

### MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 7/1993

## Partielle Differentialgleichungen

7.2. bis 13.2.1993

Die Tagung fand unter der Leitung von Herrn G. Huisken (Tübingen), Herrn L. Simon (Stanford) und Herrn M. Struwe (Zürich) statt. Die Teilnehmer kamen aus der Bundesrepublik Deutschland, den USA, Russland, Australien, Japan, Indien, Frankreich und anderen Ländern. Sie vertraten einen breiten Themenkreis aus dem Gebiet der partiellen Differentialgleichungen.

Die Ergebnisse wurden in interessanter und verständlicher Weise vorgetragen. Sicherlich gaben auch die fruchtbaren Diskussionen vielerlei Anregungen.



### Vortragsauszüge:

### Hardy Spaces and P.D.E.

by Stefan Müller (Bonn)

We consider surfaces M immersed into  $\mathbb{R}^n$  and we prove that the quantity  $\int_M |A|^2$  (where A is the second fundamental form) controls in many ways the behaviour of conformal parametrizations of M. In the case when M is complete, connected and  $\int_M |A|^2 < \infty$  we obtain a more or less complete picture of the behaviour of the immersions. In particular we prove that under these assumptions the immersions are proper. We also prove that conformal parametrizations of graphs of  $W^{2,2}$  functions on  $\mathbb{R}^2$  exist, are bi-Lipschitzian and the conformal metric is continuous. The work was inspired by recent results of T. Toro.

### Asymptotic Developments by Γ-Convergence

by Gabriele Anzellotti (Trento)

I consider the following situation: for each  $\varepsilon > 0$  let  $\mathcal{J}_{\varepsilon} \colon X_{\varepsilon} \to \overline{\mathbb{R}}$  be a functional with infima  $m_{\varepsilon} = \inf_{X_{\varepsilon}} \mathcal{J}_{\varepsilon}$  and corresponding minimizers  $u_{\varepsilon}$  of  $\mathcal{J}_{\varepsilon}$ . I am interested in the problem of describing  $m_{\varepsilon}$  and  $u_{\varepsilon}$  when  $\varepsilon \to 0$ . I propose the following definition:

DEFINITION: Let  $(X,\tau)$  be a topological space with metrizable topology. Let  $\mathcal{J}^{(0)},\mathcal{J}^{(1)},\mathcal{J}^{(2)},\mathcal{J}_{\varepsilon}\colon X\to \overline{\mathbb{R}},\ \varepsilon>0$ . We say that the  $\Gamma$ -development

(\*) 
$$\mathcal{J}_{\varepsilon} =_{\Gamma} \mathcal{J}^{(0)} + \varepsilon \mathcal{J}^{(1)} + \varepsilon^{2} \mathcal{J}^{(2)} + o(\varepsilon^{2})$$

holds, if

(i) 
$$\Gamma(\tau^{-}) - \lim_{\varepsilon \to 0} \mathcal{J}_{\varepsilon} = \mathcal{J}^{(0)}$$

(ii) 
$$\Gamma(\tau^{-}) - \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} (\mathcal{J}_{\varepsilon} - m_{0}) = \mathcal{J}^{(1)}$$



 $\odot \bigcirc$ 

(iii) 
$$\Gamma(\tau^{-}) - \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \left( \frac{1}{\varepsilon} (\mathcal{J}_{\varepsilon} - m_{0}) - m_{1} \right) = \mathcal{J}^{(2)}$$

where  $m_i = \min_X \mathcal{J}^{(i)}, i = 0, 1, 2.$ 

(Similar definitions for developments of any order. An extension is possible for  $X_{\varepsilon}$  varying with  $\varepsilon$ .) One has the following

THEOREM: Assume that (\*) holds and that  $\{u_{\varepsilon}\}_{{\varepsilon}>0}$  is a compact family of minimizers. Then one has:

(1) 
$$m_{\varepsilon} = m^{(0)} + \varepsilon m^{(1)} + \varepsilon^2 m^{(2)} + o(\varepsilon^2)$$

(2) {limit points of minimizers for  $\varepsilon \to 0$ }  $\subset U^{(2)} \subset U^{(1)} \subset U^{(0)}$ 

where  $m^{(i)} = \min_{X} \mathcal{J}^{(i)}, \ U^{(i)} = \{ u \in X \, | \, \mathcal{J}^{(i)}(u) = m^{(i)} \} = \{ \min \text{minimizers of } \mathcal{J}^{(i)} \}.$ 

These results and other applications to phase transitions are joint work with Sisto Baldo (Trento) [to appear Appl. Math. Opt.]. Other applications to fine elastic structures are joint work with S. Baldo and D. Percivale [to appear in Asympt. Anal.].

## Global Existence and Convergence of the Yamabe Flow

by Rugang Ye (Santa Barbara)

The Yamabe flow is a canonical parabolic deformation of a Riemannian metric into constant scalar curvature in its conformal class. We prove long time existence and convergence of the Yamabe flow. The key ingredient is a Harnack inequality. Its proof relies on some global arguments. No local version seems to hold.

## Computation of Surfaces of Prescribed Mean Curvature

by Gerd Dziuk (Freiburg i. Br.)

A new method to compute parametric minimal surfaces is based on the following formulation of Plateau's problem.

Let  $\Gamma$  be a Jordan curve in three dimensional space. We want to compute a minimal surface  $\bar{u} \colon B \to \mathbb{R}^3$  parametrized over the unit



disc B in  $\mathbb{R}^2$  and spanned in  $\Gamma$ . If  $\gamma \colon \partial B \to \Gamma$  is a fixed parametrization of the boundary curve we compute zeros of the derivative of the functional

$$E(u) = \frac{1}{2} \int_{B} |\nabla \bar{u}|^2$$

where  $u: \partial B \to \partial B$  is any re-parametrization and  $\bar{u}$  is the harmonic extension of  $\gamma(u)$ :

$$\Delta \bar{u} = 0$$
 in  $B$   
 $\bar{u} = \gamma(u)$  on  $\partial B$ 

This is done numerically using piecewise linear Finite Elements and Newton's method. If there are no Jacobi fields of a continuous solution  $\bar{u}$ , a unique discrete solution  $\bar{u}_h$  nearby exists and the error estimate

$$\|\bar{u}-\bar{u}_h\|_{H^1(B)}\leq c\,h$$

holds. The result is joint work with J. Hutchinson.

## Multiple Surfaces of Prescribed Mean Curvature near a Constant

by Norbert Jakobowsky (Aachen)

Let  $\Gamma$  be a rectifiable closed Jordan curve in  $B_1(0) \subset \mathbb{R}^3$  and  $H \in C^{0,1}(\mathbb{R}^3,\mathbb{R})$ . We prove the existence of at least two solutions to Plateau's problem for surfaces of prescribed mean curvature H parametrized over  $\overline{B_1(0)} \subset \mathbb{R}^2$  provided  $H = H_0 + \tilde{H}$ ,  $H_0 \in (-1,1) \setminus \{0\}$  and  $\|\tilde{H}\|_{L^{\infty}}$  is sufficiently small. This extends the corresponding results of Brezis-Coron, Struwe, Steffen and Wente concerning Rellich's conjecture.

Our proof is given in two parts: First, following Struwe, we apply minimax methods in  $W^{1,2}$  for smooth  $\Gamma$  (or polygons) and deduce a Palais-Smale condition using methods due to Struwe, Brezis-Coron and myself. Having established the existence of two different solutions in this case we then derive the statement by approximation. As an essential tool in both steps we involve a precompactness result in  $W^{1,2}(\bar{B}\setminus\{q_1,\ldots,q_n\})$ ; moreover, we use  $W^{1,2}$  and  $L^\infty$  a priori bounds for solutions due to Struwe and Grüter respectively.



#### Singular Perturbation of Dirichlet Eigenvalues

by Martin Flucher (Bonn)

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Given the solutions of the Dirichlet eigenvalue problem

$$-\Delta \varphi = \lambda \varphi \quad \text{in } \Omega$$
$$\varphi = 0 \quad \text{on } \partial \Omega$$
$$\int_{\Omega} |\varphi|^2 = 1$$

we provide asymptotic formulas for the solutions  $\lambda^r$ ,  $\varphi^r$  on the perturbed domain  $\Omega^r = \Omega \setminus \overline{B_r(x)}$ . The first order expansion with respect to the capacity  $\operatorname{cap}_{\Omega} B_r(x) = \int_{\Omega^r} |\nabla u^r|^2 \quad (u^r = 1 \text{ on } \partial B_r(x), u^r = 0 \text{ on } \partial \Omega)$  is given by

$$\lambda^{r} = \lambda + \varphi^{2}(x) \operatorname{cap}_{\Omega} B_{r}(x) + o(\operatorname{cap}_{\Omega} B_{r}(x))$$
  
$$\varphi^{r} = \varphi - \varphi(x)u^{r} + O(\operatorname{cap}_{\Omega} B_{r}(x))$$

uniformly on  $\Omega^r$ .

Using the minimax characterization of eigenvalues this is an example of a  $\Gamma$ -asymptotic development (see G. Anzellotti's talk). Together with an asymptotic formula for the capacity of small balls [1] we obtain the following approximation for the perturbed eigenvalues:

$$\lambda^{r} - \lambda \approx \begin{cases} -\frac{2\pi\varphi^{2}(x)}{\log\frac{r}{r_{\Omega}(x)}} & \text{for } n = 2\\ 4\pi r \left(1 + \frac{r}{r_{\Omega}(x)}\right) \varphi^{2}(x) & \text{for } n = 3 \end{cases}$$

where  $r_{\Omega}$  denotes the harmonic radius ([2] and [1]). Numerical experiments confirm the practical usefulness of this formula. One striking feature of the harmonic radius is that its slope at the boundary is always 2. This property is equivalent with C. Bandle's result on the boundary behaviour of maximal solutions of  $\Delta u = e^u$  and  $\Delta u = u^{\frac{n+2}{n-2}}$ .

[1] M. Flucher: An asymptotic formula for the minimal capacity among sets of equal area (Calc. Var. 1 (1993), 71–86)



[2] M. Flucher: Harmonic radius, conformal radius, and Robin constant (in preparation)

## Boundary Regularity of Area Minimizing Currents with Prescribed Volume

by Frank Duzaar (Bonn)

We consider rectifiable n-currents T in  $\mathbb{R}^n$  which minimize area subject to a volume constraint whose boundary  $\partial T$  is represented by an oriented smooth submanifolds  $\Gamma$  of dimensions n-1 in  $\mathbb{R}^{n+1}$  without boundary. We prove the following local boundary regularity result:

THEOREM: Suppose T is an integer multiplicity rectifiable n-current in  $\mathbb{R}^n$  which is area minimizing with respect to volume preserving variations in an open neighborhood of a point  $a \in \operatorname{spt} \partial T$  and  $B = \partial T$  is represented in this neighborhood by an n-1 dimensional submanifold  $\Gamma$  of class  $C^{1,\alpha}$ ,  $(0 < \alpha \le 1)$  with multiplicity one. Then either:

(1) T has density  $\frac{1}{2}$  at a and there exists r > 0 such that spt  $T \cap B_r(a)$  is a submanifold of class  $C^{1,\beta}$  in  $\mathbb{R}^{n+1}$  for any  $0 < \beta < \frac{1}{2}\alpha$  with boundary  $\Gamma \cap B_r(a)$  and  $T \sqcup B_r(a)$  is represented by multiplicity one integration over this submanifold,

or

(2) T has density  $m - \frac{1}{2}$  at a for some  $m \geq 2, m \in \mathbb{N}$  and there exists r > 0 such that  $\operatorname{spt} T \cap B_r(a)$  is a submanifold of class  $C^{1,\beta}$  in  $\mathbb{R}^{n+1}$  for any  $0 < \beta < \frac{1}{2}\alpha$  with empty boundary  $\operatorname{spt} T \cap B_r(a) \setminus \operatorname{spt} \partial T$  has exactly two components. Moreover,  $T \sqcup B_r(a)$  is represented by this submanifold taken with orientation T with multiplicity m on one of these components and with multiplicity m-1 on the other component.

In addition spt  $T \cap B_r(a)$  has constant mean curvature with respect to the orientation induced by  $\vec{T}$  at any point  $x \in \operatorname{spt} T \cap B_r(a)$ 

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### Uniqueness of Global Positive Solutions Branches of Semilinear Elliptic Problems with Symmetry

by Hansjörg Kielhöfer (Augsburg)

We consider the problem

where  $\Omega$  is some symmetric domain in  $\mathbb{R}^2$  and  $f\colon \mathbb{R}_+\to \mathbb{R}$  fulfills  $f(0)\geq 0$ . We prove: to any c>0 where  $f(c)\neq 0$  there is a unique solution  $(\lambda,u)$  of (1) with sign  $\lambda= \mathrm{sign}\, f(c)$  and  $\|u\|_\infty=c$ . This solution is on a global smooth curve  $\{(\lambda,u)\}$  of solutions of (1) which can be parametrized by  $\|u\|_\infty\in(a,b)$  with  $0\leq a< b\leq \infty$ , characterized by  $f(c)\neq 0$  for  $c\in(a,b)$ . The condition  $f(0)\geq 0$  is sharp. The behaviour at  $\|u\|_\infty=0$  is discussed as well.

#### A Nonlinear Evolution Problem in Hermitian Geometry

by Jürgen Jost (Bochum)

The following evolution problem for maps from a Hermitian manifold X with metric  $(\gamma_{\alpha,\bar{\beta}})_{\alpha,\beta=1,\dots,\dim_{\mathbb{C}}X}$  into a Riemannian manifold N with Christoffel symbols  $\Gamma_{ik}^{i}$  is studied:

$$(P) \qquad \gamma^{\alpha,\bar{\beta}}(z) \frac{\partial^2 f^i}{\partial z^\alpha \, \partial z^{\bar{\beta}}} + \gamma^{\alpha,\bar{\beta}}(z) \Gamma^i_{jk}(f(z)) \frac{\partial f^j}{\partial z^\alpha} \frac{\partial f^k}{\partial z^{\bar{\beta}}} = 0 \,,$$

where  $i=1,\ldots,\dim_{\mathbb{R}}N$ . (P) differs from the standard harmonic map evolution problem in the linear second order term. In particular, it does not have a variational or divergence structure. If N has non-positive sectional curvature, solutions of (P) exist for all time with time independent estimates. They converge to a solution of the corresponding elliptic problem unless  $\frac{\partial f}{\partial t}$  converges to a nontrivial





parallel section of the pull back of TN. In particular the elliptic problem is solvable if there are no such parallel sections. An example shows that the nonexistence of such sections is indeed necessary for the solvability of Dirichlet problems.

The results will appear in:

J. Jost - S. T. Yau: A nonlinear elliptic system for maps from Hermitian to Riemannian manifolds and rigidity problems in Hermitian geometry. Acta Math.

#### Generalized Evolution of Phase Boundaries

by Yoshikazu Giga (Sapporo)

We continue to study generalized evolutions (obtained by level set methods) of a hypersurface moved by its mean curvature. If the initial hypersurface encloses a bounded open set, it becomes extinct in a finite time. We are interested in estimating the extinction time from below by geometric quantities of the initial data. Our main result says that the extinction time is dominated by two times square of the volume of the set enclosed by the hypersurface over its area. The constant two is optimal. This estimate is easy to obtain at least formally. We adapt the formal proof to level set equations. This work is a joint work with my student K. Yama-uchi.

## Viscosity Solutions for a Parabolic Mean Curvature Equation

by Bernd Kawohl (Erlangen)

In the lecture I report on joint work with N. Kutev. We study the problem  $u_t - \operatorname{div}(Du/\sqrt{1+|Du|^2}) = 1$  in  $\Omega \times \mathbb{R}^+$ , u = 0 on the parabolic boundary, for domains  $\Omega$  for which no stationary solution exists. Otherwise u converges to those non-parametric solutions. Numerical evidence suggests (and we prove) that  $u_t$  is bounded and that the asymptotic profile of  $u_t$  attains its maximum in a set  $\Omega_*$  which solves the geometric variational problem  $\min_{G \subset \Omega} \frac{|\partial G|}{|G|} =: c$ . We perturb the differential equation by  $a - \varepsilon \Delta u$  term and show that





the solutions  $u_{\varepsilon}$  converge in a suitable sense to a weak viscosity solution in the sense of Crandall. Ishii and Lions. We derive comparison results for u which allow for a precise description of its growth on special domains  $\Omega$ , such as balls or cubes. In fact, as  $t \to \infty$ ,  $u(t,x) \approx (1-c)t$  on  $\Omega_*$ , so that in general u violates the boundary condition and goes to infinity even on parts of the boundary.

### Global Stability of Large Solutions to the Three-Dimensional Navier-Stokes Equations

by Reinhard Racke (Bonn)

We discuss the  $H^1$ -stability of mildly decaying global strong solutions to the Navier-Stokes equations in three space dimensions. Combined with previous results on the global existence of large solutions with various symmetries, this gives the first global existence theorem for large solutions with non-symmetric initial data in general domains. The stability of unforced two-dimensional flows under three-dimensional perturbations is also obtained. (Joint work with G. Ponce, T. C. Sideris, E. S. Titi)

## Local existence for solutions of fully nonlinear wave equations

by Peter Lesky jun. (Stuttgart)

Let  $\Omega \subset \mathbb{R}^n$  and  $m \in \mathbb{N}$ . We study the initial boundary value problem for

$$\begin{split} \partial_t^2 u(t,x) + A(t,x) u(t,x) + B(t,x) \partial_t u(t,x) = \\ f(t,x) + g(t,x, \partial_t^2 u, \bar{D}_x^m \partial_t u, \bar{D}_x^{2m} u) \end{split}$$

with homogeneous Dirichlet boundary condition. Here A(t,x) denotes a uniformly strongly elliptic differential operator of order 2m, B(t,x) denotes a differential operator of order m, and

$$\tilde{D}_{x}^{m} = \left\{ \partial_{x}^{\alpha} \colon |\alpha| \leqslant m \right\}.$$

We prove the existence of a T > 0 and the existence of a unique classical solution u up to T.

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## Flow Near the Equilibrium of Quasilinear Hyperbolic Equations

by Herbert Koch (Heidelberg)

The linearization of a differential equation dominates the behavior of solutions near an equilibrium—at least locally in time. A Hopf bifurcation is an example of a global in time property, which can be deduced from the properties of the linearization.

Any quasi-linear hyperbolic equation

$$U_{tt} - \partial_{\alpha} F_{\lambda}^{\alpha}(x, U, \nabla U) = w_{\lambda}(x, U, \nabla u) \quad \text{in } \Omega$$
$$U = 0 \quad \text{on } \partial\Omega$$

where  $U = \binom{u}{v}$ , with linearization

$$u_{tt} - \Delta u + 2(1+\lambda)u_t + ((1+\lambda)^2 + \varepsilon)u - 2v = 0$$
  
$$v_{tt} - \Delta v + 2(1+\lambda)v_t + ((1+\lambda)^2 + \varepsilon)v - 2u = 0$$

satisfies the assumptions on the spectrum of the generator for a Hopf bifurcation theorem. The flow, however, is not differentiable with respect to initial data.

Nevertheless it can be proved that an implicit function theorem holds, without assuming uniform continuity of the flow. This is the essential step in proving Hopf bifurcation.

#### Periodic Wigner-Poisson System

by Horst Lange (Köln)

The periodic Wigner-Poisson system is the problem to find 1-periodic solutions  $w_m(x,t)$ ,  $(x \in Q = [0,1]^3, t \in \mathbb{R})$ , of

$$(WP_{\pi}) \qquad \left\{ \begin{array}{l} \frac{\partial w_m}{\partial t} + v_m \cdot \nabla_x w_m + \Theta_h(V) w_m = 0 \\ \Delta V = 1 - n(x,t), \quad n(x,t) = \sum_{m \in \mathbb{Z}^3} w_m(x,t) \\ w_m(x,0) = w_m^0(x), \end{array} \right.$$



plus 3-dimensional 1-periodic boundary conditions on Q; here  $\Theta_h(V)$  is the pseudodifferential operator given by

$$\Theta_h(V)w_m(x,t) = \frac{-i}{\hbar} \sum_{m' \in \mathbb{Z}^3} \int_Q \left[ V(x + \frac{\hbar}{2}\eta, t) - V(x - \frac{\hbar}{2}\eta, t) \right] \times e^{2\pi i (m - m')} w_{m'}(x, t) d\eta$$

and  $v_m = 2\pi m$ ,  $m \in \mathbb{Z}^3$ .

By using the equivalence of  $(WP_{\pi})$  to an infinite order coupled system of nonlinear Schrödinger equations with potential V we can show, that  $(WP_{\pi})$  has a strong global 1-periodic solution sequence  $w = (w_m)$  in  $C^1([0,T],W)$ ,  $W = l^1(L^2(Q))$  (any T > 0), assuming initial data to be in W (no smallness condition).

## On the Solutions of Quasilinear Elliptic Equations with Boundary Blow Up

by Catherine Bandle (Basel)

Quasilinear equations of the type  $\Delta u = e^u$  or  $u^p$ , p > 1 are studied. They admit solutions tending to infinity at the boundary. The exact asymptotic behaviour is established near the singularities of such solutions.

It turns out that the solutions and their gradients are asymptotically independent of the geometry of the boundary. The method developed uses a coordinate system which flattens the boundary, and is based on a scaling argument.

### Existence and Multiplicity of Positive Solutions for Nonlinear Elliptic Problems in Exterior Domains

by Giovanna Cerami (Palermo)

The problem considered is the following one: find solutions of

$$-\Delta u + \lambda u = u^{p-1} \qquad \text{in } \Omega$$
 (P) 
$$u > 0 \qquad \text{in } \Omega$$
 
$$u \in H_0^1(\Omega)$$





where  $\Omega \subset \mathbb{R}^N$ ,  $N \geqslant 3$ , is an unbounded domain having smooth boundary  $\partial \Omega \neq \emptyset$ ,  $\mathbb{R}^N \setminus \Omega$  is bounded,  $\lambda \in (0, +\infty)$ ,  $p \in (2, \frac{2N}{N-2})$ .

More precisely the object of the interest is the study of the effect of the topology of the domain on the number of solutions of (P).

Problem (P) admits a variational formulation. In fact the solutions of (P) can be related to the critical points of the functional

$$E_{\lambda}(u) = \int_{\Omega} (|\nabla u|^2 + \lambda u^2) dx, \qquad u \in H_0^1(\Omega)$$

constrained to lie on the manifold

$$W_p = \left\{ u \in H_0^1(\Omega) \colon \int_{\Omega} |u|^p \, dx = 1 \right\}$$

and to belong to the positive cone of  $H_0^1(\Omega)$ . However, the lack of compactness of the embedding

$$j \colon H_0^1(\Omega) \hookrightarrow L^p(\Omega)$$

gives rise to serious difficulties when one tries to use variational methods in a standard way.

After a careful analysis of the compactness question, it is possible to prove the following

THEOREM: For any  $p \in (2, \frac{2N}{N-2})$  there exists  $\bar{\lambda}(p)$  such that  $\forall \lambda \geqslant \bar{\lambda}(p)$  problem (P) has at least  $(\cot_{\bar{\Omega}}[\bar{\Omega}, \mathbb{R}^N \setminus B_{\bar{\varrho}}(0)]) + 1$  distinct solutions. Moreover,  $\forall \lambda > 0$  there exists at least one solution of (P). Note that in the above statement

$$\bar{\varrho} = \inf \left\{ \varrho \in \mathbb{R} \colon \mathbb{R}^N \setminus \Omega \subset B_{\varrho}(0) \right\}$$

and

$${\rm cat}_{\bar{\varOmega}}[\bar{\varOmega},{\rm I\!R}^N\setminus B_{\bar{\varrho}}(0)]$$

denotes the relative category (in the sense of Fadell) in  $\bar{\Omega}$  of  $\bar{\Omega}$  with respect to  $\mathbb{R}^N \setminus B_{\bar{\varrho}}(0)$ .



Weak Solutions of  $-\Delta u = u^{\alpha}$  in  $\mathbb{R}^n$ , Partial Regularity Results and Existence of Solutions with Prescribed Singular Set

by Frank Pacard (Noisy-le-Grand)

First we give some a-priori regularity for positive weak solutions of  $-\Delta u = u^{\alpha}$  in bounded domains of  $\mathbb{R}^n$ . Then we introduce the notion of stationary weak solutions of  $-\Delta u = u^{\alpha}$  and prove that the Hausdorff dimension of their singular set is less than or equal to  $n - \frac{2(\alpha+1)}{(\alpha-1)}$ , if  $\frac{n+2}{n-2} \leqslant \alpha \leqslant \frac{n+1}{n-3}$ .

For exponents  $\alpha = \frac{n}{n-2}$  we build solutions with prescribed singular set and explain how to extend the results to exponents  $\alpha > \frac{n}{n-2}$ .

#### Semilinear Elliptic Equations with Neumann Boundary Conditions

by Adimurthi (Bangalore)

Let  $\Omega \subset {\rm I\!R}^n$   $(n \ge 3)$  be a bounded domain with smooth boundary. Let  $\lambda > 0$  and consider

$$-\Delta u + \lambda u = u^{\frac{n+2}{n-2}} \quad \text{in } \Omega$$

$$u > 0 \quad \text{in } \Omega$$

$$\frac{\partial u}{\partial \nu} = 0 \quad \text{on } \partial \Omega$$

The problem is to find conditions on  $\lambda$  and  $\Omega$  such that  $(\mathbb{P}_{\lambda})$  admits a non constant solution and to study the asymptotic behaviour of minimal energy solutions of  $(\mathbb{P}_{\lambda})$ . We have:

THEOREM:  $\exists \lambda^* = \lambda^*(\Omega) > 0$  such that for all  $\lambda > \lambda^*(\Omega)$ ,  $(\mathbb{P}_{\lambda})$  admits a non constant minimal energy solution.

Theorem: Let  $u_{\lambda}$  be a minimal energy solution. Then  $\exists \lambda_0 > \lambda^*$  such that  $\forall \lambda > \lambda_0$ 

- (a) There exists a unique  $P_{\lambda} \in \partial \Omega$  such that  $\max_{\bar{\Omega}} u_{\lambda} = u_{\lambda}(P_{\lambda})$ .
- (b) As  $\lambda \to \infty$ , cluster points of  $\{P_{\lambda}\}$  are points of maximal mean curvature.





#### Closed Hypersurfaces of Prescribed Curvature

by Claus Gerhardt (Heidelberg)

## Asymptotically Periodic Solutions of Conservative Wave Equations

by Fred Weissler (Villetaneuse)

A famous result of P. Rabinowitz says that the nonlinear wave equation  $w_{tt} - w_{xx} + w^3 = 0$ , where  $w(t,x) \in \mathbb{R}$ ,  $t \in \mathbb{R}$ ,  $x \in [0,\pi]$ ,  $w(t,0) = w(t,\pi) = 0$ , has nontrivial time-periodic solutions. We would like to understand the behavior of the non-periodic solutions. In particular, are they almost periodic, or recurrent; or can they decay weakly?

As a simpler model equation, we have studied solutions to  $w_{tt} - w_{xx} + ||w(t)||^2 w = 0$ , with the same boundary conditions, and where  $||\cdot||$  is the (spatial) norm in  $L^2(0,\pi)$ . For this equation we can show that no nontrivial solution can decay to 0 weakly in  $L^2(0,\pi)$ , and that trajectories  $(w,w_t)$  are always precompact in  $H_0^1 \times L^2$ . Moreover, there exist nonrecurrent solutions of the form  $w(t,x) = u(t) \sin x + v(t) \sin 2x$  where  $v(t) \not\equiv 0$  but  $v(t) \to 0$  as  $t \to \pm \infty$ .

For more general powers ( $||w||^{\alpha}w$ ), we can show the existence of solutions of the same form with  $v(t) \not\equiv 0$  but  $v(t) \to 0$  as  $t \to +\infty$ . These solutions are asymptotically periodic and hence nonrecurrent.

This is joint work with A. Haraux and T. Cazenave.

### Local Estimates and Boundary Behavior of Solutions of Elliptic Monge-Ampère Equations

by Erhard Heinz (Göttingen)

We consider solutions  $z(x,y) \in C^2(H) \cap C^0(\bar{H})$  of elliptic Monge-Ampère equations  $Ar + 2Bs + Ct + (rt - s^2) - E = 0$ ,  $D = AC - B^2 + E \ge \mu > 0$ , where  $H = \Omega \cap O$ , and  $\Omega$ , O are bounded open sets in  $\mathbb{R}^2$ . The boundary condition  $z|_{\partial H} = h$  is imposed on  $\partial \Omega \cap O$ . Furthermore, A, B, C satisfy certain structural conditions, and a





convexity condition is imposed on  $\partial\Omega\cap O$ . Set  $H(d)=\Omega\cap O_d$  and assume that  $\sup_H(|z|+|\nabla z|)\leq \gamma<\infty$ . Then it is shown that z belongs to  $C^{2,\nu}(\overline{H(d)})$  for d>0,  $\nu\in(0,1)$  and satisfies an estimate of the form  $||z||_{C^{2,\nu}(\overline{H(d)})}\leq\Theta<\infty$ , where  $\Theta$  only depends on  $\gamma,\mu,\nu,d$  and the data. A detailed presentation will appear in "Journal für die reine und angewandte Mathematik".

### Homogenization of Flow through Porous Media

by Willi Jäger (Heidelberg)

The results reported here are contained in a joint paper with A. Mikelić (to appear).

Consider a domain  $\Omega^{\varepsilon} = \Omega_1 \cup \Sigma \cup \Omega_2^{\varepsilon}$ , where  $\Omega_1$  is the "free" part,  $\Omega_2^{\varepsilon}$  is the "porous" part, and  $\Sigma$  is the interface between  $\Omega_1$  and  $\Omega_2^{\varepsilon}$ . The porous part can be generated by translations of cells of size  $\varepsilon$  containing a hole ("grain") in the center. We consider a solution  $(u_{\varepsilon}, p_{\varepsilon})$  of the Stokes equation:

$$-\Delta u_{\varepsilon} + \nabla p_{\varepsilon} = f$$
, div  $u_{\varepsilon} = 0$  in  $\Omega^{\varepsilon}$ ,  $u_{\varepsilon}|_{\partial\Omega^{\varepsilon}} = 0$ .

The limit  $\varepsilon \downarrow 0$  ("homogenization" limit) is studied and an expansion of the solution is derived in the form

$$u_{\varepsilon}(x) = u^{0}(x) + u_{b}^{1}(x, \varepsilon)\varepsilon + \left(u^{2}(x) + u_{b}^{2}(x, \varepsilon)\right)\varepsilon^{2} + O(\varepsilon^{5/2})$$
  
$$p_{\varepsilon}(x) = p^{0}(x) + O(\varepsilon)$$

with respect to  $L^2(\Omega)$ , where  $(u_{\varepsilon}, p_{\varepsilon})$  are natural extensions to  $\Omega$  and  $\Omega$  is the domain obtained from  $\Omega^{\varepsilon}$  by filling in the holes.  $(u_{\varepsilon}^{0, p_{0}})$  is a solution of the original Stokes problem in  $\Omega_1$  where  $u^0$  vanishes on  $\Sigma$ .  $p^0$  is a solution of Darcy's law in the porous part  $\Omega_2$  of  $\Omega$ :

$$\operatorname{div}(\mathcal{K}(\nabla p^0 - f)) = 0$$

and is continuous on  $\Sigma$ .  $u^2$  is a solution of a Stokes problem in  $\Omega_1$  with prescribed divergence, satisfying the boundary condition

$$u|_{\Sigma} = \nu \mathcal{C}(\nabla p^0 - f)$$
 on  $\Sigma$ ,  $\nu$  normal.

In  $\Omega_2$  holds

$$u^2 = \mathcal{K}(\nabla p^0 - f).$$





 $\mathcal{K}$  and  $\mathcal{C}$  can be computed by solving Stokes equations in the standard cell.  $u_b^1$  and  $u_b^2$  are boundary layer terms satisfying  $u_b^j(x,\varepsilon) \to 0$  exponentially for  $\varepsilon \downarrow 0$  and  $x \notin \Sigma$ .

## Existence of Solutions for Non-Isothermal Phase Separation

by Hans-Wilhelm Alt (Bonn)

We consider the following system for an order parameter u, the chemical potential v, and the inverse temperature w > 0:

$$-v + \Phi_{,u} - \nabla \cdot \Phi_{,p} = 0$$
$$\partial_t u - \nabla \cdot (m \nabla v) = 0$$
$$\partial_t E + \nabla \cdot (l \nabla w) = 0,$$

where  $\Phi(u, \nabla u, w) = wF(u, \nabla u, w)$  with Helmholtz free energy F. From the entropy principle we derive Gibb's relation

$$S + \Phi = wE$$
 with  $E = \Phi_{.w}$ 

for the internal energy E and the entropy S. We then sketch the proof for the existence of a weak solution as described in a joint paper with I. Pawlow in Adv. Math. Sci. Appl. 1, 319-409 (1992).

### Some Implicit Parabolic Free Boundary Problems

by Wolfgang Walter (Karlsruhe)

The following problem is considered:

$$(P) \begin{cases} u_{xx} - u_t = f(t, x, u, u_x) & \text{for } 0 < t \le T, \\ & \text{and } 0 < x < s(t) \end{cases}$$

$$u(t, 0) - bu_x(t, 0) = \alpha(t) & \text{for } 0 < t \le T$$

$$u(0, x) = u_0(x) & \text{for } 0 \le x \le s_0$$

$$u(t, s(t)) = u_x(t, s(t)) & \text{for } 0 < t \le T$$

Here  $a, b \ge 0$  with a+b=1, initial values  $u_0(x)$  with  $u_0(x) > 0$  in  $[0, s_0)$ ,  $u_0(s_0) = 0$ , the input rate  $\alpha(t) > 0$ , and the decay rate





 $f = f(t, x, u, u_x)$  are given. If the right hand boundary x = s(t) were given, the problem would be overdetermined. The problem is to find a "free boundary" s(t) such that a corresponding solution of (P) exists. Problem (P) is a simple physical model for the penetration of a substance into a (one-dimensional) body lying on the positive x-axis. The substance enters the body at x = 0 and is "eaten up" in the body by a physical, biological, or other process at the rate f per time unit and length unit. At time t, the part of the body on the left of x = s(t) is contaminated, while on the right of this point the body is clean.

The results include existence, uniqueness and continuous dependence on the data—in short, (P) is a well-posed problem. For the existence proof, the Rothe method (discretization in t) is employed. This procedure is also used for numerical calculations. The cases "f (weakly) decreasing in t", which results in an increasing free boundary, and also "f increasing in t", which gives a decreasing free boundary up to extinction time  $T^*$  where  $s(T^*)=0$ , are considered separately.

### Isolated Singularities of Monge-Ampère Equations

by Ralf Beyerstedt (Aachen)

Let  $\Omega$  be the unit disc in  $\mathbb{R}^2$  and  $z\in C^2(\Omega\setminus\{0\})$  a solution of the Monge-Ampère equation  $Ar+2Bs+Ct+rt-s^2=E$ . Assume that  $A,\ B,\ C,$  and E are of class  $C^{1,\mu}(\Omega)$  and satisfy  $AC-B^2+E\ge \mathrm{const}>0,\ C+r>0$ . This talk is concerned with the behaviour of the solution z at the isolated singular point x=y=0. The main result is the following: If the singularity is not removable, then  $\lim_{\tau\downarrow 0}p(\tau\cos\alpha,\tau\sin\alpha)$  and  $\lim_{\tau\downarrow 0}q(\tau\cos\alpha,\tau\sin\alpha)$  are continuous, non-constant functions of  $\alpha$  and  $\lim_{(x,y)\to(0,0)}(r+t)=+\infty$ .

# Weak Solutions to the Evolution Problem of p-harmonic Maps into Spheres

by Norbert Hungerbühler (Zürich)

Let M be a compact Riemannian manifold without boundary with



metric g. We consider the deformation of a given map  $u_0 \colon M \to S^n \subset \mathbb{R}^{n+1}$  under the heat flow related to the energy  $E(u) = \int_{M^{\frac{1}{p}}} (g^{\alpha\beta} \frac{\partial u^i}{\partial x^\alpha} \frac{\partial u^i}{\partial x^\beta})^{p/2} dM$ , i.e. we study the following evolution problem:

(\*) 
$$\begin{cases} \partial_t u + \Delta_{M,p} u = |\nabla u|^p u & \text{in } M \times \mathbb{R}_+ \\ u(\cdot,0) = u_0 & \text{on } M \\ |u(x,t)| = 1 & \text{a.e. on } M \times \mathbb{R}_+ \end{cases}$$

where

$$\Delta_{M,p} u = -\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^{\alpha}} \left( (g^{\rho\sigma} \frac{\partial u^{i}}{\partial x^{\rho}} \frac{\partial u^{i}}{\partial x^{\sigma}})^{\frac{p}{2}-1} \sqrt{g} g^{\alpha\beta} \frac{\partial u}{\partial x^{\beta}} \right) .$$

We show the existence of a global weak solution of (\*) by using Galerkin's method for the penalized energy

$$E_k(u) = E(u) + \frac{k}{4\alpha} \int_M |u^2 - 1|^{2\alpha} dM,$$

Minty's trick, a compactness result and a maximum principle for the solutions of the penalized equations.

This is joint work with Yunmei Chen and Min-Chun Hong.

## Regularity of Solutions for Parabolic Systems and some Applications

by A. I. Koshelev (Berlin)

The regularity of solutions for parabolic systems in higher dimensions depends not only on parabolicity but also on the dispersion of the spectrum of the parabolicity (ellipticity) matrix. The conditions are unavoidable and in the case of elliptic systems sharp. The sufficient conditions of regularity lead to necessary conditions for "blowing up" of the solutions. The regularity theory for general quasilinear elliptic and parabolic systems will be discussed.





#### Maximum Principles at Infinity and a Uniqueness Theorem for the Helicoid

by Friedrich Tomi (Heidelberg)

Let  $M_1, M_2$  be two properly immersed minimal surfaces in  $\mathbb{R}^3$  such that  $M_1 \cap M_2 = \emptyset$ . Then the maximum principle at infinity states that  $d(M_1, M_2) \ge \min\{d(\partial M_1, M_2), d(M_1, \partial M_2)\}$  where d(A, B) denotes the minimal Euclidean distance of subsets A, B of  $\mathbb{R}^3$ . This maximum principle has been proved by Meeks and Rosenberg (1990) under the assumption that  $\partial M_1$  and  $\partial M_2$  are compact, but without further assumptions on  $M_1$ ,  $M_2$  than already mentioned above. In a joint work with J. Ripoll I have investigated the case of noncompact boundaries. One of our results is as follows: Let us assume that  $M_1$ ,  $M_2$  are disjoint properly immersed minimal surfaces with bounded curvature, and  $M_1$  is "bandlike", i.e. the intrinsic distance of points of  $M_1$  to  $\partial M_1$  is bounded from above. Then,  $d(M_1, M_2)$  is positive, provided min $\{d(\partial M_1, M_2), d(M_1, \partial M_2)\}\$  is positive. This maximum principle can be used to prove a uniqueness theorem for certain portions of the helicoid with a double helix as boundary curve: Let S be a line segment through the origin in the (x, y)-plane with endpoints  $p_1, p_2$  where  $|p_1|, |p_2| \le \varepsilon_0 = \sinh u_0$ , and  $u_0$  is the positive root of the equation  $\cosh u_0 = u_0 \sinh u_0$ . Let  $H_S$  denote the orbit of S under the screw motion group

$$\Phi_t(x, y, z) = (x \cos t - y \sin t, x \sin t + y \cos t, z + t), \quad (t \in \mathbb{R}).$$

Then  $H_S$  is unique within the class of minimal surfaces of bounded curvature with boundary  $\partial H_S$  and satisfying the asymptotic condition

$$\frac{x^2 + y^2}{z^2} \to 0, \quad (x, y, z) \to \infty. \tag{*}$$

In particular,  $H_S$  is the only stable minimal surface with boundary  $\partial H_S$  satisfying (\*).



## Reaction-Diffusion Processes of Electrically Charged Species

by Konrad Gröger (Berlin)

Some steps in the manufacturing of semiconductor devices can be understood as reaction-diffusion processes of electrically charged species. In the lecture a class of initial boundary value problems modeling such processes will be presented. It will be shown that on the basis of an estimate for the free energy some questions concerning existence, uniqueness, and further properties of solutions can be answered. Many physically relevant questions, however, remain open. For example, in three space dimensions even existence of solutions is proved under severe restrictions with respect to the data.

### Decay Estimates for Nonlinear Parabolic Equations

by Michael Wiegner (Bayreuth)

If u solves a general nonlinear parabolic equation

$$u_t - a_{ij}(x, t, u, \nabla u)u_{ij} = f(x, t, u, \nabla u)$$

on a bounded domain  $\Omega$  with  $u \to 0$  for  $t \to \infty$ ,  $u|_{\partial\Omega\times(0,\infty)} = 0$ ,  $\frac{\partial f}{\partial u}(x,\infty,0,0) \le 0$ , then we show that without any growth restrictions the following estimates hold:

- i)  $|u(x,t)| \leq Ce^{-\lambda_0 t}$ ,  $t \geq 0$ ,  $x \in \Omega$
- ii) There is some  $T_0 > 0$  such that  $|\nabla u(x,t)| \leq Ce^{-\lambda_0 t}$  for  $t \geq T_0, x \in \partial \Omega$ .
- iii) If for  $t \geq T_1$  a gradient bound  $|\nabla u| \leq C_1$  is known, then even  $|\nabla u(x,t)| \leq Ce^{-\lambda_0 t}$ ,  $x \in \bar{\Omega}$ .

Here  $\lambda_0 > 0$  is the first eigenvalue of the linearization,  $L\Theta + \lambda_0 \Theta = 0$  with  $L\Theta = a_{ij}(x, \infty, 0, 0)\Theta_{ij} + \frac{\partial f}{\partial p_i}(x, \infty, 0, 0)\Theta_i + \frac{\partial f}{\partial u}(x, \infty, 0, 0)\Theta$ . As an application this improves decay results for nonparametric mean curvature flow by V. Oliker and N. Uraltseva.





### Quasilinear Singular Degenerate Parabolic Equations

by Alexandre V. Ivanov (St. Petersburg)

We prove existence of nonnegative Hölder continuous weak solutions of Cauchy-Dirichlet problems for some classes of singular degenerate parabolic equations. The prototypes of this class of equations are

1. the Leibenson equation

$$\frac{\partial u}{\partial t} - \operatorname{div}(\nu_0 |u|^l |\nabla u|^{m-2} \nabla u) = 0$$

$$\nu_0 > 0$$
.  $l > 0$ ,  $1 < m < 2$ ,  $n \ge 1$ ;

2. the equation of the flow through porous medium in turbulent regime

$$\frac{\partial u}{\partial t} - \operatorname{div}(\nu_0 |u|^t |\nabla u - c_0 |u|^k \vec{Z}|^{m-2} |\nabla u - c_0 |u|^k \vec{Z}|) = 0$$

$$u_0 > 0, \ l > 0, \ 1 < m < 2, \ n = 3; \text{ where } c_0 > 0, \ k \ge 1, \ \vec{Z} = (0, 0, 1).$$

Moreover, we establish Hölder estimates for weak solutions of equations of the mentioned classes as well as some uniqueness theorems of the Cauchy-Dirichlet problem.

#### Diffusion Problems in Electrolysis

by Hermann Amann (Zürich)

In the lecture, we treat questions on the wellposedness of equations occurring in the mathematical model of electrolytic processes. These equations form a system of coupled reaction-diffusion equations with nonlinear boundary conditions, which satisfies a nonlocal sidecondition. It is shown that the original system is equivalent to a system of quasilinear reaction-diffusion equations with nonlinear boundary conditions, coupled with a quasilinear elliptic boundary value problem with nonlinear boundary conditions. Under certain conditions, this "nonstandard" elliptic-parabolic system admits a local solution.



Leider waren Frank Merle und Gabriella Tarantello verhindert an der Tagung teilzunehmen; von ihnen sind aber ebenfalls Vortragszusammenfassungen eingegangen, die wir nachstehend abdrucken.

#### Blow-up Points for Nonlinear Heat Equations

by Frank Merle (Paris)

We consider the nonlinear heat equation of the type

$$u_t = \Delta u + u^p$$
  
$$u(0) = \varphi.$$

For some initial data the solution blows up at a time t = T. We are interested in the localization and the "number" of blow up points.

Existence and Multiplicity for Semilinear Elliptic Equations with Changing Sign Nonlinearities with Possible Critical Growth

by Gabriella Tarantello (Pittsburgh)

Motivated by the assigned scalar curvature problem in Riemannian geometry, we study semilinear elliptic equations with nonlinear term of the form W(x)f(u). W is a given function that changes sign. The existence of positive solutions and relative multiplicity is established under a condition on W (involving the total curvature), which turns out to be also necessary in the case f is homogeneous. Other existence and multiplicity results are also discussed.

Berichterstatter: Lutz Wilhelmy





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