operatorname{cmd}\varphi = \operatorname{imp}_{R'} S$, where the imperfection is defined by $\operatorname{imp} = \operatorname{pd} - \operatorname{grade}$.

Consider the following inequalities: (4) $\operatorname{cmd}\varphi + \operatorname{cmd}\Psi \geq \operatorname{cmd}\psi\Psi$, (5) $\operatorname{cmd}\varphi\Psi \geq \operatorname{cmd}\varphi$, (6) $\operatorname{cmd}\varphi\Psi \geq \operatorname{cmd}\Psi$. Then (4) holds always, (5) holds when $\operatorname{fd}\varphi < \infty$ and (6) holds when $\operatorname{fd}\Psi, \operatorname{fd}\varphi < \infty$.

This is joint work with L. Avramov and B. Herzog.

M. Hashimoto (Nagoya)

Determinantal ideals of skew symmetric matrices

Let $S = k[X_{ij}]_{1\leq i<j\leq n}$ be a polynomial ring over a field of characteristic p. We set $J_{2t} = S\cdot$ (all $2t$-Pfaffians of (X_{ij})), $I_t = S\cdot$ (all t-minors of (X_{ij})) where (X_{ij}) is the alternating $n \times n$ matrix given by $X_{ji} = -X_{ij}$ $(i<j)$ and $X_{ii} = 0$. We give a formula for the first Betti number $\beta_1^p(J_{2t}) = \dim_k \operatorname{Tor}_1^S(J_{2t}, k)$ which includes n, t and p. As a result, $\beta_1^p(J_{2t}) = \beta_1^0(J_{2t})$ if $p \neq 2$, while $\beta_1^2 > \beta_1^0$ when $2 \leq t \leq n/2 - 2$. In this case there is no minimal free resolution of S/J_{2t} (as an S-module) over $\mathbb{Z}$. For the case $n-2t=3$, the existence of the minimal free resolution is not known. We also give an example such that $J_{2t}^2 \not\subset I_{2t} \not\subset J_{2t}^2 (I_{2t} = J_{2t}^2$ when $p=0)$. An example when $J_{2t+2}\cdot J_{2t} \not\subset I_{2t+1} \not\subset J_{2t+2}\cdot J_{2t}$ is not known.

A. Kustin (Columbia)

Deviation two Gorenstein ideals

Let $X^{\text{alt}}_{2n\times 2n}$ and $Y_{1\times 2n}$ be matrices with entries from the ring R. Form the ideal $I = I_1(YX) + Pf(X)$. If grade $I = 2n-1$, then I is a (deviation two) Gorenstein ideal. The minmal R-resolution of $A = R/I$ is a DGΓ-algebra. Conjecture: The Poincaré series $P_A^M(z)$ is a rational function for all finitely generated A-modules M. The conjecture has been established for $n \le 3$.

C. Peskine (Paris)

Hilbert polynomials for smooth codimension 2 varieties in projective space

Let $\Sigma \subset \mathbb{P}_{n+2}$ be a hypersurface, reduced irreducible, of degree σ. Let $X \subset \Sigma$ be an irreducible variety (reduced) of dimension n. Then there exist polynomials P_σ and Q_σ with $d^\circ P_\sigma = n+1$ and $d^\circ Q_\sigma \le n$, depending only of σ, such that $P_\sigma(d^\circ X) - Q_\sigma(d^\circ X) \le (-1)^n \chi(\mathcal{O}_X) \le P_\sigma(d^\circ X)$. Furthermore X is the complete intersection of Σ and another hypersurface if and only if $(-1)^n\chi(\mathcal{O}_X) = P_\sigma(d^\circ X)$. Given χ, the varieties $X_n \subset \mathbb{P}_{n+2}$ such that $\chi(\mathcal{O}_{X_n}) = \chi$ form a finite number of components of the Hilbert scheme of dimension n varieties in $\mathbb{P}_{n+2}$.

D. Popescu (Bucharest)

Liftings and relative liftings of modules

Let (Λ, m) be a Noetherian local ring, $\kappa = (\kappa_1,\dots,\kappa_r)$ a regular system of elements of Λ and M a finite $\Lambda_1 := \Lambda/\kappa\Lambda$-module. A finite Λ-module L is a lifting of M to Λ if (i) $M \cong L/\kappa L$ and (ii) κ is a L-sequence. If Λ is complete and $\text{Ext}^2_\Lambda(M,M) = 0$ then M is liftable as Auslander-Ding-Solberg showed. Suppose now that Λ is regular, Λ_1 is nonregular and M is a maximal Cohen-Macaulay module. Then M is liftable to Λ iff it is free. Thus it is necessary to look for weaker notions of liftings which include for example the bundles when $r = \dim\Lambda$. A finite Λ-module E is a relative lifting of a finite Λ_1-module M if (i) holds and (ii′) κ is a relative sequence on E (after Fiorentini, Huneke), i.e. for all s, $1 \le s < r$ $((\kappa_1,\dots,\kappa_s)E : \kappa_{s+1})_E \cap (\kappa)E = (\kappa_1,\dots,\kappa_s)E$.

Let T_2 be an arbitrary $\Lambda_2 = \Lambda/(\kappa)^2$-module such that $T_2/(\kappa)T_2 \cong M$ and N the kernel of the canonical map $M^r \to (\kappa)T_2$ given by $(\hat{y}_1,\dots,\hat{y}_r) \to \sum_{i=1}^r \kappa_i y_i$, where $y_i \in T_2$. If Λ is complete, $\text{Ext}^1_{\Lambda_1}(N,(\kappa)T_2) = 0$, and furthermore $\text{Ext}^2_{\Lambda_1}(M,(\kappa)T_2) = 0$, then M has a relative lifting E to Λ with $\Lambda_2 \otimes E \cong T_2$.

I. Reiten (Trondheim)

Connections between Artin algebras and commutative rings

Let Λ be an Artin algebra or a commutative noetherian complete local ring. We illustrate connections between the two cases, via influence of methods and results and independent developments. Examples are provided through almost split sequences and associated criteria for finite representation type, and through connections between (co) tilting modules and dualizing modules. Contravariantly and covariantly finite subcategories (Auslander-Smalø) have been important for Artin algebras, in particular those closed under extensions. In the commutative case they have appeared in the theory of Cohen-Macaulay approximations (Auslander-Buchweitz). Let for $d > 0, \chi_d = \text{add}\, \Omega^d(\text{mod}\, \Lambda)$ be a syzygy category. With respect to the above properties we have (with Auslander) that χ_d is covariantly finite in both cases and is contravariantly finite for Artin algebras (also with Smalø). If Λ is k-Gorenstein, then χ_k is closed under extensions in both cases. For Artin algebras we have: χ_d is closed under extensions for all $d \leq k \Leftrightarrow$ in a minimal injective resolution $0 \to \Lambda_\Lambda \to I_0(\Lambda_\Lambda) \to \ldots, \text{pd}\, I_j(\Lambda_\Lambda) \leq j+1$ for $j < k$. For Λ commutative we have: χ_d is contravariantly finite and closed under extensions $\Leftrightarrow$ Λ is Cohen-Macaulay of dimension d and $\chi_d = CM(\Lambda)$ (the maximal Cohen-Macaulay modules). From this we get that if Λ is Gorenstein of dimension $d > 0$, then χ_d is not contravariantly finite for $0 < i < d$. An example of the influence from commutative rings on Artin algebras is the study of Gorenstein conditions on Artin algebras. This gives rise to two problems: If $id_\Lambda \Lambda < \infty$, is $id\Lambda_\Lambda < \infty$? If Λ is k-Gorenstein (i. e. $\text{pd}\, I_i(\Lambda) \leq i$ for $i < k$), is then Λ Gorenstein (i. e. $\text{id}_\Lambda \Lambda < \infty$ and $\text{id}\, \Lambda_\Lambda < \infty$) ? A positive answer would imply the Nakayama Conjecture. We can prove (with Auslander) that if Λ is k-Gorenstein for all k and $\text{id}_\Lambda \Lambda < \infty$, then Λ is Gorenstein.

P. Roberts (Salt Lake City)

Chern Classes of Matrices

We review the algebraic construction of the localized Chern characters of Baum, Fulton, MacPherson. If M is a module of finite length over a regular local ring A of dimension d, then M has a resolution of length d, so defines d matrices with support in the maximal ideal of A. The local Riemann-Roch formula states that $\text{length}(M) = \sum_{i=1}^{d}(-1)^{i+1}\, \text{ch}_d(M_i)$, where M_i is the matrix defining the i^{th} map of the resolution. If M is defined by monomials, that is, if $M = A/I$ for I an ideal generated by monomials in a polynomial ring, or more generally is the quotient of two such ideals, it is possible to interpret these numbers. Let $M = I/J$ be such a quotient in two variables, then the monomials of $I - J$ define a subregion of the plane of area $\text{length}(M)$. Placing this in a fourth quadrant there exists a broken line from the lower left to the upper right entirely inside the region dividing it in two parts such that $\text{ch}_2(M_1)$ is the area above the line and $(-1)\, \text{ch}_2(M_2)$ is the area below it. We ask (with not much evidence) whether the numbers $(-1)^{i+1}\, \text{ch}_d(M_i)$ are in general positive in this situation.

J. E. Roos (Stockholm)

A computer-aided study of the graded Lie algebra associated to a local commutative Noetherian ring

Let $(R, \mathfrak{m}, k)$ be a local commutative noetherian ring and $\text{Ext}_R^*(k,k)$ the corresponding Yoneda Ext-Algebra. It is well known that $\text{Ext}_R^*(k,k)$ is the enveloping algebra of an (infinite) positively graded Lie algebra $\mathfrak{g}_R = \mathfrak{g}^1 \otimes \mathfrak{g}^2 \otimes \mathfrak{g}^3 \otimes \ldots$ R is a Golod ring exactly when $\bar{\mathfrak{g}} = \mathfrak{g}^2 \otimes \mathfrak{g}^3 \otimes \ldots$ is a free graded Lie algebra. Avramov, Kustin and Miller have proved that if $\dim_k(\mathfrak{m}/\mathfrak{m}^2) \leq r$, then $\bar{\mathfrak{g}}$ contains a free Lie algebra $\mathfrak{f}$ (as an ideal) such that $\bar{\mathfrak{g}}/\mathfrak{f}$ is an abelian Lie algebra, concentrated in degree 2. More generally one can prove that for "many" rings R, the corresponding $\bar{\mathfrak{g}}$ has a finite filtration (*) $\mathfrak{g}_1 \subset \mathfrak{g}_2 \subset \ldots \subset \mathfrak{g}_\nu = \bar{\mathfrak{g}}$ such that $\mathfrak{g}_i/\mathfrak{g}_{i-1}$ are free graded Lie algebras. This is true e.g. if $R = k[X_1, \ldots, X_N]/(X_1, \ldots, X_N)^m$ by Golod (here $\nu = 1$) or if $R = k[X_1, \ldots, X_N]/$(some monomials in the X_i's) by Backelin-Fröberg. This lecture is a report about some results from computer-aided studies of $\mathfrak{g}_R$ when $R = k[X_1, \ldots, X_N]/$(quadratic forms in the X_i's). "Macaulay" and a program "Bergman" written by J. Backelin were used to find examples when (*) does not hold. If $R := k[X,Y,Z,U]/(X^2+XY, Y^2, XZ+YU, XU)$ then

$$\mathrm{Tor}^R_{i,j}(k,k) = \begin{cases} 0, & if \quad i \neq j, i \leq 5 \\ 1, & if \quad i = j = 6 \end{cases}$$

This shows that a question of Eisenbud and I. Peeva about linear resolutions has to be modified.

F.-O. Schreyer (Bayreuth)

Complete intersections of two quadrics, Clifford algebras and hyperelliptic curves

Let $X = Q_o \cap Q_1 \subset \mathbb{P}^{2g+1}$ be a smooth complete intersection of two quadrics, $S = \{\mathbb{P}^{g-1} \in \mathbb{G}(g, 2g+2) \mid \mathbb{P}^{g-1} \subset X\}$ the variety of g-dimensional isotropic subspaces and $\pi : E \to \mathbb{P}^1$ the hyperelliptic curve branched over the $2g+2$ points of the discriminant of the pencil of quadrics. A classical result of Reid and Desale-Ramanan says $J(X) \cong S \cong \mathrm{Jac}\, E$ where $J(X)$ is the intermediate jacobian of X and $\mathrm{Jac}\, E$ the jacobian of E. We give a new proof of this result based on the Clifford algebra $\mathrm{Cliff}(X)$ of the pencil of quadrics on one hand, $\mathrm{Cliff}(X) \cong \mathrm{Ext}^{\cdot}_X(k,k)$, and on the other hand the center of $\mathrm{Cliff}(X)$ is the homogeneous coordinate ring of E, and the even part an endomorphism algebra of a vector bundle on E. Thus the result follows by using the Bernstein-Gelfand-Gelfand correspondence $\mathrm{mod}\, X \to \mathrm{mod}\,\mathrm{Cliff}$ together with the Morita equivalence $\mathrm{mod}\,\mathrm{Cliff}^+ \to \mathrm{mod}\, E$.

This is joint work with R.-O. Buchweitz.

J. Stückrad (Leipzig)

A structure theory of projective varieties of degree = codim + two

Let K be an algebraically closed field of arbitrary characteristic. The aim of this talk is to present a complete "algebraic" classification of all reduced, irreducible and non-degenerate subschemes X of $\mathbb{P}^n_K$ with $\deg X = 2 + \mathrm{codim}\, X$ in terms of their homogeneous coordinate rings $A(X) = R/I(X)$, where $R := K[X_0, \ldots X_n]$ with indeterminates $X_0, \ldots, X_n$ of degree one.

Theorem. The following conditions are equivalent:

(i) $\deg X = 2 + \mathrm{codim}\, X$,

(ii) $H^i_{\mathfrak{m}}(A(X)) = 0$ for all $i \neq r := \mathrm{depth}\, A(X)$, $d := \dim A(X)$ and either (1) $r = d$ (i.e. $A(X)$ is Cohen-Macaulay) and $[H^d_{\mathfrak{m}}(A(X))]_{2-d} \cong K$ or (2) $r < d$, $H^r_{\mathfrak{m}}(A(X)) \cong K[y_0, \ldots, y_{r-2}]^{\cdot}(r-2)$ with algebraically independent elements $y_0, \ldots, y_{r-2}$ of $[A(X)]_1$ and $[H^d_{\mathfrak{m}}(A(X))]_{2-d} = 0$, where $\mathfrak{m} = (X_0, \ldots, X_n)R$.

The main tool for proving this theorem is a result which gives a complete description of the minimal free resolutions of $R/I(Y)$, where Y is an arbitrary set of $n+2$ distinct points of $\mathbb{P}_K$ spanning $\mathbb{P}^n_K$. Using generic hyperplane sections the problem is reduced (inductively on d) to this special situation. Moreover, a lot of consequences are presented.

The results are contained in a joint paper with L. T. Hua and W. Vogel.

B. Ulrich (East Lansing)

General hyperplane sections of projective curves

Let k be an algebraically closed field of characteristic zero, $S = k[X_o, \dots, X_n]$, $\mathfrak{m} = (X_o, \dots, X_n)$, $\mathfrak{p} \subset \mathfrak{m}^2$ a two-dimensional homogeneous prime ideal of S, and $B = S/\mathfrak{p}$ (the homogeneous coordinate ring of a nondegenerate reduced and irreducible curve $C \subset \mathbb{P}^n_k$). Let x be a general linear form in S, let $^-$ be reduction modulo (x), let $R = \bar{S}$, and $A = \bar{B}/H^0_{\mathfrak{m}}(\bar{B}) = R/I$ the homogeneous coordinate ring of a general hyperplane section $H \cap C \subset \mathbb{P}^{n-1}_k$). Write $b = \min\{i | [I/\bar{\mathfrak{p}}]_i \neq 0\}, g = \operatorname{ht} I$, and let $0 \to \otimes_j R(-n_{gj}) \to \dots \to \otimes_j R(-n_{1j}) \to R$ be a homogeneous minimal resolution of A.

Theorem 1. A is Gorenstein and $\mathfrak{p} \subset \mathfrak{m}^3$, or if A is a complete intersection and $n \geq 4$, then B is Gorenstein.

This generalizes a result by Strano for the case $n = 3$ and follows from:

Proposition 2. If B is not Cohen-Macaulay then $\min\{n_{gj}\} \leq b + g \leq \max\{n_{1j}\} + g$.

Theorem 3. Assume A is Gorenstein and $n \geq 4$. Then B is Buchsbaum if and only if $I/\bar{\mathfrak{p}}$ is cyclic.

This is joint work with C. Huneke.

G. Valla (Genova)

Quadrics through a set of points and their syzygies

Let $X \subset \mathbb{P}^n_k$ be a nondegenerate projective variety and $A = k[X_0, \dots, X_n]/I$ its homogeneous coordinate ring. If we consider a minimal free resolution of A or an R-module, its linear part is measured by the integers $a_i := \dim_k \operatorname{Tor}_i^R(A, k)_{i+1}$ It is easily seen that $a_i \neq 0$ if X is contained on a variety of minimal degree and dimension $n - i$ or $X \subset \mathbb{P}^k \cup \mathbb{P}^r$ with $k + r = 2n - i - 1$, $k, r < n$. As for the converse we have the following results:

1. If $a_1 \neq 0$, then $X \subset \mathbb{P}^{n-1} \cup \mathbb{P}^{n-1}$ or X is contained in an irreducible quadric.
2. If $a_n \neq 0$, then $X \subset \mathbb{P}^r \cup \mathbb{P}^k$ for some positive integer k, r such that $k + r = n - 1$.
3. If X is a set of points spanning $\mathbb{P}^n$ such that $n - 1$ of them are never on a $\mathbb{P}^{n-3}$, then $a_{n-1} \neq 0$ iff $X \subset$ a rational normal curve or $X \subset \mathbb{P}^r \cup \mathbb{P}^k, r + k = n$.
4. Let $n \geq 3$, $1 \leq p \leq n-2$. If X is a set of s points in linear general position with $s < 2n + 1 - p + (n - p - 1)/(p + 1)$, then the points are on a rational normal scroll of dimension $n - p - 1$.

W. Vasconcelos (New Brunswick)

Hilbert functions, analytic spread and Koszul homology

Let R be a Noetherian ring and let $R[It]$ be the blow up ring of an ideal I. We introduce devices (Sally module, mixed graded algebra) that permit the arithmetical study of these algebras by techniques that were previously restricted to ideals generated by d-sequences. They rely on being able to pass to a minimal reduction of I properties of the Koszul homology of I. There are applications to the behaviour of the coefficients of the Hilbert polynomial of I (in case it is primary), to restrictions on the analytic spread of I, to the number of generators of certain prime ideals, and to the Cohen-Macaulayness of $R[It]$.

K. Watanabe (Tokyo)

F-regular and F-pure rings vs. log-terminal and log-canonical singularities

The notions of F-regular and F-pure rings defined by Hochster-Huneke, Hochster-Roberts are rather mysterious, but enjoy properties similar to those of log-terminal and log-canonical singularities in characteristic 0.

(1) strongly F-regular rings in dimension 2 are quotient singularities in "char = 0", which coincides with log-terminal singularities, and the same holds for F-pure and log-canonical singularities.

(2) If you take a cyclic cover $S = \otimes_{i\in \mathbb{Z}_r} S_i$ from $S_0 = R$, which is ètale in codim 1, then S is log-terminal (resp. log-canonical), strongly F-regular (resp. F-pure) if and only if so is R.

(3) If $S = S(R, D, \mathcal{F}) = \otimes_{n\in \mathbb{Z}_r} R(iD)u^i$ with $u^* = \mathcal{F}, D = \frac{1}{r}\operatorname{div}_R(f) \in \operatorname{Div}(R) \otimes_{\mathbb{Z}} Q$ and if you put $D' = \sum \frac{s_\nu - 1}{s_\nu}\nu$ when $D = \sum \frac{t_\nu}{s_\nu}\nu, (t_\nu, s_\nu) = 1$, then S is log-terminal (resp. log-canonical) if $t(K_R + D')^\sim aD$ for some $t > 0, a \in \mathbb{Z}$, the pair (R, D') is log-terminal (resp. log-canonical) and F-regular pair (R, D') (resp. F-pure pair) can be defined such that $S = S(R, D, \mathcal{F})$ is strongly F-regular (resp. F-pure) iff so is (R, D').

(4) If (X, κ) is a normal algebraic variety over a field k of characteristic zero and if the reduction $\bmod p$ is strongly F-regular (resp. F-pure) for an infinite set of p then (X, κ) is log-terminal (resp. log-canonical). The converse should probably be true but may be difficult, as shown by the example of cones over elliptic curves, which is log-canonical but F-purity is hard to show.

J. Weyman (Boston)

The discriminants and resultants of multihomogeneous polynomials

We give a construction of determinantal expressions for the resultants of multihomogeneous polynomials. In the case of $l+1$ homogeneous polynomials in $l+1$ variables, of degree d we construct a family of complexes $\mathbb{F}(m)$ $(m \in \mathbb{Z})$ such that $\mathbb{F}(m)$ reduces to a matrix in the following cases:

(0) $d = 1, l$ arbitrary, $(m = -1, 0, 1)$, (1) $l = 1$, d arbitrary $(-1 \le m \le 2d-1)$, (3) $l = 2$, d arbitrary $(d-1 \le m \le 2d-1)$, (4) $l = 3$, d arbitrary $(2d-3 \le m \le 2d-1)$, (5) $l = 4$, $d = 2$ $(m = 2, 3)$, (6) $l = 4$, $d = 3$ $(m = 5)$, (7) $l = 5$, $d = 2$ $(m = 3)$. In each listed case the determinant of the resulting matrix is the resultant. This list includes all the determinantal expressions known to us, notably Sylvester's expression ($l = 1$, d arbitrary, $m = 2d-1$), Bezout's expression ($l = 1$, d arbitrary, $m = d-1$) and the expression given by Muir in his book on determinants ($l = 2$, d arbitrary, $m = 2d-1$). We obtain a similar result for multihomogeneous polynomials.

This is joint work with A. Zelevinski.

A. Zelewski (Hamilton)

Vanishing theorems and the syzygy problem

Let X denote a complex, smooth, projective variety. Let $F_.$ denote a complex of line bundles on X, $F_. : F_m \xrightarrow{d_m} F_{m-1} \to \dots F_1 \xrightarrow{d_1} F_0$ such that $F_k = \bigoplus_j L_j$, L_j negative line bundles for $k > 0$ and $F_0 = \mathcal{O}_X \oplus$ (anything). Suppose that the cohomology of the complex of global sections of $\bigoplus_{i=0}^{\infty} F_. \otimes H^i$ is finite dimensional over $\mathbb{C}$ for an ample vector bundle H. Then $\operatorname{rank} \operatorname{Ker} d_1 \geq n$, $\operatorname{rank} \operatorname{Ker} d_2 \geq n-1, \dots,$ $\operatorname{rank} \operatorname{Ker} d_n \geq 1$. The proof is based on Le Potier vanishing theorem. The result is an "offspring" of the Evans-Griffith syzygy theorem. The bounds for d_1, d_2, and d_3 are the best possible (there are appropiate examples with $n = 10$ for d_2).

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