MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 11 / 1993

MATHEMATISCHE STOCHASTIK

7.3. bis 13.3.1993

Die Tagung fand unter der Leitung von P. Deheuvels (Paris), A. Irle (Kiel) und J. Steinebach (Marburg) statt. Es war auch diesmal wieder ein besonderes Anliegen, das breite Spektrum innerhalb der Stochastik aufzuzeigen. Im Mittelpunkt des Interesses standen aktuelle Entwicklungen, welche geeignet sind, die Querverbindungen zwischen Wahrscheinlichkeitstheorie und Mathematischer Statistik zu fördern.

Schwerpunktthemen waren u.a.
- Banachraummethoden in der Stochastik
- statistische Inferenz stochastischer Prozesse
- empirische und Quantil-Prozesse
- nichtparametrische Verfahren, Bootstrap
- starke Grenzwertsätze und Invarianzprinzipien
- asymptotische Entwicklungen, Konvergenzraten.

Es wurden insgesamt 42 Vorträge gehalten, davon fünf Übersichtsvorträge. Neben dem Vortragsprogramm kam es zu intensiven Diskussionen und wissenschaftlichem Gedankenaustausch unter den Teilnehmern. Insbesondere hatten auch die jüngeren Stochastiker Gelegenheit, ihre Arbeit einem fachkundigen Publikum vorzustellen.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

M. ALEX :

INVARIANCE PRINCIPLES IN EXTENDED RISK MODELS

In collective risk theory, one is interested in the risk reserve $R_u(t)$, $t \geq 0$ of an insurance company, which depends on the initial reserve $u > 0$, and the time of ruin $\tau(u) := \inf\{t > 0;\ R_u(t) \leq 0\}$. In the classical case, Horváth and Willekens (1986) proved strong approximations of $P\{\tau(u) \leq u^2 x\}$, $x \geq 0$, by the distribution function of the first-exit time of a Wiener process over a straight line.
Extended risk models allowing for risk fluctuations and including interest and inflation functions are presented, based on models, which were introduced by Garrido (1987) and Grandell (1991). Using invariance principles for the composition of stochastic processes, strong approximations of $P\{\tau(u) \leq u^2 x\}$ by distribution functions of first-exit times of Wiener processes over curved boundaries can be proved in these generalized models. It is remarkable that the convergence rates in these models are the same as in the classical case.

G. ALSMEYER :

A RECURRENCE THEOREM FOR SQUARE-INTEGRABLE MARTINGALES

Let $(M_n)_{n \geq 0}$ be a zero-mean martingale with canonical filtration $(\mathcal{F}_n)_{n \geq 0}$ and stochastically L_2-bounded increments $Y_1, Y_2, \ldots$, which means that

$$P\{|Y_n| > t | \mathcal{F}_{n-1}\} \leq 1 - H(t) \text{ a.s. for all } n \geq 1, t > 0$$

and some square-integrable distribution H on $[0, \infty)$. Let $V^2 = \sum_{n \geq 1} \mathrm{E}(Y_n^2 | \mathcal{F}_{n-1})$. It is the main result that each such martingale is a.s. convergent on $\{V < \infty\}$ and recurrent on $\{V = \infty\}$, i.e. $P\{M_n \in [-c, c] \text{ i.o. } | V = \infty\} = 1$ for some $c > 0$. This generalizes a recent result by Durrett, Kesten and Lawler who consider the case of finitely many square-integrable increment distributions. As an application of our recurrence theorem, we obtain an extension of Blackwell's renewal theorem to a fairly general class of square-integrable processes with independent increments and linear positive drift function.

J. BEIRLANT :

ON THE ASYMPTOTIC NORMALITY OF L_p-NORMS OF EMPIRICAL FUNCTIONALS

A general method is presented for deriving the asymptotic normality of the L_p-norm of empirical functionals which make use of the neighboring data at any point x of interest. This technique is based on a Poisson representation for the empirical process together with a Fourier inversion technique for conditional characteristic functions. As applications of this method, the asymptotic normality is established for L_p-norms $(1 \leq p < \infty)$ of regression and density estimators. In the process previous results on this subject from the literature are extended and/or sharpened.

E. BOLTHAUSEN :

SELF-ATTRACTING RANDOM WALKS

We consider several models of random walks on $\mathbb{Z}^d$ with a self-attractive interaction of the path. The laws of these self-attractive random walks on paths of length T are given by a density proportional to $\exp\{H_T(\omega)\}$ with respect to the law of the ordinary symmetric random walk, where the "Hamiltonian" H_T is large when the path ω is untypically clumping together. We mention some examples which will be discussed during the talk:

a) $$H_T(\omega) = \frac{1}{T}\int_0^T dt \int_0^T ds\, V(\omega_s - \omega_t), \qquad V \geq 0,$$

b) $$H_T(\omega) = -N_T,$$

where N_T is the cardinality of the set of points visited up to time T.

M. BRONIATOWSKI :

LARGE DEVIATION PRINCIPLES FOR SET-INDEXED PROCESSES WITH INDEPENDENT INCREMENTS

Let $\{X_\lambda(A); A \in \mathcal{A}\}_{\lambda \geq 0}$ be a family of processes indexed by a collection of sets $\mathcal{A}$. Assume that the process X_λ has independent increments, meaning that $X_\lambda(A)$ and $X_\lambda(B)$ are independent when $A \cap B = \emptyset$. Assume that, for every fixed

$A \in \mathcal{A}$, $(X_\lambda(A))_{\lambda \geq 0}$ obeys a large deviation principle (LDP) in some Hausdorff space $\mathcal{X}$. We prove that, under suitable conditions on this marginal LDP, the processes $(X_\lambda(\cdot))_{\lambda \geq 0}$ obey an LDP in $\mathcal{X}^{\mathcal{A}}$. This is based on an extension of the approach of Lynch and Sethuraman (AP 1987) for functional LDP's for the partial sum processes.

Two applications: 1) Consider a sample $X_1, ..., X_n$ of real i.i.d. r.v.'s and let $M_n(t) = \max(X_1, ..., X_{[nt]})$, $0 \leq t \leq 1$, be the extremal process. Embedding $M_n(t)$ in the process $\mu_n(A) = \max_{i \in nA} X_i$ and applying our result, we obtain a functional LDP for M_n. Here $\mathcal{A}$ is some subset of $B[0,1]$.

2) Let $P_n(A) = n^{-1} \sum_{i \in nA} \delta_{X_i}$ be the sequential empirical measure and let $A \mapsto P_n(A)$ be the induced process. We obtain a Sanov-type functional LDP and deduce LDP's for weighted U-statistics.

(Joint work with Ph. Barbe)

E. CSÁKI :

SOME LIM INF RESULTS FOR TWO-PARAMETER RANDOM PROCESSES

Let $Z(x,y)$ be a two-parameter random process and let Δ_T be a set of (x,y) depending on T. Small deviation results are applied to study the lim inf behaviour of

$$\sup_{(x,y)\in\Delta_T} Z(x,y) \qquad \text{and} \qquad \sup_{(x,y)\in\Delta_T} |Z(x,y)|$$

for certain Gaussian processes $Z(x,y)$ such as Kiefer (empirical) process and Wiener sheet. These results are obtained via eigenfunction expansion for the Ornstein-Uhlenbeck process.

M. CSÖRGŐ :

STUDENTIZED INCREMENTS OF PARTIAL SUMS AND SELF-NORMALIZED ERDŐS-RÉNYI LAWS

Let $X, X_1, X_2, \ldots$ be i.i.d. r.v.'s with $EX = 0$, and assume that X belongs to the domain of attraction of the normal law. We, jointly with Z.-Y. Lin and Q.M. Shao, establish Studentized-Increments-versions of the Csörgő-Révész (1979, 1981) laws of large numbers for increments of partial sums of these r.v.'s. We prove that

replacing the normalizing constants $[c \log n]$ by the r.v.'s $\sum_{i=k+1}^{k+[c\log n]}(X_i^2+1)$ in the classical Erdős-Rényi strong law of large numbers, a corresponding result remains true under assuming the existence of the first moment only. Consequently, we lose the distribution-determining nature of the classical E-R law, though the result is not distribution-free. For proving our self-normalized E-R-type law of large numbers, X is not assumed to belong to the domain of attraction of the normal law.
(Joint work with Z.-Y. Lin and Q.M. Shao)

S. CSÖRGŐ :

RESOLUTION OF THE ST. PETERSBURG PARADOX

We show that the distribution functions of the suitably centered average gain of Paul in a sequence of Petersburg games and a sequence of infinitely divisible, semi-stable distribution functions merge together. The approximating distributions are taken from the family of all possible partial limits, a family with the cardinality of the continuum. We derive some theoretical results for this family and further investigate it by computer-assisted numerical methods. As a result, for any winning probabilty $p \in (0,1)$ of Peter, we propose an asymptotically precise premium formula for Paul to pay for n games. For any reasonable p, this price is greater than the "fair price" suggested by Feller's law of large numbers. Motivated by the dominating role of extremes, we also consider modifications in which Paul renounces a few of his largest principal gains. Then another merging approximation is possible based upon trimmed versions of the original subsequential limiting laws. Even if just the single largest gain is retained by Peter, the game can be made asymptotically fair in the classical sense. The results, when all put together, resolve the 279 year old paradox by thoroughly explaining the probabilistic essence of the asymptotic behavior of a sequence of Petersburg games. Graphical and numerical illustrations will be given.
(Joint work with G. Simons)

P. DEHEUVELS :

STRASSEN'S LAW IN STRONG TOPOLOGIES

The classical Strassen (1964) law of the iterated logarithm asserts that if

$\{W(t), t \geq 0\}$ is a Wiener process, then (with $LLt = \log(\log t \vee e^e)$) the set of functions $\{(2TLLT)^{-1/2}W(T\,\cdot)\}$ is almost surely relatively compact in the set $C(0,1)$ of continuous functions on $[0,1]$ endowed with the uniform topology. We give sufficient conditions for this result to hold when the uniform topology is replaced by the topology defined by an arbitrary measurable norm. Moreover, we also give necessary and sufficient conditions under small restrictions such as assuming that the norm is lower semi-continuous.
(Joint work with M.A. Lifshits)

J.H.J. EINMAHL :

MAXIMAL TYPE TEST STATISTICS BASED ON CONDITIONAL PROCESSES

A general methodology is presented for non-parametric testing of independence, location and dispersion in multiple regression. The proposed procedures are based on the concepts of conditional distribution function, conditional quantile, and conditional shortest t-fraction. Techniques involved come from empirical process and extreme value theory. The asymptotic distributions are standard Gumbel.
(Joint work with J. Beirlant)

U. EINMAHL :

RATES OF CLUSTERING IN STRASSEN'S LIL FOR PARTIAL SUM PROCESSES

We provide a detailed description of the rate of clustering in Strassen's functional LIL for partial sum processes. Necessary and sufficient conditions are given for certain convergence rates in terms of moment-type conditions. Our proof is based on a new strong approximation of sums of i.i.d. random variables.
(Joint work with D. Mason)

P. EMBRECHTS :

STOCHASTIC DISCOUNTING AND THE BOOTSTRAP

Let (Y_i) and (Z_i) be independent i.i.d. sequences of r.v.'s satisfying the moment conditions: $\exists p > 0$: $\mathrm{E}|Z_1|^p < 1$, $\mathrm{E}|Y_1|^p < \infty$. Define the so-called perpetuity variables:

$$\forall\, l \geq 1 : \; S_l = \sum_{k=1}^{l} Z_1 \ldots Z_k Y_k \,.$$

Under the above conditions, $S_l \rightarrow_{a.s.} S$ where S satisfies the random equation:

$$S \stackrel{\mathcal{D}}{=} (Y_1 + S) Z_1$$

Explicit (analytical) solutions are difficult to obtain. We present a bootstrap procedure for estimating $\mathrm{P}\{S \leq x\}$. The method of proof depends on properties of the minimal L_p-distance:

$$d_p(F, G) = \inf\{||X - Y||_p ;\; F_X = F,\; F_Y = G\}$$

For the bootstrap variable S^*_{lmn} it is shown that $d_p(S^*_{lmn}, S) \rightarrow 0$ as $\min(l, m, n) \rightarrow \infty$. Various examples showing that the method actually works in practice are given.

H. J. ENGELBERT :

ON ONE-DIMENSIONAL STOCHASTIC DIFFERENTIAL EQUATIONS: SOME RESULTS ON EXISTENCE AND UNIQUENESS

We consider the one-dimensional stochastic differential equation

$$X_t = X_0 + \int_0^t a(s, X_s) ds + \int_0^t b(s, X_s) dB_s, \; t < S(X), \qquad (1)$$

where a, b are Borel functions on $[0, \infty) \times \mathbb{R}$, (B_t) is a Brownian motion, and $S(X)$ is the explosion time of X. By absolutely continuous transformation of probability measure or space transformation (at least if a does not depend on s), equation (1) can be reduced to the equation without drift

$$X_t = X_0 + \int_0^t b(s, X_s) dB_s, \; t < S(X). \qquad (2)$$

In the present talk, we first give necessary and sufficient conditions on b for existence as well as uniqueness in law of the solution to (2) in the homogeneous case ($b(s, x) = b(x)$). Then we discuss several new results in the general, non-homogeneous case.

P. GAENSSLER :

ON SET-INDEXED PARTIAL-SUM PROCESSES WITH RANDOM LOCATIONS

The purpose of this talk is to present a unified approach to empirical and partial-sum processes by studying processes $S_n = (S_n(C))_{C \in \mathcal{C}}$ with $S_n(C)$ being defined by

$$S_n(C) := \sum_{j \leq j(n)} 1_C(\eta_{nj})\, \xi_{nj},\ C \in \mathcal{C},$$

the η_{nj}'s being random elements (random locations) in an arbitrary sample space X, the ξ_{nj}'s being real-valued random variables, and where the index family $\mathcal{C}$ is a Vapnik-Chervonenkis class of subsets of X. Our main emphasis is on asymptotic results (as the sample size n tends to infinity) for the processes S_n such as uniform laws of large numbers (the uniformity being w.r.t. $\mathcal{C}$) and functional central limit theorems containing various results for empirical and partial-sum processes as special cases.

U. GATHER :

PROPERTIES OF SHORTEST HALF-BASED SCALE ESTIMATORS

The paper deals with scale estimators for univariate distributions, which are of the type of Rousseeuw's (1988) length of the shortest half.
It is the aim to compare these estimators with other similar estimators (e.g. Grübel 1988) as well as to study their properties described by their influence function, sensitivity curve and large sample properties.

E. GINÉ :

THE NECESSITY PART OF THE CLT FOR DEGENERATE U-STATISTICS

Let $h : S^m \to \mathbb{R}$ be a measurable function (on (S^m, λ^m)), symmetric in its arguments, and let X_i be i.i.d. S-valued. We prove that if the sequence $\{n^{-m/2} \sum_{i_1 < \ldots < i_m \leq n} h(X_{i_1}, \ldots, X_{i_m})\}_{m=1}^{\infty}$ is stochastically bounded, then $Eh^2 < \infty$ and $Eh(X, x_2, \ldots, x_m) = 0$ $x_2, \ldots, x_m - a.s.$ (w.r.t. $\mathcal{L}(X)^{m-1}$). And in particular the sequence converges in distribution. The proof uses Rademacher ran-

domization and decoupling, Khintchine's inequality (reiteratively) and an argument of Paley-Zygmund. It provides, for $m = 1$, a simple proof of $EX^2 < \infty \Leftarrow$ CLT for $n^{-1/2} \sum_{i=1}^n X_i$.
(Joint work with J. Zinn)

F. GÖTZE :

ASYMPTOTIC EXPANSIONS AND THE BOOTSTRAP

Asymptotic expansions up to an error of order $O(n^{-1})$ are shown to be valid for asymptotically linear nonparametric statistics which admit a Hoeffding expansion with terms of order $O(n^{-k/2})$, $k = 0, 1, 2$. These approximations hold uniformly in the class of statistics with fourth order moments and a Cramér-type smoothness condition on the sum of r.v.'s. Here the Bootstrap approximation dominating for the standardized statistics works up to an error of order $O(n^{-1})$.
This is joint work with V. Bentkus.
For von Mises statistics of order 2 we show that for twice differentiable kernels defined on a Euclidian space and which have infinitely many eigenvalues the error in the functional limit theorem is of order $O_P(n^{-1/2})$. It is expected that iterated Bootstrap methods even yield an error of order $O_P(n^{-1})$.
This result is joint work with R. Zitikis.

V. GOODMAN :

LIMIT THEOREMS FOR SELF-SIMILAR PROCESSES

Let $W(t)$ be a standardized Wiener process and $k_t(u,v)$ be in $L^2(\mathbb{R}^2)$ for each $t \geq 0$. Processes of the form $X_t = \int\int k_t(u,v)\, dW(u)\, dW(v)$ have been studied by Taqqu, Kôno, and Mori and Odaira. M.-O. (1986) impose conditions on $k_t(u,v)$ to give processes having stationary increments and satisfying a self-similar identity: $X_{ct} \overset{\mathcal{D}}{=} c^H X_t$ for some index H, $1/2 < H < 1$.
M.-O. (1986) obtain functional laws of the iterated logarithm for the processes $n^{-H}(\log\log n)^{-1} X_{nt}$ under these assumptions. Goodman and Kuelbs (1993) relax the assumptions on the $k_t(u,v)$ for the FLIL to hold. Here, I give examples, related to the Rosenblatt process, where special choices of k_t satisfy G.-K. conditions but not M.-O. conditions. Furthermore, a rate of convergence in the

FLIL is obtained for a portion of the Rosenblatt process. The rate is shown to be $O\Big((\log\log\log n)^2(\log\log n)^{-H/(2+H)}\Big)$.

K. GRILL :

BRANCHING RANDOM WALK

The branching random walk is obtained by superimposing a random walk structure on a Galton-Watson process. We assume that the offspring distribution has finite variance, and consider only the simple random walk case. From the work of Asmussen and Kaplan we know that (even in more general cases) the ratio of the number of particles to the left of x and the total size of the generation tends to a normal distribution (always assuming that the Galton-Watson process is supercritical, and conditioned on non-extinction). We investigate the question whether large deviation results can be transferred to this setting. It turns out that for $-1 < \alpha < \alpha_0$, where α_0 is the solution of $\alpha^\alpha(1-\alpha)^{1-\alpha} = m/2$ ($\alpha_0 = -1$ for $m > 2$) there are no particles eventually at $[n\alpha]$, whereas for $0 > \alpha > \alpha_0$ we have that the ratio of the relative frequency of particles in $[n\alpha]$ and the corresponding random walk probability converges to a nondegenerate random variable $W(\alpha)$.

A. GUT :

FIRST PASSAGE TIMES FOR PERTURBED RANDOM WALKS

A perturbed random walk is a sequence $\{Z_n, n \geq 1\}$, such that $Z_n = S_n + \xi_n$, where $\{S_n, n \geq 1\}$ is a random walk (with positive drift) and $\{\xi_n, n \geq 1\}$ is a sequence of random variables, such that $\xi_n/n \stackrel{\text{a.s.}}{\to} 0$ as $n \to \infty$.
Let $\nu(t), t \geq 0$, be the first time a perturbed random walk crosses a (general nonlinear) boundary. We provide limit theorems for the first passage times, the stopped perturbed random walk and the overshoot as $t \to \infty$. In particular, these results are applied to the important case when the perturbed random walk is of the form $\{n\,g(S_n/n), n \geq 1\}$, where $\{S_n, n \geq 1\}$ is a random walk whose increments have positive, finite mean and g is positive, continuous and, possibly, has further smoothness properties.
The traditional case considered in nonlinear renewal theory is when the summands have finite variance and g is twice continuously differentiable. We also present some

results concerning existence of moments and uniform integrability. We conclude with a number of examples.

E. HAEUSLER :

ON COVERAGE PROBABILITIES OF CONFIDENCE INTERVALS BASED ON A WEIGHTED BOOTSTRAP

Efron's classical nonparametric bootstrap puts weights on the observations forming a random vector having a multinomial distribution and being independent of the observations. Typically, this bootstrap leads to two-sided confidence intervals for which the actual coverage probability converges at rate $1/n$ to the nominal level, where n is the sample size. Recently, bootstrap procedures employing much more general weights have been developed. We discuss the coverage probabilities of confidence intervals for the mean based on a specific type of a weighted bootstrap.

C. C. HEYDE :

WHEN DO WE NEED THE LIKELIHOOD ?

Recent developments in the general theory of inference suggest that likelihood-based methodology can in many cases be subsumed into a quasi-likelihood framework with considerable advantages in robustness and simplicity of derivation. Only first and second moment properties are required. Illustrations were given related to REML estimators, estimating the drift in a diffusion and estimating parameters subject to constraints. The material is from a soon-to-be-completed monograph on quasi-likelihood and its application whose principal focus is on statistical models which can be written in a semimartingale form as a signal (involving the parameter of interest) plus noise.

A. JANSSEN :

PRINCIPAL COMPONENT DECOMPOSITION OF KOLMOGOROV-SMIRNOV TYPE TESTS

The talk deals with the comparison of asymptotic power functions of non-parametric unbiased tests. The bench-mark is the power function of the best two-sided Gaussian tests (two-sided Neyman-Pearson tests). Each non-parametric unbiased test has a principal component decomposition given by a Hilbert-Schmidt operator. Thus every test has reasonable curvature only for a finite number of orthogonal directions of alternatives. As application one obtains results about the curvature of the two-sided Kolmogorov-Smirnov test. It is shown that these tests prefer for small α approximately the same direction as the two-sample median rank test, which has power only for one direction of alternatives. The consideration of the curvature can be used to establish global extrapolations of the power function. It turns out that -except of one direction- the power of the Kolmogorov-Smirnov test is flat for small α. This explains earlier numerical calculations for that power function.

J. KUELBS :

METRIC ENTROPY AND THE SMALL BALL PROBLEM FOR GAUSSIAN MEASURES

We establish a precise link between the small ball problem for a Gaussian measure μ on a separable Banach space, and the metric entropy of the unit ball of the Hilbert space H_μ generating μ. This link allows us to compute small ball probabilities from metric entropy results, and vice versa.
(Joint work with Wenbo Li)

H. R. LERCHE :

OPTIMAL CHANGE POINT DETECTION

We consider the sequential change point detection problem of Shiryayev. A process $W(t) = B(t) + \theta(t-\tau)^+$ is observed, B is a standard Brownian motion with drift 0, τ is an independent exponentially distributed random variable and θ is a positive constant. For a stopping time T of W the risk is taken as

$$R(T) = \mathrm{P}\{T < \tau\} + cE(T-\tau)^+.$$

We show that $R(T) = \int g(\pi_T)dP$ where g is a convex function with a unique minimum p^* and π_t is the posterior probability of a change before t. Then it follows immediately that

$$T^* = \inf\{t > 0;\ \pi_t \geq p^*\}.$$

Proceeding from this idea further one can derive a similar result for the case when θ is unknown. The results give an intuitive support for detecting a change of trends in a stock market by log-likelihood statistics.

F. Liese :

ASYMPTOTIC PROPERTIES OF SELECTION PROCEDURES

For k populations with distribution functions $F(x-\theta_1), \ldots, F(x-\theta_k)$ one wants to select the population with the largest θ-value by taking n independent observations in each population. For the probability of correct selection for the sequence of localized models $F(x-(\theta_0+h_1/\sqrt{n})), \ldots, F(x-(\theta_0+h_k/\sqrt{n}))$ with sample size $n \to \infty$ an asymptotic Hajek-Le Cam bound may be established. The optimal sequence of selection procedures depends on the density $f(x-\theta_0)$ which is unknown in practical situations. An adaptive and asymptotically efficient selection procedure can be constructed with the help of a kernel estimate of $f(x-\theta_0)$. For fixed parameter configuration $\theta_1, \ldots, \theta_k$ the error probabilities for many selection procedures tend exponentially to zero. The maximum exponential rate can be explicitely expressed in terms of the Hellinger integrals of the distributions $F(x-\theta_i)$, $F(x-\theta_j)$.

P. Major :

ON THE NUMBER OF LATTICE POINTS IN A RANDOM DOMAIN

The investigation of the number of lattice points in a large domain is a classical topic in number theory. Recently, such questions became interesting also because of their application in physics. The investigation of finer properties of the spectrum of the Laplace operator in a domain leads to such questions.
We discuss questions of the following type: Let A be a domain with smooth boundary in a plane and R be a randomly chosen number with uniform distribution in an interval $[1, T]$. What can be said about the distribution of the number of lattice points in the domain RA or in $\left(R+\frac{1}{R}\right)\mathrm{A}\backslash RA$ if $T \to \infty$? Some new results and open problems will be discussed together with the relation of the above

problems to physics.

D. M. MASON :

A GENERAL BOOTSTRAP

Let $X_1, X_2, \ldots$ be i.i.d. F and independent of these random variables let $W = (W_{1,n}, \ldots, W_{n,n})$, $n \geq 1$, be a triangular array of exchangeable random variables. Introduce the general weighted 'bootstrapped' empirical distribution function

$$F_{W,n}(x) := \frac{1}{n} \sum_{i=1}^{n} W_{i,n} 1_{\{X_i \leq x\}}, \quad -\infty < x < \infty.$$

The reason for the inclusion of the word 'bootstrap' in the definition of $F_{W,n}$ is that when $W = (M_{1,n}, \ldots, M_{n,n}) \overset{\mathcal{D}}{\sim} \mathrm{Mult}(n; 1/n, \ldots, 1/n)$, $F_{W,n} \overset{\mathcal{D}}{=} F_n^*$ conditioned on $X_1, \ldots, X_n$, where F_n^* denotes the usual bootstrapped empirical distribution. The asymptotic properties of this general weighted bootstrapped empirical distribution and functionals of it are described. These functionals include the general weighted bootstrapped mean and empirical process.

H.G. MÜLLER :

SEMIPARAMETRIC MODELLING OF VARIANCE FUNCTIONS

We propose a general semiparametric variance function model in a fixed design regression setting. In this model, the regression function is assumed to be smooth and is modelled nonparametrically, whereas the relation between the variance and the mean regression function is assumed to follow a generalized linear model. Almost all variance function models that were considered in the literature emerge as special cases. Least squares type estimates for the parameters of this model and the simultaneous estimation of the unknown regression and variance functions by means of nonparametric kernel estimates are combined to infer the parametric and nonparametric components of the proposed model. The asymptotic distribution of the parameter estimates is derived and is shown to follow usual parametric rates in spite of the presence of the nonparametric component in the model. This result is applied to obtain a data-based test for heteroscedasticity under minimal assumptions on the shape of the regression function.
(Joint work with P.L. Zhao)

G. NEUHAUS :

TWO SAMPLE RANK TESTS FOR CENSORED DATA : AN OVERVIEW

The development of the theory of two-sample rank tests for censored data is discussed with special emphasis on optimality questions in local asymptotic models. Two sorts of rank statistics for testing the equality of failure time distributions F_1, F_2 of both samples have been suggested by practitioners: 'score statistics' and 'observed-expected statistics'. While the first sort extends the classical linear rank statistic in a straightforward manner to censored data, the second sort is seemingly qualitatively different.

It is demonstrated that both sorts of statistics are asymptotically equivalent in local asymptotic models, the observed-expected statistics being martingale versions of the score statistics. This entails asymptotic optimality and distributional results for both classes. Moreover, following an idea of Gill (1992), the optimality properties of the observed-expected statistics under unequal censoring in both samples are explained by enlarging the usual one-parameter asymptotic model to a two-parameter model.

Finally, conditional (permutation) tests, introduced by the author, are discussed. These tests are finite sample distribution-free under $F_1 = F_2$ with equal censoring and are asymptotically equivalent to their unconditional counterparts also under unequal censoring.

R. NORVAIŠA :

A CHARACTERIZATION OF AN ASYMPTOTIC BEHAVIOUR OF DISTRIBUTIONS ON BANACH FUNCTION SPACES INDUCED BY EMPIRICAL AND PARTIAL SUM PROCESSES

We consider, as a sample path space, a Banach function space of measurable functions defined on a σ-finite measure space. One may pose the question of describing those Banach function spaces where empirical and partial sum processes have almost all their sample paths and induced distributions converge weakly. We would like to formulate a couple of this kind results proved by applying probability in Banach space techniques and/or strong approximation results.

D. PFEIFER :

PSEUDO-POISSON APPROXIMATION FOR MARKOV CHAINS

We consider the problem of approximating the distribution of a Markov chain with rare transitions in an arbitrary phase space by the corresponding pseudo-Poisson process. Sharp estimates for both first- and second-order approximations are obtained. These estimates improve also the known results in the ordinary Poisson theorem.

D. PLACHKY :

CHARACTERIZATION OF DISCRETE DISTRIBUTIONS

The aim of the talk is twofold, namely to characterize some special discrete distributions (for example the Poisson distribution by some estimation-theoretical property, the uniform distribution by stochastic ordering) and to characterize discreteness of distributions by some general properties (like unique extension, existence of regular conditional distributions, and continuity from below of inner probabilities).

F. PUKELSHEIM :

THE KIEFER ORDERING OF INFORMATION MATRICES

The Kiefer ordering of information is a superposition of the Loewner ordering and majorization,

$$A \gg B \Leftrightarrow \exists\, F : A \geq F \prec B,$$

where $F \prec B :\Leftrightarrow F \in \operatorname{conv}\{QBQ' : Q \in \mathcal{Q}\}$ is the matrix majorization when the group $\mathcal{Q} \subseteq \mathrm{GL}(k)$, compact, acts by congruence, and where $A \geq F :\Leftrightarrow A - F$ n.n.d. is the Loewner ordering. We give two examples to illustrate that this is the right way to combine the Loewner ordering and matrix majorization. One is the monotonicity of the information matrix mapping $A \gg B \Rightarrow C_K(A) \gg C_K(B)$. The other is rotatable information matrices in second order models, relating to tensor representation of classical groups (R. Brauer 1937).
(Joint work with N. R. Draper and N. Gaffke)

R.-D. REISS :

XTREMES: EXTREME DATA ANALYSIS AND ROBUSTNESS (WITH SOFTWARE DEMONSTRATION)

In many applications, extremes do not fit to the ideology of normal samples and, therefore, such data are omitted or one uses statistical procedures upon which extremes have a bounded influence. The converse attitude is to regard extremes as the important part of the data. The generalized Pareto distributions will be taken for the parametric modelling of the upper tail of the distribution.
The software package XTREMES provides graphical representations of curves such as generalized Pareto, extreme value or Cauchy-normal densities and q.f.'s. Secondly, data sets may be generated according to these densities and in non-i.i.d. models. Finally, the data may be inspected by means of parametric and nonparametric methods. An external implementation of new estimators is possible. The performance of estimators may be inspected by means of diagrams, plots and Monte Carlo simulations of the MSE.
XTREMES is menu-driven, runs on IBM-compatible PC's and comes with a user's guide-manual. The first release is scheduled for May, 1993.

P. RÉVÉSZ :

BRANCHING RANDOM WALKS

At time $t = 0$ a particle located in $0 \in \mathbb{Z}^d$ begins a random walk. It moves at time $t = 1$ with equal probabilities to one of the $2d$ neighbours of 0. Arriving at the new location it produces k offsprings with probability p_k, $(k = 0, 1, 2, ...)$ and dies. Each of the offsprings moves independently at time $t = 2$ to one of its neighbours. Arriving at the new location each of them produces independently offsprings and then dies. Repeating this procedure we obtain a branching random walk. Let $\lambda(x,t)$, $(x \in \mathbb{Z}^d, t = 0, 1, 2, ...)$, be the number of particles in x at t. We are interested in the limit properties of $\lambda(x,t)$ as $t \to \infty$. A typical result is the following: for any $x \in \mathbb{Z}^d$ and $0 < \epsilon < 1$ we have

$$\lim_{T \to \infty} T^{1-\epsilon} \left| \frac{1}{2} \left(\frac{2\pi T}{d} \right)^{d/2} \frac{\lambda(x,T)}{m^T} - B \right| = 0 \text{ a.s.}$$

where $m = \sum_{k=0}^{\infty} p_k k$ and B is a r.v., $(B \geq 0)$.

W.-D. RICHTER :

LARGE DEVIATIONS FOR GAUSSIAN, ASYMPTOTICALLY GAUSSIAN AND ELLIPTICALLY CONTOURED DISTRIBUTIONS

The influence of certain geometric properties of a large deviation domain onto the asymptotic behaviour of the respective large deviation probabilities is discussed for Gaussian and for special types of asymptotically Gaussian or spherical distributions. The talk starts with a discussion of the Osipov problem for fixed and the Khintchine problem for increasing dimensions. In a sequence of steps it follows a comparison of large deviation theorems for analytically exact known with only asymptotically known distributions. Several types of large deviation limit theorems are dealt with: starting from those concerning the logarithms of large deviation probabilities up to those even including an asymptotic expansion.

L. RÜSCHENDORF :

SCHRÖDINGER EQUATIONS AND CLOSEDNESS OF SUM SPACES

We determine sufficient conditions for the closedness of sum spaces of L^1-functions. As a consequence of Csiszar's projection theorem this implies generalizations of results of Fortet, Beurling and Hobby and Pyke on the existence and uniqueness of solutions of some nonlinear integral equations, which were introduced by Schrödinger, to describe the most probable behaviour of Brownian motions conditional on the observed initial and final state in a finite interval $(0, t_1)$. The result is also of interest for a large deviation formula for infinite dimensional Brownian motions related to Schrödinger bridges and for the construction of optimal estimators in marginal models.
(Joint work with W. Thomsen)

M. SØRENSEN :

CURVED EXPONENTIAL FAMILIES OF STOCHASTIC PROCESSES AND THEIR ENVELOPE FAMILIES

Many important statistical stochastic process models are exponential families in the sense that the likelihood function corresponding to observation of the process in the time interval $[0, t]$ has an exponential family representation of the same dimension for all $t > 0$. Most exponential families of processes are curved exponential families in the sense that the canonical parameter space forms a curved submanifold of a Euclidian space.

Several modern statistical techniques for curved exponential families use properties of the full exponential family generated by the curved model. Examples are methods based on differential geometric considerations or on approximately ancillary statistics. The interpretation of the full exponential families as stochastic process models is not straightforward and must be done for each $t > 0$ separately. Therefore, the full families are referred to as envelope families in the stochastic process setting.

A general result on how to calculate the envelope families is given. Particular attention is devoted to the question in what sense the envelope families can be interpreted as stochastic process models. For Markov processes rather explicit answers can be given. Diffusion processes and counting processes are studied in particular. As an example, the family of Ornstein-Uhlenbeck processes is considered in detail. Also an application of our theory to a goodness-of-fit test for censored observations is presented.

(Joint work with U. Küchler)

J.L. TEUGELS :

REINSURANCE AND EXTREMES FOR A RANDOMLY INDEXED SEQUENCE

The theoretical study of reinsurance treaties within the framework of mathematical risk theory is still in its infancy. One of the main reasons for this meager state of the art is that reinsurance is meant to safeguard the company from the effects of large claims while extreme value theory is almost unknown to the practicing actuary.

We try to give a flavour for the problem and indicate a few steps towards an improvement.

H. WALK :

ON AVERAGED RECURSIVE ESTIMATION IN LINEAR REGRESSION

Let the random sequence $((A_n, b_n))$ of symmetric positive semidefinite $m \times m$ matrices A_n and m-vectors b_n be stationary with $A := EA_1$ positive definite and $b := Eb_1$. For the estimates X_n of $\theta := A^{-1}b$ defined by $X_{n+1} := X_n - a_n(A_{n+1}X_n - b_{n+1})$ with gains a_n, the sequence $(\bar{X}_n)$ of arithmetic means is investigated. This joint work with L. Györfi extends results of Ruppert, Polyak and Juditzky, Pechtl to general A_n under weak dependence of $((A_n, b_n))$. For $a_n \equiv \alpha > 0$ (sufficiently small) one obtains in the ergodic case, especially under mixing conditions, a.s. convergence of $\bar{X}_n$ to $\theta + \delta_\alpha$ with $\delta_\alpha = o(1)$ or $O(\sqrt{\alpha})$ and asymptotic normality with order $1/\sqrt{n}$. For $a_n = \alpha n^{-\gamma}$ $(3/4 < \gamma < 1)$ under the assumption of a functional CLT for $b_n - A_n\theta$ with asymptotic covariance matrix S and with $E\|\sum_{k=1}^n (A_k - A)\|^2 = O(n)$ and $E\|\sum_{k=1}^n (b_k - b)\|^2 = O(n)$, an invariance principle for $\bar{X}_n - \theta$ with convergence order $1/\sqrt{n}$ and optimal asymptotic covariance matrix $A^{-1}SA^{-1}$ is obtained.

J. ZINN :

STRONG LAWS OF LARGE NUMBERS FOR QUADRATIC FORMS

Under mild regularity conditions on the normalizing sequence, γ_n, and symmetry of the distribution of the i.i.d. sequence, (X_j), necessary and sufficient conditions are given for

$$\frac{1}{\gamma_n} \sum_{i \neq j \leq n} X_i X_j \to 0 \text{ a.s.}.$$

Preliminarily, one gives necessary and sufficient conditions for

$$\frac{1}{\gamma_n} \max_{i \neq j \leq n} |X_i X_j| \to 0 \text{ a.s.}$$

under only $\gamma_n \nearrow \infty$.

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