

#### MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/1993

## Analysis auf lokalsymmetrischen Räumen

21. - 27.03.1993

Die Tagung fand unter der Leitung von J. Brüning (Augsburg) und W. Müller (Bonn) statt. P. Sarnak (Princeton), der ursprünglich der Tagungsleitung angehörte, mußte kurzfristig absagen.

An der Konferenz nahmen 27 Mathematiker teil, die aus Deutschland, England, Frankreich, Kanada, der Schweiz, USA sowie aus den Ländern der GUS kamen. Im Mittelpunkt des Interesses standen Spektraltheorie, automorphe Formen sowie asymptotische Aspekte der Analysis auf lokalsymmetrischen Räumen. Die Anordnung der Vorträge ließ genug Zeit für ausgiebige und fruchtbare Diskussionen. Dies und die angenehme Atmosphäre im Mathematischen Forschungsinstitut Oberwolfach trugen wesentlich zum Gelingen der Tagung bei.

## **VORTRAGSAUSZÜGE**

#### ERICH BADERTSCHER:

## The Pompeiu problem on locally symmetric spaces

Let X be a symmetric space, G its isometry group. For which bounded open subsets  $\Omega$  of X is the Pompeiu transform  $P_{\Omega}f(g) := \int\limits_{g\Omega} f(x)dx$  a one-to-one

map from C(X) into C(G)?

We define the transform  $P_{\Omega}$  on arbitrary complete Riemannian manifolds X. If X is locally isotropic, we completely characterize the balls  $\Omega$ , for which  $P_{\Omega}$  is not one-to-one on C(X) (in terms of the spherical functions on  $\hat{X}$  and the full Laplace spectrum of X). Nothing is known for balls on more general spaces X, not even on globally symmetric spaces of higher rank.

We show that in a locally isotropic space X, the collection of sets  $\Omega$ , such that  $P_{\Omega}$  is one-to-one on C(X) as well as its complement is dense in the collection of all bounded open subsets of X.

If  $\hat{X}$  is not compact, then it is not known if there exists a Jordan domain  $\Omega$  (not a ball), such that  $P_{\Omega}$  is not one-to-one on C(X), not even in the case  $X = \mathbb{R}^2$ . There are, however, many interesting such examples in the case X in locally isotropic and  $\hat{X}$  compact.

#### WILLIAM A. CASSELMAN:

## Franke's theorem for rank one groups

Let G be a reductive group defined over  $\mathbb{Q}, \Gamma \subseteq G$  an arithmetic subgroup, X the symmetric space  $G(\mathbb{R})/K$ . Around 1974 Borel conjectured that the inclusion

$$\Omega_A^* \hookrightarrow \Omega^*(\Gamma \backslash X)$$

induces an isomorphism of cohomology. Here  $\Omega_A^*$  is the complex of differential forms on  $\Gamma \backslash X$  which are also automorphic forms. Wigner's lemma can be used to modify this to replace  $\Omega_A^*$  by  $\Omega_{A,0}^*$ , the subspace of forms  $\omega$  annihilated by some power of  $n_o$ , the augmentation ?? of Z(g). Franke has now proven this conjecture.

In the proof he introduces another space  $\Omega_{temp}^*$  of differential forms which are tempered in the sense of Harish-Chandra, and proves also the analogue of Borel's conjecture for these forms.

He also considers the inclusion of  $\Omega^*_{temp}$  in  $\Omega^*$ , and in general extends this to a filtration of  $\Omega^*$ . But for rank one groups the filtration has degree two, and the quotient  $\Omega^*/\Omega^*_{temp}$  may be identified with part of the cohomology of the boundary.

In rank one, the proof of Borel's conjecture can be deduced from the analogue for tempered cohomology and some simple arguments about boundary cohomology. The proof for tempered cohomology depends on a remarkable theorem about the existence of multipliers on the tempered spectrum.

#### JENS FRANKE:

## On the cohomology of S-arithmetic groups

In a joint paper with Blasius and Grunewald we give a description in terms of automorphic forms of the cohomology of S-arithmetic subgroups of algebraic groups over a number field, where S is a non-empty set of non-archimedean

primes, generalising a result of Borel in the Q-anisotropic case. The analogous results for function fields were obtained by Harder fifteen years ago. The result turns out to be relatively simple, compared with the problem of the analytical description of the cohomology of usual congruence subgroups for number fields, because in the S-arithmetic case the problems caused by the existence of residues of Eisenstein series disappear. More precisely, Eisenstein series which depend on parameters give no contribution to the cohomology of S-arithmetic groups. For simple groups, the only contributions to the cohomology come from cusp forms which are "Steinberg" at the places in S and from constant function (which for groups of rank > 0 is a residue of Eisenstein series). It turns out that the decomposition of S-arithmetic cohomology into the Steinberg summand and the summand coming from invariant forms is rational. Products in S-arithmetic cohomology and restriction to Tarithmetic subgroups for  $T_S$  can be described explicitly. As a consequence, one gets the fact that in the cohomology of ordinary arithmetic subgroups, the subspace generated by invariant forms is the image of the restriction map from the cohomology of an S-arithmetic group, which is a rational subspace. For groups of equal rank, it is also possible to describe the rational structure on the space of invariant forms explicitly. Thus, up to a few difficult questions which are related ot Beilinson's conjecture for algebraic number fields, our understanding of S-arithmetic cohomology is fairly complete.

## RICHARD FROESE (joint work with P. Hislop and P. Perry):

# Eisenstein series for hyperbolic manifolds with irrational cusps

Let  $M = \Gamma \backslash H^n$  be an infinite volume geometrically finite hyperbolic manifold. The Laplace operator  $\Delta$  on M has continuous spectrum in the half line  $[(n-1)^2/4,\infty]$ . The Eisenstein series  $E(x,\cdot;r)$  for  $\Gamma$  are continuum eigenfunctions of  $\Delta$ . They are initially defined for r in a half plane. The goal of this work is to prove that  $E(x,\cdot;r)$  admit a meromorphic continuation to all of  $\mathbb C$ . The strategy of the proof is to obtain a functional equation  $E(x,\cdot;r)=S(r)E(x,\cdot;-r)$ , where S(r) is the scattering operator, and then show that S(r) has a meromorphic inverse in a half plane. The scattering operator S(r) is a pseudo-differential operator acting on sections of line bund-





les over  $B = \partial_{\infty} M$ . When M has cusps of non-maximal rank, B is no longer compact. In previous work, we showed how to invert S(r) for the case of rational cusps. When these are irrational cusps, this method breaks down because S(r) has a dense set of thresholds near zero. To handle this case, we introduce a Banach space which measures the singularities of a mixed Hankel-Fourier transform of  $E(x, \cdot, r)$ .

#### URSULA HAMENSTÄDT:

## Harmonic measure, eigenfunctions and rigidity

Let  $\tilde{S}$  be a rank-1 symmetric space of noncompact type and let  $h = \lim_{R \to \infty} \frac{1}{R} \log volS(x, R)$  be the volume entropy of  $\tilde{S}$  where S(x, R) is the distance sphere of radius R about  $X \in \tilde{S}$ . Then

$$-\frac{h^2}{4} = -inf \bigg\{ \frac{\int |\nabla f|^2}{\int f^2} \ \bigg| \ 0 \not\equiv f \in C_0^\infty(\tilde{S}) \bigg\}$$

is the least upper bound of the  $L^2$ -spectrum of the Laplacian  $\Delta$  on  $\tilde{S}$  (here  $\Delta$  is negative definite). Moreover,  $\Delta + \frac{h^2}{4}$  admits a Green's function and its Martin boundary can naturally be identified with the ideal boundary  $\partial \tilde{S}$  of  $\tilde{S}$ . Namely for  $\xi \in \partial \tilde{S}$  let  $\theta_{\xi}$  be a Busemann function at  $\xi$ ; then  $e^{-h\theta_{\xi}/2}$  is (up to normalization) the unique minimal positive solution of  $\Delta + \frac{h^2}{4} = 0$  with pole at  $\xi$ .

Let now M be an arbitrary compact Riemannian manifold of negative curvature with universal covering  $\tilde{M}$ . Let  $\phi^i$  be the geodesic flow on the unit tangent bundle  $T^1M$  of M leaving invariant the stable foliation  $W^s$  and the strong unstable foliation  $W^{su}$ . If f is a continuous function  $\tilde{f}$  on  $T^1\tilde{M}$  and for  $\xi \in \partial \tilde{M}$  the restriction of  $\tilde{f}$  to the stable manifold in  $T^1\tilde{M}$  determined by  $\xi$  projects to a continuous function  $\tilde{f}_{\xi}$  on M. Denote by h the topological entropy of the geodesic flow on  $T^1M$ , let  $\varrho_0 \geq -\frac{h^2}{4}$  be the least upper bound for the  $L^2$ -spectrum of  $\tilde{M}$  and let  $\delta$  be the Lebesgue-Liouville measure on  $T^1M$ .

THEOREM: Assume that either

- i) The bundles  $TW^s$ ,  $TW^{su}$  are of class  $C^{1,1}$  or
- ii)  $\lim_{t\to\infty} \frac{1}{t} \log \det(d\phi^t | TW^{su}(v) = h$  for  $\delta$  almost every  $v \in T^1M$ .

Then there is a continuous function  $f: T^1M \to (0, \infty)$  such that for every  $\xi \in \partial M$  the function  $f_\xi e^{-h\theta_\xi/2}$  is the unique minimal positive  $\Delta - \varrho_0$ -harmonic function on M with pole at  $\xi$ .

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A. Ancona: Negatively curved manifolds, elliptic operators, and the Martin boundary. Ann. Math. 125 (1987), 495-536.

U. Hamenstädt: Harmonic measures for compact negatively curved manidolfds and rigidity.

#### SIGURDUR HELGASON:

Fourier transforms and differential equations on symmetric spaces

Let X=G/K be a symmetric space of the noncompact type. The definition of the Fourier transform  $f\to \tilde{f}$  on X [Bull. AMS 1965] is suggested by the analogy between horocycles in X and hyperplanes in  $\mathbb{R}^n$ . Let G=NAK, g=m+a+k be Iwasawa decompositions, g=nexpA(g)k  $(A(g)\in M)$ , put  $M=K^A, B=K/M$  and define the "inner product"  $A:X\times B\to M$  by  $A(gK,kM)=A(k^{-1}g)$ . Then  $f\to \tilde{f}$  is defined by

$$\tilde{f}(\delta,b) = \int_{x} f(x)e^{-i\delta+\varrho(A(x,b))}dx \ \lambda \in a^{\bullet}, 2\varrho = sum \ of \ roots > 0.$$

The range D(X) can be described explicitly [Ann. Math. 1973]. In the lecture we discuss application of this to differential equations: Solvability criteria in the spaces  $C^{\infty}(X), D(X), \mathcal{E}'(X)$  and the Schwart'z space  $\mathcal{S}'(X)$ . Also a new proof of the extension to X of "Asgeirsson's" mean value theorem [Math. Ann. 1937] and criteria for Huygens' principle for the wave equation on X.



#### WERNER HOFFMANN:

## Weighted orbital integrals for groups of real rank one

In the Selberg-Arthur trace formula certain non-invariant distributions on a reductive group G occur. These are the weighted orbital integrals  $J_M(\gamma,f)$  and the weighted characters  $J_M(\pi,f)$ , which are parametrized by the Levi subgroups M of G, i.e., the Levi components of parabolic subgroups P of G. In particular,  $J_G(\gamma,f)$  is the ordinary orbital integral over the G-orbit through  $\gamma \in G$ , and  $J_G(\pi,f)$  is the ordinary character of the tempered irreducible representation  $\pi$  of G. Using his local trace formula, J. Arthur has calculated the Fourier transform of  $J_M(\gamma,f)$  on the discrete part of the Schwartz space C(G). Moreover, he has defined invariant distributions  $I_M(\gamma,f)$  inductively on M and proved an invariant form of the trace formula in terms of them.

In the case when G is a semisimple Lie group of real rank one, we calculate explicitly the Fourier transform of  $I_M(\gamma, f)$ . The result can be expressed in terms of hypergeometric series F(s, 1; s + 1; z).

#### ANDREAS JUHL:

## Zeta-functions and secondary characteristic classes

A functional equation for the Ruelle zeta function of the geodesic flow of a compact locally symmetric space X of rank 1 is discussed from the point of view of hyperbolic dynamics and index theory. The zeta function  $Z_R$  is defined as the infinite product

$$Z_R(s) = \prod_{\epsilon} (1 - \ell^{-s\ell_{\epsilon}})^{-1}, Re(s) > h(h = \text{topological entropy of the flow})$$

over the prime periodic orbits.  $\ell_c$  denotes the length of the closed geodesic in X corresponding to the orbit c. Then  $Z_R$  admits a meromorphic continuation to the complex plane and if  $\dim(X)$  is even it satisfies the equation.

$$Z_R(s) \cdot Z_R(-s) \cong ((1 - \ell^{\mu is})(1 - \ell^{-i\mu s}))^{M_o}$$

where  $\cong$  denotes equality up to an exponential polynominal. Here  $\mu \in \mathbb{R}$  is the length of the periodic geodesics in the compact dual symmetric space

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 $\hat{X}^d$ . Moreover, the integer  $M_o$  can be written in the form

$$M_o = \int_{S(X)} D ,$$

where D is a differential form which is canonically associated to the flow-invariant foliations of S(X). D represents a top-degree secondary characteristic class. A conjecture is that  $\cong$  can be replaced by =.

### GERHARD KNIEPER:

## Asymptotic geometry on compact manifolds of negative curvature

The geometry of compact manifolds of negative sectional curvature is to a large extend determined by its behaviour at infinity. This idea has been extensively used like in the famous Mostow rigidity theorem.

In this talk we discuss a selection of asymptotic quantities like spherical means of function on the unit tangent bundle and the Patterson-Sullivan measure at infinity. If  $\varphi:SM\to\mathbb{R}$  is a continuous function on the unit tangent bundle of a manifold of negative curvature, the spherical mean is defined by

$$M_r \varphi(p) = \frac{1}{vol S_r(p)} \int_{S_r^+(p)} \varphi d\lambda_r$$

where  $S_r^+(p) = \phi^r S_p M$  is the image of the fiber  $S_p M$  of the unit tangent bundle SM under the geodesic flow  $\phi^r: SM \to SM$  and  $d\lambda_r$  is the density of the corresponding geodesic sphere of radius r in the universal cover. Using the unique ergodicity of the horospherical foliation we show that the limit

$$\lim_{r\to\infty}M_r(\varphi)(p)=\int\limits_{SM}\varphi d\mu_H$$

exists and is independent of  $p \in M$ . We call  $\mu_H$  the horospherical measure which in dimension 2 is the invariant measure for the horocycle flow. Furthermore, we obtain:

$$\int\limits_{SM} \varphi d\mu_H = \frac{1}{\int a(p) dvol} \int\limits_{M} \int\limits_{S_{pM}} \varphi d\mu_p \, dvol(p)$$

where  $d\mu_p$  is the Patterson-Sullivan measure at infinity radially projected onto the fibers, and

$$a(p) = \lim_{r \to \infty} \frac{1}{e^{hr}} \operatorname{vol} S_r(p)$$

denote the Margulis' asymptotic function, h the topological entropy of the geodesic flow.

As an application we show for the number of lattice points

$$N_{\Gamma}(p,q,t) = \{ \gamma \in \Gamma \mid d(p,\gamma,q) \le t \}$$

of the group  $\Gamma$  of covering transformations that

$$\lim_{t\to\infty}\frac{N_{\gamma}(p,q,t)}{e^{ht}}=\frac{a(p)a(q)}{h\int\limits_{M}a\,dvol}.$$

We show that in dimension 2 this limit does not depend on p and q if and only if the manifold has constant negative curvature.

#### JEAN-PIERRE LABESSE:

## Stabilization of the trace formula and $L^2$ -index for locally symmetric spaces

We consider "arithmetic" locally symmetric spaces  $\Gamma \backslash G/K$  with  $\Gamma$  arising from an adilic situation. We discuss the Hecke-equivariant  $L^2$ -index of Dirac operators, from the point of view of representation theory. We give a geometric expression for the index in the  $L^2$ -discrete spectrum using the Selberg trace formula in the form developped by J. Arthur. A completely explicit expression of the various contributions of conjugacy classes, even for the non semisimple ones, is conjecutally obtained using the stabilization of the trace formula which is in progress. Some low dimensional examples can be treated completely.

## JONATHAN MIDDLEBROOK HUNTLEY:

## Analysis on $GL(3, \mathbb{R})$

Several problems in the spectral theory of the Laplacian in  $SL_3(z)\backslash PGL_3(\mathbb{R})/$ 

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 $SO_3(\mathbb{R})$  are considered, as are several generalizations. Joint work with Bump shows that, in an elementary way, the Fourier expansion of a cusp form is determined by local factors, by exploiting a new theorem on asymptotic expansions of Whittaker functions. We also show, in joint work with Tepper, that the Selberg conjecture is true for  $SL(3,\mathbb{Z})$ . Finally, we show that Weyl's laws hold for lattices in  $PGL_3(R)$ . The last result, based on the Lax-Phillips method, holds in much more generality.

### LEONID PARNOVSKI:

## Spectral asymptotics of the Laplace operators on the surfaces with hyperbolic ends

We consider the asymptotics of the sum of the counting functions of continuous and discrete spectrum of the surfaces with hyperbolic cusps. Three terms of this asymptotics are obtained. Also the counting function of the resonance set of these surfaces is considered and the known estimate of the remainder term is improved.

### ZEÉV RUDNICK:

## Integer points on symmetric spaces

Counting problems for homogeneous spaces of reductive Q-algebraic groups are considered. The solution is achieved by an equidistribution result."

THEOREM: (Duke-Rudnick-Sarnak)

Let G be a semisimple Q-algebraic group,  $H \subset G$  a semisimple Q-subgroup. and  $\Gamma \subset G(\mathbb{R})$  an arithmetic group commensurable with  $G(\mathbb{Z})$ . Then for all  $f \in C_{c}(\Gamma \backslash G(\mathbb{R}))$  we have

$$\frac{1}{vol(\Gamma\cap H\backslash H(\mathbb{R}))}\int\limits_{\Gamma\cap H\backslash H(\mathbb{R})}f(hg)dh\underset{H\backslash G}{\longrightarrow}\frac{1}{vol(\Gamma\backslash G(\mathbb{R})}\int\limits_{\Gamma\backslash G(\mathbb{R})}f(x)dx$$

provided  $H\backslash G$  is a semisimple symmetric space.

#### LESLIE SAPER:

## Intersection cohomology for the seductive Borel-Serre compactification

Let X=G/K be a Hermitian symmetric space of noncompact type and let  $\Gamma$  be an arithmetically defined group of automorphisms. Consider two compactification of  $\Gamma \backslash X$  in the noncompact case. The reductive Borel-Serre compactifications  $\Gamma \backslash X$  has a rather clear local structure, but is only a real analytic space. On the other hand, the Baily-Borel-Satake compactification  $\Gamma \backslash X^{\bullet}$  is a natural compactification as a normal projective algebraic variety, but has rather complicated singularities. There is a natural projection map  $\Gamma \backslash X \to \Gamma \backslash X^{\bullet}$ . It was conjectured by Rapoport and independently by Goresky and MacPherson that there is a natural isomorphism of (middle perversity) intersection cohomology,

$$IH^*(\Gamma \backslash \hat{X}; \mathbb{E}) \cong IH^*\Gamma \backslash X^*; \mathbb{E}$$

where E is a local system coming from a finite dimensional representation of G.

The interest in the conjecture is that it would allow one to formulate a Lefschetz fixed point formula for  $IH(\Gamma\backslash X^*;\mathbb{E})$  in terms of local data on the simpler space  $\Gamma\backslash \hat{X}$ . In view of the isomorphism  $IH^*(\Gamma\backslash X^*;\mathbb{E})\cong H^*_{(2)}(\Gamma\backslash X;\mathbb{E})$  ("Zucker's conjecture", proved by Saper-Stern and Looijenga), one can hope to relate this formula to the Arthur-Selberg frace formula.

We have verified the conjecture over the three highest dimension singular strata of  $\Gamma\backslash X^*$ , and have a general argument to prove the conjecture, however, some details still need to be checked in the latter. (Over the top dimension singular stratum, this was joint work with Stern.) The analogous result for the "weighted cohomology"  $WH^*(\Gamma\backslash \hat{SX}; \mathbb{E})$  has been proven by Goresky-Harder-Mac-Pherson.

#### RAINER SCHIMMING:

## Helmholtz operators on harmonic manifolds

The Helmholtz operator  $\Delta + \delta$  on a harmonic manifold of even dimension  $n = 2m + 2 \ge 4$  admits a logarithm-free elementary solution if and only if  $\delta$  is a root of a certain polynominal of degree m. The roots are explicitly determined for the isotropic homogeneous spaces, including the two-point

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homogeneous spaces and the Lorentzian symmetric spaces. As a consequence in the latter case, Huygens' principle holds for the operator  $\Delta + \delta$  precisly for these m exceptional values of  $\delta$ .

#### JOACHIM SCHWERMER:

## Arithmetic groups, and a decompositon of spaces of autormorphis forms

Let G be a connected reductive algebraic gropu  $/\mathbb{Q}$ , let  $A_G$  be the maximal  $\mathbb{Q}$ -split torus in the center of G, and let  $(\tau, E)$  be a finite dimensional algebraic representation of G(c). If  $A_E \subset C_u^\infty mg(G(\mathbb{Q})A_G(\mathbb{R})^\circ\backslash G(\mathbb{A}))$  denote the space of functions of uniform moderate growth which are annihilated by a power of J. The annihilator of the dual representation  $E^v$  in the center of the universal enveloping algebra of Lie (h), one ha, following Langlands [1972] a direct true decomposition  $A_E = \bigoplus A_{E,\{P\}}$  ranging over the set  $\ell$  of classes of associate parabolic  $\mathbb{Q}$ -subgroups of G. In joint work with J. Franke, a refinement

$$A_E = \frac{\Theta}{\{P\} \in \ell} \quad \oplus \quad A_{E,\{P\},\varphi}$$

of this decomposition was defined where the second sum ranges over the set  $\phi_{E,\{P\}}$  of classes of associate irreducible cuspidal automorphic representations of the Levi components of elements of  $\{P\}$  subject to some technical conclusions.

This give for the cohomology of arithmetic subgroups of G, dealt with in the inclusive limit

$$H^{\star}(G,E) = H^{\star}(m_G, K_{\infty}; C^{\infty}(G(\mathbb{Q})A_G(\mathbb{R})^{\circ}\backslash G(\mathbb{A})) \otimes E)$$
  
=  $H^{\star}(m_G, K_{\infty}; A_E \otimes E)$ 

a corresponding decomposition. The individual summand  $Eis_{E,\{P\},\varphi}:=H^*(m_G,k_\infty,A_{E,\{P\},\varphi}),\{P\}\neq\{G\}$  generated by suitable Eisenstein cohomology classes reflects the contribution of the class  $\{P\}$  and the representations of type  $\varphi$  of the Levi components of P to the cohomology "at infinity" in  $H^*(G,E)$ . The internal structure of the spaces  $Eis_{E,\{P\},\varphi}$  depends on arithmetical data (e.g. automorphic L-functions attached to  $\varphi$ .



### BIRGIT SPEH:

## Analytic torsion and representation theory

We proved that the analytic torsion of locally symmetric spaces  $G/\Gamma$  is zero unless G has a simple factor with a cuspitial parabolic subgroup P=MAN with  $\dim A=1$ . Then we gave an interpretation of the torsion as the density on the unitary spectrum.

#### ROBERT J. STANTON:

## Geometric zeta functions and secondary invariants

Let  $\tilde{X} = G/K$  be a globally symmetric space of noncompact type and with Hermitian structure. Let  $\Gamma$  be a discrete torsionfree subgroup of G with  $X = \Gamma \backslash \tilde{X}S$  compact. Let  $(\tau, V_{\lambda})$  be a finite dimensional representation of K and V the associated holomorphic bundle.

To any connected component  $X_{\gamma}$  of the geodesic flow on SX we associated certain characteristic classes depending on  $\mathbb{V}$  and the sturcture of the normal bundle to the image of  $\tilde{X}_{\gamma}$  in the boundary of the compactification of  $\tilde{X}, \bar{X} \subseteq G_{\mathbb{C}}/K_{\mathbb{C}}P_{-}$ . Denoting these classes by

$$\frac{\mathbb{V}_{ga}}{\Lambda_{-1}(\mathcal{N}_{+}(\gamma)\oplus\mathcal{V}_{+}(\gamma)\oplus\mathcal{U}_{\mathbb{C}}(\gamma))}$$

and letting  $D_{\gamma}$  be an appropriate elliptic operator of mixed deRham type, we define

$$X_{\gamma}(\mathbb{V}) = \left\langle [D_{\gamma}], \frac{\mathbb{V}_{\gamma}}{\Lambda_{-1}(\mathcal{N}_{+}(\gamma) \oplus \mathcal{V}_{+}(\gamma) \oplus \mathcal{U}_{\mathbb{C}}(\gamma))} \right\rangle.$$

If  $\ell_{\gamma}$  denotes the length of geodesics in the class  $X_{\gamma}$  and  $\mu_{\gamma}$  the generic multiplicity, and  $(\phi, F)$  a finite dimensional unitary representation of  $\Gamma$ , we define the geometric zeta function

$$Z_{\lambda,\varphi}(z) = \exp{-\sum_{[\gamma] \neq e} Tr \phi(\gamma) \chi_{\gamma}(\mathbb{V})} \frac{e^{-z\ell \gamma}}{\mu_{\gamma}}.$$

THEOREM: (Moscovici-Stanton)  $Z_{\lambda,\varphi}$  has a meromorphic continuation to  $\mathbb{C}$ . Moreover,  $Z_{\lambda,\varphi}(z) = T_h^2 C(g,\tilde{X}) z^r +$ 

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 $0(z^r)$  where  $T_h$  is the holomorphic torsion of  $\mathbb{V}, C(g, \tilde{X})$  a constant independent of  $\Gamma$  and

$$r = 2d + 2\tilde{T}d([X])ch(\mathbb{V})[X]$$

for an integer d and the derived Todd class  $\tilde{T}d$ .

A consequence of this obtained from Hirzebruch probability is that for example for compact Hermitan locally symmetric spaces of  $\mathbb{C}$ -dimension 2, Td([X]) is divisible by 8.

### PETER ZOGRAF:

## Families index theorem for puncted Riemann surfaces

A local index theorem for families of  $\bar{\partial}$ -operators acting on k-differentials on Riemann surfaces of genus g with n>0 punctures is formulated. For the first Chern form of the determinant line boundle  $\lambda_k$  on the Teichmüller space  $T_{g,n}$  with Quillen's metric (where the determinant of the corresponding Laplacian  $\Delta_k$  is defined as Selberg's Z(k)). The following formula holds:

$$c_1(\lambda_k) = \frac{6k^2 - 6k + 1}{12\pi^2} \omega_{WP} - \frac{1}{q} \omega_{cusp};$$

here  $\omega_{WP}$  is the Weil-Petersson Kähler form on  $T_{g,n}$ , and  $\omega_{cusp}$  is the Kähler form of a new Kähler metric on  $T_{g,n}$  defined in terms of Eistenstein series. The result differs from the case of compact Riemann surfaces (when n=0) by ab additional term (cuspidal defect) equal to  $-\frac{1}{g}\omega_{cusp}$ .

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