MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Topics in Pseudo-Differential Operators

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The meeting was organized by
H. O. Cordes (Berkeley)
B. Gramsch (Mainz)
B. W. Schulze (Berlin)
and H. Widom (Santa Cruz)

In 33 lectures recent results and methods were presented and discussed with emphasis on the following topics:

> Spectral asymptotics of Schrödinger operators, boundary value problems on singular domains, pseudodifferential operators with singularities, microlocal analysis and operator algebras.

A variety of applications was indicated, including

> quantum mechanics, reflection and diffraction of waves, crystal elasticity and N–body problems, to mention only a few.

44 mathematicians from Belgium, Denmark, France, Germany, Great Britain, Hong Kong, Israel, Italy, Japan, Netherlands, Poland, Russia, Sweden, Turkey and the United States enjoyed the pleasant atmosphere of the "Mathematische Forschungsinstitut Oberwolfach". The lectures and many interesting discussions stimulated further research and contributed to a fruitful exchange of ideas from various branches of pseudodifferential analysis.

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ABSTRACTS

E. L. Basor
Asymptotics of tau–functions and connections to Toeplitz determinants with singular symbols

(Joint work with C. A. Tracy)
Define the tau function $\tau(t;\theta,\lambda)$ by the formula

$$\tau(t;\theta,\lambda) = \exp\left(-\sum_{k=1}^{\infty}\frac{\lambda^{2k}}{k}e_{2k}(t;\theta)\right),$$

where

$$e_{2k}(t;\theta) = \int_0^\infty dx_1 \cdots \int_0^\infty dx_{2k} \prod_{j=1}^{2k} \frac{\exp(-\frac{1}{2}t(x_j + x_j^{-1}))}{x_j + x_{j+1}} \left(\frac{x_1 x_3 \cdots x_{2k-1}}{x_2 x_4 \cdots x_{2k}}\right)^{\theta},$$

$$x_{2k+1} = x_1, \quad t > 0, \quad -1 < \theta < 1 \text{ and } |\lambda| < \frac{\cos \pi\theta/2}{\pi}.$$

We describe the asymptotics for $\tau(t;\theta,\lambda)$ as $t \to 0^+$. The asymptotics are of the form

$$\tau(t;\theta,\lambda) \sim \tau_0(\theta,\lambda)t^{\frac{1}{2}(\sigma^2-\theta^2)},$$

where $0 \leq \Re\sigma < 1$ and σ satisfies $\pi^2\lambda^2 = \sin\frac{\pi}{2}(\sigma+\theta)\sin\frac{\pi}{2}(\sigma-\theta)$. The constant τ_0 is given by

$$\tau_0(\theta,\lambda) = 2^{-2(\alpha^2-\beta^2)}\frac{G(1+\alpha+\beta)G(1+\alpha-\beta)G(1-\alpha+\beta)G(1-\alpha-\beta)}{G(1+2\alpha)G(1-2\alpha)}$$

and here $2\sigma = \alpha$, $2\theta = \beta$ and $G(1+z)$ is the Barnes G–function satisfying $G(1+s) = \Gamma(s)G(s)$. The asymptotic formula is computed using the Widom operator. This operator was fundamental in describing the asymptotics of Toeplitz determinants with singular generating functions.

M. Sh. Birman
The discrete spectrum in the gaps of the continuous one: Remarks on corresponding ψdo

The analysis of the discrete spectrum of a Schrödinger operator in a gap of the continuous spectrum leads to the analysis of spectral asymptotics of ψdo 's of negative order. We discuss the case of Weyl type asymptotics as well as non Weyl type situations. In the Weyl type case the behaviour of the symbol for large impulses is sufficient for the main term of the asymptotics. Otherwise the main contribution to the distribution function is given by the singularities of the symbol (for finite ξ). This are limit effects, local in ξ. We also consider problems, when the spectral asymptotics, corresponding to local singularities in ξ and to the behaviour for large ξ have the same order. Such a situation we meet, e.g., in the theory of Dirac operators. In the case of perturbed periodic Schrödinger operators the symbol of the corresponding ψdo contains a periodic in x factor. The corresponding effects will be discussed.

R. Brummelhuis
Semi-classical spectral estimates around a critical energy

(Joint work with T. Paul (Ceremade) and A. Uribe (Ann Arbor))
Let $A = A(x, \hbar D)$ be an elliptic ψdo on a compact manifold (for instance a Schrödinger operator). One studies the distribution of the eigenvalues $E_j(\hbar)$ near an energy E by studying the asymptotics, as $\hbar \to 0$, of the sums

$$(1) \quad \gamma_\hbar(\varphi) = \sum_j \varphi\left(\frac{E_j(\hbar) - E}{\hbar}\right), \quad \hat{\varphi} \in C_c^\infty(\mathbb{R}).$$

If E is a regular value of $H(x,p)$ = principal symbol of A, the expansion of (1) in increasing integer powers of $\hbar$, together with formulas for the leading coefficients, is known as the semi-classical trace formula (Guillemin–Uribe–Paul–Brummelhuis). We have studied the expansion of (1) if $E = E_c$, a critical value of H, under the assumption that the set Θ of critical points of H is a non-degenerate manifold in the sense of Morse theory. The main results are:

- In the expansion of (1) logarithmic terms $\log \hbar^{-1}$ and half-integer powers of $\hbar$ will appear.
- If codim $\Theta = 2$, the leading term is $C\hbar^{-(n-1)} \log \hbar^{-1}$. If codim $\Theta > 2$, the log appears later in the expansion, but the coefficient of the leading log term can still be computed from the Hamilton flow of H.

The main tools are the singular Lagrangian distributions of Guillemin-Melrose-Uhlmann and a stationary phase lemma for oscillatory integrals with cubic phases.

V. Buslaev
Semiclassical integral operators with discontinuous symbols

The initial object is a semiclassical operator of the form $A = \theta(\hat{x})a(\frac{\varepsilon}{i}\frac{d}{dx})\theta(\hat{x})$, where $(\hat{x}f)(x) = xf(x)$, $x \in \mathbb{R}$, θ is the characteristic function of the interval $[-1,+1]$, a is some bounded <u>discontinuous</u> function. The operator A is considered as an operator acting on functions $f : [-1,+1] \to \mathbb{C}$. The problem is what is the asymptotical behaviour of A^{-1} when $\varepsilon \to 0$. We can give a regular asymptotic expansion of A^{-1} in terms of operators: $a(\frac{\varepsilon}{i}\frac{d}{dx})$ on all the axis, on half-axises $(-\infty,+1],[-1,+\infty)$ and $b(\frac{\varepsilon}{i}\frac{d}{dx})$ on $[-1,+1]$, where a/b is a smooth function. The corresponding geometrical procedure can be described as some generalization of the Schwartz alternating method. Special classes of symbols a are indicated for which this procedure can be realized explicitely and justified as the asymptotic one.

L. Coburn
Berezin–Toeplitz quantization

The main analytic properties of the Berezin–Toeplitz quantization of $\mathbb{R}^{2n}$ are discussed. This quantization is given by a map

$$f \to T_f^{(h)}$$

for f in an appropriate algebra of functions on "phase space" $\mathbb{C}^n = \mathbb{R}^{2n}$, with $T_f^{(h)}$ a Toeplitz operator with "symbol" f on a Bergman-type Hilbert space of holomorphic functions $H^2(\mathbb{C}^n, d\mu_h)$. Here $d\mu_h$ is family of Gaussian probability measures on $\mathbb{C}^n$,

$$d\mu_h(z) = (h\pi)^{-n} e^{-|z|^2/h} dv(z).$$

In a recent paper with C. A. Berger, we obtained sharp estimates on the norms $\|T_f^{(h)}\|_{(h)}$. More recently, I have obtained the estimates needed to justify the description of $f \to T_f^{(h)}$ as a "first order quantum deformation" of the algebra $AP + C_0$ in the sense of F. A. Berezin, M. A. Rieffel, S. Klimek and A. Lesniewski.

M. Demuth
Stochastic spectral analysis for selfadjoint Feller operators

(In collaboration with J. van Casteren, Antwerp)
Introducing basic assumption for transition density functions free Feller operators are defined as generators of semigroups associated to strong Markov processes satisfying the Feller property. Regular and singular perturbations are introduced via the Feynman–Kac–formulae. The spectrum of the perturbed Feller operators is studied by estimations of resolvent and semigroup differences. That yields Hilbert–Schmidt and trace class criteria, continuity with respect to Kato–Feller norms, large coupling behaviour of spectral data.

Yu. V. Egorov

On estimates of the negative spectrum of the Schrödinger operator

Let $H = -\Delta - V(x)$ be the Schrödinger operator in the space $H_0^1(\Omega)$, $\Omega \subset \mathbb{R}^n$. The potential $V(x)$ is real but it can change its sign.
To estimate the number N_- of the negative eigenvalues of H one can use a partition of Ω in a sum of cubes $\Omega = \bigcup Q_j$ of different sizes.
Theorem. Let $V_\pm(x) = \max\,(0, \pm V(x))$. Let $\Omega = \bigcup Q_j$ and

1) $\int_{Q_j} V_+(x)^p |x - x_0|^{2p-n} dx \le a_{p,n}$ for some $p \ge n/2$, $j = 1, \cdots, k$;

2) $\int_{Q_j} V_+(x)dx \le \frac{1}{2} \int_{Q_j} V_-(x)dx$ for $j = N+1, \cdots$

3) $$\sum_{k=1}^{n} \max_{\substack{x_1 \cdots x_{k-1} \\ y_{k+1} \cdots y_n}} \int_{\substack{x \in Q_j \\ y \in Q_j}} |x_k - y_k| V_+(x) V_-(y)\, dy_1 ... dy_k\, dx_k ... dx_n \le \frac{1}{2} \int_{Q_j} V_-(x)dx$$

for $j = N+1, \cdots$

Then $N_- \le N$.
Conditions, sufficient for the absence of negative eigenvalues and two-sided estimates of the first negative eigenvalue are obtained in a similar way.

A. Erkip

Fredholm criteria for ψdo's and some (elliptic) boundary value problems that are not normal solvable

For a system of ψdo's with symbol in the Grushin class $\tilde{S}^m(\mathbb{R}^n)$ (slowly varying as $x \to \infty$) we derive a necessary and sufficient condition for having one sided Fredholm inverse. The condition is closely related to the concept of md–ellipticity of Cordes. $a \in \tilde{S}^m$ is md–elliptic if for some positive c, R, $|a(x,\xi)| \ge c(1+|\xi|)^m$ for all $|x| + |\xi| \ge R$. Several authors have shown that this is necessary and sufficient for the ψdo $\mathrm{Op}(a)$ to be Fredholm. Our proof on the other hand has a local character that enables us to consider certain non–compact manifolds and boundary value problems. We show that if P is elliptic in the classical sense, but not md–elliptic as $x \to \infty$ and if Ω is a "large"domain, then no boundary value problem related to P on Ω is normally solvable for L^2–Sobolev spaces. The condition for Ω being large turns out to be equivalent to Poincaré's inequality being valid on Ω and characterizes domains for which zero is in the spectrum of the Dirichlet problem for the Laplace operator.

B. Gramsch

Fréchet algebras in the microlocal analysis and the propagation of singularities

A theory of special Fréchet operator algebras is used to give an approach to the Weyl-lemma related to elliptic and microlocal regularity. The operational calculus for Ψ^*-algebras leads with pseudoinverses and localizations to an abstract regularity result using locally some semi-Fredholm operators. Specializing we get e.g. the following remark: Let Ω be a region in $\mathbb{R}^2, A \in \Psi^0_{cl,x}(\Omega), A$ elliptic in $U = \{x \in \Omega : |x - x_0| < r\} \subset \Omega$ and X an arbitrary compactly supported C^∞-vector field on U. For $u \in L^2(\Omega)$ we assume $Au \in D^\infty := \{\varphi \in L^2(\Omega) : X^\nu \varphi \in L^2(\Omega)\ \forall \nu \in \mathbb{N}\}$. Then $u \in D^\infty$ holds. The operator algebra methods lead to a microlocal version. Connected to some results of Cordes, Helton and M. Taylor a general definition of a wave front is discussed. The dynamical aspect of the propagation of singularities can be described by using special Fréchet algebras on appropriate spaces. For other aspects of the theory of "Ψ^*-algebras" see the talks of E. Schrohe and W. Kaballo at this conference and in Operator Theory: Adv. Appl. vol. 57, 71 - 98 (1992).

G. Grubb

Weakly parametric ψdo-s and Atiyah–Patodi–Singer boundary problems

(Joint work with R. T. Seeley, Univ. of Massachusetts, Boston)

Consider a first order elliptic differential operator $P : C^\infty(X, E_1) \to C^\infty(X, E_2)$ between vector bundles E_1 and E_2 over a compact n-dimensional manifold X with boundary $\partial X = X'$ (e.g. a Dirac operator). Assume that over a collar nbd. $X' \times [0, c]$ of $X', P = \sigma(\partial_{x_n} + A + x_n P_1 + P_0)$, were σ is a unitary morphism from $E_1|_{X'}$ to $E_2|_{X'}, A$ acts in $E_1|_{X'}$ and is selfadjoint, and the P_j are of order j. Consider the realization P_B of P defined by the boundary condition $B(u|_{X'}) = 0$, where $B = \Pi_> + B_0, \Pi_>$ being the orthogonal projection onto the positive eigenspace of A, and B_0 being an orthogonal projection rangeing in the eigenspace for eigenvectors in $[-R, R]$ (some R) and commuting with A. Then P_B is Fredholm, with index

$$\text{index } P_B = \text{Tr } e^{-t\Delta_1} - \text{Tr } e^{-t\Delta_2},\ \Delta_1 = P_B^* P_B,\ \Delta_2 = P_B P_B^*.$$

Atiyah, Patodi and Singer determined the index in case $P_1 = P_0 = 0$. The speaker extended the formula to the general case in Comm. P.D.E 17 (1992) by studying Tr $e^{-t\Delta_i}$ from the point of view of her 1986 book, showing that

$$\text{Tr } e^{-t\Delta_i} = e_{i,-n} t^{-n/2} + \cdots + e_{i,-1} t^{-1/2} + e_{i,0} - (-1)^i \frac{1}{4}\eta_A + R_i(t),$$

with $R_i(t) = O(t^{3/8})$ for $t \to 0$. Using another parameter-dependent calculus we can improve $R_i(t)$ to a full expansion

$$R_i(t) \sim \sum_{\ell \geq 1} (c_{i,\ell} + c'_{i,\ell} \log t) t^{\ell/2},$$

with coefficients $c'_{i,\ell}$ locally determined and $c_{i,\ell}$ globally determined.
This is shown via an analysis of the resolvents $(\Delta_i + \mu^2)^{-1}$. Here the parameter μ enters in special expansions for $\mu \to \infty$ (a kind of Taylor expansions in $z = \frac{1}{\mu}$), besides the usual polyhomogeneous symbol expansions. Playing on both types of expansions, we obtain asymptotic trace formulas in decreasing powers of μ, some of them multiplied by $\log \mu$.
For the APS problem, the application relies on a precise description of $(\Delta_i + \mu^2)^{-1}$ in the Boutet de Monvel calculus, where the Poisson and trace operator ingredients are shown to be better behaved (strongly polyhomogeneous) than the entering ψdo–s in the boundary (weakly polyhomogeneous).

N. Jacob
On pseudo–differential operators generating Markov–processes

By a theorem of Ph. Courrège any generator of a Feller semigroup on $\mathbb{R}^n$ which maps $C_0^\infty(\mathbb{R}^n)$ into $C(\mathbb{R}^n)$ is restricted to $C_0^\infty(\mathbb{R}^n)$ of the form

$$(*) \qquad Au(x) = -(2\pi)^{-n/2} \int_{\mathbb{R}^n} e^{ix\xi} a(x,\xi) \hat{u}(\xi) d\xi,$$

where $a : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{C}$ is a continuous function which is for any fixed x negative definite with respect to ξ. Using a scale of anisotropic Sobolev spaces it is possible under further conditions on the symbol $a(x,\xi)$ to show that certain operators of the type (*) do extend to a generator of a Feller semigroup. Moreover, it is possible to determine properties of the corresponding Markov–process $(X_t)_{t \geq 0}$ only by using the symbol $a(x,\xi)$.

W. Kaballo
Multiplicative decompositions of analytic Fredholm functions

This lecture was a report on joint work with B. Gramsch. We study families $T(z) \in \Phi$ of Fredholm operators on Banach spaces or of elliptic operators in submultiplicative Ψ^*–algebras Ψ of pseudo differential operators, which depend analytically on $z \in \Omega$; here Ω denotes a Stein manifold.
Assuming that T is homotopic in $C(\Omega, \Phi)$ to an operator function in $C(\Omega, \Psi^{-1})$, we construct multiplicative decompositions

$$T(z) = A(z)(I + S(z)),$$

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where $A : \Omega \to \Psi^{-1}$ and $S : \Omega \to J$ are analytic; here J is any given "arbitrarily small" (F)-ideal in Ψ.

H. Komatsu
Pseudodifferential operators on boundaries of strictly pseudoconvex domains

Let $\Theta = \{z; \rho(z) < 0\}$ be a strictly pseudoconvex (local) domain defined by a strictly plurisubharmonic real analytic function $\rho(z, \bar{z})$. The "microfunctions" on the boundary $\sum = \{z; \rho(z) = 0\}$ is defined by

$$\mathcal{C}(\Sigma) := \mathcal{O}(\Theta)/\mathcal{O}(\Theta \cup \Sigma).$$

Let $\rho(z, \bar{w})$ be an analytic continuation of $\rho(z, \bar{z})$ in a neighborhood of the antidiagonal $\Delta_{\sum} = \{(z, \bar{z}); z \in \sum\}$ in $\mathbb{C}^n \times \bar{\mathbb{C}}^n$. A (local) Toeplitz kernel is a function of the form

$$T(z, \bar{w}) = \sum_{j=-m-n}^{\infty} a_j(z, \bar{w})\rho^j + \sum_{j=0}^{\infty} b_j(z, \bar{w})\rho^j \log \rho.$$

The integral operator with kernel T defines a microlocal operator $T : \mathcal{C} \to \mathcal{C}$ independent of the representative in $\mathcal{O}(\Theta)$. If Θ is the unit tube $D\mathbb{R}^n := \{x + iy \in \mathbb{C}^n; |y| < 1\}$, then these local Toeplitz operators are identified with the microdifferential operators (= classical pseudodifferential operators with analytic coefficients) on the cosphere bundle $S^*\mathbb{R}^n$.

A. Kozhevnikov
Diagonalization of Douglis–Nirenberg elliptic systems on manifolds with boundary

It is known, that even the integer powers of the Douglis–Nirenberg elliptic systems (DNE systems) are not systems of the same kind. This fact and also a non–homogeneity of the principal symbol's eigenvalues are two main difficulties for constructing the resolvent and the complex powers of DNE systems. For the partial differential systems of a single order the full theory has been constructed by R. Seeley.

For the DNE systems on a compact manifold without boundary the above mentioned difficulties were overcome. It has been proved that a DNE system with elliptic principal minors is similar to some diagonal system and thus the case of the DNE systems was reduced to the case of single order systems.

The aim of this work is to prove a theorem on the diagonalization of the DNE systems in the case of a manifold with boundary. This result is an analog of the corresponding theorem for the case of a boundaryless manifold. In other words it has been established a theorem on the similarity of the DNE system with elliptic principal minors in the case of a manifold with boundary to the corresponding block–diagonal system.

A. Laptev
Overdetermined system of functions and spectral asymptotics

We study a second order selfadjoint nonnegative differential operator H_0 in $L_2(\mathbb{R}^n)$. We assume that $H = H_0 - \mu^2 V$, μ^2 is a large coupling constant and V is a potential decreasing at infinity. Using a special type of the FBI transform we represent H as an operator of multiplication in the space $L_2(\mathbb{R}^{2n})$. Combining this representation with the Berezin inequality for the trace of a convex function of a selfadjoint operator we obtain a Weyl type asymptotic formula for the negative spectrum of H when $\mu^2 \to \infty$.

P. Laubin
Distributions associated to a 2–microlocal pair of Lagrangian manifolds

Let X be a C^∞ manifold. The purpose of the talk is to define a class $I^{m,p}(X, \Lambda_0, \Lambda_1)$ of distributions associated to a pair (Λ_0, Λ_1) of Lagrangian submanifolds of the cotangent bundle T^*X. It is expected they play the same role with respect to the second wave front set as the Lagrangian distributions do with respect to the singular wave front set.
The intersection of the Lagrangian submanifolds does not need to be clean but is described by a conic Lagrangian submanifold of $T_{\Lambda_0}T^*X$. They can have rather general intersection.
When the intersection is clean, we get the class defined by Melrose–Uhlmann and used to construct parametrices to the Cauchy problem for hyperbolic operators. Another application is concerned with the solution of a strictly diffractive boundary value problem. We show in the example of Friedlander–Melrose that the solution belongs to a class $I^{m,p}$ in the interior, modulo some Lagrangian distributions.

O. Liess
Decay estimates for the system of linear crystal elasticity

We study decay estimates of the form $|u(x,t)| \le c\, t^{-\alpha}, \alpha > 0$, for solutions $u = (u_1(x,t), u_2(x,t), u_3(x,t))$ of the system of crystal elasticity

$$\frac{\partial^2}{\partial t^2}\, u_p = \sum_{q,r,s} c_{pqrs} \frac{\partial^2}{\partial x_r \partial x_s}\, u_q \quad , p = 1, 2, 3$$

for the case of cubic crystals. At a technical level the main difficulties come from the study of some oscillatory integrals defined on the wave surface of the system. When we localize an integral near a smooth point of the surface where the total curvature is nonvanishing then it is classical that we get a contribution of type $0(t^{-1})$ and for the smooth part it is also easy to see that the estimate will be at least of type $c\,t^{-1/6}$. Our main results refer to the contributions near singular points. The singular points can be conic or uniplanar. In the first case one gets estimates of the form $c\,t^{-1/2}$ and for the uniplanar case of the form $c\,t^{-3/4}$, provided that one is not too far from the isotropic case.

E. Meister
Some multiple–part Wiener–Hopf–problems with applications in mathematical physics

After formulating the 4-wedge transmission problem arising from the scattering of time harmonic scalar waves (E-polarization vertical to the xy–plane!) by different materials filling the wedges a "four-part Wiener–Hopf functional equation" in the L^q-space of $2D$–Fourier transformed functions Φ_j, restricted to Q_j, the cross-section of the $j-th$ wedge arises. Reviewing the concepts of a Toeplitz and 2-part Wiener–Hopf operator with arbitrary continuous projectors P and bijective linear A's on a B–space $\mathcal{X}$, necessary and sufficient conditions for their invertibility are given in the context of strong operator factorization w.r.t. the two invariant subspaces of $\mathcal{X}$. The sufficient conditions for "strong ellipticity" of A is generalized for the N–part–problem with A_j elliptic. By means of factorization a reduction to an "$(N-2)$–alternating Wiener–Hopf system" is possible. In the special case of $N = 3$ this is equivalent to an equation with the product of two Toeplitz operators with inverse generating A. The case of "strip diffraction" leads to inverse symbols with almost bounded symbols for A.

M. Nagase
Spectra of relativistic Schrödinger operators with magnetic vector potentials

(Joint work with T. Umeda)
Let us consider the operator $h^W(X,D)$ defined by

$$h^W(X,D)u(x) = \frac{1}{(2\pi)^n}\iint e^{i(x-y)\cdot\xi} h\left(\frac{x+y}{2},\xi\right) u(y)dy\,d\xi,$$

where $h(x,\xi) = \sqrt{|\xi - a(x)|^2 + m^2}$. The operator $h^W(X,D)$ is the Weyl quantized Hamiltonian of a relativistic spinless particle of mass $m > 0$ with a magnetic vector potential $a(x) = (a_1(x),\cdots,a_n(x))$. If the magnetic potential $a(x)$ is smooth, then the operator $h^W(X,D)$ is essentially self-adjoint on $C_0^\infty(I\!R^n)$. Let H be the closure of the operator $h^W(X, D)$. The problem is to investigate the spectral property of the operator H and $H_1 = H + V(x)$. Our main theorem is

Theorem. We assume that $a(x)$ and $V(x)$ satisfy

(i) $a(x)$ is bounded

(ii) $|\partial_x^\alpha a(x)| \to 0$ $(|x| \to \infty)$ for $\alpha \neq 0$

(iii) $V(x) < D >^{-1}$ is a compact operator on $L^2(\mathbb{R}^n)$.

Then we have

$$\sigma_{ess}(H_1) = \sqrt{\sigma_{ess}(T)}$$

where $T = \sum_{j=1}^n \left(D_{x_j} - a_j(x)\right)^2 + m^2$.

More detailed results on H and H_1 can be obtained from this theorem.

V. M. Petkov

Fourier integral operators related to degenerate reflecting rays

Let $\Omega \subset \mathbb{R}^n, n \geq 3$, n odd be a domain with C^∞ boundary $\partial\Omega$ and bounded complement $K = \mathbb{R}^n \setminus \Omega \subset \{x : |x| \leq \rho_0\}$. Let $s(t, \theta, \omega) \in \mathcal{S}'(\mathbb{R}_t \times S^{n-1} \times S^{n-1})$ be the scattering kernel related to the wave equation in $\mathbb{R} \times \Omega$ with Dirichlet boundary condition on $\partial\Omega$. For fixed $\theta \neq \omega$ we have the relation

$$\operatorname*{sing\ supp}_t \ s(t, \theta, \omega) \subset \{-T_\gamma \ : \ \gamma \in \mathcal{L}_{\omega,\theta}\},$$

where T_γ is the sojourn time of γ and $\mathcal{L}_{\omega,\theta}$ is the union of all (generalized) (ω, θ)-rays in $\bar{\Omega}$ incoming with direction $\omega \in S^{n-1}$ and outgoing with direction $\theta \in S^{n-1}$. If $-T_{\gamma_0}$ is an isolated point of $s(t, \theta, \omega)$ and if there exists only one reflecting (ω, θ)-ray γ_0 with sojourn time T_{γ_0} one can prove that $-T_{\gamma_0} \in$ sing supp $s(t, \theta, \omega)$, provided γ_0 non-degenerate, that is the differential cross-section of γ_0 is non-vanishing.
We study in this talk the case when γ_0 is a (ω, θ)-degenerate reflecting ray under the assumption that there exists $\delta > 0$ such that all (ω, θ)-rays γ having sojourn times $T_\gamma \in [T_{\gamma_0} - \delta, T_{\gamma_0} + \delta]$ are reflecting and all such γ lie in a sufficiently small neighbourhood of γ_0. By using a class of Fourier integral operators with well arranged lower order symbols, we apply a result of Soga for the asymptotics of oscillatory integrals with degenerate phase function. Finally, we show that for γ_0 with the properties described above we have $-T_{\gamma_0} \in$ sing supp $s(t, \theta, \omega)$.

B.A. Plamenevsky

On classification of extensions related to pseudodifferential operators with singularities

Pseudodifferential operators with discontinuous symbols on a compact manifold M without boundary give rise to the short exact sequence

$$0 \to \mathcal{K}L_2(U) \to \Psi \to \mathfrak{S} \to 0 \qquad (1)$$

where $\mathfrak{S}$ is a noncommutative rather complicate algebra of operator-valued symbols, Ψ being the algebra of ψdo. In reasonable situations (piecewise smooth symbols, piecewise smooth manifold) the algebra Ψ turns out to be solvable, i.e. there exists the composition series $0 = I_0 \subset I_1 \subset \cdots \subset I_{l+1} = \Psi$ such that $I_{j+1}/I_j \simeq C_0(X_j) \otimes \mathcal{K}$, where X_j is a local Hausdorff space, $\mathcal{K}$ the compact operator ideal. It allows us to form the chain of short extensions

$$0 \to I_j/I_{j-1} \to I_{j+1}/I_{j-1} \to I_{j+1}/I_j \to 0$$

and introduce with the help of Joneda product the corresponding long extensions. The talk was devoted to the classification of such extensions and extensions of the form (1) for some situations. Some index formulas are given in analytic terms and in terms of the cyclic homology $HC(\mathfrak{S})$ for the symbolic algebra $\mathfrak{S}$.

D. Robert

Pointwise semi–classical asymptotics for total scattering cross sections in N–body problems

(Joint work with X.P. Wang)

Let us consider N–particles of mass $m_i > 0, \quad 1 \leq i \leq N$ in the Euclidian space $\mathbb{R}^\nu$ interacting through pair potentials $V_{ij}(x^i - x^j)$ where $x^i \in \mathbb{R}^\nu$ is the coordinate system for the particle i. The motion is described in the center of mass frame:

$$X = \{(x^1, \cdots, x^N) \in \mathbb{R}^N \ : \sum_{i=1}^{N} m_i x^i = 0\}.$$

X is equipped with the metric: $< x, y >= \sum_{i=1}^{N} 2m_i x^i \cdot y^i$. The quantum Hamiltonian of the total system is given by:

$$P = -h^2 \Delta_x + \sum_{i<j} V_{ij}(x^i - x^j) \text{ in } L^2(X).$$

The V_{ij} are assumed to be decreasing fast enough at infinity. Let a be a cluster decomposition of $\{1, 2, \cdots, N\}, \#a = 2$, and a channel scattering: $\alpha = (a, E, \phi)$

($P^a\phi = E\phi, \phi \in L^2(X^a), P^a$ being the internal Hamiltonian in the internal configuration space X^a). Let $\mathcal{C}$ be the set of threshold of P. Then using an idea of Enss–Simon (1980) we can define the total scattering cross section $\sigma_\alpha(\lambda,\omega;h)$ for the incoming scattering channel α as a continuous function in $\lambda \in]E,+\infty[\backslash\mathcal{C}$ and $\omega \in \mathbb{S}_a$, the unit sphere of $X_a = X^{a\perp}$ (configuration space of the relative motions of clusters in a).
Using the completeness property for wave operators proved by Sigal–Soffer (1989), we can prove the optical theorem for N–body problems:

$$\sigma_\alpha(\lambda,\omega,h) = \frac{1}{hn(\lambda)} \Im < (P-\lambda-i0)^{-1} I_a e_\alpha, I_a e_\alpha > \qquad (*)$$

where: $e_\alpha = \phi(x^a)\exp\left(i\frac{n(\lambda)}{h}x_a\cdot\omega\right), n(\lambda) = \sqrt{\lambda - E}$, x^a , x_a are the projections of x in X^a, X_a ; I_a is the intercluster potential, $\omega \in \mathbb{S}_a$.
With $(*)$ and a non trapping assumption we can prove a semi classical asymptotic for σ_α :

$$(**) \qquad \sigma_\alpha(\lambda,\omega;h) = 4\cdot \int_{X_a\cap\{u_a\cdot\omega=0\}} \sin^2\left\{\frac{1}{4n(\lambda)}\int_{-\infty}^{+\infty} I_a(0,u_a+s\omega)ds\right\} du_a$$
$$+O\left(h^{-\frac{n_a-1}{\rho-1}+\delta_0}\right)$$

for some $\delta_0 > 0$, "O" being locally uniform in λ, uniform in ω. (n_a =dim X_a).
For $N=2$, $(**)$ was proved before by Yafaev and Robert–Tamura. Recently, Itoh–Tamura proved $(**)$ in the distributional sense in (λ,ω), without any non trapping assumption. But for pointwise estimates like $(**)$ a non trapping condition is necessary in general.

E. Schrohe

A characterization of boundary value problems by commucators and submultiplicativity of Boutet de Monvel's algebra

In 1977, R. Beals gave a characterization of pseudodifferential operators by the behaviour of their iterated commutators on Sobolev spaces.
In the same spirit, I characterized the crucial elements in Boutet de Monvel's calculus, namely the pseudodifferential operators with the transmission property and the singular Green operators, by the behaviour of their iterated commutators with multipliers and vector fields tangential to the boundary on the wedge Sobolev spaces introduced by B.-W. Schulze.
Both characterizations together show that the algebra of operators of order and type zero in Boutet de Monvel's calculus is submultiplicative.
In connection with results of B. Gramsch and C. Phillips this has applications for Oka principle, non-abelian cohomology and K–theory.

B. W. Schulze

Continuous asymptotics in the edge pseudo–differential algebra

The edge problems are analogously to operators in Boutet de Monvel's algebra of the form

$$\mathcal{A} = \begin{pmatrix} A & K \\ T & Q \end{pmatrix} : \begin{matrix} \mathcal{W}^{s,\gamma}(W) \\ \oplus \\ H^s(Y, \mathbb{C}^{N_-}) \end{matrix} \to \begin{matrix} \mathcal{W}^{s-\mu,\gamma-\mu}(W) \\ \oplus \\ H^{s-\mu}(Y, \mathbb{C}^{N_+}) \end{matrix}$$

where W is the given manifold with edge Y and $\mathcal{W}^{s,\gamma}(W)$ are the weighted Sobolev spaces over W . There are two leading symbolic levels

$$\sigma^\mu_\psi(\mathcal{A})(t,x,y,\tau,\xi,\eta), \text{ the interior symbol}$$

$$\sigma^\mu_\wedge(\mathcal{A})(y,\eta), \text{ the edge symbol .}$$

This refers to local coordinates $(t,x,y) \in \mathbb{R}_+ \times X \times \Omega$ close to Y , where $\mathbb{R}_+ \times X = X^\wedge$ is the open stretched model cone, $\Omega \subseteq Y$ an open set. The ellipticity is determined by the bijectivity of both symbol components. The singular terms of the continuous edge asymptotics have the form

$$F^{-1}_{\eta\to y}\{<\eta>^{\frac{n+1}{2}} \omega(t<\eta>) < \hat{\zeta}(\eta), (t<\eta>)^{-z} >\}$$

for a certain

$$\zeta(y) \in H^s(\mathbb{R}^q, \mathcal{A}'(K, C^\infty(X))),$$

$K \subset \{\Re z < \frac{n+1}{2} - \gamma\}$ a compact set, $\mathcal{A}'(K, C^\infty(X))$ the space of $C^\infty(X)$–valued analytic functionals, carried by K, in the canonical (nuclear) Fréchet topology, $\omega(t)$ a cut–off function, i.e. $\omega(t) \in C_0^\infty(\bar{\mathbb{R}}_+), \omega(t) = 1$ near $t = 0$.

<u>Theorem</u>. Let $\mathcal{A}$ be elliptic, then $\mathcal{A}u = f \in \mathcal{W}^{s,\gamma-\mu}(W) \oplus H^{s-\mu}(Y, \mathbb{C}^{N_+}), u \in \mathcal{W}^{-\infty,\gamma}(W) \oplus H^{-\infty}(Y, \mathbb{C}^{N_-})$, and f having continuous edge asymptotics implies $u \in \mathcal{W}^{s+\mu,\gamma}(W) \oplus H^s(Y, \mathbb{C}^{N_-})$, again with continuous edge asymptotics.

J. Sjöstrand

Asymptotics and estimates for operators and integrals in large dimensions

(Partly joint work with B. Helffer)

We describe various results, uniform with respect to the dimension m and sometimes we give limits when $m \to \infty$. The methods are based on direct WKB expansions and on the maximum principle.

1) Uniform WKB–expansions for a non-degenerate point well. More precisely we discuss $-\frac{h^2}{2}\Delta + V(x)$ when V is holomorphic in $B_{\ell^\infty,\mathbb{C}^m}(0,1)$ with $|\nabla V|_{\ell^\infty} = O(1)$, $V(0) = 0$, $\nabla V(0) = 0$, $V''(0) = D + A$, D diagonal, $\|A\|_{\mathcal{L}(\ell^\infty,\ell^\infty)} \le r_1 < r_0 \le D$.

2) Extension to the C^∞ case via the notion of 0–standard functions.

3) Formal stationary phase: Under suitable assumptions we show that

$$\int e^{-\varphi(x,y)/h} dy = e^{-\Phi(x;h)/h} \text{ with } \varphi \text{ 0–standard } \Rightarrow \Phi \text{ 0–standard.}$$

4) Some applications:
a) $\exp -t(-\frac{h^2}{2}\Delta + V)$
b) $\lim_{m\to\infty} \frac{\lambda(m;h)}{m} = \Lambda(\nu;h) \sim h\sum_0^\infty \Lambda_j(\nu)h^j$ if $0 \leq \nu < \frac{1}{4}$ and $\lambda(m;h)$ = smallest eigenvalue of $-h^2\Delta + V^{(m)}(x)$, $V^{(m)}(x) = \frac{x^2}{4} - \sum_{j=1}^m \log\cosh\sqrt{\frac{\nu}{2}}(x_j + x_{j+1})$, "$m+1=1$" (with Helffer).

5) Use of the maximum principle, inspired by Singer–Wong–Yau–Yau. Gives estimates on the higher derivatives of the log of the 1:st eigenfunction for convex potentials. Ex: for $\nu < \frac{1}{4}$ we get

$$\left|\frac{\lambda(m;h)}{m} - \Lambda\right| \leq C_\kappa(h+h^2)e^{-\kappa m} \text{ if } \kappa < \cosh^{-1}\left(\frac{1-2\nu}{2\nu}\right)$$

6) Exponential decay of correlations for certain "Laplace" integrals (with Helffer).

7) Power decay of correlations in a limiting case.

T. Tang
C*–algebras of pseudodifferential operators

We study some C*–subalgebras $\mathcal{C}$ of $\mathcal{L}(L^2(\mathbb{R}^n))$ generated by certain 0^{th}–order pseudodifferential operators. They have compact commutators. The study has implications on the Fredholm Theory and spectral theory of differential operators. It is along the line of research of Cordes and others on similar algebras on more general noncompact manifolds, and in particular that of Sohrab on some algebras on $\mathbb{R}^n$. Our goal is to have a complete description of the maximal ideal space of $\mathcal{C}/\mathcal{K}$ ($\mathcal{K}$ = { compact operators on $L^2(\mathbb{R}^n)$}), and the symbol map on $\mathcal{C}$ (canonical proj. followed by the Gelfand transform). The difficulty lies in determining these objects at ∞. Sohrab overcomes this for his algebras by constructing the multiplicative linear functionals at ∞ explicitly. By a variation of one of his lemmas, we are able to study algebras that are related to differential operators with more severe singularities at ∞.

N. N. Tarkhanov
The P–Neumann problem

1. Koszul's complexes.
2. Ellipticity condition.
3. Local exactness.
4. The P–Neumann problem.
5. The main subelliptic estimate in the P–Neumann problem.
6. (Strongly) $M^{(\cdot)}$–convex domains.
7. Solvability of the P–Neumann problem.
8. Uniqueness of solutions of the P–Neumann problem.
9. The Poisson formula for generalized solutions of the P–Neumann problem.
10. Solvability of the P–Neumann problem in the Hardy classes.
11. Approximate solving the P–Neumann problem.
12. The particular case $i = 0$.
13. The Lefshetz fixed point formula for Koszul's complexes on compact manifolds with boundary.
14. Conclusive remarks.

H. Triebel
Eigenvalue distributions of degenerate elliptic PDO's: an approach via entropy numbers

1. If T is compact from A (Banach space) into itself, then $|\mu_k| \leq \sqrt{2} e_k$, where μ_k are the eigenvalues and e_k the related entropy numbers.
2. If $\Omega \subset \mathbb{R}^n$ is bounded, then $e_k \sim k^{-s/n}$ for the compact embedding of the fractional Sobolev space $H_p^s(\Omega)$ into $L_q(\Omega)$, $s > 0$, $1 < p \leq q < \infty$, $s - \frac{n}{p} > -\frac{n}{q}$.
3. Based on 1. and 2. one can study the distribution of eigenvalues of operators of the type $B = b_1(x)(-\Delta)^{-1} b_2(x)$, where $b_j(x) \in L_{\nu_j}(\Omega), -\Delta$ stands for the Laplacian with vanishing Dirichlet data. If B is invertible, then $B^{-1} = b_1^{-1}(x)(-\Delta) b_2^{-1}(x)$ has the classical behaviour of the eigenvalues $\lambda_k \sim k^{2/n}$.
4. Based on 1. and 2. and the Birman-Schwinger principle one can estimate the "negative" spectrum of operators of Schrödinger type.

A. Unterberger

Pseudodifferential analysis and representation theory

In recent years, we have developed the Klein–Gordon symbolic calculus of operators as part of a program linking pseudodifferential analysis to elementary particle theory. As far as the Dirac set of equations is concerned, we suggest (in view of the troublesome separation of the space of solutions as the sum of its positive–frequency and negative–frequency parts) to first deform the relevant representation of the Poincaré group into a certain representation of some De Sitter group. In the one (space–) dimensional case, the family of representations we get is nothing but the principal series of representations of $SL(2,\mathbb{R})$. The relevant quantization rule was introduced in some recent joint work with J. Unterberger: We describe its main (rather exotic) features, that are related to the fact that it is the projective, not the linear, structure on $\mathbb{R}$ that plays a major role in the whole concept.

D. Vassiliev

Construction of the wave group for boundary value problems

Let A be an elliptic positive self–adjoint differential operator of order $2m$ acting on a compact n–dimensional manifold with boundary. The work is devoted to the effective (modulo C^∞) construction of the Schwartz kernel $u_P(t,x,y)$ of the operator $e^{-itA}P$, where P is a pseudodifferential cut–off. An abstract lemma characterizing properties of distributions of the type $u_P(t,x,y)$ is formulated. This lemma allows to state a well–posed problem for the equation $D_t^{2m}u_P = Au_P$. Then u_P is constructed as a finite sum of Fourier integral operators with complex phase functions.

H. Widom

Random matrices and integral operators

In this lecture we describe how Fredholm determinants of integral operators arise in the theory of random matrices. One that arises in many matrix models by "scaling in the bulk of the spectrum" is det $(I-K)$ where K is the operator on $(0,s)$ with kernel $\sin(x-y)/\pi(x-y)$. We describe the asymptotics as $s\to\infty$ of this and related quantities by three approaches:

(1) discretization and the use of Toeplitz matrices;

(2) the connection with a Painlevé differential equation;

(3) eigenvalue asymptotics of the operator.

The kernel $(\mathrm{Ai}(x)\mathrm{Ai}'(z)-\mathrm{Ai}'(x)\mathrm{Ai}(z))/(x-y)$ arises by "scaling at the edge of the spectrum" and analogous results are obtained. (Almost all the "results" described are heuristic and so not actually proved.) Finally mention is made of a conjecture (not the speaker's) relating in a precise way, the spacing between eigenvalues of the Laplacian on a "general" domain and the spacing between eigenvalues of a random matrix.

M. Yamazaki
Besov type function spaces based on the Morrey spaces and the Navier–Stokes equation

We construct new function spaces in the same way as Besov spaces, taking the Morrey spaces in place of the standard L^p–spaces as the basis. By taking the parameters suitably, the spaces become strictly larger than the Morrey spaces, yet enjoy heat kernel estimates similar to those of the Morrey spaces. On the other hand, our spaces behave well for the pullback by orthogonal projections in the base spaces, while the Besov spaces do not have the corresponding property in general.
We also show that the Cauchy problem for the Navier–Stokes equation on the n-dimensional Euclidean space admit unique time-global strong solutions, satisfying the initial condition in a suitable manner, for small initial data belonging to some function spaces constructed above. As the initial data we can take distributions which are not Radon measures.

B. Ziemian
Asymptotic behaviour of solutions to elliptic corner operators

A generalization of multiple Laplace integrals is proposed in order to study the asymptotic behaviour of solutions to elliptic equations at infinity. In the constant coefficient case a representation of solutions as a finite sum of Laplace integrals is derived in the case of 2 variables, and for a class of elliptic operators in higher dimensions.
The proof rests on the extension of the Leray residue formula, a study of Nilsson type integrals and on Phragmén–Lindelöf type theorems for entire functions.
Perspective applications to the study of nonlinear elliptic equations are indicated.

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