MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 2/1993

Extensions of buildings and geometries

3. bis 9.1.1993

The meeting was led by A. Beutelspacher (Giessen), F. Buekenhout (Bruxelles) and D.R. Hughes (London).
In the centre of interest where questions of classification and characterization of geometries belonging to particular (extended) diagrams, embeddings of geometries and buildings, and geometries related to Coxeter groups. The meeting was very succesful; some of the lectures stimulated other participants to prove new results which they could also present at the conference.
There was a common interest with the meeting on foundations of geometry, which was held at the same time, and a joint session was organized.

<u>Vortragsauszüge</u>

B. BAUMEISTER:
<u>Flag-transitive rank 3 geometries, which are locally complete graphs</u>

Let G be a flag-transitive group on a geometry Γ of type $\underset{P}{\circ}\overset{c}{—}\underset{L}{\circ}\overset{c^*}{—}\underset{F}{\circ}$ (P: point, L: line, F: plane), supposing that the stabilizer of a point has no regular normal subgroup. It is already known that G is finite. Furthermore we have $K_P = K_F = 1$ and the groups G_P and G_F are isomorphic (also as 2-transitive permutation groups on $\mathrm{Res}(P)$, $\mathrm{Res}(F)$ respectively). We get the nice criterion: If $Z(B) = 1$ and $G_P \not\cong L_2(11), A_7, L_2(8) : 3$ of degree 11, 15, 28 then Γ is the hypercube.
With help of this criterion and the Todd-Coxeter-algorithm we are able to show: If one point is incident to at most 20 planes then Γ and G are known or $G \cong 2 \cdot M_{22}$ with $G_P \cong A_7$. We can give a simple geometrical description of this example.
Finally assuming that one point is incident with at least 20 planes we get Γ is a hypercube or G_P is a group of Lie-type of rank 1.

A. BEUTELSPACHER:
Extensions of semiaffine linear spaces

For a non-incident point-line pair (P, l) of a linear space S denote by $\pi(P, l)$ the number of lines of S on P that do not intersect l. The linear space S is called H-semiaffine, if H is a set of integers containing all of $\pi(P, l)$.
The finite $\{0, 1\}$- and $\{0, 1, 2\}$-semiaffine linear spaces have been characterized by Dembowski and Hauptmann, Olanda-Lo Re, respectively.
We consider finite incidence structures S such that for any point P of S, the derivation S_P is a $\{0, 1, 2\}$-semiaffine linear space. We prove that any such structure can be embedded in a Möbiusplane or an extension of a projective plane. (Joint work with D. Olanda, Napoli.)

J. van BON:
Some extended generalized hexagons

A few years ago R. Weiss studied geometries of type 

where the hexagon is known and either thick or point thin, under the condition that there exist three mutually collinear points that are not on a circle. In the case where the generalized hexagon is the one associated to $PSL_3(2) : 2$ he also assumed that the stabilizer of a point P induces this group on the residue of P.
In this talk we discuss the the situation where the stabilizer of P induces a group isomorphic to $7 : 6$ on its residue.

A.E. BROUWER:

(1) discussion of recent results on $\mathbb{Z}_4$-duality for binary codes

(2) the only generalized hexagons of order 3 such that its subgeometry far away from the flags (p, L_o) have three connected components, is the well-known one.

(3) discussion of Weetman's results.
Theorem. Let Γ be a Taylor graph, not the hexagon. Then locally Γ graphs have bounded diameter.

F. BUEKENHOUT:
Generalizing the Alexandrov theorem on spacetime in special relativity

The Alexandrov theorem for a Minkowski space M_n of dimension n states that a permutation of the points preserving 0-distances is an automorphism of M_n. The result has been extended to arbitrary fields, the metric being defined by a quadratic form of index ≥ 1 and infinite dimensional spaces being allowed (J. Lester (1985), E. Schröder (1990)). Our viewpoint departs from metric and from automorphisms. We consider any affine space A and a set B of points at infinity of A, generating a hyperplane at infinity. The Alexandrov space Alex(A, B) is the set of points of A equipped with the collection of sets $C(p, B)$ where p is a point of A and and $C(p, B)$ is the union of all lines of A containing p and a point $b \in B$. We get sufficient conditions on B in order that Alex(A, B) does uniquely determine all lines of A and the parallelism. One such condition is that B is a nondegenerate polar space of index ≥ 1, embedded in the hyperplane at infinity and such that all tangent hyperplanes have an empty intersection.

P.J. CAMERON:

Ovoids, spreads and flat geometries

If an infinite incidence structure has the properties
(a) # points per block = # blocks (= α, say)
(b) given a block B and a set S of fewer than α points disjoint from B. Then α points on B are collinear with no point in S,
then the point set can be partitioned into ovoids.
The class of structures with these properties includes polar spaces over infinite fields, and infinite near polygons (including generalized polygons, and dual polar spaces over infinite fields except for type O^+_{2n}).

Applications include
(a) a theorem of Shult and Thas on hyperplanes of dual polar spaces doesn't extend to infinite fields;
(b) a slight generalization of a construction of Kantor gives many geometries with star diagrams and residues of the above form (including Tits geometries).

C. HUYBRECHTS:

The commutativity of the ground division ring of a D_n-geometry

If Γ is a thick and residualy connected D_n-geometry, $n \geq 4$, it is well known that Γ is defined over a unique ground division ring which is commutative. A footnote of Tits (1964) suggests to give an elementary proof of this fact. It is easy to show that it suffices to tread the case of D_4 geometries. The main step of the proof is to build a null polarity in the 3-dimensional projective subspaces of Γ (i.e. a polarity π such that only point p is incident with $\pi(p)$).
Here is how to construct the null polarities. Consider two end nodes of the diagram D_4 and call elements of these types respectively points and blocks. A block has the structure of a 3-dimensional projective geometry. For any disjoint blocks B and B', we find a natural duality $\delta_{B,B'}$ of the residues Γ_B onto $\Gamma_{B'}$. Next, we show that for every block B, there are blocks B' and B'' such that B, B' and B'' are pairwise disjoint. The mapping $\delta_{B'',B} \circ \delta_{B',B''} \circ \delta_{B,B'}$ is a null polarity in Γ_B.

A.A. IVANOV:
Nonabelian representations of geometries

Let $S = (P, L)$ be a point-line incidence system with 3 points on a line. A group H is a *representation group* of S if it is generated by a set of (non-necessary distinct) elements x_p indexed by the points $p \in P$ such that (i) $x_p^2 = 1$; (ii) $x_p x_q x_r = 1$ if p, q, r are distinct and collinear. If in addition (iii) $[x_p, x_q] = 1$ for all $p, q \in P$ then H is a *representation module* of S. It is clear how to define the universal representation group and the universal representation module in terms of generators and relations. Let $G \cong J_4, F_2$ or F_1 and $\mathcal{G}(G)$ be the 2-local geometry having the central involutions as points. It is known that $\mathcal{G}(G)$ does not have a nontrivial representation module, but G is obviously a representation group of $\mathcal{G}(G)$.
Conjecture. The universal representation groups of $\mathcal{G}(G)$ for $G \cong J_4, F_2$ and F_1 are isomorphic to $J_4, 2 \cdot F_2$ and F_1, respectively.
A considerable progress in proving the conjecture was recently done in my joint work with D.V. Pasechnik and S.V. Sphectorov.

A. IVIĆ WEISS:
Chiral Polytopes

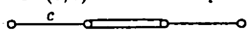
Abstract polytopes are discrete geometric structures which generalize the classical notion of a convex polytope. Chiral polytopes are those abstract polytopes which have maximal symmetry by rotation, in contrast to the abstract regular polytopes which have maximal symmetry by reflection. Chirality is a fascinating phenomenon which does not occur in the classical theory. We give the basic theory of chiral polytopes and present some general results. We furthermore discuss the existence of chiral polytopes in higher dimensions.

O.H. KING:
CP^* geometries

The geometries in this talk will have diagram o—c—o—P^*—o (nodes 0, 1, 2), they will have a flag-transitive automporphism group G and will satisfy condition (LL), that two points lie on at most one line. They will also be residually connected. o—c—o—P^*—o is a special case of o—L—o—P^*—o which is otherwise o———o—P^*—o . The latter is more commonly consider in it dual form o—P—o———o and all (both) geometries are known. In o—c—o—P^*—o G must act imprimitively on 2-elements.
The embedding of the Petersen graph in vector spaces of dimension 4,5 and 6 over GF(2) leads to geometries on 16, 32, 32 and 64 points with full automorphism groups $2^4 : S_5$, $2^5 : S_5$, $2^5 : A_5$ and $2^6 : S_5$ respectively.

Consider the collineation graph $\overline{\Gamma}$ of a geometry Γ. The neighbourhood graph of a point has valency 4, 6, 8, 12 or 14. Only a small number of graphs arise except in the case of valency 4. In that case $\overline{\Gamma}$ is locally the line graph of the Petersen graph and any triangle of Γ lies in a circle (2-element).
We note that the examples above (and an example with group $3.A_6$) arise as quotients of truncations of quotients of (thin) Coxeter geometries of rank 4.

M. KITAZUME:
<u>Some non-split extensions of orthogonal groups</u>

My purpose is to give an explicit construction of the non-split extensions $3^7 \cdot O(7,3)$ and $3^8 \cdot O^-(8,3)$ which are the point-stabilizer of their 3-local geometries belonging to the diagram . This purpose is not established yet, but some subgroups of $3^7 \cdot O(7,3)$ has been constructed by using the following function $g(x,y) : \mathbb{F}_3^3 \times \mathbb{F}_3^3 \longrightarrow \mathbb{F}_3^3$ due to R.L. Greiss,

$$g(x,y) = (x_1 - y_1)(x_2 y_3 - x_3 y_2)$$

where $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3) \in \mathbb{F}_3^3$. These subgroups are corresponding to maximal parabolic subgroups of $O(7,3)$.

P. MCMULLEN:
<u>Changing generators of reflection groups</u>

A group $G = \langle \rho, ..., \rho_{n-1} \rangle$ generated by involutions ρ_j which satisfy the intersection property

$$\langle \rho_i | i \in I \rangle \cap \langle \rho_i | i \in J \rangle = \langle \rho_i | i \in I \cap J \rangle$$

for all $I, J \subseteq N := \{0, ..., n-1\}$ underlies a flag-transitive thin geometry. Choosing new generators for G will lead to different geometries. We discuss systematic ways of changing generators, with particular reference to the case $n = 3$.

B. MÜHLHERR:
<u>Embeddings of buildings</u>

Polarities of projective spaces provide polar spaces embedded in these spaces. This situation is also well known for other buildings. Examples are the hexagons fixed by a triality in D_4 or the octagons fixed by a polarity in F_4.
In the case of thin buildings - the Coxeter complexes - one gets also other embeddings which do not come from an automorphism of a diagram. These are provided by admissible

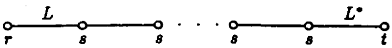
partitions. So the question is, whether there exist also thick examples of such embeddings. The main result about this question is the following:

Theorem. Every n-gon ($3 \leq n < \infty$) embedded in a building of rank at least 3 of irreducible type is Moufang.

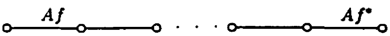
A. PASINI: (joint work with A. Del Fra)

Linear-dual-linear extensions of projective geometries

We consider flag-transitive geometries belonging to the following diagram of rank $m \geq 4$:



With $r \leq t < s < \infty$. By a theorem of Delandsheer, the possibilities that can arise are the following:

(1) 

(2) 

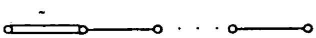
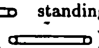
(3) 

It is known that (3) characterizes a unique geometry for HS. We prove that (2) is impossible. We also obtain some partial results on (1), towards a proof of the following conjecture: (1) characterizes geometries obtained from $PG(m, q)$ (for some prime power q) by deleting a hyperplane and the star of a point and, possibly, by taking quotients.

S.V. SHPECTOROV:

Classification of the flag-transitive tilde geometries of symplectic type

A tilde geometry is a geometry which belongs to the diagram

 with  standing for the triple cover of the generalized quadrangle of order (2,2) (so that Aut () $\cong 3 \cdot S_6$). By now a complete classification of flag-transitive tilde geometries is available in a series of papers of several authors. Starting from a particular characterization of an infinite series of tilde geometries (joint work with G. Stroth), some ideas of the general classification will be presented.

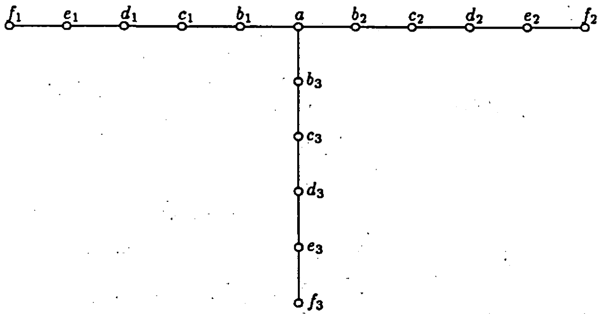
J.A. THAS:
The affine plane AG(2,q), q odd, has a unique one point extension

Let I be a finite inversive plane of odd order q. If for at least one point p of I the internal affine plane I_p is Desarguesian, then I is Miquelian. Other formulation: the finite Desarguesian affine plane of odd order q has a unique one point extension; this extension is the Miquelian inversive plane of order q. It follows that there is a unique inversive plane of order q, with $q \in \{3,5,7\}$.

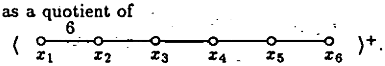
S.V. TSARANOV:
A Coxeter subsystem for $^3D_4(2)$ of the Coxeter system

for the Thompson group

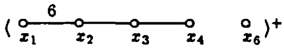
Y-generators of the Bimonster give rise to a Coxeter group



admitting the automorphism π of order 3 that rotates 3 branches. But the additional relation $(ab_1c_1ab_2c_2ab_3c_3)^{10} = 1$ which defines the Bimonster doesn't admit π. Nevertheless in the Monster π can be interpreted as an inner automorphism induced by $3C$-element of the Monster. Its centralizer in the Monster is the Thompson group so that it may be considered as a quotient of



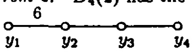
We prove the following results.

Theorem 1. The subgroup $\langle$ 

$\rangle^+$ is isomorphic to ${}^3D_4(2)$.

Denote $y_i = x_i x_6$, $1 \leq i \leq 4$.

Theorem 2. ${}^3D_4(2)$ has the following presentation:



$((y_1y_2)^2y_3)^6 = 1$, $y_5 = (y_4(y_1y_2)^3y_3)^7$, $(y_4y_5)^4 = 1$, $y_5^2 = y_1$.

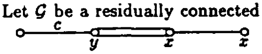
H. Van MALDEGHEM:
<u>Folding diagrams and filling apartments</u>

Consider a building with diagram Δ. We know that certain foldings of Δ give rise to lower rank buildings (see B. Mühlherr's talk). A lot of examples related to Generalized Polygons are known. We present a heuristic argument for the converse: given a generalized polygon S, under what condition can one construct a building of higher rank it naturally lives in? This can be obtained by considering the apartment of S and appropriately filling it up. This way, for example, we construct the 24-cell out of the usual octagon.

S. YOSHIARA:
<u>Some flag-transitive extensions of polar spaces of non-classical type</u>

The following result is described together with constructions and characterizations of three simply connected $c.C_2$-geometries with point-residues non-classical thick generalized quadrangles.

Theorem. Let $\mathcal{G}$ be a residually connected thick geometry belonging to the diagram



admitting a flag-transitive group G. If the residue $\mathcal{G}_p$ at a point p is not the dual of a classical polar space, the one of the following holds:
(1) $\mathcal{G}_p$ is the sporadic A_7-geometry, $G = \mathrm{Aut}(\mathcal{G}) \cong 2^4 : A_7$ and $\mathcal{G}$ is isomorphic to a geometry on 16 points constructed in terms of the Steiner system $S(24, 8, 5)$ with specified pair of point and octad.
(2) $G = \mathrm{Aut}(\mathcal{G})$ acts regularly on the chambers and $y + 2 = 2^e$ or 3^e for some $e \geq 1$.

It is conjectured that the case (2) does not occur.

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