MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 12/1993

Gewöhnliche Differentialgleichungen
14.-20.03.1993

The conference was organized by H.W. Knobloch (Würzburg), J. Mawhin (Louvain-la-Neuve), and K. Schmitt (Salt Lake City).

Following a now established tradition the conference focused upon one area of current research interest in ODE. The organizers had chosen

Asymptotic behavior of solutions and the structure of attractors.

as the conference title and they were fortunate to have a positive answer from a very distinguished selection of the world experts in this area, as revealed by the list of participants and the titles of the lectures. A few defections were caused by the severe weather conditions on the East coast of the USA.

Forty-five scientists from ten different countries followed the Institute's invitation to attend the conference. There were a total of thirty-eight lectures presented during the week. Most of them were closely related to the conference title and described recent important progress in the theory and in the applications.

Several of the lectures were devoted to *chaos theory* and in particular to recent progress in shadowing, links with knot theory, chaos and discretization, homoclinic and heteroclinic orbits, fractals as iterated functions systems, global analysis of cellular automata, structural stability, Hausdorff dimension of the attractors, chaos in retarded differential equations.

There were several contributions to *bifurcation theory*, in particular to secondary bifurcation of periodic solutions in retarded differential equations, bifurcation with symmetry, bifurcation from singular solutions in elliptic equations, bifurcation in reaction-diffusion systems and bifurcation from the continuous spectrum.

Progress was also reported on *reaction-diffusion systems and population dynamics*, with emphasis on the structure of the global attractor, permanence and the influence of spatial heterogeneity.

The state of the art for important *conjectures in dynamical systems and ODE* like Hilbert sixteenth problem, Dulac, Markus-Yamabe and Jacobian conjectures was described, as well as recent progress in *fundamental techniques of the theory of ordinary differential equations* like integral manifolds, nearly integrable Haamiltonian systems and linear differential equations and systems.

Some contributions to *boundary value problems and periodic solutions* were also presentend, with special emphasis upon dry friction problems, singular second order functional differential equations, subharmonic solutions, oscillating potentials and radially symmetric solutions of quasilinear elliptic equations.

Many of the above mentioned talks were motivated by some *applications* but further lectures concentraded more on this applied side and in particular to cardiology and to control theory.

To accomodate so many lectures, it was necessary to plan a few ones in the evening, which were chosen for their historical, entertaining or general character. Intense discussions were generated of course by all the lectures, and were continued late in the evening in the traditional Oberwolfach unique spirit.

Such a spirit is of course mostly due to the kind and efficient service provided by the Institute's administration and staff which has to be acknowledged and congratulated.

The meeting was adjourned at 3.00 p.m., on Friday, March 19, 1993.

Vortragsauszüge

P. BRUNOVSKY :

An example of diffusion driven oscillations of two 2D systems

An example is presented in which it can be shown analytically that if one of the diffusion coefficients is increased the initially stable homogeneous equilibrium loses its stability in a pitchfork bifurcation and afterwards a stable periodic orbit bifurcates from the nonhomogeneous equilibrium. Up to now this phenomenon has been established numerically only. This example indicates strongly that the periodic orbit bifurcates into a stable invariant torus.

S. CANTRELL :

Reaction-diffusion models for mathematical ecology

We model the interactions of two theoretical populations which are allowed to move at random throughout a bounded habitat via systems of two weakly coupled reaction-diffusion equations. The reaction terms in these systems involve parameters which are subject to biological interpretation and which are assumed to be spatially dependent. We examine the effet of spatial heterogeneity on the long-term viability of each of the populations, whith the aim of quantifying the effect in terms of the biological parameters in the models. To this end, we employ the dynamic concept of permanence of the interacting populations, conditions for which lead directly to the spectral theory for linear elliptic boundary value problems, so that the long-term viability of the populations can be expressed in terms of eigenvalues depending on the biological parameters of the models in directly quantifiable ways. We give a number of examples, and demonstrate for the first time via reaction-diffusion equations that spatial heterogeneity can lead to coexistence in situations wherein extinction would result, were the habitat spatially homogeneous.

K. DEIMLING :

Periodic solutions of dry friction problems

Experiments show that in certain cases dry friction has to be modelled by functions of the velocity which are discontinuous at zero. this leads to discontinuous ODEs, the quantitative theory of which can be successfully developed by means of the corresponding theory for associated multivalued differential equations. We consider in particular

$$x'' + \alpha x' + \mu \mathrm{sgn}(x') + \beta x = \sin t,$$

and give a complete picture for the 2π-periodic solutions in case $\alpha = 0$. In the resonance case ($\beta = 1$) we find such solutions (with deadzones) which are globally asymptotically stable, provided $\mu \in (\pi/4, 1)$.

P. DORMAYER :

Floquet Eigenwerte und Sekundärverzweigung periodischer Lösungen von Funktionaldifferentialgleichungen

Wir betrachten die verzögerte Differentialgleichung $x'(t) = -\alpha f(x(t-1)), \alpha > 0$ wobei vorausgesetzt wird daß $f : \mathbb{R} \to \mathbb{R}$ glatt und ungerade ist, und $xf(x) > 0$ für $0 < x < a$ und einem $a > 0$ gilt. Es ist bekannt, daßes zu jedem $z \in]0, a[$ ein $\alpha = \alpha(z)$ und ein $x = x(., z) : \mathbb{R} \to \mathbb{R}$ gibt, so daß x eine periodische Lösung mit Amplitude z und Periode 4 ist. Wir betrachten die Floqueteigenwerte dieser periodischen Lösungen und geben Bedingungen für Sekundärverzweigung periodischer Lösungen an. Unser Hauptergebnis ist, daßz.B. für $f = \sin$ eine Folge $\alpha_k = \alpha(z_k), k \in \mathbb{N}$, existiert, für die gilt: (i) $(\alpha_k, x(., z_k))$ ist ein Verzweigungspukt, (ii) $\alpha_k \to \infty$ für $k \to \infty$ und (iii) $\alpha_{k+1} - \alpha_k \to \pi$ für $k \to \infty$. Damit ist gezeigt, daßFunktionaldifferentialgleichungen unendlich viele Sekundärverzweigungspunkte haben können.

L. ERBE :

Boundary value problems for singular second order functional differential equations

We consider BVPs of the form

$$(1) \qquad y''(x) + f(x, y(\tau(x))) = 0,\ 0 \le x \le 1$$

$$(2) \qquad \alpha y(x) - \beta y'(x) = \mu(x),\ x \in [a, 0],\ \gamma y(x) + \delta y'(x) = \nu(x),\ x \in [1, b].$$

Here $f : (0,1) \times (0, \infty) \to (0, \infty)$ is continuous and decreasing in y for each fixed x and integrable on $[0,1]$ in x for each fixed y and satisfies

$$\lim_{y \to 0+} f(x, y) = \infty \text{ uniformly on compact subsets of } (0,1),$$

$$\lim_{y \to \infty} f(x, y) = 0 \text{ uniformly on compact subsets of } (0,1).$$

The function $\tau(x)$ is continuous on $[0,1]$ and satisfies $\inf_{[0,1]} \tau(x) = 1$ and $\sup_{[0,1]} \tau(x) > 0$. The functions $\mu(x), \nu(x)$ are defined on the intervals $[a, 0]$ and $[1, b]$ respectively, where $a = \min\{0, \inf \tau(x)\}, b = \max\{1, \sup \tau(x)\}$ with $\mu(0) = \nu(1) = 0$. Existence and uniqueness results are obtained via a monotone fixed point theorem. The prototype is the equation involving a function $f(x, y)$ of the form $f(x, y) = p(x) y^{-\lambda}, \lambda > 0$. (Joint with Qingkai Kong).

W.N. EVERITT :

On a property of the Titchmarsh-Weyl m-coefficient of second-order ordinary linear differential equations

The Titchmarsh-Weyl m-coefficient is a Nevanlinna function of the complex spectral parameter λ associated with the Sturm-Liouville differential equation

$$-(py')' + qy = \lambda wy \text{ on } [a, b). \qquad (^*)$$

Each m-coefficient is associated uniquely with a self-adjoint operator generated by (*). The m-coefficients of (*) can be considered as limits of m-coefficients of the differential equation considered on the compact interval $[a, \beta]$, as $\beta \to b$. The lecture discusses the properties of this limit process.

B. FIEDLER :

The global attractor of semilinear parabolic equations

We consider dissipative equations

$$u_t = u_{xx} + f(x, u, u_x),\ 0 < x < 1,$$

with Neumann boundary conditions. Let $\mathcal{A}_f$ denote the global attractor. By a gradient structure, $\mathcal{A}_f$ consists of the set E_f of equilibria, and of orbits connecting equilibria. Let the equilibria $\{v_1, \ldots, v_N\} = E_f$ be all hyperbolic. Comparing the orderings of $v_1(x), \ldots, v_N(x)$ at $x = 0, x = 1$, resp., defines a permutation $\pi \in S_N$. Using this ODE input information encoded in π determines the attractor $\mathcal{A}_f$: given π, there is a constructive procedure to determine whether or not $v, w \in E_f$ are connected by a PDE trajectory. This result generalizes earlier work by P. Brunovsky and the author. It was obtained jointly with C. Rocha.

A.M. FINK :

An example where the dynamics are determined by the averages

Consider the problem:

$$(1) \qquad x' = k_1 x(\sigma - x - y) - \beta_1 x - cx, \; y' = k_2 y(\sigma - x - y) - \beta_2 y + cx,$$

where all constants are positive and one is looking at the dynamics in the quadrant $x, y \geq 0$. The quantities $k_2\sigma - \beta_2$ and $\Delta = \frac{\beta_2}{k_2} - (\frac{\beta_1}{k_1} + c)$ determine the dynamics according to their signs. If the β_i and c are replaced by positive almost periodic functions, then the dynamics are similar to the system (1) where the constants are the mean values of the functions β_i and c. A slight generalization is also considered.

D. FLOCKERZI :

Integral manifolds in nonlinear control theory

By means of examples of qualitative control problems like modal synthesis, asymptotic stabilization and the synthesis of stable periodic solutions, we show how dynamic feedback can be used to generate integral manifolds that solve the respective control problem. On the other hand we present disturbance attenuation and tracking problems where an appropriate feedback turns given manifolds in the extended phase space into integral manifolds with the properties of normal attractivity and asymptotic phase. Thereby the standard state and error feedback regulator problems can be solved for nonlinear affine control problems.

A. FONDA :

Subharmonic solutions for some second order differential equations

Some existence results on subharmonic solutions of both scalar equations and conservative systems are reviewed. In particular, the existence of subharmonics for a simplified suspension bridge model is illustrated, as well as for some equations modeling the motion of an electric charge in a Coulombian field.

G. FREILING :

Nonsymmetric matrix Riccati equations

We consider matrix Riccati differential equations

$$W' = B_{21} + B_{22}W - WB_{11} - WB_{12}W \qquad \text{(RDE)}$$

and the corresponding algebraic Riccati equations

$$0 = B_{21} + B_{22}W - WB_{11} - WB_{12}W, \qquad \text{(ARE)}$$

where $W, B_{11}, B_{12}, B_{21}, B_{22}$ are matrices of dimensions $m \times n, n \times n, n \times m, m \times n$ and $m \times m$ respectively. It is known that matrix Riccati equations are playing an important role in many branches of applied mathematics and in particular in systems theory; nonsquare matrix Riccati equations appear for example in Nash and Stackelberg control problems, where the solutions of Riccati equations are used to determine the optimal open loop strategies.

In the first part of the lecture we present a general representation formula for all solutions of (RDE) with constant coefficients B_{ij} and we show that a similar formula can be obtained if these coefficients are T-periodic functions or are polynomially dependent on t or on a complex parameter λ. Further we explain how our representation formula can be used for the description of the phase portrait of (RDE) and for a parametrization of all solutions of (ARE).

In the second part of our talk we apply our results to the investigation of coupled matrix Riccati equations appearing in open-loop differential games.

P. HABETS :

A nonlinear BVP with potential oscillating around the first eigenvalue

In this talk, we consider the Dirichlet boundary value problem

$$u'' + u + f(t,u) = 0, \; u(0) = u(\pi) = 0, \qquad (1)$$

assuming that the potential

$$F(t,u) = \int_0^u f(t,s)\,ds$$

satisfies the conditions

$$\limsup_{a\to+\infty} \int_0^\pi F(s, a\sin s)\,ds = +\infty,$$

$$\liminf_{a\to+\infty} \int_0^\pi F(s, a\sin s)\,ds = -\infty.$$

We prove there exist sequences (u_n) and (u_n^*) of positive solutions of (1) such that

$$\lim_{n\to\infty} \phi(u_n) = +\infty, \quad \lim_{n\to\infty} \phi(u_n^*) = -\infty,$$

where

$$\phi(u) = \int_0^\pi \left[\frac{u'^2(t)}{2} - \frac{u^2(t)}{2} - F(t, u(t))\right] dt$$

is the functional associated with (1), whose singular points are solutions of (1). The variational approach we use extends to different boundary value problems such as the periodic or the Dirichlet problem for elliptic PDE.

M.W. HIRSCH :

Shadowing, asymptotic phase, and stochastic perturbation

A new "exponential shadowing" theorem is applied to asymptotic phase and to stochastic perturbations. Let X be a complete metric space, $f : X \to X$ a continuous map, $K \subset X$ a positively invariant closed set. The distance in X is written $\|x-y\|$. The *expansion constant* $EC(f, X, K)$ is the supremum of all numbers $\nu > 0$ such that there exists $\rho^* \geq 0$ for which

$$x \in K, 0 \leq \rho \leq \rho^* \Rightarrow f(B(\rho, x)) \supset B(\nu\rho, f(x)),$$

where $B(\rho, x) = \{y \in X : \|x-y\| \leq \rho\}$. Define the *expansion rate*

$$\mathcal{E}(X, f, K) = \sup_{n>0} EC(f^n, X, K)^{1/n}.$$

Theorem 1. *Assume $f|K$ is uniformly continuous. Let $\{a_k\}$ be a $\lambda-$pseudo-orbit in K : this means $0 \leq \lambda < 1$ and*

$$\limsup_{n\to\infty} \|f(a_n) - a_{n+1}\|^{1/n} \leq \lambda.$$

Suppose $\mathcal{E}(f, X, K) = \mu$ and $\lambda < \min\{1, \mu\}$. Then there is a unique orbit $\{f^n y\}$ in K which is λ-shadowed by $\{a_k\}$; that is : for some $l \geq 0$,

$$\limsup_{n\to\infty} \|f^n y - a_{n+l}\|^{1/n} \leq \lambda.$$

Asymptotic phase. Suppose $x \in X$ is attracted to K at rate $\lambda, 0 \leq \lambda < 1$:

$$\limsup_{n\to\infty} \text{dist}\,(f^n(x), K)^{1/n} \leq \lambda.$$

Theorem 2. *Assume f is Lipschitz. If $\lambda < \mu = \mathcal{E}(f, X, K)$. Then x has an asymptotic phase in K: there is a unique orbit $\{f^n y\}$ in K such that for some $l \geq 0$:*

$$\limsup_{n\to\infty} \|f^{n+l}(x) - f^n(x)\|^{1/n} < \lambda.$$

Stochastic perturbations. Let $f : \mathrm{R}^d \to \mathrm{R}^d$ be a Lipschitz vector field with flow $\varphi = \{\varphi_t\}_{t\in\mathrm{R}}$. Consider a stochastic process

$$X_{n+1} = X_n + \gamma_n(f(X_n) + U_{n+1}),$$

where the X_n, U_{n+1} are random variables on a probability space $(\Omega, \mathcal{F}, P)$ with values in R^d, and $\gamma_n \to 0$ in R_+. The goal is to find conditions ensuring that $\{X_n\}$ approaches a solution to $\frac{dx}{dt} = f(x)$. Assume there is a compact set $K \subset \mathrm{R}^d$ positively invariant under φ, such that almost surely $X_n \in K$ for all $n \geq 0$. Suppose further that $E(U_{n+1}|\mathcal{F}_n) = 0$, where $\mathcal{F}_n$ denotes the σ-field of events up to time n.

Theorem 3. *(Michel Benaim). Assume further :*
(a) $\sup \|U_{n+1}\| < \infty$ *almost surely.*
(b) $\limsup_{n\to\infty} \frac{\log \gamma_n}{\sum_{i=0}^{n} \gamma_n} = -\alpha < 0$,
(c) $e^{-\alpha/2} < \mu = \sup_{t>0} \mathcal{E}(\varphi_t, \mathrm{R}^d, K) = \sup_{t>0} \inf_{x\in K} \|D\varphi_{-t}(\varphi_t x)\|^{1/t}$,
(d) $\sum \lambda_n = \infty$, $\sum \lambda_n^2 < \infty$.
Then there exists a random variable $Y : \Omega \to \mathrm{R}^d$ *such that almost surely*

$$\lim_{n\to\infty} \|X_n - \varphi_{\gamma_1+\ldots+\gamma_n}(Y)\| = 0.$$

The hypotheses on γ_n hold if $\gamma_n = c/n$ for sufficiently small $c > 0$, or if $\gamma_n = \frac{1}{n \log n}$.

Note : Benaim shows the limit set of $\{X_n\}$ in R^d is a compact, connected φ-invariant subset of the chain recurrent set of φ.

To appear in : "Control theory, Dynamical Systems and Geometry of dynamics", K.D. Elworthy, W.N. Everitt, E.B. Lee ed., Marcel Dekker.

J. HOFBAUER :

Stability of heteroclinic cycles

Robust heteroclinic cycles arise in dynamical systems with symmetry and in ecological differential equations

$$x_i' = x_i f_i(x_1, \ldots, x_n),\ i = 1, \ldots, n,\ x_i \geq 0.$$

We associate to such a heteroclinic cycle a "characteristic matrix" A_1 consisting of the external eigenvalues at the fixed points on the cycle. Instead of Poincaré maps we use "average Lyapunov functions" of the form $P = \prod x_i^{p_i}$ to analyze their stability. This leads to systems of linear inequalities of the form $p > 0, A_p > 0$. Matrices A with this property are called "semipositive". For "simple" heteroclinic cycles (dim $W^u = 1$ at each point) this leads to a characterization of (in)stability in terms of M-matrices. A special case are planar heteroclinic cycles where $\det A = \prod \lambda_i - \prod \mu_i <$ or > 0 is the condition for (in)stability. More general (multiple) heteroclinic cycles are repelling iff $A^{-1} \geq 0$.

V. HUTSON :

Asymptotics for reaction-diffusion systems

Reaction diffusion equations are frequently used to model problems in genetics, ESS theory and ecology, when species are both interacting and dispersing. A problem of major importance in practice is to determine conditions under which all species 'coexist' in the long term. A definition (called permanence) of the term 'coexistence' is now often taken to mean that species densities are repelled uniformly by the boundary (corresponding to zero species densities), and it is not necessary that the densities should approach stationary states asymptotically. A number of techniques have been developed recently for tackling this problem, and they have some mathematical interest in their own right.

In this talk, one of these techniques will be described, and an application given to problems with zero Dirichlet conditions. Conditions for permanence involve the signs of eigenvalues of certain linear operators. As a bonus, by means of a fixed point theorem, we obtain some of the standard conditions for the existence of stationnary interior coexistence states.

F. KAPPEL :

An ODE model for fundamental regulation processes in the cardiovascular system

Based on the four compartment model by Grodins we develop a model for the response of the cardiovascular system to a short term submaximal workload. Basic mechanisms included in the model are the Frank-Starhing law of the heart, the Bowditch effect and autoregulation in the peripheral regions. A fundamental assumption is that the action of the feedback control is represented by the baroreceptor loop and minimizes a quadratic cost functional. Simulation results show that the model provides a satisfactory description of data obtained in bicycle ergometer tests.

H. KIELHOFER :

Uniqueness of global positive solution branches of semilinear elliptic problems with symmetry

We consider the problem

$$\Delta u + \lambda f(u) = 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega,\ u > 0 \text{ in } \Omega \quad (1)$$

where Ω is some symmetric domain in $\mathbb{R}^2$ and $f = \mathbb{R}_+ \to \mathbb{R}$ fulfills $f(0) \geq 0$. We prove : To any $c > 0$ where $f(c) \neq 0$, there is a unique solution (λ, u) of (1) with

$\|u\|_\infty = c$. This solution is on a global smooth curve $\{(\lambda, u)\}$ of solutions of (1) with sign λ = sign $f(c)$. This curve can be parametrized by the amplitude $\|u\|_\infty \in (a, b)$ with $0 \le a < b \le \infty$, characterized by $f(c) \ne 0$ for $c \in (a, b)$. The condition $f(0) \ge 0$ is sharp. The behavior at $\|u\|_\infty = 0$ is discussed as well.

N. KOKSCH :

Comparison systems and properties of special integral manifolds

In this talk we consider the ordinary differential system

$$x_1' = f_1(t, x_1, x_2),\ x_2' = f_2(t, x_1, x_2) \qquad (*)$$

with $f \in C^1(F, X_1 \times X_2)$, $F \subseteq [0, \infty) \times X_1 \times X_2$, $X_i = \mathbb{R}^{n_i}$. Let $s : W_0 \to X_2$, $W_0 \subseteq X_1$, be a given C^1-function. We introduce the integral manifold M which is generated by the solutions of (*) passing through $x_2 = s(x_1)$ at $t = 0$. Let W be the projection of M into $[0, \infty) \times X_1$. Our aim is to find conditions which guarantee that M is the graph of a C^1-function $S : W \to M$. Moreover, we investigate the attractivity of M and the existence of asymptotic phases in M. These problems can be solved by using of nonlocal assumptions, quadratic Lyapunov functions and comparaison systems.

T. KUPPER :

On the bifurcation structure of nonlinear perturbations of Hill's equation at boundary points of the continuous spectrum

Nonlinear perturbations of Hill's equations have been studied as a first application of a general operator theoretic approach to treat bifurcation at boundary points of the continuous spectrum. It has been established that there is bifurcation into the gap at distinguished boundary points. Moreover for fixed parameters in the gap there are m distinct solutions where m can be characterized by the number of negative eigenvalues of an associated linear eigenvalue problem. For a class of nonlinear Hill's equations with a nonlinearity concentrated on a finite interval $[-N, N]$ we are able to reduce the problem to an auxiliary nonlinear Sturm-Liouville problem with parameter dependent boundary conditions. The reduction is based on the knowledge of the stable/unstable spaces of the linearized problem. Although the reduced problem is of a complicated nature we can analyze its bifurcation structure by a modified Lyapunov-Schmidt procedure. In that way we provide a detailed analysis of both the reduced and the original problem and we can explain various phenomena which occur in connection with bifurcation from the continuous spectrum. In particular we detect global effects of the presence of continuous spectrum and we provide a mechanism to understand results on the various numbers of solutions.

A. LASOTA :

From fractals to differential equations

Our starting point is the observation (Barnsley) that fractals can be defined as the attractors of iterated function systems (IFS). A classical IFS is given by a finite sequence of transformations $S_i : X \to X$, $i = 1, \dots, N$ where X is a locally compact metric space and by a probability vector $p_i : X \to [0,1]$, $\sum_i p_i(x) = 1$ for $x \in X$. Every IFS defines a Markov operator $P : \mathcal{M} \to \mathcal{M}$ acting on the space $\mathcal{M}(X)$ of Borel measures on X. We generalize the definition of IFS by replacing the finite sequences S_i, p_i, $i = 1, \dots, N$ by families $S_t : X \to X, p_t : X \to [0,1], t \in T$, where T is a measure space and $\int_T p_t(x)\, dt = 1$ for $x \in X$. Again a generalized IFS defines a Markov operator $P : \mathcal{M} \to \mathcal{M}$. We proved (A. Lasota, J.A. Yorke) a convergence theorem which can be applied to this class of operators. In particular it is applied to Poisson driven differential equation (A. Lasota, J. Traple) of the form

$$dx = b(x)dt + \sigma(x)d\xi_x, \; x \in \mathrm{R}^d,$$

where ξ_x is a Poisson process with an intensivity $\lambda(x)$ depending on x.

N.G. LLOYD :

Centres and limit cycles in two-dimensional systems

Let the number of limit cycles of the system

$$x' = P(x,y), \; y' = Q(x,y) \qquad (s)$$

be $r(s)$. Part of Hilbert's sixteenth problem is to obtain information about

$$H_n = \sup\{r(s) : P, Q \text{ are polynomials of degree } \le n\}.$$

When P and Q are polynomials, it is known that $r(s) < \infty$, but it remains unproved that $H_n < \infty$, even for $n = 2$. We have recently shown that $H_n \ge 0(n^2 \log n)$.

The talk is designed to highlight the interplay between obtaining information about the number of limit cycles which can bifurcate, from, e.g., a critical point or from the set of orbits forming a centre, and conditions for the integrability of the system. Two systems will be used as illustrative examples :

(1) $x' = y, \; y' = -x + a_1x^2 + a_2xy + a_3y^2 - a_4x^3 + a_5x^2y + a_6xy^2 + a_7y^3,$

(2) $x' = y(x+1), \; y' = -x - a_1x^2 - a_2x^3 - a_4x^4 - a(1+ux)xy - wy^2.$

Necessity of conditions for a centre is proved by computing focal values, and the bifurcation of limit cycles can be investigated at the same time. Sufficiency is proved by constructing a Dulac function which is a product of powers of invariant polynomials. Both make intensive use of Computer Algebra. In case (1) necessity is particularly demanding of resources. In (2) not all the cases of a centre are found by the invariant curve construction, and the method of Cherkar, involving transformation to a Liénard system, is used. There are instances in which invariant curves do not exist.

S. MAIER

Convergence for radially symmetric solutions of quasilinear elliptic equations is generic

We prove that radially symmetric solutions $w = w(x)$, $x \in \mathbb{R}^n$ and $n > 1$, of

$$\Delta w + f(w) = 0,$$

and of

$$\operatorname{div}(A(|Dw|)Dw) + f(w) = 0,$$

have a definite limit at infinity under reasonable assumptions on the coefficient function $A \in C^1(\mathbb{R}_0^+)$ and for all nonlinearities $f \in C^1(\mathbb{R})$ with a regular value zero.

L. MARKUS :

Hybrid control theory and nonlinear evolution equations

Certain types of nonlinear evolution equations, arising from hybrid control theory, are proved to be globally asymptotically stable. The hybrid control systems consist of an infinite dimensional dynamical system (P.D.E. of a wave or an elastic vibration), linked at the boundary to a finite dimensional dynamics (O.D.E. of a nonlinear oscillator), through which an appropriate feedback controller is applied. The methods involve energy inequalities and Lyapunov functionals. Problems concerning rate of decay are difficult, and remain unsolved in some cases.

A. MIELKE :

Chaos and knots

We give a small introduction to knot theory and braids as far as they concern ODEs in 3 dimensions. For a continuous mapping $f : D \to D$ (= closed unit disc in $\mathbb{R}^2$) with a periodic orbit $\{x_1, \ldots, x_n\}$ (where $x_{i+1} = f(x_i)$) we introduce the stretching factor as follows. Let γ be a loop in $\tilde{D} = D \setminus \{x_1, \ldots, x_n\}$ and $[\gamma] \in \Pi_1(\tilde{D})$ the loop class. With the length $l([\gamma]) = \inf\{\text{length}(\tilde{\gamma}) : \tilde{\gamma} \in [\gamma]\}$, we define $G_f([\gamma]) = \limsup_{n\to\infty} \frac{1}{n} \log l([f^n(\gamma)])$ and the stretching factor $G(f; \tilde{D}) =$

$\sup\{G_f([\gamma]) : [\gamma] \in \Pi_1(\tilde{D})\}$. According to Manning (1975) the topological entropy $h(f, D)$ always exceeds $G(f; \tilde{D})$. We apply this theory to a Hamiltonian system with an orbit being homoclinic to a saddle-center. Close to the homoclinic orbit we construct nontrivial knots implying positive entropy in the associated return map.

C. OLECH :

Global asymptotic stability and the Jacobian conjecture

G.H. Meisters and the author (1988) proved that the polynomial autonomous system on the plan is globally asymptotically stable if the Jacobian matrix of the right-hand side is stable for each point. This result constitutes a confirmation of Markus-Yamabe conjecture (1960) in the case $n = 2$ and polynomial vector field, as well as a contribution to the real Jacobian conjecture. The latter states that the not vanishing of Jacobian of a polynomial map implies injectivity. This and other relations between O.D.E. and injectivity was discussed. In particular the following problem was mentioned : Is the system $x' = -x + H(x) + a$ globally asymptotically stable for any constant vector a if H is homogeneous polynomial map and the Jacobian matrix $H'(x)$ is nilpotent. This is a special case of Markus-Yamabe conjecture and the positive answer to this problem for H of degree three will lead to the solution of the Jacobian conjecture (Keller, 1939), that the polynomial map with constant nonzero Jacobian is invertible; that is the inverse map is also polynomial.

K. PALMER :

Shadowing orbits of differential equations

A new notion of shadowing of a pseudo-orbit, an approximate solution, of an autonomous system of ordinary differential equations by an associated newly true orbit is introduced. Then a general shadowing theorem for finite time, which guarantees the existence of shadowing in ordinary differential equations and provides error bounds for the distance between the true and the pseudo-orbit in terms of computable quantities, is proved. The use of this theorem in numerical computation of orbits is illustrated on the Lorenz equations.

H.O. PEITGEN :

Cellular automata, attractors and dynamical systems

The aim of this paper is to develope tools for the global analysis of cellular automata. We will relate cellular automata (CA) with several associated dynamical systems, such as matrix substitution systems (MSS) and hierarchical iterated function systems (HIFS). In several ways the idea is to use methods and concepts from dynamical systems theory to discuss and understand problems of scaling and self-similarity features of a large class of CA which includes linear cellular automata (LCA) in all dimensions and of all degrees. In particular we discuss the following problems:

- What is the formal description of rescaling procedures ?
- Which are the classes of CA that are scaling ?

For those CA which are scaling we introduce the rescaled evolution set, which captures the global evolution of a CA. The next set of problems for which we prove answers is :

- What are the geometrical invariants of rescaled evolution sets ?
- How can one describe and decipher the self-similarity features of the rescaled evolution set ?
- What is the Hausdorff dimension of the rescaled evolution set ?

The paper is joined work with Fritz von Haeseler and Gencho Skordev from the University of Bremen.

V.A. PLISS

A class of periodic systems with hyperbolic non-wandering set

The system of differential equations

$$\mu\frac{dx}{dt} = X(x) + f(t) \qquad (1)$$

is considered, $x, X, f \in \mathrm{R}^n$, $\mu > 0$ is a small parameter, $X \in C^1(\mathrm{R}^n), f(t) = \overline{f}(t - m\omega)$, where m is arbitrary integer and $\omega > 0, \overline{f} \in C^1([0,\omega])$ and $\overline{f}(0) \neq \overline{f}(\omega)$. Let $|f| \leq M$. Assumptions :

I. There exists a function $v(x)$ with properties $v \in C^1(\mathrm{R}^n)$, $v(x) \to +\infty$ as $|x| \to \infty$ and $\frac{\partial v}{\partial x}X + |\frac{\partial v}{\partial x}|\mu < 0$ for $|x| \geq r$ with some positive r.

Consider the family of autonomous systems

$$\frac{dx}{dt} = X(x) + f(\tau), \qquad (2)$$

where $\tau \in [0,\omega]$ is a parameter.

II. All systems (2) are systems of Morse-Smale type without periodic solutions. Let $f_i(\tau)(i = 1,\ldots,k)$ be a solution of equation $X(x) + f(\tau) = 0$. In accordance with II the real parts of the eigenvalues of $\frac{\partial X(f_i(\tau))}{\partial x}$ are never zero. Denote by $f_{kj}(\tau)$ those $f_i(\tau)$ for which the matrix $\frac{\partial X(f_i(\tau))}{\partial x}$ has k eigenvalues with negative real parts, $k = 0,1,\ldots,n$, $j = 1,\ldots,s_k$, $s_0+\ldots+s_n = K$. Denote by $W^s(f_i(\tau))$ and $W^u(f_i(\tau))$ the stable and unstable manifolds of the rest point $f_i(\tau)$ of the system (2).

III. $f_i(0) \notin \bigcup_{k=1}^{n-1}\bigcup_{j=1}^{s_k} W^s(f_{kj}(\omega)) \bigcup W^u(f_{kj}(\omega))$,

$$f_i(\omega) \notin \bigcup_{k=1}^{n-1}\bigcup_{j=1}^{s_k} W^s(f_{kj}(0)) \bigcup W^u(f_{kj}(0)),\ i = 1,\ldots,K.$$

IV. All intersections

$$W^s(f_i(0)) \bigcap W^u(f_j(\omega)), W^u(f_i(0)) \bigcap W^s(f_j(\omega))$$

are transversal.

Theorem. *If assumptions I-IV are satisfied then there exists a $\mu_0 > 0$ such that if $\mu \leq \mu_0$ then the system (1) is structurally stable.*

V. REITMANN :

Introduction of a metric tensor in Hausdorff dimension estimates of attractors

The Hausdorff dimension of a compact invariant set of dynamical systems on Riemannian manifolds is considered. Using a special singular value decomposition for the linear operators arising from the linearization of flows, upper bounds for the Hausdorff dimension of invariant sets are given.

H. RUSSMANN

Integration in the presence of small divisors

We extend Kolmogorov's theorem on the preservation of quasi periodic motion under small perturbations of the Hamiltonian to those cases in which the frequency vector $\omega = (\omega_1, \ldots, \omega_n) \in \mathbb{R}^n$ of the quasi periodic motion satisfies the condition

$$| < k, \omega > |^{-1} \leq \Omega(|k|), \ k = (k_1, \ldots, k_n) \in \mathbb{Z}^n \setminus \{0\},$$

where $< k, \omega >= k_1\omega_1 + \ldots k_n\omega_n$, $|k| =< k, k >^{1/2}$, and $\Omega : [1, \infty[\rightarrow]0, \infty[$ is a non decreasing continuous function with $\int_1^\infty \log \Omega(t) \frac{dt}{t^2} < \infty$. This condition is equivalent to the condition formulated by A.D. Bruno for differential equations and mappings holomorphic near a singular point.

References : A.D. Bruno, Local Methods in Nonlinear Differential Equations, Springer Verlag

H. Rüssmann, On the frequencies of quasi periodic solutions of analytic nearly integrable Hamiltonian systems, Proc. Euler Intern. Math. Institute St. Petersburg, Birkhäuser Verlag.

R. SCHAAF :

Bifurcation from singular solutions

Regular positive solutions of a parameter dependent stationary diffusion problem with a fast growing source term can approximate singular solutions. More precisely we look at

$$(^*) \qquad \Delta u + \lambda^2 f(u) = 0 \ (B^n), \ u = 0 \ (\partial B^n),$$

where B^n is the unit ball in $\mathbb{R}^n$ and

$$f(u) = (1+u)^p + \text{l.o.t.}, \ p > \frac{n+2}{n-2}, \ n \geq 3,$$

$$f(u) = (1-u)^{-p} + \text{l.o.t.}, \ p > 0, \ n \geq 2,$$

$$f(u) = e^u + \text{l.o.t.}, \ n \geq 3,$$

$f(u)/u \geq c > 0$ for $u > 0$.
Under (more or less) these assumptions a singular solution $(\lambda_\infty, u_\infty)$ exists, which under a transformation turns out to correspond to the unstable manifold of a hyperbolic stationary point of an autonomous ODE in 3 variables. This manifold attracts solutions coming from the regular regime in fast time. Time estimates are possible by exploiting a Lie group structure of the problem in the limit $u \to \infty$. A more refined asymptotic analysis shows that (*) has infinitely many solutions for $\lambda = \lambda_\infty$ which become more and more unstable or else $(\lambda_\infty, u_\infty)$ is approximated by stable solutions monotically in λ.

J. SCHEURLE

Discretization of autonomous equations and homoclinic orbits (joint with B. Fiedler)

One-step discretizations of order p and step size ϵ of ordinary differential equations $x' = f(\lambda, x)$ can be viewed as time-ϵ maps of

$$x' = f(\lambda, x) + \epsilon^p g(\epsilon, \lambda, \frac{t}{\epsilon}, x), x \in \mathbb{R}^n, \lambda \in \mathbb{R},$$

where g has period 1 in t/ϵ. This is a rapidly forced nonautonomous system. We study the behaviour of a homoclinic orbit $\Gamma, \epsilon = 0, \lambda = 0$ under discretization. Under generic assumptions, Γ turns out to break and the perturbed stable and unstable invariant manifolds turn out to intersect transversally for small positive ϵ, which gives rise to chaotic behaviour. However the transversality effects can be estimated from above to be exponentially small in ϵ, if f is analytic. For example, the length $l(\epsilon)$ of the parameter interval λ for which the local invariant manifolds have nonempty intersections can be estimated by $l(\epsilon) \leq Ce^{-2\pi\eta/\epsilon}$, where C, η are positive constants. The factor η is related to the minimal distance from the real axis of the poles of $\Gamma = \Gamma(t)$ in the complex time plane.

K. SCHNEIDER

Melnikov's techniques for singularly perturbed systems

The existence of a transversal homoclinic orbit in a dynamical system implies the existence of an invariant Cantor set on which the dynamics is topologically equivalent to the Bernoulli shift. Melnikov's method is an analytical tool to prove the existence of transversal homoclinic orbits for systems of the type (*) $dx/dt = f(x) + \epsilon g(x,t)$, where ϵ is sufficiently small. In order to be able to apply Melnikov's method we need the knowledge of a homoclinic orbit $\gamma(t)$ of $dx/dt = f(x)$ and a solution $\psi(t)$ of the system $dy/dt = -f'(\gamma(t))^T y$ which is bounded on $\mathbb{R}$ (and $\neq 0$). Additionally we have to show that the Melnikov integral $\int_{-\infty}^{\infty} \psi(t)^T g(\gamma(t), t + t_0)\, dt$ has a simple zero. If f is higher-dimensional these assumptions can be verified analytically only in a few cases. In our talk we present problems which lead to singularly perturbed systems of the type

$$(^{**}) dx/dt = f_0(x,y) + \epsilon f_1(x,y,t), \epsilon dy/dt = g_0(x,y) + \epsilon g_1(x,y,t).$$

We show that by applying results from the theory of invariant manifolds (**) can be reduced to a lower order system (*). This approach permits the non-autonomous perturbation to be almost periodic and can be extended to systems with delays in the fast subsystem. We present an example related to pattern formation and put some open problems (systems with singular homoclinic orbits).

G.R. SELL

Network Dynamics and cardiac modeling

We study the dynamics of the pacemaker behaviour in the sino-atrial node of a mammalian heart. For each individual cell in the sinus node (i.e., the pacemaker node) we use the Noble model, which is a 14-dimensional system of ordinary differential equations representing the voltage and chemical reactions. For the full 2D (or 3D) network we allow the parameters in the individual cells to vary with space, and we introduce a "nearest-neighbor" diffusion for the voltage, with a diffusion rate $r > 0$. A calculation of the behavior of the sino-atrial node was performed on the Connection Machine CM-5 and a video produced, both at the AHPCRC at the University of Minnesota. This video suggests that the voltage u in the sinus node is nearly homogeneous, i.e., independent of spatial dependence. We introduce the spatial average operator M where $u = u^M + u^S, u^M = Mu$ does not depend on space and $Mu^S = 0$. We then show that

$$\|u^S(t)\|^2 \leq e^{-crt}\|u_0^S\|^2 + \frac{Q^2}{Cr^2},\ t \geq 0,$$

where $C > 0, Q > 0$, and $\|.\|$ denotes an L^2-norm over the sinus network. One then has $\|u^S(t)\|^2 \to 0$, as $r \to \infty$. While $\|u^S(t)\|^2$ is small, the video shows a surprising change with respect to time t. In fact the largest values of $u^S(t)$ seems to occur at those times t where $\partial_t u^M(t)$ assumes its largest values. It would be nice to develop a theory which predicts this phenomenon. If one sets $u^S = 0$ in the equations for the network dynamics of the sinus node, one obtains the equations for an "average" cell. While one would like to have a rigorous proof that the single cell model of a sinus cell does have a stable periodic orbit, the numerical evidence of this property is convincing. We then see that period of the pacemaker node is (approximately) equal to the period of the average cell, provided r is large.

This is an ongoing research project, and the collaborators include D. Noble (Oxford), R. Winslow (John Hopkins), A. Varglase (Minnesota), and M. Marion (Lyon).

A. VANDERBAUWHEDE

A general reduction result for periodic solutions near equilibria

In this talk we describe and discuss a general reduction principle for periodic solutions near an equilibrium in autonomous systems. The result combines the normal form approach with the Liapunov-Schmidt method, and is particularly useful when the linearization at the equilibrium has purely imaginary eigenvalues which are in resonance or which have nilpotencies. Also, the reduced problem keeps the particular structure (such as equivariance, reversibility, Hamiltonian structure, etc) of the original system. For Hamiltonian systems the reduction result can be, roughly, described as follows. Consider a Hamiltonian system, depending on a parameter λ, and such that the origin is an equilibrium for all λ. Given some λ_0 and some $T_0 > 0$, the problem is to describe all small T-periodic solutions for T near T_0 and λ near λ_0. The result says that such periodic solutions are in 1-1 relation to the small T-periodic solutions of a reduced Hamiltonian system, which is S^1-equivariant and which has as phase space the generalised eigenspace corresponding to the purely imaginary eigenvalues which are in resonance with the period T_0. Moreover, the reduced Hamiltonian can be calculated by bringing the original one in normal form. We also discuss an application to the Hamiltonian Hopf bifurcation.

H.O. WALTHER

On attractors of differential delay equations

Consider the equation

$$(1) \qquad x'(t) = -\mu x(t) + f(x(t-1)),$$

($\mu \geq 0, f\ C^1, f(0) = 0$ and $xf(x) < 0$ for $x \neq 0$) which models a system with one steady state ($x = 0$), governed by delayed negative feedback. A solution is called slowly oscillating (s.o.) if its zeros are spaced at distances greater than the delay (> 1).

Theorem. *(Work in progress). Suppose $\mu = 0$ and $f'(x) < 0$ for all $x \in \mathbb{R}$. Then the set of initial data $\varphi \in C = C([-1,0],\mathbb{R})$ so that the corresponding solution is* *s.o. on some unbounded interval $[t_\varphi, \infty)$ is open-dense.*

This proves an old conjecture (see e.g. a paper by Kaplan and Yorke in SIAM J. Math. Anal. 6 (1975)) on the importance of s.o. solutions. The phase curves $t \mapsto x_t \in C$ (where $x_t(s) = x(t+s)$ for $t \geq 0, -1 \leq s \leq 0$) of s.o. solutions belong to the positively invariant set $S \subset C$ of data $\varphi \neq 0$ with at most one change of sign.

Theorem. *Suppose $\mu \geq 0$ and $f'(x) < 0$ for all $x \in \mathbb{R}$. Then the global attractor $A \subset \overline{S}$ of the restricted semiflow on $\overline{S}$ is either trivial ($A = \{0\}$), or a 2-dimensional Lipschitz continuous graph in C which is homeomorphic to a disk and bordered by the orbit of a s.o. periodic solution. Phase curves in A converge to 0 or to periodic orbits as $t \to \pm\infty$.*

This applies, e.g. to Wright's equation $x'(t) = -\alpha x(t-1)[1+x(t)]$.

Theorem. *(Joint work with B. Lani-Wayda). There exist smooth C^1-functions $f : \mathbb{R} \to \mathbb{R}, xf(x) < 0$ for $x \neq 0$, so that $A \subset \overline{S}$ contains chaotic motion.*

These C^1-functions have local extrema, of course, and $\mu = 0$ here. Chaos is caused by a transversal homoclinic point of a Poincaré map $P : H \supset \Omega \to H$ associated with a hyperbolic unstable s.o. periodic solution y. The proof makes use of the following transversality criterion ("Transversality by oscillation"). A vector $\chi \in H$ is transversal to the local stable manifold $\omega^s \subset H$ of P at y_0 (i.e. $\chi \in H \setminus T_\varphi \omega^s$ for $\varphi \in \omega^s$) if and only if every nontrivial linear combination $az' + b\omega : [0, \infty) \to \mathbb{R}$ (z the solution of (1) generated by φ, ω the solution of the variational equation $v'(t) = f'(z(t-1))v(t-1)$ generated by χ) is s.o. on some unbounded interval $[t_{a,b}, \infty)$.

W. WALTER

The simplicity axiom with applications

Following the principle "Make it simple", a counterexample due to G.W.F. Hegel (from his thesis of 1801, Universität Iena) and three examples were given.

I. A simple proof due to Ray Redheffer and the author is given of the following

Theorem. *Let $f : G \to \mathbb{R} \cup \{+\infty\}$ ($G \subset \mathbb{R}^n$ open and convex) be lower semicontinuous and $|Df(x)| \leq K$ in G. Then $|f(x) - f(y)| \leq K|x-y|$ in G. Here*

$$Df(x) = \{p \in \mathbb{R}^n : \liminf_{y \to x} \frac{f(y) - f(x) - p.(y-x)}{|x-y|^2} > -\infty\}$$

$$\subset \{p \in \mathbb{R}^n : \liminf_{y \to x} \frac{f(y) - f(x) - p.(y-x)}{|x-y|^2} \geq 0\} = f'(x),$$

($f'(x)$ is the usual subgradient).

II. A simple proof is given of the following

Theorem. *Let A be a $n \times n$ matrix and $\beta < \Re\lambda_i < \alpha$ for the eigenvalues of A. Then there exists a Hilbert norm $\|.\|$ such that*

$$e^{\beta t} \leq \|e^{At}\| \leq e^{\alpha t} \text{ for } t \geq 0.$$

In the case $\alpha = -\beta = \delta$, the norm is generated by $< .,. >$,

$$< c, d >= \int_{-\infty}^{\infty} e^{-2\delta|t|}(e^{At}c, e^{At}d)\, dt.$$

III. Another approach to the maximum principle for second order ODEs is given, based on the following

Theorem. *Assume that $p(x) > 0$ a.e. in J and $u, pu' \in AC_{loc}(J)$. If u has a nontrivial local minimum in J, then $(pu')' > 0$ on a set of positive measure in J.*

The reasoning is different from the usual proof of the maximum principle.

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