# Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 1/1993

## Grundlagen der Geometrie

3.1. bis 9.1.1993

Die Tagung fand unter der Leitung von W. Benz (Hamburg) statt. Im Mittelpunkt des Interesses standen Fragen zur Theorie der Möbius- und Minkowski-Ebenen (miquelsch und auch nicht-miquelsch), der kinematischen Gruppen, der Spiegelungsgeometrie, der Konstruktion von Spreads, der Netze, der fraktalen Struktur von Kreisgeometrien. Die neuerdings interessierenden Probleme über die Grundlagen der Lösungen der Einsteinschen Feldgleichungen wurden stark diskutiert. Insbesondere zur Einsteinschen Zylinderwelt und zur de Sitter-Geometrie wurden Vorträge gehalten. Wie bei allen Tagungen über Grundlagen der Geometrie interessierten auch dieses Mal Fragen der Verbindung von Schul- und Universitätsgeometrie.

Die Qualität aller Vorträge war sehr hoch. Allgemein bedauert wurde, daß wegen der nötigen Begrenzung der Teilnehmerzahl auf 25, etwa 50 Interessenten abgewiesen werden mußten.

### Vortragsauszüge

W. BENZ:

**Characterizations of distances in Einstein's Cylinder Universe**

The following Theorem can be proved:

Let $C^n$ be the set of points of $n$-dim Einstein's Cylinder Universe and let $d$ be a function from $C^n \times C^n$ into the set $\mathbb{R}_{\geq 0}$ of non-negative reals such that

(i) $d$ is a 2-point-invariant of the group of motions of $C^n$,

(ii) $d$ is additive on admissible point triples,

(iii) $d$ is locally Lorentz-Minkowskian.

Then $d^2(x,y) = \mid [\arccos \sum_{i=1}^n x_i y_i]^2 - (x_{n+1} - y_{n+1})^2 \mid$ for all $x, y \in C^n$ such that the angle between $(x_1, \ldots, x_n)$, $(y_1, \ldots, y_n)$ is less than $\pi$. Here arccos has to be chosen in $[0, \pi]$.

V.G. BOLTYANSKI

### On the Hanner numbers

A compact, convex body $M \subset \mathbb{R}^d$ is said to possess the $(p,q)$–intersection property ($p < q$), if the following assertion is true: For every collection $M_1, \ldots, M_q$ of translates of $M$, if every $p$ of the bodies $M_1, \ldots, M_q$ have a common point, then $M_1 \cap \ldots \cap M_q \neq \emptyset$. O Hanner (a student of Professor B. Szökefalvi–Nagy) has found all the bodies in $\mathbb{R}^d$ with (2,3)–inversection property. For example, in $\mathbb{R}^3$ only affine images of the cube and the regular otcahedron possess this property. Some other results in this direction were obtained by Lindenstrauss, Hansen, Lima. So, Lima has established that if $M \subset \mathbb{R}^d$ is a centrally symmetric, compact, convex body with (3,4)–intersection property, then it possesses the $(3,\infty)$–intersection property, i.e., (in accordance with a result of the talker). $M$ is a direct vector sum of convex sets, each of which has a dimension $\leq 2$. In the talk the general necessary and sufficient condition for $(p, p+1)$–intersection property will be formulated (not published yet). It follows from this condition that Lima's Theorem can be generalized to non–symmetric cases, i.e., if a body $M \subset \mathbb{R}^d$ had the (3,4)–intersection property, then it has $(3,\infty)$–intersection property. All bodies of such a kind were described in a resent (1992) paper of the talker.

R. FRITSCH

### Remarks on Bodenmiller's Theorem

The original version of Bodenmiller's Theorem states that the three circles with the diagonals of a complete quadrilateral as diameters intersect in the same two points. We provide a simple proof of an abstract version of this theorem containing several variations of the classical result as special cases: The radical axes of the three Bodenmiller spheres of a complete quadrilateral in a (pseudo–)euclidean space coincide.

H. HAVLICEK

### Spreads of Right Quadratic Skew Field Extensions

Let $L/K$ be a right quadratic (skew) field extension and let $\tilde{\mathcal{P}}$ be a 3–dimensional projective space over $K$ which is embedded in a 3–dimensional projective space $\mathcal{P}$ over $L$. Moreover let $\mathcal{J}$ be a line of $\mathcal{P}$ which carries no point of $\tilde{\mathcal{P}}$. The main result is that — even when $L$ or $K$ is a skew field — the following holds true: A desarguesian spread of $\tilde{\mathcal{P}}$ is given by the set of all lines of $\tilde{\mathcal{P}}$ which are indicated by the points of $\mathcal{J}$. A spread of $\tilde{\mathcal{P}}$ arises in this way if, and only if, there exists an isomorphism of $L$ onto the kernel of the spread such that $K$ is elementwise invariant. Furthermore a geometric characterization of right quadratic extensions with a left degree other than two and of quadratic Galois extensions is given.

H. KARZEL

### Generalized kinematic groups

Let $(G, \cdot, \mathcal{F})$ be a kinematic space with the line set $\mathcal{L} := \{xF \mid x \in G, F \in \mathcal{F}\}$, let $* : G \to G; x \to x^*$ be an involutory antiautomorphism of $(G, \cdot)$ and let $P \subset \text{Fix}\, *$ such that $\forall x \in G : xPx^* = P$. Then $(G, \cdot, \mathcal{F}, *, P)$ is called a generalized kinematic group. $(G, \cdot, \mathcal{F}; *, P)$ is a kinematic group in the sense of H. Hotje if $*$ is the inverse map. By "$(a,b) \equiv (c,d) :\Leftrightarrow \exists x, y \in G : \{c,d\} = \{xay, xby\}$" we define a congruence relation on $G$. With respect to the trace structures, $P$ becomes an "absolute space"

$(P, \mathcal{L}_p, \equiv_P)$ and $G^\square := \{g^\square = g_l \circ (g^*)_r \mid g \in G\}$ is a motion group of $(P, \mathcal{L}_P, \equiv_P)$. All absolute planes and the 3-dimensional hyperbolic space can be derived in this way from a generalized kinematic group. If further $1 \in P$ and $(*) \forall p \in P\, \exists_1 \sqrt{p} \in P : \sqrt{p}^2 = p$, then $(P, \oplus)$ with $a \oplus b := \sqrt{a} b \sqrt{a}$ is a $K$–loop in the sense of W. Kerby and H. Wefelscheid and $G^\square = P^\oplus \rtimes_q Q^\square$ with $Q := \{x \in G \mid x^* = x^{-1}\}$ is a quasidirect product (cf. e.g. [1]). This result can be applied on the proper orthochronous linear Lorentzgroup $G$, where $P$ is a set of pure boosts and $Q$ the group of all rotations of the 3–dim. euclidean space.

[1] Karzel, H. and Thomsen, M.J. Near–fields, Generalizations, Near-rings with regular elements and Application, a Report. Contributions to General Algebra 8 (1992) 91–110.

B. KLOTZEK

**Spiegelungsgeometrie unendlich dimensionaler Räume**

Nachdem in mehreren Jahrzehnten bedeutende Beiträge zur Spiegelungsgeometrie metrischer Ebenen erschienen waren und von Bachmann in einer Monographie zusammengefaßt wurden, begann die Übertragung der Methoden auf den $n$–dimensionalen Fall (Ahrens 1959, Kinder 1965). Besondere Probleme bestanden beim spiegelungsgeometrischen Aufbau unendlich dimensionaler Räume, zu deren Lösung vom Vortragenden zwei Varianten vorgestellt wurden: 1) Zulassung gewisser unendlicher Produkte von Erzeugenden, 2) Verwendung geteilter Erzeugenden–Systeme (Anwendung auf affine Räume beliebiger Dimension 1971). Daran schloß sich die entsprechende Kennzeichnung euklidischer und nichteuklidischer Räume beliebiger Dimension mittels eines geteilten Erzeugenden–Systems (Ewald 1974; vgl. auch Smith 1974/75) sowie pseudoeuklidischer Räume (1983). Neuere Untersuchungen beziehen sich u.a. auf die Tragfähigkeit der oben genannten Varianten und andere modelltheoretische Fragestellungen (vgl. auch Quaisser); darüber wird abschließend berichtet.

A. KREUZER

**Examples of $K$–loops**

A $K$–loop $(F, \oplus)$ is a loop with the following properties for $a, b, c \in F$: there exists an automorphism $\delta_{a,b}$ with $a \oplus (b \oplus c) = (a \oplus b) \oplus \delta_{a,b}(c), \delta_{a,b} = \delta_{a,b \oplus a}, \delta_{a,b} = \mathrm{id}$ if $a \oplus b = 0$ and $\ominus(a \oplus b) = (\ominus a) \oplus (\ominus b)$.

These are the properties of the additive structure of a near domain $(F, \oplus, \cdot)$, which was introduced by H. Karzel in order to discribe sharply 2–transitively groups. A. Ungar showed 1988 that $\mathbb{R}^3_c := \{V \in \mathbb{R}^3 : |v| < c\}$ ($c$ = speed of light) with the relativistic velocity addition is a $K$–loop. Further examples are:

1. For a field $(K, +, \cdot)$ let $F = K^4$ and for $x, y \in F$ let $(x_1, x_2, x_3, x_4) \oplus (y_1, y_2, y_3, y_4) := (x_1 + y_1 + (x_2 + 2y_2)(x_3 y_4 - x_4 y_3), x_2 + y_2, x_3 + y_3, x_4 + y_4)$. Then $(F, \oplus)$ is a $K$–loop.

2. For a map $\lambda : \mathbb{R} \to \mathbb{R}^*$, $x \to \lambda(x) =: \lambda_x$ with $\lambda_x \lambda_y = \lambda_{x+y} + \lambda_{x-y}$ let for $(x, y), (z, w) \in \mathbb{R}^2$ $(x, y) \oplus (z, w) := \left(x + z, \varphi(x, z) y + \psi(x, z) w\right)$ with $\varphi(x, z) = \lambda_{2z} \lambda_z^{-1} \lambda_{z+2z} \lambda_{2x+2z}^{-1}$, $\psi(x, z) = \lambda_{2z} \lambda_{2x+2z}^{-1}$. Then $(\mathbb{R}^2, \oplus)$ is a $K$–loop.

3. K.H. Robinson and H. Niederreiter gave for odd primes $p, q$ with $q$ dividing $p^2 - 1$ examples for Bruck–loops $(L, \oplus)$ with $|L| = pq$. It turns out that these loops are $K$–loops.

H.-J. KROLL

## A characterization of miquelian Minkowski planes

An automorphism $\alpha$ of a Minkowski plane $\mathcal{M}$ is called a $(pq)$-homothety if the restriction of $\alpha$ to the affine derivation $\mathcal{A}(q)$ at the point $q$ is a dilatation with fixed point $p$. The Minkowski plane $\mathcal{M}$ is called $(p,q)$-transitive, if the group $\Gamma(p,q)$ of all $(p,q)$-homotheties of $\mathcal{M}$ acts transitively on $C \setminus \{p,q\}$ where $C$ is a cycle with $p,q \in C$. $\mathcal{M}$ is called strongly $q$-transitive if it is $(p,q)$-transitive for every point $p$ of $\mathcal{A}(q)$. In 1982 E. Hartmann characterized the miquelian Minkowski planes $\mathcal{M}$ by the property

$$(*) \quad \mathcal{M} \text{ is strongly } q\text{-transitive for all points } q.$$

We will weaken this condition and obtain the Theorem: A Minkowski plane $\mathcal{M}$ is miquelian if and only if there is a generator $G$ such that $\mathcal{M}$ is strongly $q$-transitive for all $q \in G$.

A.V. KUZ'MINYKH

## Isometric and similarity mappings characterization and fractals with paradoxical geometric properties

1) Let $\Omega$ be a countable subset of the set of positive numbers, $a,b \in \Omega$, $a \neq b$; and let $f : \mathbb{R}^n \to \mathbb{R}^n$, $n \geq 3$, be an injective mapping such that the condition $d(X,Y) \in \{a,b\}$ (where $X,Y \in \mathbb{R}^n$, $d$ — Euclidean metric) always implies that $d\big(f(X), f(Y)\big) \in \Omega$. Then the mapping $f$ is a similarity. (If $\Omega$ is the set of all prime numbers, then $f$ is an isometry.)

2) There is a curve $\mathcal{M} \subset \mathbb{R}^n$ (i.e., a subset of $\mathbb{R}^n$ which is homeomorphic to a straight line) which has the following property: for every straight line $\lambda \subset \mathbb{R}^n$ the intersection $\lambda \cap \mathcal{M}$ has the cardinality of the continuum.

3) There is a connection between the characterization of isometric (and similarity) mappings (see 1) and fractals which are analogous to the curve $\mathcal{M}$ (see 2).

J. LESTER

## Centre functions of triangles

Identify the Euclidean plane with the complex numbers $\mathbb{C}$; then for any triangle $\triangle abc$, the number $[\infty, a; b, c]$ (brackets denote a cross ratio) is called the *shape* of $\triangle abc$, and determines $\triangle abc$ up to (direct) similarity. Shapes may be used to provide simple analytic proofs of many theorems about similar triangles.

With respect to a fixed reference triangle $\triangle abc$, the triangle coordinate of any point $p$ is $[p, a; b, c]$. If $p$ is a special point of the triangle (i.e. if it is defined in terms of the vertices and is similarly situated in similar triangles) then (Theorem) its triangle coordinate is a function of its shape. If the special point is a centre, then its function satisfies several symmetry conditions.

These functions, together with the various properties of cross ratio, may be used to prove theorems about special points. Example: the circumcentre, the nine–point centre and the two isogonic centres lie on a circle.

Complex triangle functions can be used to develop a theory of centres; the problem of finding all triangle centres reduces to the problem of finding all functions satisfying a certain set of functional equations.

H. MÄURER

**Non miquelian inversive planes with inversions for all circles**

Let $K$ be a field, $\sigma_1, \sigma_2 \in \text{Aut } K$ with $\langle\sigma_1, \sigma_2\rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$ and with $F_i = \text{Fix } \sigma_i (i = 1, 2)$. If $U := F_1^\bullet \cdot F_2^\bullet \cdot \{k^2 \mid k \in K^\bullet\}$ is a proper subgroup of $K^\bullet$, then

$$\mathcal{F} := \{F_1 \cdot u \mid u \in U\} \cup \{F_2 \cdot u' \mid u' \in K^\bullet \setminus U\}$$

is spread of $K$ and

$$\Big(K \cup \{\infty\}, \{\sigma(f \cup \{\infty\}) \mid \sigma \in PSL(2, K), f \in \mathcal{F}\}, \in\Big)$$

is a non miquelian inversive plane with an inversion for every circle.

M. MARCHI

**Automorphisms of Incidence Loops**

A fibered incidence loop with parallelism $(P, \mathcal{L}, \|, \cdot)$ is an incidence space with parallelism $(P, \mathcal{L}, \|)$ together with a loop structure $(P, \cdot)$ such that the set of left multiplications $P^{\cdot} := \{a^{\cdot} \mid a^{\cdot} : P \to P; x \to ax, \forall a \in P\}$ is a set of translations acting regularly on $P$. If suitable conditions are fulfilled, a family $J$ of subsets of $P$, named "strings" (namely: $J := \{Mx \mid M \in L(1), x \in P\}$) can be defined such that any two distinct points belong to exactly one string and an equivalence relation "$\hat{\|}$", named "string–parallelism", fulfilling the Euclid's axiom, is allowed. This fact leads us to consider also the set of right mappings ${}^{\cdot}P := \{{}^{\cdot}a \mid {}^{\cdot}a : P \to P; x \to xa, \forall a \in P\}$ which is still acting regularly on $P$.

Now many different properties of the set $P^{\cdot}$, as a subset of the group Aut $(P, L, \|)$, and of the set ${}^{\cdot}P$, as a subset of the group Aut $(P, J)$, are studied.

U. OTT

**Nets and Generalized Quadrangles**

We give a short report about a joint project with D. Ghinelli.
Let $N = (\Omega, \Lambda, I)$ be a net with the parameters $s$ and $t$. Thus $N$ admits $s^2$ points and each point contains exactly $1 + t$ lines. We choose a basepoint $O$. Let $K$ be a field.

**Definition 1** *A function $F : \Omega \times \Omega \to K$ is called an area function on $N$ with values in $K$ if the following conditions hold*

*A1 if the points $O, A, B$ are collinear then $F(A, B) = 0$*

*A2 for collinear points $A, B$ we have $F(A, B) = -F(B, A)$*

*A3 if $A, B, C$ are the points of a triangle then the equation $F(A,B) + F(B,C) + F(C,A) = 0$ is equivalent to the fact that the points $A, B, C$ are collinear.*

In this notation we get the following theorem and conjecture

**Theorem 1** *If there exists an area function on a net with parameters $s$ and $t$ (degree $t+1$) with values in $GF(t)$, then there is a generalized quadrangle with parameters $s, t$.*

**Conjecture 1** *Let $N = A$ be an affine plane of prime order. If there is an area function with values in $GF(p)$ then the plane is desarguesian and the area function is (in a certain sense) equivalent to the symplectic form.*

We have the question whether a translation plane of order $q$ can have an area function with values in $GF(q)$.

P. PLAUMANN

### The lattice of connected subgroups of an algebraic group

In joint work with K. Strambach and G. Zacher we have studied the lattice $\Lambda(G)$ of all closed connected subgroups of an algebraic group (over some algebraically closed filed). Lattice theoretical properties which interest us are those which are shared by projective spaces, like modularity, atomicity, complementarity, the Jordan–Dedekind–condition etc. We classify the groups $G$ for which $\Lambda(G)$ satisfies this properties; in prime characteristic some interesting phenomena arise. Here I want to mention the following

**Theorem:** For an algebraic connected group $G$ over an algebraically closed field its lattice $\Lambda(G)$ is complementary if and only if $G$ splits over its solvable radical $R$ and $R$ is a vector group.

S. PRIESS

### Examples of spherically complete spaces

(joint work with Paulo Ribenboim)

Let $(\Gamma, \leq)$ be a partially ordered set with $0 < \gamma$ for each $\gamma \in \Gamma$ and let $X \neq \emptyset$ be a set. A mapping $d : X \times X \to \Gamma \cup \{0\}$ is called an ultrametric, if it has formally the same properties as a metric, but instead of the triangle inequality, the following one for all $x, y \in X$, $\gamma \in \Gamma$: if $d(x,y) \leq \gamma$, $d(y,z) \leq \gamma$, then also $d(x,z) \leq \gamma$. The set $B\gamma(a) = \{x \in X \mid d(x,a) \leq \gamma\}$ is a ball of $X$, and $X$ is spherically complete if $\bigcap \mathcal{B} \neq \emptyset$ for every chain $\mathcal{B} \neq \emptyset$ of balls. In a spherically complete space $X$, one has a Banach–like Fixed Point Theorem (cp. Priess–Crampe, Ribenboim, Abhandlungen Math. Sem. Hamburg 1993). Therefore, one is interested in examples of spherically complete spaces. Outside of valuation theory, complete Boolean algebras (with the symmetric difference as a distance) and function spaces $R^X$ (with $d(f,g) = \{x \in X \mid f(x) \neq g(x)\}$) are such spaces.

E. QUAISSER

### Investigations to the mapping geometrical representation

At first are proposed conditions at the conception of the mapping geometrical representation. Then are given some results of the "complete" representation of the plane affine geometry (and the affine geometry with arbitrary dimension $\geq 2$) on basis of affine reflections resp. non involutory shears resp. axial dilatations resp. skew reflections resp. $m$-reflections. In last case the necessary condition for the representability results from the reflection geometrical language.

In second part is given a characterization theorem for the respresentability of the plane parabolic geometry: This theory is representable if all models are translation planes with char $\neq 2$ and $(\iota,\iota)$-transitivity (with reference to incidence structure).

H.-J. SAMAGA

**Are there fractal structures in circle geometries?**

For $z_n, w \in \mathbb{C}$, the iteration $z_{n+1} = z_n^2 + w$ leads to the well known Mandelbrot set $M_{\mathbb{C}}$ and filled in Julia sets $J_{\mathbb{C}}(w)$. From a geometrical point of view $\mathbb{C}$ fixes the miquelian Möbius plane. There are further circle geometries over $\mathbb{R}$ of dimension 2 and 3; to each of them belongs an algebraic structure. These structures are $\mathbb{D} := \{a + b\varepsilon \mid a,b \in \mathbb{R} \not\ni \varepsilon, \varepsilon^2 = 0\}, \mathbb{R}\times\mathbb{R}, \mathbb{C}\times\mathbb{R}, \mathbb{R}\times\mathbb{R}\times\mathbb{R}, \mathbb{D}\times\mathbb{R}$, $\mathbb{R}[X]/_{\langle X^3\rangle}, \mathbb{R}[X,Y]/_{\langle X^2, XY, Y^2\rangle}$.

Problem: What are the shapes of $M_L, J_L(w)$, if $L \in \{\mathbb{D}, \mathbb{R}\times\mathbb{R}, \ldots\}$? Answer (roughly): Look at a homeomorphic image of the "Feigenbaum"-function $f(x) = x^2 + c$ in the right way and you know (nearly) everything you want!

In detail, 2- and 3-dimensional Mandelbrot sets and all 2-dimensional Julia sets are illustrated by theorems and/or (computer) graphics.

M. SCAFATI-TALLINI

**Topics on hypervector spaces**

We define hypervector space over a field $K$ the quadruplet $(V,+,\circ,K)$, where $(V,+)$ is an abelian group and

$$\circ : K \times V \to P'(V)$$

is a mapping of $K \times V$ into the set $P'(V)$ of non-empty subsets of $V$, such that the following conditions hold:

(1) $\forall a,b \in K,\ \forall x \in V,\ (a+b)\circ x \subseteq (a\circ x) + (b\circ x)$,

(2) $\forall a \in K,\ \forall x,y \in V,\ a\circ(x+y) \subseteq (a\circ x) + (a\circ y)$,

(3) $\forall a,b \in K,\ \forall x \in V,\ a\circ(b\circ x) = (ab)\circ x$

(4) $\forall a \in K,\ \forall x \in V,\ a\circ(-x) = (-a)\circ x = -(a\circ x)$,

(5) $\forall x \in V,\ x \in 1\circ x$.

Here we explain various properties of such spaces, the geometric structures that we associate with them, and the cathegory of such spaces, once given in a suitable way the notions of factor space and homomorphism.

S.E. SCHMIDT

## Foundations of a generalized affine geometry

(A) **Concept:** We develop an axiomatic structure theory of affine geometry which induces the geometry of all affine submodules of unitary modules (over associative rings). We point out that it is not intended to include "weak affine" geometries (as introduced by E. Sperner and generalized by H.-J. Arnold), "skew-affine" geometries (as studied by J. André) or "generalized (partial) affine" geometries (as investigated by W. Leißner and F. Radó) — nevertheless the latter occur as partial substructures in our set-up.

(B) Main results:

(1) Completion of "affine line systems" to "affine closure systems".

(2) Algebraization of affine spaces (in our sense) by unitary modules.

E.M. SCHRÖDER

## On 0-distance-preserving permutations of affine and projective quadrics

Let $(V, \mathbb{F}, q)$ be a metric vector space. The set $\mathcal{F}_\pi := \{\mathbb{F}x \mid x \in V \backslash \{0\} \wedge q(x) = 0\}$ is the quadric wrt. $q$ of the projective space $\Pi := \Pi(V, \mathbb{F})$. In the affine space $A := A(V, \mathbb{F})$, the set $\mathcal{F}_A := \{x \in V \mid q(x) + \kappa(x) + \alpha = 0\}$ ($\kappa : V \to \mathbb{F}$ linear, $\alpha \in \mathbb{F}$) is an affine quadric wrt. $q$, and $\mathcal{F}_A^\pi := \mathcal{F}_A \cup \{(\mathbb{F}x) \| \; \big| \; x \in V\backslash\{0\} \wedge q(x) = 0\}$ is a quadric of the projective closure $\Pi(A)$ of $A$.

Assume $\mathcal{F}_\pi$ resp. $\mathcal{F}_A^\pi$ contains lines, but no double points.

For $\mathcal{F} := \mathcal{F}_\pi$ resp. $\mathcal{F} := \mathcal{F}_A$, points $X, Y \in \mathcal{F}$ are called 0-distant (or conjugate or parallel), written $X \approx Y$, iff the connecting line $\overline{X,Y}$ is contained in $\mathcal{F}$.

Now assume $4 \leq \dim \Pi \leq \infty$ in case of $\mathcal{F} = \mathcal{F}_\pi$ resp. $4 \leq \dim A \leq \infty \wedge |\mathbb{F}| \geq 4$ in case of $\mathcal{F} = \mathcal{F}_A$.

Then the following theorem holds true.

If $\varphi$ is a permutation of $\mathcal{F}$ with the property

$$(*) \qquad X \approx Y \Leftrightarrow X^\varphi \approx Y^\varphi \quad \forall X, Y \in \mathcal{F},$$

then there exists an $\alpha \in \mathbb{F}\backslash\{0\}$ and a semi-linear bijection $(\sigma, \varrho)$ of $(V, \mathbb{F})$ such that $q \circ \sigma = \alpha \cdot \varrho \circ q$ and $X^\varphi = X^\sigma \; \forall X \in \mathcal{F}_\pi$ resp. $X^\varphi = 0^\varphi + X^\sigma \; \forall X \in \mathcal{F}_A$.

G. TALLINI

## $(n)$-varieties in linear spaces

A $(n)$-variety $H$ in a linear space $(P, L)$ is a subset of $P$ such that:

$$\forall l \in L, \text{ either } l \subset H \text{ or } |l \cap H| \in \{0, 1, n\}$$

and $n$-secant lines do exist.

$H$ is called non-singular quasiregular if $\forall x \in H$ the union of the lines through $x$ either tangent or contained in $H$ is a subspace $\tau(x) \neq P$ (regular, strongly regular respectively if $\tau(x)$ is either a hyperplane or a prime). Here we outline the foundations of the theory of $(n)$-varieties, in particular of quasiregular ones, in a linear space $(P, L)$, pointing out the properties of a linear space containing a $(n)$-variety and conversely. We explain several results and open problems on this subject.

H. TECKLENBURG

**Low Order Projective Planes**

It is well-known that there exists exactly one projective plane of order $n$ for $n = 2, 3, 4, 5, 7$ (up to isomorphism), while a projective plane of order 6 cannot exist. Pure geometrically with short unified proofs we show the uniqueness and existence or non-existence resp. of such planes.

Remark: Using a computer, M. Hall, J.D. Swift, and R.J. Walker (1956) proved the uniqueness of the projective plane of order 8. It is assumed that this case can be treated directly with our method so that the use of a computer is no more necessary.

B. WERNICKE

**Möbius planes and geometry of reflections**

At the beginning we refer to results of P. Dembowski, H. Mäurer, and H. Karzel. Geometry of reflections in Möbius planes was also investigated by E. Molnár (over pythagoreian fields) and K. Lang (over fields of char $\neq 2$). We give a group theoretical system of axioms for a miquelian Möbius plane (independent of characteristic). We need for the proof of the "Berührsatz" one of the two orthogonality axioms. So we have a system of three simple "incidence" axioms and one weak "orthogonality" axiom for a group so that the group plane is a Möbius plane with a reflection in each circle (H. Mäurer)

H. ZEITLER

**A. Conjecture of A. ROSA, concerning systems $S(2,4,v)$**

A report about joint works with Shen Hao and furthermore an extension of a lecture given 14.12.1992 in Oberwolfach:

(1) Exactly $\forall v \equiv 40, 49 (36)$ there exist systems $S(2,4,v)$ with *exactly one* sub system $S\Big(2,4,r = \frac{1}{3}(v-1)\Big)$.

(2) It's possible that within the complement of $S(2,4,r)$ there exists an *affine plane* $AG(2,3)$. Adding a line from $S(2,4,r)$ to $AG(2,3)$ in the usual way a *projective plane* $PG(2,3)$ is obtained.

Some results of Rees-Stinson are used.

(3) For all admissible numbers $v, v \neq 13$ there exists a $S(2,4,v)$ *without any* subsystem $S(2,4,r)$. (The conjecture of A. Rosa.)

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