MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 5/1993

Optimale Steuerung partieller Differentialgleichungen

24.01. bis 30.01.1993

Die Tagung wurde organisiert von K.-H. Hoffmann (München) und J. Sprekels (Essen).

Im Mittelpunkt des wissenschaftlichen Programms standen 33 Vortragsbeiträge zur mathematischen Theorie, Numerik und Anwendung der Kontrolltheorie partieller Differentialgleichungen.
Dabei wurden folgende Problemkreise behandelt: optimale Kontrolle von Gleichungen der Fluiddynamik, Feedbackkontrolle (Hamilton-Jacobi-Bellman Gleichung), Parameteridentifikation, Simulation und Steuerung von Phasenübergängen, insbesondere mit Phase-Field-Modellen, numerische Verfahren zur Berechnung optimaler Steuerungen sowie Fragestellungen im Bereich Shape Optimization und Homogenisierung.
Die Tagung wurde mitgeprägt durch eine intensive Diskussion der Vorträge und angrenzender Problemstellungen, die insbesondere auch während der Abendzeit im kleinen Kreis stattfanden.
Die angenehme Atmosphäre der Tagung, die nicht zuletzt der exzellenten Betreuung durch die Mitglieder des Instituts zu verdanken ist, sei noch besonders erwähnt. Im Namen der Tagungsteilnehmer danken wir Herrn Prof. Dr. M. Barner und seinen Mitarbeitern herzlich dafür.

Vortragsauszüge

H. W. Alt:

Mathematical models of nonisothermal phase transitions I

We consider phase transition models with a given free energy $\varphi(u, w, \nabla u)$, where u is the order parameter and w the inverse absolute temperature. For example (further examples in Part II) dynamics are given by

$$\begin{aligned} \partial_t u + \nabla \cdot \vec{j} &= 0, \\ \partial_t e + \nabla \cdot \vec{q} &= 0, \end{aligned}$$

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where $e(u,w,\nabla u)$ is the internal energy. We postulate an entropy inequality

$$\tau := \partial_t s + \nabla \cdot \vec{\psi} \geq 0$$

for all solutions of the above system. Here $\vec{j}, \vec{q}, s, \vec{\psi}$ are functions of $u, v, w, \partial_t u, \nabla u, \nabla v, \nabla w$ with chemical potential $v = \frac{\delta\varphi}{\delta u}$. Using the approach of rational thermodynamics we prove that there exists a function $\lambda = \lambda(w) > 0$ such that after normalization

$$s + \lambda\varphi = we \quad \text{with } e = \frac{\partial}{\partial w}\big(\lambda\varphi\big) \quad (\underline{\text{Gibbs relation}}),$$

and $\vec{j} = \vec{j}_0$,

$$\vec{q} = \vec{q}_0 - \vec{p}_e \quad , \quad \vec{p}_e = \frac{\partial\lambda}{\partial w}\vec{p},$$

$$\vec{\psi} = \vec{\psi}_0 + \vec{p}_s \quad , \quad \vec{p}_s = \Big(\lambda - w\frac{\partial\lambda}{\partial w}\Big)\vec{p}.$$

The flux $\vec{p} = \partial_t u \frac{\partial\varphi}{\partial\nabla u}$ we call <u>interface flux</u>. Here the index 0 denotes the part of the fluxes which vanish if $\nabla v = 0$, $\nabla w = 0$. Moreover

$$\tau = -\nabla(\lambda v)\cdot\vec{j}_0 + \nabla w \cdot \vec{q}_0 \geq 0.$$

We compare these models with those existing in literature, where $\lambda = 1$ or $\lambda = w$.

L. Bittner:

Bemerkungen zur Steuerung quasilinearer partieller Differentialgleichungen erster Ordnung

Gegenstand des Vortrags sind Differentialgleichungen 1. Ordnung, die durch Charakteristikenmethoden in Integralgleichungen transformiert werden. Durch die Charakteristiken kommen die "äußeren" Argumente $t, x, ..$ auf der rechten Seite unmittelbar in die Argumente der Steuerungen hinein, so daß sich die Unstetigkeiten der Steuerungen sofort auswirken und zu unstetigen Zustandsfunktionen führen können. Im Vortrag wird daher ein (Produktsummen –) Typ $\sum_k u_k(t)v_k(x,..)$ von Steuerungen vorgeschlagen, der sichert, daß die Zustandsfunktionen, wie gewünscht, stetig bleiben. Mit Hilfe von Optimalitätsaussagen über ein "Modell – Problem" des Vortragenden wird, des Beispiels wegen, für ein u.a. von Brokate behandeltes quasilineares hyperbolisches Steuerproblem ein "starkes" Maximumprinzip angegeben.

M. Brokate

On the Mróz model

We present a formal description and a mathematical analysis of the constitutive stress–strain law for rate–independent plastic flow due to Mróz. We are particularly interested in its memory structure, since for the uniaxial situation, the buildup and deletion of memory is intimately related to the rainflow counting method used for fatigue analysis and the estimation of damage. We then show how the rainflow method can be generalized to the tensor situation on the basis of the Mróz model.

(joint work with K. Dreßler and P. Krejčí)

E. Casas:

Optimal Control of some equations appearing in fluid mechanics

We consider the problem of controlling the turbulence behaviour of viscous, incompressible three-dimensional flows. The control variables are the body forces or the heat flux through the boundary of the domain occupied by the fluid. The state is the velocity of the fluid and the cost functional involves the norm of the vorticity of the fluid. This norm gives a good measure of the turbulence within the flow. The relation between the control and the state, that is, the state equation, is described by the evolution Navier-Stokes equations, coupled with the heat equation when the control is the heat flux. Our goal is to formulate an optimal control problem, to study the existence of a solution and to derive the conditions for optimality. To derive the optimality conditions we must consider the so-called strong solutions of the Navier-Stokes equations. The uniqueness of these solutions is known long time ago, however the existence is still an open problem. To overcome this difficulty, we make a suitable formulation of the control problem, which allows us to prove the existence of a solution, under some reasonable hypotheses, and to obtain the optimality conditions.

P. Cannarsa:

Hamilton-Jacobi equations in finite and infinite dimensions and applications to optimal control

The talk will be concerned with existence, uniqueness and regularity properties of solutions of infinite dimensional Hamilton-Jacobi equations. In particular, the linear-convex approach and the viscosity solution approach will be described.
The main focus will be on recent results for equations related to optimal control problems with state constraints and to optimal boundary control problems.

Z. Chen:

Optimal boundary controls for a phase field model

The phase field model describes the phase transitions between two different phases, e.g. solid and liquid. In this work we consider first a general boundary control problem which is governed by the phase field model. A necessary condition for optimality is given. Then two special cases are studied. Under some conditions we prove the uniqueness of the optimal solutions for a boundary control problem without constraints. For an optimal boundary control problem with constraints we show a result concerning the so-called "Bang-Bang-Principle".

G. Da Prato:

Some results on periodic control problems

We consider a dynamical system governed by the evolution equation in a Hilbert space

H,

$$y' = Ay + Bu + f(t).$$

Here A is the generator of a C_0–semigroup, u is the control, $B \in \mathcal{L}(V;H)$ is a bounded operator from the controls space V into H and f is a 2π–periodic function from $\mathbb{R}$ into H. We want to minimize the cost

$$J(y,u) = \frac{1}{2\pi}\int_0^{2\pi} \left(|y(s)|^2 + |u(s)|^2\right) ds$$

where $u \in L^2_\#(V)$ and

$$(y,u) \in W = \{(y,u) : y' = Ay + Bu + f, \quad y \ 2\pi\text{–periodic}\}.$$

We solve, under suitable assumptions, this problem, using Dynamic Programming arguments, and show the relation with Ergodic Control problems.

C. Fabre:

Approximate controllability of the semilinear heat equation

We consider a non linear heat equation (in a bounded and regular open set Ω in $\mathbb{R}^n$) when the nonlinearity is globally Lipschitz. We consider the case of boundary Dirichlet conditions and with controls acting in an open subset ω of Ω. For an initial data fixed in $L^p(\Omega)(p \geq 1)$ (respectively in $C_0(\Omega)$), we consider the reachable set at time $T > 0$ when the controls discribe $L^\infty(\omega \times (0,T))$, and we force that the reachable set is dense in $L^p(\Omega)(p \geq 1)$ (respectively in $C_0(\Omega)$). For this, we apply a fixed point argument and then study the linear heat equation perturbed by a potential for which we characterize a control with minimal $L^\infty(\omega \times (0,T))$–norm.

M. Falcone:

Recent results in the approximation of optimal control problems

We report on some new results in the approximation of viscosity solutions of Hamilton–Jacobi–Bellman equations related to deterministic and stochastic optimal control problems. The common framework of these results is Dynamic Programming. The numerical schemes available for HJB equations all have low order of convergence and this is a difficulty when applying the dynamic progamming approach to the solution of real control problems. A class of *high order methods* for deterministic control problems can be obtained coupling accurate one–step schemes for ordinary differential equations with quadrature formulae, the main result is that if the data of the control problem are sufficiently smooth (say C^p) and the optimal open–loop control is piecewise smooth the schemes are of order p. Other results deal with the approximation of *stochastic control problems* where the evolution is governed by a diffusion process which can eventually degenerate and of deterministic problems with *state constraints*. For all the above problems, we establish convergence and obtain approximate feedback optimal controls. We present also the results of some numerical experiments.

H. O. Fattorini:

Relaxed controls in fluid flow

We study control problems for fluid flow described by the Navier–Stokes equations. The equation can be fitted with ordinary controls or relaxed controls (probability measure–valued controls). The latter provide automatic existence theorems for control problems. The main subject of this talk was to show that trajectories corresponding to relaxed controls can be uniformly approximated by trajectories corresponding to ordinary controls. This shows that the relaxed system is closely related to the original system.

J. Haslinger:

Optimization of composite materials

Assume a material, made from 2 constituents having 2 different heat conductivities. We study the dependence of the homogenized coefficients on the shape of the inclusion. Our aim is to recover the shape of the inclusion in such a way that the resulting homogenized coefficients have à–priori given properties. The main attention is devoted to the numerical realization of the problem. We present a new approach, based on the primal and the dual variational formulation, enabling us to state the two sided estimates of the heat conductivities and the coefficients of the homogenized material.

M. Heinkenschloß:

A multilevel method for the numerical solution of a semilinear parabolic control problem

In this talk we investigate the numerical solution of an optimal control problem governed by the so–called phase field model, which arises in the description of phase change problems and which is given as a system of two semilinear parabolic equations. For the solution of the optimal control problem we reformulate the necessary optimality conditions as a compact fixed point problem and apply a multilevel Newton method due to Atkinson and Brakhage for its solution. The multilevel newton method works on a sequence of grids and uses cheap coarse grid information to obtain good approximations for the Newton step. This yields a fast linearly convergent method which considerably reduces the amount of work per iteration and also requires fewer data to handle. Other advantages of this approach are that the reformulation as a compact fixed point problem yields a decoupling of the phase field model and generates functions that only involve the solution of linear parabolic equations. The work per iteration of the resulting algorithm is linear in the number of variables, if fast methods for the solution of the linear PDEs are used.

D. Hömberg:

Phase transitions in eutectoid carbon steel: existence and optimal control

A mathematical model for the austenite–pearlite and austenite– martensite phase change in eutectoid carbon steel, based on Scheil's Additivity Rule and the Koistinen and Marburger formula is presented. Existence and uniqueness results are established. Necessary conditions for optimality are derived for a control problem which is related to the heat treatment of steel.
To test the model, the Jominy end quench test has been calculated numerically for the carbon steels C 1080 and C 100 W1. The results of the numerical calculations are in good agreement to measured physical data.

W. Horn:

The Penrose–Fife model for Ising ferromagnets

We consider the system of nonlinear PDEs

$$\begin{aligned} \varphi_t - \Delta\varphi + s_0'(\varphi) + \frac{\lambda(\varphi)}{\theta} &= 0, \\ \theta_t + \Delta\Big(\frac{1}{\theta}\Big) - \lambda(\varphi)\varphi_t &= g, \end{aligned}$$

$(x,t) \in \Omega \times [0,T], \Omega \subset \mathbb{R}^n, n \leq 3$ with

$$s_0(\varphi) = c_1(\varphi\log\varphi + (1-\varphi)\log(1-\varphi)) - \frac{c_2}{2}\varphi^2 + (c_3 - c_1)\varphi + c_4$$

and suitable initial and boundary conditions. A global existence and uniqueness theorem is stated and the proof is being sketched. We pay special attention to the parts of the proof which give uniform L^∞ estimates on the solutions and show that there exist constants a, b such that $0 < a \leq \varphi(x,t) \leq b < 1$. This system can be used to describe Ising ferromagnets. At the end we make some remarks on the connected Optimal Control Problem.
(joint work with J. Sprekels and S. Zheng)

N. Kenmochi:

An extended Penrose–Fife model for phase transitions

An extended Penrose–Fife model for phase transitions in the non–conserved case is discussed. The model is described as a coupled system of two nonlinear parabolic PDEs:

$$\begin{aligned} \frac{\partial}{\partial t}\big[\rho(u) + \lambda(w)\big] - \Delta u &= f(t,x), \quad \text{in } Q_T := (0,T)\times\Omega, \\ \frac{\partial w}{\partial t} - k\Delta w + \beta(w) + g(w) &\ni \lambda'(w)u \quad \text{in } Q_T \end{aligned}$$

with initial and boundary conditions, where $\rho(u) \approx -\frac{1}{u}$ (the Kelvin temperature) and w is the order parameter; λ, g are smooth functions from $\mathbb{R}$ into $\mathbb{R}$ and β is a maximal monotone graph in $\mathbb{R} \times \mathbb{R}$ with $\overline{D(\beta)} = [\sigma_*, \sigma^*]$ ($[\sigma_*, \sigma^*]$ is the domain of the order parameter). The existence–uniqueness result is proved as well as some results on the asymptotic stability of the solution.

R. Klötzler:

Optimal control of currents

Referring to L. C. Young usual problems of optimal control can be replaced by problems of optimal control of currents resp. flows in a domain Ω. These again are representable as a general transportation problem (P) subject to additive vector–valued set–functions of bounded variation. The dual problem (D) of (P) is a deposit problem.
It is shown that $\max(D) = \min(P)$ holds and that this strong duality allows a concept of approximate solutions of these problems on the basic of a simplicial partition of Ω and piece–wise constant flows.

B. Krause:

Regularity properties for state and adjoint state of nonlinear control problems

We study necessary optimality conditions for the solution of control problems with the nonlinear evolution eqution

$$\dot{y} + \nu Ay + B(y) = u, \quad \gamma_0 y = y_0$$

and the cost functional

$$J(y,u) = \int_0^T (\|y(t) - y_d(t)\|_k^p + \|u(t) - u_d(t)\|_l^q)dt \longrightarrow \inf$$

where A is a positive selfadjoint operator and the control u belongs to a set U_{ad} of admissible controls. Assuming that for the state equation there exists at least one admissible element (y, u) one can prove the existence of an optimal control $(\hat{y}, \hat{u})$. Since the cost functional and the estimate

$$\|B(y)\|_n \leq c(1 + \|y\|_m^r)$$

of the nonlinear operator B give additional a priori information about the solution $\hat{y}$ one can get conditions for the parameters p, q, r, k, l, m, n such that $\hat{y} \in W_{p,q}^{k,l} = \{y \in L_p(0,T; H^k(\Omega)) : \dot{y} \in L_q(0,T; H^l(\Omega))\}$. In the regular case ($p \geq 2, q \geq 2$) with the assumption

$$\|B'(y)z\|_n \leq c(1 + \|y\|_m^{r-1})\|z\|_m$$

it is shown that the adjoint state of the control problem is an element of $W_{q',p'}^{-l,-k}$.
The results are used to characterize the solution of a control problem for the periodic three–dimensional NAVIER–STOKES system.

K. Kunisch:

Estimation of state–dependent coefficients using convex analysis techniques

In the first part of this lecture I revisite linear parameter estimation problems formulated as bilinear optimal control problems. Under the general assumption that the residue at the minimum is small I argue

(i) uniqueness of regularized solutions,

(ii) convergence of a parallel implementation of the auxiliary optimization problem arising in the first order augmented Lagrangian method,

(iii) quadratic convergence of augmented Lagrangian–SQP methods.

In the second part we consider the estimation of state dependent coefficients in partial differential equations. As a specific example one may consider the estimation of $a(.)$ in

$$div(a(\nabla y)) = f$$

from measurements y_0 for $y(a^*)$ with a^* the 'true' coefficient. The admissible coefficients are restricted to lie in a subset of $K = \{a = \partial j \,|\, j : \mathbb{R}^n \to \mathbb{R} \text{ convex}\}$. Based on the conjugacy formula the estimation is cast as a penalty type optimization problem. Existence and wellposedness is asserted. Subsequently the problem is reformulated as an optimal control problem, amenable to computer implementation.
The second part is joint work with V. Barbu.

G. Leugering:

Control of dynamic elastic networks of strings and beams – theory and numerical simulation

Models of elastic networks of strings, various beams are going to be presented along with concepts of controlling the transient behaviour of such multilink structures. Numerical simulations are designed to underline the significance of these models.

T. Lohmann:

Numerical computation of parameter estimates in nonlinear implicit models

Nowadays the use of simulation and optimization techniques to analyse and control real life processes on computers increases rapidly in science as well as in industry. The basic model equations have to describe the investigated process qualitatively correctly. But in many cases model coefficients (parameter constants or parameter functions) are unknown. They have to be estimated by observations.
In our talk we present a new numerical algorithm which is able to compute parameter estimates in nonlinear implicit models like ordinary or partial differential equations. The O.D.E.–identification problem is discretized by a boundary value problem approach like multiple shooting. The P.D.E.–identification problem is formulated as a variational problem. The space–dependent part of the solution and the model coefficients are discretized by finite elements. For partial differential equations of parabolic type this approach also yields an O.D.E.–identification problem which depends on the unknown discretization parameters. In each case we get a finite dimensional constrained least squares problem. This problem is solved numerically by a Gauss–Newton method. Local and global convergence can be proved. Furthermore, a convenient statistical analysis of the computed parameter

estimates is given. On the basis of the proposed algorithm we developed a numerical procedure which has already been tested on some examples.

W. Merz:

Modelling, analytical and numerical treatment of the oxidation process of silicon

Der Oxidationsprozeß von Silizium ist ein wesentlicher Schritt bei der Herstellung von Mikrochips. Bei Temperaturen zwischen 700 und 1200 ^{O}C wird in oxidierender Atmosphäre Silizium in Siliziumdioxid umgewandelt. Die entstehenden Oxidbereiche dienen hauptsächlich als isolierende Schichten in integrierten Halbleiterschaltungen.
Es existiert eine Vielzahl von mathematischen Modellen zur Beschreibung der physikalischen und chemischen Eigenschaften dieses Prozesses. Ausgehend von der allgemeinen Idee, daß ein Gemisch verschiedener Substanzen vorliegt, die teilweise miteinander chemisch reagieren, werden eine Reihe von Modellvorschlägen präsentiert. Impuls- und Massenbilanzen sowie Diffusionsvorgänge führen zu einem umfassenden System von nichtlinearen partiellen Differentialgleichungen.
Es werden neben Existenz- und Eindeutigkeitsresultaten auch numerische Ergebnisse der Simulation des Oxidationsprozesses vorgestellt. Die numerische Auswertung erfolgte mit Hilfe des Prozeß-Simulators DIOS.

M. Niezgòdka:

State-constrained problems arising from non-isothermal diffusive phase transitions

Systems of strongly coupled evolution equations are considered that extend the Penrose-Fife models onto the case of state-constrained problems. This provides the possibility of accounting for pure phases and applying control action by means of some external fields. The constraints are admitted to be explicitly time-dependent. For the conserved order parameter w (binary case), with $\rho(u)$ corresponding to Kelvins temperature, the governing equations assume then the form

$$\begin{aligned} \rho(u)_t + \lambda(w)_t - \Delta U &= h, \\ w_t - \Delta\{-\nu\Delta w + \xi + g(w) - \lambda_0(w)u\} &= 0, \quad \nu > 0, \\ \xi &\in \beta^t(w) \end{aligned}$$

in $(0,T)\times\Omega$, $\Omega \subset \mathbb{R}^N (N \leq 3)$, where $\beta^t(.) \subset \mathbb{R}\times\mathbb{R}$ maximal monotone, $\rho(.), \lambda(.), \lambda_0(.)$, $g(.)$ locally Lipschitz, ρ monotone. Results on the existence, uniqueness and asymptotic behaviour (as $t \to \infty$) of solutions are given along with an analysis of the continuity of system trajectories with respect to perturbations of the thermodynamic potential (β^t, g). (joint work with N. Kenmochi, Chiba)

I. Pawlow:

Mathematical models of nonisothermal phase transitions II

We propose a class of Landau-Ginzburg type models for nonisothermal phase transitions which is consistent with the entropy principle. The model formulation applies in particular to processes of diffusive phase seperation, solidification problems and phase transitions in shape memory alloys. For the first two groups of problems the governing equations are

$$\begin{aligned} \partial_t u + \nabla.\vec{j}_0 &= 0, \quad \text{or} \quad \partial_t u = -\alpha v, \quad \alpha > 0, \\ \partial_t e + \nabla.(\vec{q}_0 - \vec{p}_e) &= 0 \end{aligned}$$

where u is the order parameter, e the internal energy, v the chemical potential, $\vec{j}_0$ the mass flux, $\vec{q}_0$ the energy flux, and $\vec{p}_e$ the nonequilibrium phase interface flux.
For the shape memory alloys the models reads as

$$\begin{aligned} \partial_t^2 u - \nabla.\left(\frac{\lambda}{w}\sigma\right) &= 0, \\ \partial_t(e + \frac{1}{2}(\partial_t u)^2) + \nabla.(\vec{q}_0 - \vec{p}_e - \frac{\lambda}{w}\sigma.\partial_t u) &= 0 \end{aligned}$$

where u is the displacement vector, σ the total stress tensor, and the function λ as in Part I.
For the class of problems under consideration we study the Lyapunov property and the existence of absorbing sets.

S. Pickenhain:

Stability and maximum principle in mathematical control theory

Two variants of a maximum principle for multidimensional control problems of Dieudonné–Rashevsky–type are shown. The first variant uses smoothness assumptions of the objective functional in the state variables and convexity assumptions in the control as well as the linearity of the state constraints. It is applicable to relaxed control problems. The second variant assumes stability result with respect to a suitable dual problem.

J.-P. Puel:

Boundary stabilization for the full system of dynamical Von Kármán equations

We consider a thin elastic plate, the vibrations of which are modelized by the full system of dynamical Von Kármán equations. This model takes into account the in–plate acceleration and a term of rotational inertia, and it is written in terms of physical variables without any introduction of an Airy function. On a part of the boundary we introduce suitable feedbacks so that the energy of the system is strictly decreasing. We prove first the existence of (at last) one solution of finite energy for the complete system (including feedbacks). Uniqueness for these weak solutions is not known. We also prove that for more regular initial data, there exists a unique strong solution and this result relies on a careful study of the underlying linear operator. Then, using the method of perturbed energy, we prove a stabilization result for strong solutions, namely that their energy is exponentially decreasing in time, the rate of decay being uniform for bounded initial data in the energy norm.

T. Roubiček:

Optimal control of a microstructure

An optimal control problem for a microstructure (describing a multiphase system) governed by a nonconvex steady-state variational problem is studied. The controlled variational problem requires a fine relaxation not to loose information about the microstructure we are just interested in, therefore a technique of (generalized) Young measures must be used. The question about uniqueness of the solution to the relaxed problem, which is especially important in context of control, is discussed. An approximation theory is developed and some numerical experience will be mentioned, as well.

E.W. Sachs:

Reduced SQP–methods for nonlinear heat conduction control problems

A problem is considered which arises in the control of heat conduction processes governed by nonlinear diffusion equations. We present the discretized form of such a problem and apply a reduced SQP–method for the numerical solution of the optimization problem. This method makes use of the sparsity and offers the advantage to approximate second order information by a quasi–Newton update which is practicable with regard to storage. The convergence result is a local 1–step q–superlinear convergence rate which is an improvement over results achieved previously. The algorithm is tested on the parabolic control problem and the results document a sizeable reduction in the number of iterations.

J. Sokolowski:

Shape sensitivity analysis of thin shells

Classical results from control theory and from material derivative method are used in order to optimize the shape (middle surface and thickness) of an elastic general thin shell. The attention is concentrated on the general continuous formulation of the problem. Thus very general results are obtained which can be used in the forthcoming works to analyze the approximation by finite element method and to realize some industrial applications. The general formulation of the linear thin shell equtions of KOITER are used. The derivatives of the different functionals concerned by the optimization problems were computed. In addition, the form of the second derivative of cost is obtained.

S.S. Sritharan:

Optimal control of viscous flow

We will report on the recent progress on feedback control of Navier–Stokes equations. Main results are

(1) existence of ordinary and Young Measure Valued optimal control,

(2) necessary conditions (Pontryagin Maximum Principle) for the case with target constraints,

(3) feedback analysis using infinite dimensional Hamilton–Jacobi equation (Viscosity Solution technique).

Current research directions include stochastic dynamic programming of Navier–Stokes equations using infinite dimensional second order Bellman equation.

D. Tiba:

State constrained control problems; optimal design problems

In this paper (written in cooperation with M. Bergonnieux – Univ. of Orléans, France and T. Männikkö – Univ. of Jyväskylä, Finland) we study a new form of necessary and sufficient optimality conditions for state constrained parabolic control problems. We weaken the standard Slater hypothesis and we study also the case of empty interior constraints. The applications concern a bang–bang type result and an augmented Lagrangian algorithm. Numerical results are also commented.

F. Tröltzsch:

Convergence of the Lagrange–Newton method for parabolic boundary control problems

The Lagrange–Newton method (or SQP method) solves nonlinear programming problems through a sequence of quadratic problems.
Under the assumption of second order sufficient optimality conditions the quadratic convergence of the method in finite dimensional spaces is known some years.
The proof of convergence was extended recently to infinite dimensional Hilbert spaces and, under additional requirements, also to Banach spaces.
These techniques were applied to optimal control problems governed by nonlinear systems of ordinary differential equations. In contrast to this, the behaviour of the method remained open for nonlinear parabolic boundary control problems.
In the lecture, a first proof of convergence together with an estimate of the corresponding order is presented for this class of problems.

L. v. Wolfersdorf:

Optimal control problems for memory kernels in heat conduction theory

A linear integrodifferential equation describing the hest flow in a material with memory is considered. This equation contains a pair of time–dependent convolution kernels that are unknown. Such kernels are determined as solutions of an optimal control problem by using additional data obtained from measurements of average temperature around some fixed points of the domain over some finite time interval. We show the existence of an optimal solution of this problem and derive optimality conditions for it.

M. Yamamoto:

Unique determination of coefficients in one–dimensional wave equations from boundary data

We consider an initial/boundary–value problem for a one–dimensional wave equation describing a vibration of a string in a viscous fluid, given by

$$\rho(x)\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\Big(\sigma(x)\frac{\partial u}{\partial x}\Big) - k(x)\frac{\partial u}{\partial t}; \quad 0 < x < 1, -T < t < T,$$

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 \quad (\text{ or } u(0,t) = u(1,t) = 0); \quad -T < t < T,$$

$$u(x,0) = a(x), \quad \frac{\partial u}{\partial t}(x,0) = b(x); \quad 0 < x < 1.$$

Here ρ and σ are the mass–density and the tension, respectively, which are functions of the space variable x. The term $-k(x)\frac{\partial u}{\partial t}$ corresponds to viscous drag.
Our problem is: Can we uniquely determine ρ, σ, k, a, b from observation of u (or $\frac{\partial u}{\partial x}$) at boundary points $(0,t)$ and $(1,t)$?
Our result is: Under some assumptions, the boundary data determine:

$\sigma, k, \frac{\partial a}{\partial x}, b$, if ρ is known,

$\rho, k, \frac{\partial a}{\partial x}, b$, if σ is known,

$\rho, \sigma, \frac{\partial a}{\partial x}, b$, in the interval where $k \neq 0$, if k is known.

The key technique is the Gel'fand–Levitan approach.

Berichterstatter: D. Hömberg

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